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一类不确定非线性离散系统的模糊自适应控制器设计

范永青†, 王文庆, 江祥奎, 刘 颖

(西安邮电大学 自动化学院，西安 710121)

摘 要: 针对一类不确定非线性离散系统,提出一种带有自动可调伸缩因子的模糊自适应控制方法. 该控制器设计方法的优点是模糊逻辑系统的逼近精度不再依赖于模糊逻辑系统的结构和规则数目,参数自适应律调节与被逼近函数的特征和逼近精度有关,因此能有效减少在线估计的参数数目,且设计方法能够保证闭环系统的所有状态半全局一致终极有界. 最后,通过数值仿真算例表明所提出方法的有效性.

关键词: 不确定离散系统；模糊逻辑系统；自适应控制；半全局一致终极有界

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Fuzzy adaptive control design for a class of uncertain nonlinear discrete-time systems

FAN Yong-qing†, *WANG Wen-qing*, *JIANG Xiang-kui*, *LIU Ying*

(School of Automation，Xi’an University of Posts and Telecommunications，Xi’an 710121，China)

Abstract: A fuzzy adaptive controller with scaler is proposed for a class of uncertain nonlinear discrete-time systems in this paper. The advantage of the design method is that the approximation of fuzzy logic systems does not depend on the structure and the number of the rules in fuzzy logic systems, and the number of updated laws is related to the character and approximation of the approximated functions, which can not only reduce the number of on-line parameters, but also guarantee the states of systems semi-global uniformly ultimately bounded (UUB). Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: uncertain nonlinear discrete-time systems；fuzzy logic systems；adaptive control；semi-global uniformly ultimately bounded

0 引 言

模糊逻辑系统和神经网络系统具有万能逼近性质,在对非线性系统中的不确定项处理时具有举足轻重的作用[1-5]. 近年来,采用神经网络控制器和模糊自适应控制的设计方法得到了众多学者的关注[6-18]. 在文献 [6, 10-18] 中,模糊自适应控制器设计的特点是将被逼近的未知项表示成某些模糊基函数的线性组合形式,然后利用自适应技术估计基函数的线性组合系数和逼近精度来设计自适应控制器,因此,采用这类方法的结论是,自适应参数的多少完全由模糊规则的数目决定. 对于模糊逻辑系统而言, 大量模糊规则会导致过多的在线调节自适应参数,这类设计方法会使系统的控制过程发生延迟继而产生失控现象.

在如何减少模糊自适应控制中的在线调节参数的问题上, 文献 [19-22] 采用向量范数把模糊逻辑系统的输出后件参数归一化以减少自适应参数的数目,但是这种方法的缺点是模糊逻辑系统的输入变量的数目会增多. 文献 [23] 提出了广义模糊双曲正切模型,该模型的模糊规则中每个模糊变量的模糊集简化为两个,且在每个模糊规则的输出部分省略了输入变量,使得模糊规则减少的同时在线调节自适应参数的数量也相应减少;对一类带有控制方向未知反对象滞后多输入多输出系统的自适应预定义控制器设计问题,给出一种基于广义模糊双曲正切模糊

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作者简介: 范永青 (1978−), 女, 副教授, 博士, 从事模糊控制、自适应控制、非线性控制理论等研究；王文庆 (1964−), 男, 教授, 博士后, 从事控制理论与工程等研究.

†通讯作者. E-mail: yongq-f@163.com.

自适应控制方法[24],但是这种模糊模型对系统的描述能力变差. 在这些研究成果中,自适应律的构造是建立在模糊逻辑系统的输出具有线性化参数的基础上,而对于其他不能表示成基函数的多种不同形式的模糊逻辑系统,如非规则的“推理模糊逻辑系统”[25]、“三I形式的模糊逻辑系统”[26]和“正规模糊逻辑系统”[27],上述文献所给出的控制方法则无能为力. 由此可知,有必要进一步研究其他理论方法.

在离散系统的模糊自适应控制器设计过程中,难点之一体现在被控系统中非线性函数的状态需要事先假设落入一有界集中才能满足模糊逻辑系统逼近的假设条件[28],但在许多实际控制中这一假设并不能得到保证. 文献[29]中,引入了带有参数的伸缩器和饱和器,加载在普通模糊逻辑系统的两端以改造模糊逻辑系统,通过理论分析,该模糊自适应控制器设计方法大大改善了其他文献中的缺陷,但是仅对连续系统做了分析. 由于在分析方法和所得结果上,离散系统与连续系统有着本质的不同[30],其主要体现在离散时间的Lyapunov函数不再具备连续时间Lyapunov函数的某种线性,导致离散系统与连续系统的所得结果不同. 而且当今的控制器大都采用计算机实现,因此,如何设计不确定离散系统的模糊自适应控制也是值得深入研究的问题.

