

随机时滞系统的神经网络输出反馈动态面控制

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摘 要: 针对一类具有未知控制方向的随机时滞系统设计自适应神经输出反馈控制器. 首先, 利用状态观测器估计不可测量的系统状态; 其次, 选择合适的 Lyapunov-Krasovskii 函数消除未知延迟项对系统的影响, 利用 Nussbaum-type 函数处理系统的未知控制方向问题, 通过神经网络逼近未知的非线性函数, 以及用动态表面控制(DSC)解决控制器设计中出现的复杂性问题; 最后, 通过 Lyapunov 稳定性理论, 构造一个鲁棒自适应神经网络输出反馈控制器, 可以保证闭环系统中所有信号在二阶或四阶矩意义下一致最终有界, 跟踪误差能收敛到零值小的领域内. 仿真实例验证了所提出方法的有效性.

关键词: 神经网络; 输出反馈; 反推; 随机非线性系统; 动态表面控制

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Output-feedback neural DSC control for stochastic nonlinear time-delay systems

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Abstract: This paper deals with the problem concerned with output-feedback adaptive neural control for a class of stochastic nonlinear time-delay systems with unknown control directions. Firstly, the state observer is established for estimating the unmeasured states. Then, in the design procedure, an appropriate Lyapunov-Krasovskii functional is used to compensate the unknown time-delay terms, the Nussbaum-type gain function is used to handle the unknown control directions, the neural network is employed to approximate the unknown nonlinearities, and the dynamic surface control(DSC) technique is used to avoid the the problem of 'explosion of complexity' in control design. Finally, a robust adaptive neural output feedback control scheme is constructed via the Lyapunov stability theory. It is shown that the designed controller can ensure that all the signals in the closed-loop system are 4-Moment (or 2 Moment) semi-globally uniformly ultimately bounded(SGUUB) and the tracking error converges to a small neighborhood of the origin. The example is presented to verify the effectiveness of the proposed scheme.

Keywords: neural networks; output feedback; backstepping; stochastic nonlinear systems; dynamic surface control(DSC)

0 引 言

随机扰动在工业控制、物理电路系统中客观存在且不可避免. 在控制器设计中随机因素的存在使所建立的模型无法精确描述被控对象, 会增大系统误差, 影响系统性能. 时间延迟也是在实际的工程系统中常见的现象, 电力系统、化学反应、网络传输等都会出现时滞现象^[1]. 因此, 研究随机非线性时滞系统的控制问题是一个具有挑战而且有意义的工作. 早期的随机非线性研究需要非线性系统已知或者可

以线性参数化^[2-3], 限制了实际应用范围. 具有逼近能力的神经网络和模糊逻辑可以避免花费大量精力对系统进行建模^[4-7]. 最近, 基于神经网络和模糊逻辑的自适应 backstepping 方法也应用在随机非线性时滞系统的控制器设计中. 文献[8]针对严格反馈系统利用神经网络逼近特性和 Lyapunov-Razumikhin 函数提出了时滞随机系统的跟踪控制方法. 文献[9]引入一个变量分离方法并提出了模糊逻辑的自适应控制方法, 解决一类非严格反馈随机时滞系统的跟踪控制问

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题. 文献[10]研究了具有未知输入死区特性的时滞随机系统的控制. 上述的控制器设计方法^[9-10]是针对全状态反馈的随机非线性系统, 当状态不可测量或者不可获得时, 文献[11]利用基于神经网络逼近特性和backstepping方法, 设计了时滞随机系统的输出反馈控制器. 文献[12]提出了切换控制的随机时滞系统的模糊自适应跟踪控制.

控制方向未知是实际工程系统中普遍存在的问题, 如船舶航行、自主机器人和航空航天等控制系统中. 最近已有文献针对未知控制方向的随机系统提出了自适应模糊逻辑和神经网络的backstepping控制设计方法. 文献[13]提出了自适应模糊逻辑控制策略, 用以解决未知控制方向的随机非线性系统. 文献[14]引入一个Razumikhin Nussbaum定理来解决具有未知控制增益的时滞随机系统的神经网络控制问题. 文献[13-14]的结果同样只能应用于状态完全可测而无法解决输出反馈控制的问题. 文献[15]解决了具有未知控制方向的延迟随机系统的输出反馈问题.

当用神经网络或者模糊逻辑去逼近未知的非线性函数时, 由于backstepping设计过程中重复对虚拟函数等非线性函数进行微分, 导致复杂性爆炸. 为了避免出现该问题, 文献[16]利用动态表面控制(DSC)技术在传统的backstepping设计中引入一个一阶低通滤波器. 在文献[17]中, 利用DSC技术和backstepping方法研究了延迟系统的神经网络控制. 随着将时滞系统推广到模糊逻辑的输出反馈控制^[18], 文献[19]将DSC技术应用到了随机非线性系统的神经网络控制中.

本文研究一类具有未知控制方向的随机非线性时滞系统的自适应神经网络输出反馈控制问题. 利用RBF神经网络逼近未知的非线性函数, 构造一个状态估计器估计不可测量的系统状态; 利用Lyapunov-Krasovskii函数处理系统输出的时滞项. 基于动态表面控制(DSC)技术和backstepping方法设计的RBF神经网络控制器避免了复杂性问题; 利用Lyapunov稳定性理论证明了整个闭环系统的所有信号在二阶或者四阶矩意义下是有界的.

1 问题描述及预备知识

1.1 预备知识

考虑如下随机系统:

$$dx = f(x, t)dt + h(x, t)d\omega, \quad \forall x \in \mathbf{R}^n. \quad (1)$$

其中: $x \in \mathbf{R}^n$ 为状态向量; $f: \mathbf{R}^n \times \mathbf{R}^+ \rightarrow \mathbf{R}^n$, $h: \mathbf{R}^n \times \mathbf{R}^+ \rightarrow \mathbf{R}^n$ 为连续函数, 满足局部Lipschitz条

件; ω 为定义在完备概率空间 $(\Omega, \mathcal{F}, P)$ 上的 r 维相互独立的标准维纳过程向量, Ω 为样本空间, $\mathcal{F}$ 为 σ -代数族, P 为概率测度.

定义1 对于随机系统(1), 给定任意的 $V(x, t) \in C^{2,1}$, 定义如下的微分算子:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} \text{Tr} \left\{ h^T \frac{\partial V}{\partial x} h \right\}, \quad (2)$$

其中 $\text{Tr}(A)$ 代表矩阵 A 的迹.

定义2 $N(\cdot)$ 是连续Nussbaum类型的函数, 具有如下性质:

$$\begin{cases} \limsup_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty, \\ \liminf_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty. \end{cases} \quad (3)$$

由定义2可以看出, 很多连续的偶函数满足这些条件, 例如 $\exp(\zeta^2) \cos \zeta$, $\ln(\zeta + 1) \cos \sqrt{\ln(\zeta + 1)}$ 和 $\zeta^2 \cos \zeta$. 本文采用 $\zeta^2 \cos \zeta$ 作为Nussbaum类型函数来设计控制器.

