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# 离散异构多自主体系统时变编队-合围控制

韩娜妮[1†], 罗小元[2], 朱亚锟[3]

(1. 陇东学院 电气工程学院，甘肃 庆阳 745000；2. 燕山大学 电气工程学院，河北 秦皇岛 066004；
3. 东北大学秦皇岛分校 控制工程学院，河北 秦皇岛 066004)

**摘 要:** 针对高阶离散异构多自主体系统的时变编队-合围控制问题,考虑时变时延,提出分布式编队-合围控制协议. 首先,在合理假设的基础上,通过模型转变和状态空间分解,将编队-合围控制问题转化为子系统的稳定性问题,再利用Lyapunov-Krasovskii函数,以LMIs的形式给出协议有效的充分条件,并指出LMIs的个数与系统中自主体的个数无关;然后,给出编队参考函数的具体形式,证明指出编队参考函数不受时变时延的影响;最后,通过固定和时变编队-合围仿真验证所设计的协议的有效性.

**关键词:** 编队-合围；异构多自主体系统；离散系统；时变时延

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# Time-varying formation-containment control of discrete-time heterogeneous multi-agent systems

*HAN Na-ni*[1†], *LUO Xiao-yuan*[2], *ZHU Ya-kun*[3]

(1. College of Electrical Engineering，Longdong University，Qingyang 745000，China；2. School of Electrical Engineering，Yanshan University，Qinhuangdao 066004，China；3. School of Control Engineering，Northeastern University at Qinhuangdao，Qinhuangdao 066004，China)

**Abstract:** To investigate the formation-containment problem of discrete-time heterogeneous multi-agent systems, the formation-containment protocols with time-varying delay are presented. Firstly, based on reasonable hypothesises, by model transformation and state space decomposition, the problem of formation-containment is transformed into stability problem of subsystems. By using the Lyapunov-Krasovskii function, the sufficient conditions that guarantee the effectiveness of the proposed protocols are presented in the form of LMIs, and it is pointed out that the number of LMIs is independent of that of agents. Then, the formation reference function is also described in detail, and the proof illustrates that the formation reference function is not affected by time-varying delay. Finally, the simulation results demonstrate the effectiveness of the designed protocols.

**Keywords:** formation-containment；heterogeneous multi-agent system；discrete-time system；time-varying delay

## 0 引 言

近年来,由于多自主体系统在多领域的广泛应用,受到了众多学者的关注. 截止目前,关于多自主体系统的一致性问题已涌现出大量的研究成果[1-3]. 当多自主体网络中有一个leader时,一致性问题也称为跟踪问题或leader-following一致性问题[1-2]. 若系统中的每个自主体具有不同的数学模型,则称此系统为异构多自主体系统[3].

当多自主体网络中有多个leader时,系统一致性问题变为合围控制问题,其目标是使follower状态趋于leader自主体形成的凸集. 文献[4-5]分别是同构和异构多自主体系统的合围控制成果.

在合围控制中都是假设leader之间没有通信,但在实际应用中,leader也要求形成一定的编队,同时follower趋于leader形成的区域中,即编队-合围控制,其在军事和民用领域中有着广阔的应用前景. 文献[6-7]分别是一阶、二阶多自主体组成的无向多自主体网络的编队-合围控制成果. 对于二阶离散多自主体系统,Zheng等[8]采用位置信息设计了两种不同的编队-合围控制协议;Dong等[9-10]分别研究了存在时延的高阶多自主体系统的状态和输出编队-合围控制,并以LMIs的形式给出了系统实现控制目标的充

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†通讯作者. E-mail: hannani@yeah.net.

分条件.

本文针对异构离散多自主体系统,考虑时变时延,设计多自主体系统的时变编队-合围控制协议. 利用图论和Lyapunov-Krasovskii理论,给出系统实现时变编队-合围控制的充分条件. 本文的创新点在于系统是离散异构高阶多自主体系统.

## 1 图论和问题描述

### 1.1 图 论[11]

多自主体系统的网络拓扑用图$G=\{V,E,A\}$表示. 其中:$V$是顶点集;$E=\{e_{ij}\}$;$A=[a_{ij}]_{N\times N}$是邻接矩阵,若$e_{ij}\in E$,则$a_{ji}>0$,否则$a_{ji}=0$,并且对于所有的$i$,有$a_{ii}=0$. 矩阵$L=[l_{ij}]_{N\times N}$, $l_{ii}=\sum\limits_{j=1,j\neq i}^{n}a_{ij}$且$i\neq j$时, $l_{ij}=-a_{ij}$. $N_i$表示节点$i$的邻接集. 若图$G$中至少有一节点,从此节点到其他节点之间都存在一条有向路径,则称此图含有生成树.

### 1.2 问题描述

考虑由$N$个自主体组成的系统,其中$M(M<N)$个自主体为follower,其余为leader. 自主体的模型描述为

$$x_i(k+1)=A_ix_i(k)+B_iu_i(k). \tag{1}$$

其中: $x_i(k)$, $u_i(k)$分别是自主体$i$在$k$时刻的状态和控制输入; $A_i, B_i$是已知常数矩阵.