综合以上分析, 作为解决上述问题的一个尝试,本文探索如何在引入伸缩器和饱和器的模糊逻辑系统中,为一类不确定离散系统设计模糊自适应控制器,使得控制器的在线参数调节律独立于模糊逻辑系统的输出形式,减少在线调节参数的数目,同时又能保证模糊逻辑系统的语言可解释性. 最后,通过仿真算例验证所提出方法的有效性.

1 系统与预备知识

1.1 系统描述与假设

考虑如下形式的离散时间非线性系统:

$$\begin{cases} x_1(k+1)=x_2(k), \\ \quad\vdots \\ x_{n-1}(k+1)=x_n(k), \\ x_{n+1}(k+1)=f(x(k))+g(x(k))u(k)+\tau_d(k), \\ y(k)=x_1(k). \end{cases} \tag{1}$$

其中: $x(k)=[x_1(k),\cdots,x_n(k)]^{\mathrm{T}}\in R^n$为状态向量,且完全可测;非线性函数$f(x(k))$和$g(x(k))$是未知的; $u(k)\in R$为控制输入; $\tau_d(k)$为外界干扰,并满足$|\tau_d(k)|\leqslant\overline{\tau}$, $\overline{\tau}$为已知常数; $y(k)\in R$为系统的输出.

给定一理想跟踪信号$y_d(k)$,跟踪误差定义为$e(k)=e_1(k)=y(k)-y_d(k)$,令误差向量$E(k)=[e_1(k),e_2(k),\cdots,e_n(k)]^{\mathrm{T}}$,其中$e_i(k)=x_i(k)-y_d(k+i-1)$, $i=1,2,\cdots,n$. 假设误差向量是可测量的,则系统(1)可写成如下形式:

$$E(k+1)=AE(k)+B[f(x(k))+g(x(k))u(k)-y_d(k+n)+\tau_d(k)]. \tag{2}$$

其中:矩阵$A=\begin{bmatrix}O & I_{n-1}\\ 0 & O^{\mathrm{T}}\end{bmatrix}$, $B=[O^{\mathrm{T}}\ \ 1]^{\mathrm{T}}$, O代表元素全部为0的$n-1$阶列向量;控制器$u(k)$是根据下面控制目标设计的.

控制目标: 1) 设计控制器$u(k)$使得系统(1)的输出$y(k)$与跟踪期望信号$y_d(k)$之间的跟踪误差收敛到零的一个小邻域内; 2) 闭环系统的所有信号保持半全局一致有界.

假设1 1) 未知非线性函数$f(x(k))$满足有界条件,即:在$x(k)\in R^n$上,满足$|f(x(k))|\leqslant\overline{f}$, $\overline{f}$是已知的正常数; 2) 未知非线性函数$g(x(k))$是有界的,即存在已知常数$\underline{g}>0,\overline{g}>0$,使得$\underline{g}\leqslant|g(x(k))|\leqslant\overline{g}$成立.

假设2 参考信号$y_d(k)$是光滑有界的,并满足$|y_d(k)|\leqslant\overline{y}_d$, $\overline{y}_d$为已知的正常数.

本文中,由于非线性函数$f(x(k))$未知,需要采用模糊逻辑系统来逼近,首先给出如下假设:

假设3[31] 对于$\forall X,Y\in R^n$,非线性函数$f(x(k))$满足如下Lipschitz条件:

$$\|f(X)-f(Y)\|\leqslant L\|X-Y\|, \tag{3}$$

其中$L>0$为函数$f(x(k))$的Lipschitz常数.

注1 对于已知函数$f(x(k))$,其Lipschitz常数L在仿真中可以通过求函数在定义域上的微分值上界得到[31-32]. 但在本文中,非线性函数$f(x(k))$未知,此常数不能通过求微分值获得,所以在实际工程应用中,未知非线性函数的Lipschitz参数值L一般不容易获得. 在本文中,假设此参数未知,考虑设计关于此参数的自适应律来解决参数难以获得的问题.

1.2 模糊逻辑系统

本文中,采用如下带p条If-Then规则形式的Mamdani型模糊逻辑系统,其第l条规则为

$$R^l:\ \text{If } x_1 \text{ is } \phi_1^{(l)} \text{ and } x_2 \text{ is } \phi_2^{(l)} \text{ and}\cdots\text{and } x_n \text{ is } \phi_n^{(l)}, \\ \text{Then } y_f \text{ is } y_f^{(l)},\ l=1,2,\cdots,p. \tag{4}$$

其中: $\phi_i^{(l)}$为第l条规则中x_i对应的模糊子集, $y_f^{(l)}$为第l条规则的输出.