引理1^[20] 考虑随机系统, 设 $V(x, t) \geq 0$ 且存在平滑函数 $\zeta: \mathbf{R}^+ \rightarrow \mathbf{R}$, 同时存在一个偶的平滑Nussbaum类型函数 $N(\zeta(t))$, 如果有如下不等式成立:

$$V(x, t) \leq c_0 + e^{-\kappa t} \int_0^t (gN(\zeta(\tau))\dot{\zeta} + \dot{\zeta}) e^{\kappa\tau} d\tau, \quad (4)$$

则 $V(t)$, $\zeta(t)$, $\int_0^t (gN(\zeta(\tau))\dot{\zeta} + \dot{\zeta}) d\tau$ 是有界的. 其中 g 为一个非零函数, κ 为正的常数, c_0 为一个相应合适的参数.

1.2 RBF神经网络

RBF神经网络^[21]在本文被用来逼近未知函数. 对于给定的连续函数 $f(x): \mathbf{R}^m \rightarrow \mathbf{R}$, 在紧集 $\Omega_Z \subset \mathbf{R}^m$, 和任意值 $\epsilon > 0$, 存在 $W^T S(Z)$, 使 $\sup |f(Z) - W^T S(Z)| \leq \epsilon$, 其中 $W \in \mathbf{R}^l$ 为神经网络权值, $l > 1$ 为神经网络个数, $Z \in \mathbf{R}^m$ 为神经网络输入, $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]$ 为径向基函数. 本文采用高斯函数如下:

$$s_i(z) = \exp\left(\frac{-(z - \mu_i)^T(z - \mu_i)}{\eta_i^2}\right), \quad i = 1, 2, \dots, l, \quad (5)$$

其中 μ_i 和 η_i 分别为神经元中心和高斯函数的宽度. 根据神经网络逼近原理, 连续函数 $f(Z)$ 可以逼近为

$$f(Z) = W^{*T} S(Z) + \epsilon(Z). \quad (6)$$

其中: W^* 为理想神经网络权值向量, $\epsilon(Z)$ 为逼近误差.

理想权重可表达为

$$W^* := \arg \min_{\hat{W} \in \mathbf{R}^l} \left\{ \sup_{Z \in \Omega_Z} |f(Z) - \hat{W}^T S(Z)| \right\}, \quad (7)$$

其中 $\hat{W}$ 为 W^* 的估计值.

通常 W^* 是未知常值向量, 需要用 $\hat{W}$ 作为估值.

然而,本文不直接利用估计值 $\hat{W}$,而是借助 W^* 的范数. $\|W^*\|^2$ 是一个未知的常值,存在未知常值 θ^* ,即 $\|W^*\|^2 = b\theta^*$,其中参数 b 是相关的正常数. $\hat{\theta}$ 作为 θ^* 的估计值,其中 $\tilde{\theta} = \hat{\theta} - \theta^*$.

注1 $W^* \in R^l$ 在计算过程中都需要估值,会引起计算负荷的增加,导致神经网络控制器存在过多的参数.而本文只需要估计一个参数 θ^* ,文献[22-23]同样利用了这种方式.

引理2^[24] 考虑高斯RBF神经网络(6),定义 $\varrho := \frac{1}{2} \min_{i \neq j} \|\xi_i - \xi_j\|$, q 为神经网络输入 Z 的维数, η 为高斯函数(如式(5)所示)的宽度,可以得出 $\|S(Z)\|$ 的上界

$$\|S(Z)\| \leq \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{-2\varrho^2 k^2 / \eta^2} := s^*, \quad (8)$$

其中 s^* 是与神经网络输入 Z 和权值 W 维数 l 无关的值.

1.3 系统描述

考虑如下具有随机非线性的时滞系统:

$$\begin{cases} dx_i = (g_i x_{i+1} + \phi_i(\bar{x}_i) + f_i(t, y(t), y(t - \tau_i)))dt + \\ \quad h_i(t, y(t), y(t - \tau_i))d\omega, \quad i = 1, 2, \dots, n-1; \\ dx_n = (g_n u + \phi_n(\bar{x}_n) + f_n(t, y(t), y(t - \tau_n)))dt + \\ \quad h_n(t, y(t), y(t - \tau_n))d\omega; \\ y(t) = x_1(t). \end{cases} \quad (9)$$

其中: $x_i \in R$ ($i = 1, 2, \dots, n$), $u \in R$ 和 $y \in R$ 分别为系统的状态,系统输入和系统输出;控制增益 g_i ($i = 1, 2, \dots, n$)为未知的常值; $\phi_i(\cdot)$ 为未知的平滑函数; $f_i(\cdot): R^n \rightarrow R$, $\psi_i(\cdot): R^n \rightarrow R^r$ 为未知的局部Lipschitz平滑函数, $f_i(t, 0, 0) = 0$; $h_i^T(t, 0, 0) = 0$; ω 为定义在完备概率空间 (Ω, F, P) 上的 r 维标准布朗运动, Ω 为样本空间, F 为 σ 代数, P 为概率测度; τ_i 为未知的常值时滞,满足 $\tau_i \leq \tau$, τ 为正常值.

本文的控制目标是设计针对系统(9)的神经网络控制器,使得系统输出 $y(t)$ 跟踪理想的轨迹 $y_d(t)$,并且保证闭环系统中所有信号依概率是有界的.

假设1 理想的轨迹 $y_d(t)$ 及其 n 阶导数都是连续有界的.

假设2^[25] 对于非线性函数 f_i 和 h_i ,存在非负光滑函数 f_{i1}, f_{i2}, h_{i1} 和 h_{i2} ,使得如下不等式成立:

$$\begin{aligned} |f_i(y, y(t - \tau_i))| &\leq f_{i1}(y) + f_{i2}(y(t - \tau_i)), \\ |h_i(y, y(t - \tau_i))| &\leq h_{i1}(y) + h_{i2}(y(t - \tau_i)). \end{aligned} \quad (10)$$

注2 系统(9)中输出延迟的存在使得控制器设计变得复杂.由假设2可知, $f_i(t, y, y(t - \tau_i))$ 和

$h_i(t, y, y(t - \tau_i))$ 满足分离规则.根据假设2,可以进一步得出如下不等式成立:

$$\begin{aligned} |f_i(y, y(t - \tau_i))|^p &\leq 2^{p-1}(f_{i1}^p(y) + f_{i2}^p(y(t - \tau_i))), \\ |h_i(y, y(t - \tau_i))|^p &\leq 2^{p-1}(h_{i1}^p(y) + h_{i2}^p(y(t - \tau_i))), \end{aligned} \quad (11)$$

其中 $p \geq 1$.