在本文中用$F=\{1,2,\cdots,M\}$和$E=\{M+1,M+2,\cdots,N\}$分别表示follower和leader自主体集合, $h_E(k)=[h_{M+1}^{\mathrm{T}}(k),h_{M+2}^{\mathrm{T}}(k),\cdots,h_N^{\mathrm{T}}(k)]$是leader自主体的期望编队.

**定义1** 对于任意初始状态,若存在$r(k)$和非负常数$c_{i,j}(i\in F,j\in E)$, $\sum\limits_{j=M+1}^{N}c_{i,j}=1$同时满足

$$\lim_{k\to\infty}(x_i(k)-h_i(k)-r(k))=0, \tag{2}$$

$$\lim_{k\to\infty}\left(x_i(k)-\sum_{j=M+1}^{N}c_{i,j}\right)=0, \tag{3}$$

其中$r(k)$为编队参考函数,则称多自主体系统(1)实现了编队-合围控制.

## 2 问题转化

根据自主体邻接集的特点,矩阵$L$可分解为$L=[L_1,\ L_2;0,\ L_3]$,其中$L_1,L_2$和$L_3$分别表示follower、follower和leader、leader之间的邻接信息. 用$G_E$表示leader自主体的网络拓扑图.

**假设1** 对于每个follower自主体,至少存在一个leader与其之间存在一条有向路径.

**假设2** 图$G_E$中含有生成树.

**引理1** 若图$G$满足假设1,则$L_1$的特征值均具有正实部,矩阵$-L_1^{-1}L_2$每行的元素和为1,并且每一元素值非负[12].

**引理2** 若图$G_E$满足假设2,则矩阵$L_3$有一个特征值为0,且其余特征值均具有正实部[13].

用$\lambda_i(i\in E)$表示$L_3$的第$i$个特征值,其中$\lambda_{M+1}=0$, $0<\mathrm{Re}(\lambda_{M+2})\leqslant\cdots\leqslant\mathrm{Re}(\lambda_N)$. 令$U_E^{-1}L_3U_E=J_E$, $U_E^{-1}=[\tilde{u}_{M+1}^H;\tilde{U}_E]$, $J_E=\mathrm{diag}\{0,\overline{J}_E\}$.

考虑时变时延,设计如下编队-合围控制协议:

$$u_i(k)=K_{1i}x_i(k)+K_{2i}\sum_{j\in N_i}a_{ij}(x_i(k-\tau_k)-x_j(k-\tau_k)),\ i\in F; \tag{4}$$

$$u_i(k)=K_{1i}x_i(k)+K_{3i}\sum_{j\in N_i}a_{ij}(x_i(k-\tau_k)-x_j(k-\tau_k)-(h_i(k-\tau_k)-h_j(k-\tau_k))),\ i\in E. \tag{5}$$

其中: $K_{1i},K_{2i},K_{3i}$是常数矩阵; $\tau_k$是时变时延,且满足$\tau_1\leqslant\tau_k\leqslant\tau_2$, $\tau_1,\tau_2$是已知常数.

**假设3** 存在矩阵$K_{1i}(i=1,2,\cdots,N)$, $K_{2j}(j\in F)$, $K_{3l}(l\in E)$使$A_i+B_iK_{1i}=A+BK_1$, $B_jK_{2j}=BK_2$, $B_lK_{3l}=BK_3$,其中$A,B,K_1,K_2,K_3$为满足假设的任意矩阵.

将协议(4)和协议(5)应用于系统(1),在假设3成立时,令$z_i(k)=x_i(k)-h_i(k)(i\in E)$, $\tilde{z}_E(k)=(U_E^{-1}\otimes I)z_E(k)=[\beta_E^{\mathrm{T}}(k),\ \eta_E^{\mathrm{T}}(k)]^{\mathrm{T}}$, $I$为单位阵,则有如下等式:

$$\beta_E(k+1)=(A+BK_1)\beta_E(k)+(\tilde{u}_{M+1}^H\otimes(A+BK_1))h_E(k)-(\tilde{u}_{M+1}^H\otimes I)h_E(k+1), \tag{6}$$

$$\eta_E(k+1)=(I\otimes(A+BK_1))\eta_E(k)-(\tilde{U}_E\otimes I)h_E(k+1)+(\overline{J}\otimes BK_3)\eta_E(k-\tau_k)+(\tilde{U}_E\otimes(A+BK_1))h_E(k). \tag{7}$$

**引理3** 若下式成立:

$$\begin{cases}\lim\limits_{k\to\infty}\eta_E(k)=0,\\ \lim\limits_{k\to\infty}\varphi_F(k)=0,\end{cases} \tag{8}$$

则异构多自主体系统(1)在协议(4)和(5)的作用下实现了编队-合围控制. 其中: $\varphi_i(k)=\sum\limits_{j\in N_i}a_{ij}(x_i(k$

$-\tau_k)-x_j(k-\tau_k)),i\in F$.