如果采用单点模糊化、直积运算与加权平均法

解模糊化,则含有规则(4)的模糊逻辑系统的输出为

$$y_f(x)=\frac{\sum\limits_{l=1}^{p}y_f^l\Big(\prod\limits_{i=1}^{n}\mu_{F_i^j}(x_i)\Big)}{\sum\limits_{l=1}^{p}\Big(\prod\limits_{i=1}^{n}\mu_{F_i^j}(x_i)\Big)}. \tag{5}$$

引理1[5] 对于任意定义在紧集U上的连续函数$\varphi(x)$及任意$\varepsilon>0$,总存在形如式(5)的模糊逻辑系统$F(x)$,使得

$$\sup_{x\in U}|\varphi(x)-F(x)|\leqslant\varepsilon. \tag{6}$$

在式(6)中引入非零时变参数$\rho=\rho(k)$,可得

$$y_f\Big(\frac{x}{\rho}\Big)=\frac{\sum\limits_{l=1}^{p}y_f^l\Big(\prod\limits_{i=1}^{n}\mu_{F_i^j}\Big(\frac{x_i}{\rho}\Big)\Big)}{\sum\limits_{l=1}^{p}\Big(\prod\limits_{i=1}^{n}\mu_{F_i^j}\Big(\frac{x_i}{\rho}\Big)\Big)}. \tag{7}$$

注2 时变参数$\rho(k)$的变化会影响整个模糊逻辑系统的输出,因此,可以通过调整时变参数$\rho(k)$使系统的输出按照期望的目标变化.

本文的控制目标是设计控制器$u(k)$使得闭环系统(2)中的所有信号一致有界. 因此,采用形如式(7)的自适应模糊控制器来完成控制任务.

由引理1可得出下面的逼近引理2.

引理2 考虑在紧致域$U\in R^n$上满足Lipschitz条件的离散不确定函数$\phi(x(k))$,其中Lipschitz常数为ϑ(未知),如果存在一个模糊逻辑系统$F(x(k))$使得引理1成立,则在紧致域$\widetilde{V}=\{E(k)=\|E(k)\|\leqslant\alpha|\rho(k)|,E(k)\in R^n\}$上,下面逼近性质成立:

$$\sup\Big|\phi(x(k))-F\Big(\frac{x(k)}{\rho(k)}\Big)\Big|\leqslant\alpha\vartheta(k)|\rho(k)-1|+\varepsilon. \tag{8}$$

证明 因为函数$\phi(x(k))$满足Lipschitz条件,所以$\Big|\phi(x(k))-\phi\Big(\frac{x(k)}{\rho(k)}\Big)\Big|\leqslant\vartheta\Big\|x(k)-\frac{x(k)}{\rho(k)}\Big\|$成立. 令$\tilde{y}_d(k)=[y_d(k),y_d(k+1),\cdots,y_d(k+n-1)]^{\mathrm{T}}$,在紧致域$\widetilde{V}$上,有$\|x(k)\|\leqslant\|E(k)\|+\|\tilde{y}_d\|\leqslant\alpha|\rho(k)|+\bar{y}_d\sqrt{n}$成立,则

$$\begin{aligned}&\Big|\phi(x(k))-F\Big(\frac{x(k)}{\rho(k)}\Big)\Big|\leqslant\\&\Big|\phi(x(k))-\phi\Big(\frac{x(k)}{\rho(k)}\Big)\Big|+\Big|\phi\Big(\frac{x(k)}{\rho(k)}\Big)-\\&F\Big(\frac{x(k)}{\rho(k)}\Big)\Big|\leqslant\vartheta\Big|1-\frac{1}{\rho(k)}\Big|(\alpha|\rho(k)|+\bar{y}_d\sqrt{n})+\varepsilon.\end{aligned} \tag{9}$$

引理2得证. □

由以上分析可知,下面不等式成立:

$$\Big|\Delta(x(k))-F\Big(\frac{E(k)}{\rho(k)}\Big)\Big|\leqslant L\Big|1-\frac{1}{\rho(k)}\Big|(\alpha|\rho(k)|+\bar{y}_d\sqrt{n})+N. \tag{10}$$

其中:$\Delta(x(k))=\dfrac{\bar{g}}{g(x(k))}f(x(k))$,$L$为Lipschitz常数,$N$为逼近误差.