控制增益 g_i 的值和符号是未知的,实现输出反馈控制有些困难.因此,将原始系统(9)通过线性变换得到一个新的系统,定义 $\xi_i = \rho_i x_i$,其中 $\rho_i = 1 / \prod_{j=i}^n g_j$, $i = 1, 2, \dots, n$,新系统如下:

$$\begin{cases} d\xi_i = \\ (\xi_{i+1} + \rho_i \phi_i(\bar{x}_i) + \rho_i f_i(t, y(t), y(t - \tau_i)))dt + \\ \rho_i h_i(t, y(t), y(t - \tau_i))d\omega, \quad i = 1, 2, \dots, n-1, \\ d\xi_n = \\ (u + \rho_n \phi_n(\bar{x}_n) + \rho_n f_n(t, y(t), y(t - \tau_n)))dt + \\ \rho_n h_n(t, y(t), y(t - \tau_n))d\omega. \end{cases} \quad (12)$$

给出观测器

$$\begin{cases} \dot{\hat{\xi}}_i = \hat{\xi}_{i+1} + l_i(\xi_i - \hat{\xi}_i), \quad i = 1, 2, \dots, n-1, \\ \dot{\hat{\xi}}_n = u + l_n(\xi_n - \hat{\xi}_n). \end{cases} \quad (13)$$

其中: $\xi_1 = 1 / \left(\prod_{j=1}^n g_j \right) x_1$, l_j 为第 j 个观测器增益.

联立式(12)和(13),可以得到观测误差系统

$$d\tilde{\xi} = (A_0 \tilde{\xi} + \Phi + F)dt + H d\omega. \quad (14)$$

其中

$$\begin{aligned} F &= [\rho_1 f_1, \dots, \rho_n f_n]^T, \quad H = [\rho_1 h_1^T, \dots, \rho_n h_n^T]^T, \\ K &= [k_1, \dots, k_n]^T, \quad A_0 = \begin{bmatrix} -l_1 & & & \\ \vdots & I_{n-1} & & \\ -l_n & 0 & \dots & 0 \end{bmatrix}, \end{aligned} \quad (15)$$

A_0 为Hurwitz矩阵,因此给定矩阵 $Q > 0$,存在矩阵 $P > 0$ 使得

$$A_0^T P + P A_0 = -Q. \quad (16)$$

2 输出反馈控制器设计

给出神经网络自适应输出反馈控制的设计过程.在设计过程中用到了Lyapunov-Krasovskii方法和DSC技术,针对控制增益未知的问题利用了一个Nussbaum增益函数.

定义 $V_{\xi} = b(\tilde{\xi}^T P \tilde{\xi})^2 / 2$,对其求导可得

$$\dot{V}_{\xi} = b(\tilde{\xi}^T P \tilde{\xi}) \tilde{\xi}^T (A_0 P + P A_0) \tilde{\xi} +$$

$$2b(\tilde{\xi}^T P \tilde{\xi}) \tilde{\xi}^T P \Phi(x) + 2b(\tilde{\xi}^T P \tilde{\xi}) \tilde{\xi}^T P F(x) + 2b \text{Tr}\{H^T(2P \tilde{\xi} \tilde{\xi}^T P + \tilde{\xi}^T P \tilde{\xi} P)H\}. \quad (17)$$

利用 Young 不等式并参考假设2, 可得

$$b(\tilde{\xi}^T P \tilde{\xi}) \tilde{\xi}^T (A_0 P + P A_0) \tilde{\xi} \leq -b\lambda_{\min}(P)\lambda_{\min}(Q)\|\tilde{\xi}\|^4, \quad (18)$$

$$2b(\tilde{\xi}^T P \tilde{\xi}) \tilde{\xi}^T P \Phi \leq 2b\|\tilde{\xi}\|^3\|P\|^2\|\Phi\| \leq \frac{3}{2}bc_1^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|\tilde{\xi}\|^4 + \frac{1}{2c_1^4}\|\Phi\|^4, \quad (19)$$

$$2b(\tilde{\xi}^T P \tilde{\xi}) \tilde{\xi}^T P F \leq 2b\|\tilde{\xi}\|^3\|P\|^2\|F\| \leq \frac{3}{2}bc_2^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|\tilde{\xi}\|^4 + \frac{1}{2c_2^4}\|F\|^4 \leq \frac{3}{2}bc_2^{\frac{4}{3}}\|P\|^{\frac{8}{3}}\|\tilde{\xi}\|^4 + \frac{4}{c_2^4}n \sum_{j=1}^n (\rho_j^4 F_{j1}^4(y) + \rho_j^4 F_{j2}^4(y(t - \tau_j))), \quad (20)$$

$$2b \text{Tr}\{H^T(2P \tilde{\xi} \tilde{\xi}^T P + \tilde{\xi}^T P \tilde{\xi} P)H\} \leq 3bn^{\frac{3}{2}}c_3\|P\|^4\|\tilde{\xi}\|^4 + \frac{3bn^{\frac{3}{2}}}{c_3}\|H\|^4 \leq 3bn^{\frac{3}{2}}c_3\|P\|^4\|\tilde{\xi}\|^4 + \frac{8bn^{\frac{5}{2}}}{c_3} \sum_{j=1}^n (\rho_j^4 H_{j1}^4(y) + \rho_j^4 H_{j2}^4(y(t - \tau_j))). \quad (21)$$

联立式(18)~(20)并代入(17)中, 可得

$$\begin{aligned} \dot{V}_{\xi} \leq & -\left(b\lambda_{\min}(P)\lambda_{\min}(Q)\|\tilde{\xi}\|^4 - \frac{3}{2}bc_1^{\frac{4}{3}}\|P\|^{\frac{8}{3}} - \frac{3}{2}bc_2^{\frac{4}{3}}\|P\|^{\frac{8}{3}} - 3bn^{\frac{3}{2}}c_3\|P\|^4\right)\|\tilde{\xi}\|^4 + \\ & \frac{1}{2c_1^4}\|\Phi\|^4 + \frac{4}{c_2^4}n \sum_{j=1}^n (\rho_j^4 F_{j1}^4(y) + \rho_j^4 F_{j2}^4(y(t - \tau_j))) + \frac{8bn^{\frac{5}{2}}}{c_3} \sum_{j=1}^n (\rho_j^4 H_{j1}^4(y) + \rho_j^4 H_{j2}^4(y(t - \tau_j))). \end{aligned} \quad (22)$$

定义

$$\begin{aligned} z_1 &= y - y_d; \\ z_i &= \hat{\xi}_i - \alpha_{i-1,f}, \quad i = 2, 3, \dots, n-1; \\ e_{i+1} &= \alpha_{i,f} - \alpha_i, \quad i = 1, 2, \dots, n-1. \end{aligned} \quad (23)$$

其中: α_i 为虚拟控制器, $\alpha_{i,f}$ 为通过一阶低通滤波得出的控制函数, 即

$$\varpi_{i+1}\dot{\alpha}_{i,f} + \alpha_{i,f} = \alpha_i, \quad \alpha_{i,f}(0) = \alpha_i(0), \quad (24)$$

ϖ_{i+1} 为时间常值.