注3 由式(10)可知,改造后带有参数的模糊逻辑系统,其逼近精度可通过参数值α和$\rho(k)$实现,使得逼近精度不再受传统方法中的模糊规则数目的约束,且不再受限于传统模糊逻辑系统的输出形式.

记$\hat{L}(k)$和$\hat{N}(k)$分别为L和N的估计值,则估计误差记为$\tilde{L}(k)=\hat{L}(k)-L(k)$和$\tilde{N}(k)=\hat{N}(k)-N(k)$.

2 模糊自适应控制器设计

针对控制目标,本节给出如下控制器设计形式:

$$u(k)=\begin{cases}0,\ \|E(k)\|>\alpha|\rho(k)|;\\-\dfrac{1}{\bar{g}}F\Big(\dfrac{E(k)}{\rho(k)}\Big),\ \|E(k)\|\leqslant\alpha|\rho(k)|.\end{cases} \tag{11}$$

自适应律为

$$\rho(k+1)=\begin{cases}\Big(-\dfrac{2(\lambda+\pi_1)}{\alpha^2}+\rho^2(k)\Big)^{\frac{1}{2}},\ \|E(k)\|>\alpha|\rho(k)|;\\(\rho^2(k)-\gamma\pi_2)^{\frac{1}{2}},\ \|E(k)\|\leqslant\alpha|\rho(k)|.\end{cases} \tag{12}$$

$$\hat{L}(k+1)=\begin{cases}0,\ \|E(k)\|>\alpha|\rho(k)|;\\(1-\mu)\hat{L}(k)+\mu\phi_1,\ \|E(k)\|\leqslant\alpha|\rho(k)|.\end{cases} \tag{13}$$

$$\hat{N}(k+1)=\begin{cases}0,\ \|E(k)\|>\alpha|\rho(k)|;\\(1-\sigma)\hat{N}(k)+\sigma\phi_2,\ \|E(k)\|\leqslant\alpha|\rho(k)|.\end{cases} \tag{14}$$

其中

$$\begin{aligned}\pi_1=&\frac{1}{2}\|A\|^2\|E(k)\|^2+\frac{1}{2}\|B\|^2[\bar{f}^2+\\&\bar{y}_d^2+\bar{\tau}^2]+\|B\|(\bar{f}\bar{y}_d+\bar{f}\bar{\tau}+\bar{y}_d\bar{\tau})+\\&\|A\|\cdot\|B\|\cdot\|E(k)\|(\bar{f}+\bar{y}_d+\bar{\tau}),\\\pi_2=&(\bar{\tau}+\bar{y}_d)^2+\Big\{\Big[\alpha|\rho(k)-1|+\\&2(\bar{\tau}+\bar{y}_d)+\bar{y}_d\sqrt{n}\Big|1-\frac{1}{\rho(k)}\Big|\Big]-\\&\mu\phi_1\Big\}\hat{L}(k)+2(\bar{\tau}+\bar{y}_d)(1-\sigma)\hat{N}(k),\\\phi_1=&2(\bar{\tau}+\bar{y}_d)\Big[\alpha|\rho(k)-1|+\bar{y}_d\sqrt{n}\Big|1-\frac{1}{\rho(k)}\Big|\Big],\\\phi_2=&(\bar{\tau}+\bar{y}_d),\end{aligned}$$

参数λ、α、γ、μ、σ均为设计的正实数.

定理1 若假设1～假设3成立,则系统(1)在控

制器(11)及自适应律(12)～(14)的作用下,输出信号和跟踪信号的误差可以收敛到零的一个小邻域内,且闭环系统中的所有信号是半全局一致终极有界的.

证明 以下分为两种情况设计控制器.

情形1: $\|E(k)\| > \alpha|\rho(k)|$.

在此情形下,采用开环控制器 $u(k) = 0$,并令 $s(k) = \|E(k)\| - \alpha|\rho(k)| + \eta^{-1}\tilde{L}^2(k) + \delta^{-1}\tilde{N}^2(k)$,很明显 $s(k) > 0$. 考虑正定函数 $\bar{V} = \frac{1}{2}s^2(k)$,则函数 $\bar{V}$ 的微分为

$$\Delta\bar{V}(k) = \frac{1}{2}[s^2(k+1) - s^2(k)]. \tag{15}$$

因为 $\tilde{L}(k+1) = 0, \tilde{N}(k+1) = 0$,故

$$\begin{aligned}&s^2(k+1) = \\ &A^2E^2(k) + B^2f^2(x(k)) + B^2y_d^2(k+n) + \\ &B^2\tau_d^2(k) - 2Bf(x(k))y_d(k+n) + 2Bf(x(k))\tau_d(k) - \\ &2By_d(k+n)\tau_d(k) + 2AE(k)B[f(x(k)) - y_d(k+n) + \\ &\tau_d(k)] + \alpha^2(\rho(k+1))^2 - 2\alpha|\rho(k+1)|\cdot\|E(k+1)\|.\end{aligned} \tag{16}$$

有下面不等式成立:

$$\begin{aligned}\Delta\bar{V}(k) \leqslant\ & \frac{1}{2}\|A\|^2\|E(k)\|^2 + \frac{1}{2}\|B\|^2(\bar{f}^2 + \\ & \bar{y}_d^2 + \bar{\tau}^2) + \|B\|(\bar{f}\bar{y}_d + \bar{f}\bar{\tau} + \bar{y}_d\bar{\tau}) + \\ & \|A\|\cdot\|B\|\cdot\|E(k)\|(\bar{f} + \bar{y}_d + \bar{\tau}) + \\ & \frac{1}{2}\alpha^2\rho^2(k+1) - \frac{1}{2}\alpha^2\rho^2(k) = -\lambda.\end{aligned} \tag{17}$$

由式(12)知 $\Delta\bar{V}(k) < 0$,这就意味着系统(2)在有限时间内可以到达滑模面 $s(k) = 0$[33].

情形2: $\|E(k)\| \leqslant \alpha|\rho(k)|$.

Step 1: 定义跟踪误差 $\xi_1(k) = x_1(k) - y_d(k)$,选取Lyapunov方程

$$V_1(k) = \frac{1}{\bar{g}}\xi_1^2(k), \tag{18}$$

则式(18)的微分为

$$\triangle V_1(k) = -\frac{1}{\bar{g}}\xi_1^2(k) + \frac{1}{\bar{g}}[\xi_2(k) + \beta_1(k) - y_d(k+1)]^2. \tag{19}$$

其中: $\xi_2(k) = x_2(k) - \beta_1(k)$, $\beta_1(k)$ 为实际控制信号. 如果令 $\beta_1(k) = y_d(k+1)$,则可得

$$\Delta V_1(k) = -\frac{1}{\bar{g}}\xi_1^2(k) + \frac{1}{\bar{g}}\xi_2^2(k). \tag{20}$$

Step 2: 选取Lyapunov函数

$$\Delta V_2(k) = V_1(k) + \frac{1}{\bar{g}}\xi_2^2(k), \tag{21}$$

则式(21)沿(20)的微分为

$$\Delta V_2(k) = -\frac{1}{\bar{g}}\xi_1^2(k) + \frac{1}{\bar{g}}[\xi_3(k) + \beta_2(k) - \beta_1(k+1)]^2. \tag{22}$$

其中: $\xi_3(k) = x_3(k) - \beta_2(k)$, $\beta_2(k)$ 为实际控制信号. 令 $\beta_2(k) = \beta_1(k+1) = y_d(k+2)$,则有下式成立:

$$\Delta V_2(k) = -\frac{1}{\bar{g}}\xi_1^2(k) + \frac{1}{\bar{g}}\xi_3^2(k). \tag{23}$$

Step i: $2 \leqslant i \leqslant n-1$ 时,令 $\xi_i(k) = x_i(k) - \beta_{i-1}(k)$, $\beta_i(k) = y_d(k+i)$,选择如下Lyapunov函数:

$$V_i(k) = V_{i-1}(k) + \frac{1}{\bar{g}}\xi_i^2(k), \tag{24}$$

则式(24)的导数为

$$\Delta V_i(k) = -\frac{1}{\bar{g}}\xi_1^2(k) + \frac{1}{\bar{g}}\xi_{i+1}^2(k). \tag{25}$$

Step n: 当 $\xi_n(k) = x_n(k) - \beta_{n-1}(k)$ 时, 有

$$\begin{aligned}\xi_n(k+1) = f(x(k)) + g(x(k))u(k) + \\ \tau_d(k) - y_d(k+n).\end{aligned} \tag{26}$$

选取Lyapunov函数

$$V_n = V_{n-1} + \frac{1}{\bar{g}}\xi_n^2(k) + \frac{1}{\gamma}\rho^2(k) + \frac{1}{\mu}\tilde{L}^2(k) + \frac{1}{\sigma}\tilde{N}^2(k). \tag{27}$$