给出

$$\begin{aligned} dz_1 &= \dot{y} - \dot{y}_d = \\ & (\bar{g}(\hat{\xi}_2 + \tilde{\xi}_2) + \phi_1 + f_1)dt + h_1^T d\varpi - \dot{y}_d = \end{aligned}$$

$$(\bar{g}(z_2 + e_2 + \alpha_1 + \tilde{\xi}_2) + \phi_1 + f_1)dt + h_1^T d\varpi - \dot{y}_d; \quad (25)$$

$$\begin{aligned} dz_i &= \dot{\hat{\xi}}_i - \dot{\alpha}_{i-1,f} = \\ & \hat{\xi}_{i+1} + l_i(\xi_1 - \hat{\xi}_1) - \dot{\alpha}_{i-1,f} = \\ & z_{i+1} + e_{i+1} + \alpha_i + l_i \tilde{\xi}_1 + \frac{e_i}{\varpi_i}, \quad i = 2, 3, \dots, n-1; \end{aligned} \quad (26)$$

$$dz_n = \dot{\hat{\xi}}_n - \dot{\alpha}_{n-1,f} = u + l_n \tilde{\xi}_1 + \frac{e_n}{\varpi_n}. \quad (27)$$

定义

$$\begin{aligned} \alpha_1 &= N(\zeta)\eta; \quad \dot{\zeta} = z_1^3\eta; \\ \eta &= c_1 z_1 + \frac{\hat{\theta}}{2a} S^T(Z)S(Z)z_1^3; \end{aligned} \quad (28)$$

$$\alpha_i = -c_i z_i - l_i \tilde{\xi}_1 - \frac{e_i}{\varpi_i}, \quad i = 2, 3, \dots, n-1; \quad (29)$$

$$u = -c_n z_n - l_n \tilde{\xi}_1 - \frac{e_n}{\varpi_n}. \quad (30)$$

其中: $N(\zeta)$ 为 Nussbaum 增益函数, c_i 和 a 为设计常值.

利用 RBF 神经网络 $W^{*T}S(Z)$ 逼近未知函数, 定义 $\theta = \|W^*\|^2$, $\tilde{\theta}$ 为估计误差, $\tilde{\theta} = \hat{\theta} - \theta$, $\hat{\theta}$ 为 θ 的估计值, 有

$$\dot{\hat{\theta}} = \frac{r}{2a} S^T(Z)S(Z)z_1^6 - \sigma\hat{\theta}, \quad (31)$$

r, a 和 σ 为正的的设计常值.

$$\begin{aligned} de_2 &= \dot{\alpha}_{1f} - \dot{\alpha}_1 = \\ & -\frac{e_2}{\varpi_2}dt - L\alpha_1 dt - \frac{\partial \alpha_1}{\partial y} h_1^T d\varpi, \end{aligned} \quad (32)$$

$$\begin{aligned} de_{i+1} &= \dot{\alpha}_{i+1f} - \dot{\alpha}_{i+1} = \\ & -\frac{e_{i+1}}{\varpi_{i+1}}dt - L\alpha_i dt - \frac{\partial \alpha_i}{\partial y} h_1^T d\varpi, \end{aligned} \quad (33)$$

$$\begin{aligned} L\alpha_1 &= \frac{\partial \alpha_1}{\partial y} (\bar{g}(\hat{\xi}_2 + \tilde{\xi}_2) + \phi_1 + f_1) + \\ & \frac{1}{2} \frac{\partial^2 \alpha_1}{\partial y^2} h_1^T h_1 + \frac{\partial \alpha_1}{\partial \zeta} \dot{\zeta} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}}, \end{aligned} \quad (34)$$

$$\begin{aligned} L\alpha_i &= \frac{\partial \alpha_i}{\partial y} (\bar{g}(\hat{\xi}_2 + \tilde{\xi}_2) + \phi_1 + f_1) + \\ & \frac{1}{2} \frac{\partial^2 \alpha_i}{\partial y^2} h_1^T h_1 + \frac{\partial \alpha_i}{\partial \zeta} \dot{\zeta} + \frac{\partial \alpha_i}{\partial \hat{\theta}} \dot{\hat{\theta}} + \\ & \sum_{j=2}^i \frac{\partial \alpha_i}{\partial \hat{\xi}_j} (\hat{\xi}_{i+1} + l_i(\xi_1 - \hat{\xi}_1)). \end{aligned} \quad (35)$$

定义 Lyapunov 函数和 Lyapunov-Krasovskii 函数如下:

$$V = V_{\xi} + V_z + V_e + V_0 + \frac{1}{2r} \tilde{\theta}^2, \quad (36)$$

$$V_z = \frac{1}{4} \sum_{i=1}^n z_i^4, \quad (37)$$

$$V_e = \frac{1}{4} \sum_{i=2}^n e_i^4, \quad (38)$$

$$V_0 = e^{-\nu t} \frac{1}{1-\tau} \int_{t-\tau}^t e^{\nu s} \Psi ds. \quad (39)$$

其中: ν 和 τ 为正的设计常值, Ψ 为正的非线性函数.

基于 Itô 公式, $V_z + V_e$ 的 L 运算为

$$\begin{aligned} LV_z + LV_e = & z_1^3(\bar{g}(z_2 + e_2 + \alpha_1 + \tilde{\xi}_2) + \phi_1 + f_1 - \dot{y}_d) + \\ & \frac{3}{2} z_1^2 h_1^T h_1 + \sum_{i=2}^{n-1} z_i^3 (z_{i+1} + e_{i+1} + \alpha_i + l_i \tilde{\xi}_1 + \\ & \frac{e_i}{\varpi_i}) + z_n^3 (u + l_n \tilde{\xi}_1 + \frac{e_n}{\varpi_n}) - \sum_{i=2}^n \frac{e_i^4}{\varpi_i} - \\ & \sum_{i=2}^n e_i^3 L \alpha_{i-1} + \frac{3}{2} \sum_{i=2}^n e_i^2 \left(\frac{\partial \alpha_{i-1}}{\partial y} h_1 \right)^2. \end{aligned} \quad (40)$$

将式(29)和(30)代入(40), 整理可得

$$\begin{aligned} LV_z + LV_e = & z_1^3(\bar{g}(z_2 + e_2 + \alpha_1 + \tilde{\xi}_2) + \phi_1 + f_1 - \dot{y}_d) + \\ & \frac{3}{2} z_1^2 h_1^T h_1 + \sum_{i=2}^{n-1} z_i^3 (z_{i+1} + e_{i+1} - c_i z_i) - \\ & c_n z_n^4 - \sum_{i=2}^n \frac{e_i^4}{\varpi_i} - \sum_{i=2}^n e_i^3 L \alpha_{i-1} + \\ & \frac{3}{2} \sum_{i=2}^n e_i^2 \left(\frac{\partial \alpha_{i-1}}{\partial y} h_1 \right)^2. \end{aligned} \quad (41)$$

利用 Young 不等式, 可以得到如下不等式:

$$\begin{aligned} \frac{3}{2} z_1^2 h_1^T h_1 & \leq \frac{3}{4} z_1^4 + \frac{3}{4} h_1^4 \leq \\ \frac{3}{4} z_1^4 + 6 h_{11}^4(y) + 6 h_{12}^4(y(t - \tau_1)), \end{aligned} \quad (42)$$

$$\begin{aligned} z_1^3 f_1 & \leq \frac{3}{4} z_1^4 + \frac{1}{4} f_1^4 \leq \\ \frac{3}{4} z_1^4 + 2 f_{11}^4(y) + 2 f_{12}^4(y(t - \tau_1)), \end{aligned} \quad (43)$$

$$\begin{aligned} \frac{3}{2} \sum_{i=2}^n e_i^2 \left(\frac{\partial \alpha_{i-1}}{\partial y} h_1 \right)^2 & \leq \\ \sum_{i=2}^n \frac{3}{4} e_i^4 \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^4 + \frac{3(n-1)}{4} h_1^4 & \leq \\ \sum_{i=2}^n \frac{3}{4} e_i^4 \left(\frac{\partial \alpha_{i-1}}{\partial y} \right)^4 + 6(n-1) h_{11}^4(y) + \\ 6(n-1) h_{12}^4(y(t - \tau_1)), \end{aligned} \quad (44)$$

$$\begin{aligned} - \sum_{i=2}^n e_i^3 \frac{\partial \alpha_i}{\partial y} f_1 & \leq \\ \frac{3}{4} \sum_{i=2}^n e_i^4 \left(\frac{\partial \alpha_i}{\partial y} \right)^4 + \frac{n-1}{4} f_1^4 & \leq \end{aligned}$$

$$\begin{aligned} \frac{3}{4} \sum_{i=2}^n e_i^4 \left(\frac{\partial \alpha_i}{\partial y} \right)^4 + 2(n-1) f_{11}^4(y) + \\ 2(n-1) f_{12}^4(y(t - \tau_1)), \end{aligned} \quad (45)$$

$$\begin{aligned} - \frac{1}{2} \sum_{i=2}^n e_i^3 \frac{\partial^2 \alpha_i}{\partial y^2} h_1^T h_1 & \leq \\ \frac{1}{4} \sum_{i=2}^n e_i^6 \left(\frac{\partial^2 \alpha_i}{\partial y^2} \right)^2 + \frac{n-1}{4} h_1^4 & \leq \\ \frac{1}{4} \sum_{i=2}^n e_i^6 \left(\frac{\partial^2 \alpha_i}{\partial y^2} \right)^2 + 2(n-1) h_{11}^4(y) + \\ 2(n-1) h_{12}^4(y(t - \tau_1)), \end{aligned} \quad (46)$$

$$\begin{aligned} - \sum_{i=2}^n e_i^3 \frac{\partial \alpha_1}{\partial y} (\bar{g} \tilde{\xi}_2) & \leq \\ \frac{3}{4} \sum_{i=2}^n e_i^4 \left(\frac{\partial \alpha_1}{\partial y} \bar{g} \right)^4 + \frac{(n-1)}{4} \tilde{\xi}_2^4, \end{aligned} \quad (47)$$

$$\begin{aligned} - \sum_{i=2}^n e_i^3 \sum_{j=2}^i \frac{\partial \alpha_i}{\partial \hat{\xi}_j} l_i \tilde{\xi}_1 & \leq \\ \sum_{i=2}^n \frac{3}{4} e_i^4 \left(\sum_{j=2}^i \frac{\partial \alpha_i}{\partial \hat{\xi}_j} l_i \right)^4 + \frac{n(n-1)}{8} \tilde{\xi}_1^4, \end{aligned} \quad (48)$$

$$z_1^3 \bar{g}(z_2 + e_2) \leq \frac{3}{2} \bar{g}^{\frac{4}{3}} z_1^4 + \frac{1}{4} z_2^4 + \frac{1}{4} e_2^4, \quad (49)$$

$$z_1^3 \bar{g} \tilde{\xi}_2 \leq \frac{3}{4} \bar{g}^{\frac{4}{3}} z_1^4 + \frac{1}{4} \tilde{\xi}_2^4, \quad (50)$$

$$z_1^3 (\phi_1 - \dot{y}_d) \leq \frac{3}{2} z_1^4 + \frac{1}{4} \phi_1^4 + \frac{1}{4} \bar{y}_d^4, \quad (51)$$

$$\begin{aligned} \sum_{i=2}^{n-1} z_i^3 (z_{i+1} + e_{i+1}) & \leq \\ \sum_{i=2}^{n-1} \frac{3}{2} z_i^4 + \sum_{i=2}^{n-1} \frac{1}{4} z_{i+1}^4 + \sum_{i=2}^{n-1} \frac{1}{4} e_{i+1}^4, \end{aligned} \quad (52)$$

$$\begin{aligned} - \sum_{i=2}^n e_i^3 \frac{\partial \alpha_1}{\partial y} (\bar{g} \hat{\xi}_2 + \phi_1) & \leq \\ \sum_{i=2}^n \frac{3}{2} e_i^4 + \frac{(n-1)}{4} \left(\frac{\partial \alpha_1}{\partial y} \bar{g} \hat{\xi}_2 \right)^4 + \frac{(n-1)}{4} \left(\frac{\partial \alpha_1}{\partial y} \phi_1 \right)^4, \end{aligned} \quad (53)$$

$$\begin{aligned} - \sum_{i=2}^n e_i^3 \left(\frac{\partial \alpha_1}{\partial \zeta} \dot{\zeta} + \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) & \leq \\ \sum_{i=2}^n \frac{3}{2} e_i^4 + \sum_{i=2}^n \frac{1}{4} \left(\frac{\partial \alpha_i}{\partial \zeta} \dot{\zeta} \right)^4 + \sum_{i=2}^n \frac{1}{4} \left(\frac{\partial \alpha_i}{\partial \hat{\theta}} \dot{\hat{\theta}} \right)^4, \end{aligned} \quad (54)$$

$$\begin{aligned} - \sum_{i=2}^n e_i^3 \sum_{j=2}^i \frac{\partial \alpha_i}{\partial \hat{\xi}_j} \hat{\xi}_{i+1} & \leq \\ \sum_{i=2}^n \frac{3}{4} e_i^4 + \sum_{i=2}^n \sum_{j=2}^i \frac{1}{4} \left(\frac{\partial \alpha_i}{\partial \hat{\xi}_j} \hat{\xi}_{i+1} \right)^4; \end{aligned} \quad (55)$$