因为 $\beta_{n-1}(k+1) = y_d(k+n)$,所以有

$$\begin{aligned}&\Delta V_n = \\ &-\frac{1}{\bar{g}}\xi_1^2(k) + \frac{1}{\bar{g}}\xi_n^2(k+1) + \frac{1}{\gamma}\rho^2(k+1) - \frac{1}{\gamma}\rho^2(k) + \\ &\frac{1}{\mu}\tilde{L}^2(k+1) - \frac{1}{\mu}\tilde{L}^2(k) + \frac{1}{\sigma}\tilde{N}^2(k+1) - \frac{1}{\sigma}\tilde{N}^2(k).\end{aligned} \tag{28}$$

因为

$$\xi_n^2(k+1) \leqslant \Big[\Big|\Delta(x(k)) - F\Big(\frac{x(k)}{\rho(k)}\Big)\Big| + \bar{\tau} + \bar{y}_d\Big]^2, \tag{29}$$

由不等式(10)可知

$$\begin{aligned}&\xi_n^2(k+1) \leqslant \\ &\alpha^2L^2(k)|\rho(k) - 1|^2 + nL^2(k)\bar{y}_d^2\cdot\Big|1 - \frac{1}{\rho(k)}\Big|^2 + \\ &N^2(k) + (\bar{\tau} + \bar{y}_d)^2 + 2\alpha L^2(k)\bar{y}_d\sqrt{n}|\rho(k) - 1|\times \\ &\Big|1 - \frac{1}{\rho(k)}\Big| + 2L(k)\bar{y}_d\sqrt{n}\Big|1 - \frac{1}{\rho(k)}\Big|N(k) + \\ &2\alpha L(k)|\rho(k) - 1|\times N(k) + \\ &2\alpha(\bar{\tau} + \bar{y}_d)L(k)|\rho(k) - 1| + \\ &2(\bar{\tau} + \bar{y}_d)\Big[N(k) + \bar{y}_d\sqrt{n}\Big|1 - \frac{1}{\rho(k)}\Big|L(k)\Big].\end{aligned} \tag{30}$$

记 $h(k) = \bar{y}_d\sqrt{n}\Big|1 - \frac{1}{\rho(k)}\Big| + \alpha|\rho(k) - 1|$, 因为不等式满足条件

$$2h(k)L(k)N(k) \leqslant dh^2(k)L^2(k) + \frac{1}{d}N^2(k), \tag{31}$$

故有

$$\xi_n^2(k+1) \leqslant$$
$$\Big(\frac{1}{d}+1\Big)N^2(k) + \Big\{|1-\rho(k)|^2\Big[\alpha^2 + \frac{n\bar{y}_d^2}{|\rho(k)|} + \frac{2\alpha\bar{y}_d\sqrt{n}}{|\rho(k)|}\Big] + dh^2(k)\Big\}L^2(k) + (\bar{\tau}+\bar{y}_d)^2 + 2(\bar{\tau}+\bar{y}_d)\Big[\alpha|\rho(k)-1| + \bar{y}_d\sqrt{n}\Big|1-\frac{1}{\rho(k)}\Big|\Big]\hat{L}(k) + 2(\bar{\tau}+\bar{y}_d)\hat{N}(k) - 2(\bar{\tau}+\bar{y}_d)\Big[\alpha|\rho(k)-1| + \bar{y}_d\sqrt{n}\Big|1-\frac{1}{\rho(k)}\Big|\Big]\tilde{L}(k) - 2(\bar{\tau}+\bar{y}_d)\tilde{N}(k). \tag{32}$$

因为$\tilde{L}(k+1) = \tilde{L}(k) + \mu[\phi_1 - \hat{L}(k)]$, $\tilde{N}(k+1) = \tilde{N}(k) + \sigma[\phi_2 - \hat{N}(k)]$,以及下面等式:

$$\begin{cases}-2\tilde{L}(k)\hat{L}(k) = -\tilde{L}^2(k) - \hat{L}^2(k) + L^2(k),\\ -2\tilde{N}(k)\hat{N}(k) = -\tilde{N}^2(k) - \hat{N}^2(k) + N^2(k),\end{cases} \tag{33}$$

故有

$$\frac{1}{\mu}\tilde{L}^2(k+1) - \frac{1}{\mu}\tilde{L}^2(k) - 2\phi_1\tilde{L}(k) = \mu\phi_1^2 - (1-\mu)\hat{L}^2(k) + L^2(k) - \tilde{L}^2(k) - 2\mu\phi_1\hat{L}(k), \tag{34}$$

$$\frac{1}{\sigma}\tilde{N}^2(k+1) - \frac{1}{\sigma}\tilde{N}^2(k) - 2\phi_2\tilde{N}(k) = \sigma\phi_2^2 - (1-\sigma)\hat{N}^2(k) + N^2(k) - \tilde{N}^2(k) - 2\sigma\phi_2\hat{N}(k). \tag{35}$$