定义

$$-l_\xi = -\left(b\lambda_{\min}(P)\lambda_{\min}(Q)\| + \frac{3}{2}bc_1^{\frac{4}{3}}\|P\|^{\frac{8}{3}} + \frac{3}{2}bc_2^{\frac{4}{3}}\|P\|^{\frac{8}{3}} + 3bn^{\frac{3}{2}}c_3\|P\|^4 + \zeta_0\right)\|\tilde{\xi}\|^4; \quad (56)$$

$$-l_{z1} = -c_1 + \frac{3}{4}c_0; \quad (57)$$

$$-l_{zi} = -c_i + \frac{7}{4}, \quad i = 2, 3, \dots, n-1; \quad (58)$$

$$-l_{zn} = -c_n + \frac{1}{4}, \quad i = n; \quad (59)$$

$$-l_{ei} = -\frac{1}{\varpi_i} + 4, \quad i = 2, 3, \dots, n-1. \quad (60)$$

其中: $\zeta_0 = \text{diag}\left[\frac{n(n-1)}{8}, \frac{(n-1)}{4}, 0, \dots, 0\right]$, c_0 和 c_i 为正的设计常值. 对 Lyapunov-Krasovskii 函数求导得

$$\dot{V}_0 = -\nu V_0 + \frac{1}{1-\tau}\Psi(y) - e^{-\nu\tau}\Psi(y(t-\tau)). \quad (61)$$

定义

$$\Psi(y(t-\tau)) =$$

$$e^{-\nu\tau} \frac{4}{c_2^4} n \sum_{j=1}^n \rho_j^4 F_{j2}^4(y(t-\tau_j)) +$$

$$e^{-\nu\tau} \frac{8bn^{\frac{5}{2}}}{c_3} \sum_{j=1}^n \rho_j^4 H_{j2}^4(y(t-\tau_j)) +$$

$$2ne^{-\nu\tau} f_{12}^4(y(t-\tau_1)) +$$

$$(8n-2)e^{-\nu\tau} h_{12}^4(y(t-\tau_1)); \quad (62)$$

$$H =$$

$$\sum_{i=2}^n \frac{3}{4} e_i^4 \left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^4 + \frac{1}{2c_1^4} \|\Phi\|^4 +$$

$$\frac{3}{4} \sum_{i=2}^n e_i^4 \left(\frac{\partial \alpha_i}{\partial y}\right)^{\frac{4}{3}} + \frac{1}{4} \sum_{i=2}^n e_i^6 \left(\frac{\partial^2 \alpha_i}{\partial y^2}\right)^2 +$$

$$\frac{3}{4} \sum_{i=2}^n e_i^4 \left(\frac{\partial \alpha_1}{\partial y} \bar{g}\right)^{\frac{4}{3}} + \sum_{i=2}^n \frac{3}{4} e_i^4 \left(\sum_{j=2}^i \frac{\partial \alpha_i}{\partial \hat{\xi}_j} l_i\right)^{\frac{4}{3}} +$$

$$\frac{1}{4} \phi_1^4 + \frac{(n-1)}{4} \left(\frac{\partial \alpha_1}{\partial y} \bar{g} \hat{\xi}_2\right)^4 + \frac{(n-1)}{4} \left(\frac{\partial \alpha_1}{\partial y} \phi_1\right)^4 +$$

$$\sum_{i=2}^n \frac{1}{4} \left(\frac{\partial \alpha_i}{\partial \zeta} \dot{\zeta}\right)^4 + \sum_{i=2}^n \frac{1}{4} \left(\frac{\partial \alpha_i}{\partial \hat{\theta}} \dot{\hat{\theta}}\right)^4 +$$

$$\sum_{i=2}^n \sum_{j=2}^i \frac{1}{4} \left(\frac{\partial \alpha_i}{\partial \hat{\xi}_j} \hat{\xi}_{i+1}\right)^4 + \frac{1}{1-\tau} \Psi(y); \quad (63)$$

$$F_h = \frac{9}{4} \bar{g}^{\frac{4}{3}} z_1 + 3z_1 + \frac{H}{z_3^3}. \quad (64)$$

式(64)中 H/z^3 在 $z_1 = 0$ 处不连续, 无法利用神经网络去逼近. 引入双曲正切函数 $\tanh(z_1/\iota)$, ι 为正的设计参数 (注意 $\lim_{z_1 \rightarrow 0} (16/z_1^3) \tanh^2(z_1/\iota) H$ 存在), 有

$$F_{NN} = \frac{9}{4} \bar{g}^{\frac{4}{3}} z_1 + 3z_1 + \frac{16}{z_1^3} \tanh^2\left(\frac{z_1}{\iota}\right) H, \quad (65)$$

$$LV = LV_\xi + LV_z + LV_e + \dot{V}_0 + \frac{\tilde{\theta}}{r} \dot{\hat{\theta}} \leq$$

$$-l_\xi \|\tilde{\xi}\|^4 + z_1^3 (\bar{g} \alpha_1 + F_{NN}) - \sum_{i=2}^{n-1} l_{zi} z_i^4 -$$

$$l_{zn} z_n^4 + \sum_{i=2}^n l_{ei} e_i^4 + \frac{1}{4} \bar{y}_d^4 - \nu V_0 +$$

$$\frac{\tilde{\theta}}{r} \dot{\hat{\theta}} + \left(1 - 16 \tanh^2\left(\frac{z_1}{\iota}\right)\right) H. \quad (66)$$

利用 RBF 神经网络去逼近未知非线性函数

$$F_{NN} = W^{*T} S(Z) + \epsilon. \quad (67)$$

其中: $\epsilon \leq \epsilon^*$ 为神经网络逼近误差, $Z = [x_1, \hat{\xi}_2, \dots, \hat{\xi}_n, e_2, \dots, e_n, \hat{\theta}, y_d, \dots, y_d^{(n)}]^T$.

利用 Young 不等式可以得到

$$z_1^3 F_{NN} = z_1^3 (W^{*T} S(Z) + \epsilon) \leq$$

$$\frac{\tilde{\theta}}{2a} S^T(Z) S(Z) z_1^6 + \frac{a}{2} + \frac{3}{4} c_0^{\frac{4}{3}} z_1^4 + \frac{1}{4c_0} \epsilon^{*4}, \quad (68)$$

其中 c_0 为设计常值. 并考虑

$$-\frac{\sigma}{r} \tilde{\theta} \dot{\hat{\theta}} \leq -\frac{\sigma}{r} \tilde{\theta}^2 + \frac{\sigma}{r} \tilde{\theta} \theta^* \leq -\frac{\sigma \tilde{\theta}^2}{2r} + \frac{\sigma \theta^{*2}}{2r}, \quad (69)$$

整理最终可得

$$LV = LV_\xi + LV_z + LV_e + \dot{V}_0 + \frac{\tilde{\theta}}{r} \dot{\hat{\theta}} \leq$$

$$-l_\xi \|\tilde{\xi}\|^4 + z_1^3 \bar{g} \alpha_1 + \frac{\tilde{\theta}}{2a} S^T(Z) S(Z) z_1^6 + \frac{a}{2} +$$

$$\frac{3}{4} c_0 z_1^4 + \frac{1}{4c_0} \epsilon^{*4} - \sum_{i=2}^{n-1} l_{zi} z_i^4 - l_{zn} z_n^4 +$$

$$\sum_{i=2}^n l_{ei} e_i^4 + \frac{1}{4} \bar{y}_d^4 - \nu V_0 + \frac{\tilde{\theta}}{2a} S^T(Z) S(Z) z_1^6 -$$

$$\frac{\sigma}{r} \tilde{\theta} \dot{\hat{\theta}} + \left(1 - 16 \tanh^2\left(\frac{z_1}{\iota}\right)\right) H \leq$$

$$-\alpha V + [\bar{g} N(\zeta) + 1] \dot{\zeta} + D +$$

$$\left(1 - 16 \tanh^2\left(\frac{z_1}{\iota}\right)\right) H. \quad (70)$$

其中: $\alpha = \min\left\{\frac{2l_\xi}{b\lambda_{\max}^2(P)}, 4l_{z1}, 4l_{zi}, 4l_{ei}, \nu, \sigma\right\}$, $i = 2, 3, \dots, n$, $D = \frac{1}{4} \bar{y}_d^4 + \frac{1}{4c_0} \epsilon^{*4} + \frac{a}{2}$. 这里需要选择适当的参数保证 $\alpha > 0$.

引理3^[26] 对于 $\iota > 0$, 考虑集合 A , $A := \{z_1 | z_1 | \leq 0.2554\iota\}$. 对于 $z_1 \notin A$, 满足不等式 $1 - 16 \tanh^2(z_1/\iota) < 0$.

定理1 针对随机非线性时滞系统(9)在满足假设1、假设2的情况下, 与设计的控制器(30), 虚拟控制函数(28)和(29), 自适应更新率(31)和观测器(13)构成的闭环系统, 则在任意有界的初始条件下, 闭环系统的所有信号在二阶或四阶矩意义下半全局一致有界, 误差收敛到紧集范围内.

证明 1) 当 $z_1 \notin A$ 时, $|z_1| > 0.2554\iota$. 由 H 的定义可以看出, H 为非负数. 由引理3可知, $(1 -$

$16 \tanh^2(z_1/l)H < 0$, 因此式(70)可重写为

$$LV \leq -\alpha V + [\bar{g}N(\zeta) + 1]\dot{\zeta} + D. \quad (71)$$

由引理1可知, $V, \zeta, z_i, \hat{\theta}$ 都是有界的.

对式(71)两边积分, 可得

$$0 \leq V(t) \leq e^{-\alpha t}V(0) + e^{-\alpha t} \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} e^{\alpha \tau} d\tau + \frac{D}{\alpha}, \quad (72)$$

有

$$EV(t) \leq EV(0) + \frac{D}{\alpha} + \sup \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} d\tau, \quad (73)$$

最终可得

$$\begin{aligned} & \frac{b\lambda_{\min}^2(P)}{2} E(\tilde{\xi}^4) + \frac{1}{4} E\left(\sum_{i=1}^4 z_i^4\right) + E \sum_{i=2}^4 e_i^4 \leq \\ & EV(t) \leq \\ & EV(0) + \frac{D}{\alpha} + \sup \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} d\tau. \end{aligned} \quad (74)$$

定义

$$G_1 = EV(0) + \frac{D}{\alpha} + \sup \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} d\tau,$$

存在紧集

$$\begin{aligned} \Omega := \left\{ \xi, e_i, z_i \mid E(\tilde{\xi}^4) \leq \frac{2}{b\lambda_{\min}^2(P)} G_1; \right. \\ \left. E\left(\sum_{i=1}^n z_i^4\right) \leq 4G_1; E\left(\sum_{i=2}^n e_i^4\right) \leq 4G_1 \right\}. \end{aligned}$$

2) 当 $z_1 \in \Lambda$ 时, $|z_1| \leq 0.2554l$, 因此 z_1 是有界的. 根据式(31), 自适应更新率 $\hat{\theta}$ 是有界的, 并且 θ^* 是常值, 所以 $\tilde{\theta}$ 有界. 最终可以得出闭环系统中所有信号有界.

根据 H 的定义, 可以得出 H 是有界的, 即 $0 \leq H \leq H^*$. 由引理3可得如下不等式成立:

$$0 \leq \left(1 - 16 \tanh^2\left(\frac{z_1}{l}\right)\right) \leq 1, \quad (75)$$

即

$$0 \leq \left(1 - 16 \tanh^2\left(\frac{z_1}{l}\right)\right) H^* \leq H^*. \quad (76)$$

因此式(70)可重写为

$$\dot{V} \leq -\alpha V + [\bar{g}N(\zeta) + 1]\dot{\zeta} + D^*, \quad (77)$$

其中 $D^* = D + H^*$, 闭环系统中所有信号有界.

根据 $E((z_1)^4)/4 \leq (0.2554l)^4$, 对式(71)两边积分, 可得

$$\begin{aligned} & 0 \leq V(t) \leq \\ & e^{-\alpha t}V(0) + e^{-\alpha t} \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} e^{\alpha \tau} d\tau + \\ & \frac{D^*}{\alpha} + (0.2554l)^4, \end{aligned} \quad (78)$$

有

$$\begin{aligned} EV(t) \leq EV(0) + \frac{D^*}{\alpha} + (0.2554l)^4 + \\ \sup \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} d\tau, \end{aligned} \quad (79)$$

最终可得

$$\begin{aligned} & \frac{b\lambda_{\min}^2(P)}{2} E(\tilde{\xi}) + \frac{1}{4} E(z_1^4) + \\ & \frac{1}{4} E\left(\sum_{i=1}^4 z_i^4\right) + E \sum_{i=2}^4 e_i^4 \leq EV(t) \leq \\ & EV(0) + \frac{D^*}{\alpha} + (0.2554l)^4 + \\ & \sup \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} d\tau. \end{aligned} \quad (80)$$

定义

$$\begin{aligned} G_1 = EV(0) + \frac{D^*}{\alpha} + (0.2554l)^4 + \\ \sup \int_0^t [\bar{g}N(\zeta) + 1]\dot{\zeta} d\tau. \end{aligned}$$

存在紧集

$$\begin{aligned} \Omega_z := \left\{ \xi, e_i, z_i \mid E(\tilde{\xi}^4) \leq \frac{2}{b\lambda_{\min}^2(P)} G_1; \right. \\ \left. E\left(\sum_{i=1}^n z_i^4\right) \leq 4G_1; E\left(\sum_{i=2}^n e_i^4\right) \leq 4G_1 \right\}. \end{aligned}$$