令

$$\theta = \Big(\frac{1}{d}+2\Big)N^2(k) + \Big\{|1-\rho(k)|^2\cdot\Big[\alpha^2 + \frac{n\bar{y}_d^2}{|\rho(k)|} + \frac{2\alpha\bar{y}_d\sqrt{n}}{|\rho(k)|}\Big] + dh^2(k) + 1\Big\}L^2(k) + \mu\phi_1^2 + \sigma\phi_2^2,$$

由自适应律(12)、等式(34)和(35)可知

$$\Delta V_n(k) \leqslant -\frac{1}{\bar{g}}\xi_1^2(k) - \tilde{L}^2(k) - \tilde{N}^2(k) - (1-\mu)\hat{L}^2(k) - (1-\sigma)\hat{N}^2(k) + \theta. \tag{36}$$

当参数满足条件$1 > \mu, 1 > \sigma, \xi_1^2(k) > \bar{g}\theta$时,则有$\Delta V_n(k) < 0$成立. 因此,闭环系统中的所有信号都是半全局一致有界收敛. □

3 仿真算例

考虑如下形式的非线性离散系统:

$$\begin{cases}x_1(k+1) = x_2(k),\\ x_2(k+1) =\\ -\dfrac{3}{16}\times\dfrac{x_1(k)}{1+x_2^2(k)} + x_2(k) + u(k) + \tau_d(k).\end{cases} \tag{37}$$

非线性函数

$$f(k) = -\frac{3}{16}\frac{x_1(k)}{1+x_2^2(k)} + x_2(k)$$

未知,已知控制增益的上下界为$\bar{g} = 1.5, \underline{g} = 0.1$. 选定输入论域$U = I_1 \times I_2 = [-10,10]\times[-10,10]$,将输入论域作以下模糊划分: $I_1 = \{$负大(NB),负中(NM),负小(NS),正小(PS),正中(PM),正大(PB)$\}$; $I_2 = \{$负大(NB), 负中(NM),负小(NS),正小(PS),正中(PM),正大(PB)$\}$; 模糊逻辑系统F的规则如下:

If $x_1(k)$ is PB and $x_2(k)$ is PB, Then $f(k)$ is NB;

If $x_1(k)$ is NB and $x_2(k)$ is PM, Then $f(k)$ is PM;

If $x_1(k)$ is NM and $x_2(k)$ is NB, Then $f(k)$ is NS;

If $x_1(k)$ is PS and $x_2(k)$ is NS, Then $f(k)$ is PS;

If $x_1(k)$ is PB and $x_2(k)$ is NM, Then $f(k)$ is PM.

其模糊隶属函数分别为

$$\mu_{\rm NB}(x_i) = {\rm e}^{-\frac{(x_i(k)+10)^2}{2}},\ \mu_{\rm NM}(x_i) = {\rm e}^{-\frac{(x_i(k)+5)^2}{2}},$$
$$\mu_{\rm NS}(x_i) = {\rm e}^{-\frac{(x_i(k)+0.1)^2}{2}},\ \mu_{\rm PS}(x_i) = {\rm e}^{-\frac{(x_i(k)-0.1)^2}{2}},$$
$$\mu_{\rm PM}(x_i) = {\rm e}^{-\frac{(x_i(k)-5)^2}{2}},\ \mu_{\rm PB}(x_i) = {\rm e}^{-\frac{(x_i(k)-10)^2}{2}}.$$

参数分别选取为: $\alpha = 10, \lambda = 10, \gamma = 0.001, \mu = 0.1, \sigma = 0.02$.

1) 外界干扰为$\tau_d(k) = 1.5\sin(2\pi k)$时,已知$\bar{\tau} = 1.5$,理想跟踪信号为$y_d(k) = \dfrac{\pi}{3}\sin k$,系统的初始状态为$(1,0)^{\rm T}$, $\rho(0) = 0.6, \hat{L}(0) = 0.2, \hat{N}(0) = 0.3$. 其仿真结果如图1、图2和图3所示.