定理成立. $\square$

3 仿真结果

为了进一步说明本文方法的有效性, 这里利用文献[15]给出的倒立摆例子, 即

$$ml\ddot{q} = -mg \sin q - klq + \frac{1}{l}u. \quad (81)$$

其中: u 为倒立摆的输入扭矩, q 为杆与垂直坐标轴的夹角, g 为重力加速度, k 为摩擦系数, l 为杆的长度, m 为杆的质量. 参考文献[15]引入随机扰动和时间延迟, 定义 $x_1 = q/k$, $x_2 = ml\dot{q} + klq$, 则系统(81)可表达成如下形式:

$$\begin{cases} dx_1 = \left(g_1 x_2 - 0.5 \frac{k}{m} x_1 - 0.5 x_1(t - \tau_1)\right) dt + \\ \quad 0.5 x_1(t - \tau_1) d\omega, \\ dx_2 = (g_2 u - smg \sin(kx_1) + 0.5 \sin(x_1(t - \\ \quad \tau_2))) dt + 0.5 x_1^2(t - \tau_2) d\omega, \\ y = x_1. \end{cases} \quad (82)$$

其中: $g_1 = 1/(mkl)$ 和 $g_2 = 1/l$ 是未知的控制增益, 参考式(2)的线性状态变换, $\xi_1 = x_1/(g_1 g_2)$, $\xi_2 = x_1/g_2$. ω 选一维高斯白噪声, 其均值为零、方差为1. 控制设计的目标是设计一个自适应神经网络控制器使得所有信号有界, 系统输出 y 跟踪参考轨迹 $y_d = 0.5(\sin t + \sin(0.5t))$.

设计如下观测器:

$$\begin{cases} \dot{\hat{\xi}}_1 = \hat{\xi}_2 + l_1 \tilde{\xi}_1, \\ \dot{\hat{\xi}}_2 = u + l_2 \tilde{\xi}_1. \end{cases} \quad (83)$$

定义误差 $z_1 = y - y_d, z_2 = \hat{\xi}_1 - \alpha_{1,f}, e_2 = \alpha_{1,f} - \alpha_1$. 类似控制器的推导过程, 对应的虚拟控制信号 α_1 和信号 ζ 为 $\alpha_1 = N(\zeta)\eta$ 和 $\dot{\zeta} = z_1^3\eta$.

$$\eta = c_1 z_1 + \frac{\hat{\theta}}{2a} S^T(Z) S(Z) z_1^3, \quad (84)$$

$$u = -c_2 z_2 - l_2 \tilde{\xi}_1 - \frac{e_2}{\varpi_2}. \quad (85)$$

自适应更新率为

$$\dot{\hat{\theta}} = \frac{r}{2a} S^T(Z) S(Z) z_1^6 - \sigma \hat{\theta}. \quad (86)$$

参数设置如下: Nussbaum 函数选为 $N(\zeta) = \zeta^2 \cos \zeta, l_1 = 1.5, l_2 = 2, c_1 = 15, c_2 = 14, a = 1, \lambda = 1, \varpi = 0.5, \sigma = 0.5$. 基于设计的控制器, RBF 神经网络用来逼近未知的系统动态, RBF 神经网络 $\hat{W}^{*T} S(Z)$ 包含 5^4 个神经元, 均匀分布在 $[-2, 2] \times [-2, 2] \times [-2, 2] \times [-2, 2]$, 其神经网络间距为 1, 宽度设为 0.95. $x_1(0) = 0.2, x_2(0) = 0.3, \hat{x}_1(0) = 0.4, \hat{x}_2(0) = 1.2, \tau_1 = 0.5, \tau_2 = 1.2, \zeta(0) = 0, \hat{\theta}(0) = 0$.

图1~图5给出了仿真结果.

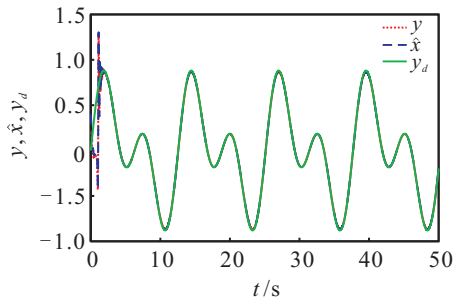


图1 控制器的跟踪性能

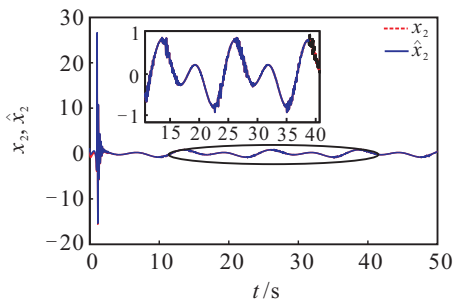


图2 系统状态 x_2 和估计 $\hat{x}_2$

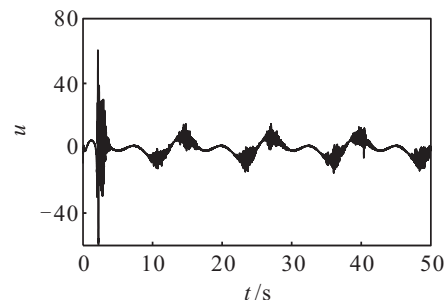


图3 系统控制输入 u

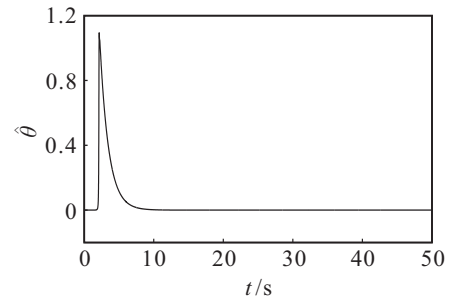


图4 自适应更新率 $\hat{\theta}$

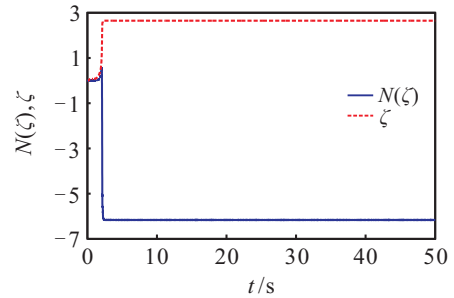


图5 Nussbaum 函数: $N(\zeta)$ 和 ζ

由图1可以看出理想的跟踪性能, 其中 y 为系统输入, y_d 为参考轨迹, $\hat{x}$ 为时间轨迹. 图2为系统状态 x_2 和观测器状态 $\hat{x}_2$ 的时间轨迹, 表明状态是有界的. 图3为系统控制输入 u , 图4表明自适应更新率 $\hat{\theta}$ 是有界的, 图5展示了 Nussbaum 函数的轨迹.

4 结论

本文研究了具有未知控制方向随机非线性时滞系统的控制问题. 利用 RBF 神经网络逼近未知的非线性函数, 利用状态观测器估计随机系统中不可测量的状态. 基于 backstepping 方法和 DSC 技术, 构造了自适应神经网络控制器, 避免了复杂性爆炸的问题. 通过 Lyapunov 函数证明了所提出的控制器可以保证闭环系统中所有信号在二阶或四阶矩意义上是半全局一致最终有界的, 观测器误差和系统输出误差收敛到零值小的范围内.

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