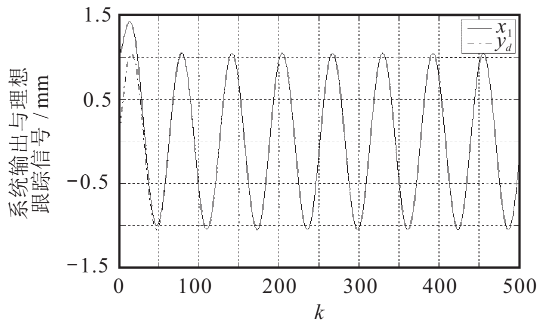

图 1 系统的输出与理想信号的时间响应(1)

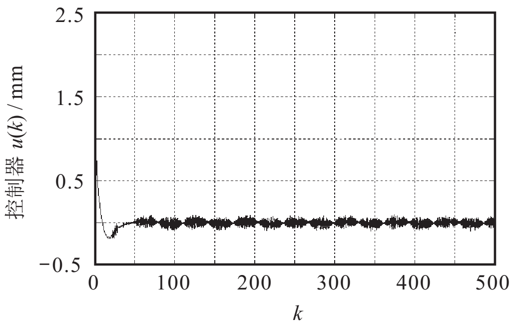

图 2 控制信号的时间响应(1)

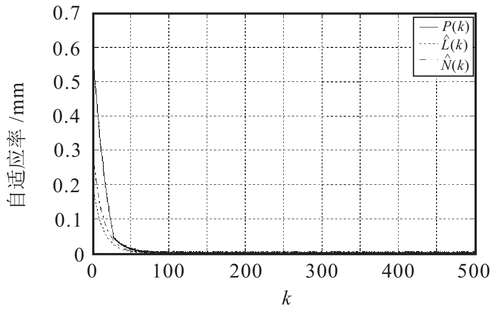

图 3　自适应参数时间响应(1)

2) 当外部干扰项为白噪声时,假设理想跟踪信号为$y_d = 0.5\sin(0.5k) + 0.5\sin(2k)$,系统的初始值为$(0.17, 0)^{\mathrm{T}}, \rho(0) = 0.8, \hat{L}(0) = 0.7, \hat{N}(0) = 0.5$,其仿真结果如图4、图5和图6所示.

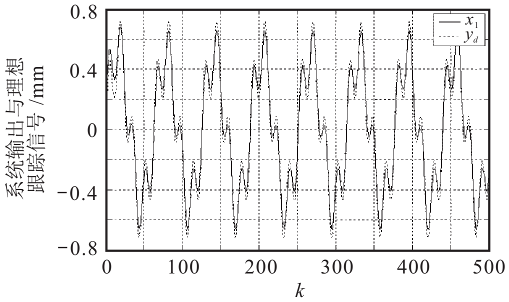

图 4　系统的输出与理想信号的时间响应(2)

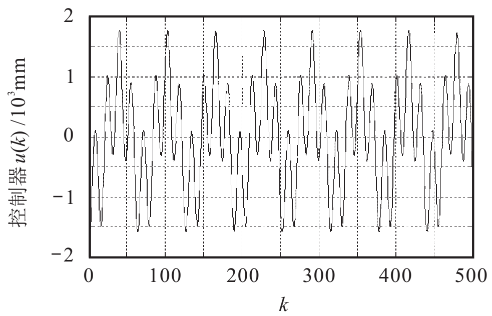

图 5　控制信号的时间响应(2)

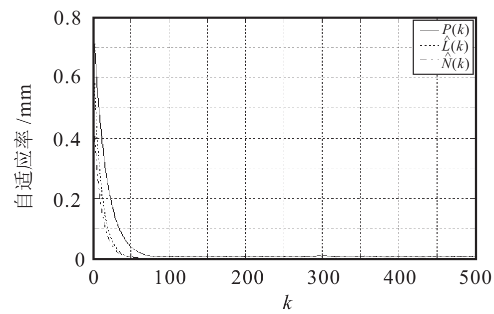

图 6　自适应参数时间响应(2)

从仿真结果可以看出,不确定离散非线性在本文设计的控制器的作用下,能够保证系统(37)的所有状态半全局一致有界.

4　结　论

本文通过在普通模糊逻辑系统中引入参数,给出了一类非线性离散系统的跟踪模糊自适应控制器设计方法. 不确定非线性函数的逼近精度可以通过自适应律在线自动调节,且构造的模糊逻辑系统的规则数目的多少不再影响逼近精度,从而减少控制器的运算负担. 这在一定程度上保证了规则具有较高的语言可解释性,同时,更适合于工程实际应用. 该方法为进一步研究其他多种形式的模糊逻辑系统提供了理论指导. 输出模式为非线性基函数的模糊逻辑系统的自适应控制器设计将是今后探索的一个内容.

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