**证明**　令 $z_{\mathrm{EC}}(k)=(U_E\otimes I)[\beta_E(k);0]$,则 $z_{\mathrm{E\overline{C}}}=z_E-z_{\mathrm{EC}}=(U_E\otimes I)[0;\eta_E(k)]$. 若 $\lim\limits_{k\to\infty}\eta_E(k)=0$,则 $\lim\limits_{k\to\infty}(z_E(k)-z_{\mathrm{EC}}(k))=0$.

令 $\varphi_i(k)$ 如式(8)所示,则 $\varphi_F(k)=(L_1\otimes I)x_F(k-\tau_k)+(L_2\otimes I)x_E(k-\tau_k)$. 其中: $\varphi_F\to 0$ 时, $x_F(k-\tau_k)-(-(L_1^{-1}L_2\otimes I))x_E(k-\tau_k)=0$. 由引理1和定义1可知,当式(8)成立时,系统(1)在协议(4)和协议(5)的作用下实现了编队-合围控制. □

**注1**　通过模型变换和状态空间分解,系统(1)的编队-合围控制问题转换成为子系统的稳定性问题.

## 3　编队-合围分析

令 $\gamma_\lambda=[\mathrm{Re}(\lambda),-\mathrm{Im}(\lambda);\mathrm{Im}(\lambda),\mathrm{Re}(\lambda)]$, $\varLambda_R=\mathrm{diag}\{R,R\}$. $\lambda_i(i\in F)$ 表示 $L_1$ 的特征根,定义 $\overline{\lambda}_{1,2}=\min\{\mathrm{Re}(\lambda_i)\}\pm \mathrm{j}\overline{\mu}_F$, $\overline{\lambda}_{3,4}=\max\{\mathrm{Re}(\lambda_i)\}\pm \mathrm{j}\overline{\mu}_F$, $\tilde{\lambda}_{1,2}=\mathrm{Re}(\lambda_{M+2})\pm \mathrm{j}\tilde{\mu}_E$, $\tilde{\lambda}_{3,4}=\mathrm{Re}(\lambda_N)\pm \mathrm{j}\tilde{\mu}_E$. 其中: $\overline{\mu}_F=\max\{\mathrm{Im}(\lambda_i),i\in F\}$, $\tilde{\mu}_E=\max\{\mathrm{Im}(\lambda_i),i\in E\}$.

**定理1**　1) 存在如下等式:

$$(A+BK_1)(h_i(k)-h_j(k))-(h_i(k+1)-h_j(k+1))=0,i\in E,j\in N_i;\tag{9}$$

2) 对于 $\tilde{\lambda}_i(i=1,2,3,4)$,存在实正定对称矩阵 $P_E,Q_{qE}(q=1,2,3),R_{jE}(j=1,2)$,满足

$$\begin{bmatrix}\varGamma_{11} & 0 & R_{1E} & 0 & \varLambda_{A+BK_1}^{\mathrm{T}}P_E & \tau_1(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{1E} & \tau_{12}(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{2E}\\ & \varGamma_{22} & 2R_{2E} & R_{2E} & (\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}P_E & \tau_1(\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}R_{1E} & \tau_{12}(\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}R_{2E}\\ & & \varGamma_{33} & 0 & 0 & 0 & 0\\ & & & \varGamma_{44} & 0 & 0 & 0\\ & & & & -P_E & 0 & 0\\ & & & & & -R_{1E} & 0\\ & & & & & & -R_{2E}\end{bmatrix}<0,\tag{10}$$

$$\begin{bmatrix}\varGamma_{11} & 0 & R_{1E} & 0 & \varLambda_{A+BK_1}^{\mathrm{T}}P_E & \tau_1(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{1E} & \tau_{12}(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{2E}\\ & \varGamma_{22} & R_{2E} & 2R_{2E} & (\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}P_E & \tau_1(\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}R_{1E} & \tau_{12}(\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}R_{2E}\\ & & \varLambda_{33} & 0 & 0 & 0 & 0\\ & & & \varLambda_{44} & 0 & 0 & 0\\ & & & & -P_E & 0 & 0\\ & & & & & -R_{1E} & 0\\ & & & & & & -R_{2E}\end{bmatrix}<0,\tag{11}$$

其中: $\varGamma_{11}=-P_E+Q_{1E}+Q_{2E}+(\tau_{12}+1)Q_{3E}-R_{1E}$, $\varGamma_{22}=-Q_{3E}-3R_{2E}$, $\varGamma_{33}=-Q_{1E}-R_{1E}-2R_{2E}$, $\varGamma_{44}=-Q_{2E}-R_{2E}$, $\varLambda_{33}=-Q_{1E}-R_{1E}-R_{2E}$, $\varLambda_{44}=-Q_{2E}-2R_{2E}$, $\tau_{12}=\tau_2-\tau_1$.

3) 对于 $\overline{\lambda}_i(i=1,2,3,4)$,存在实正定对称矩阵 $P_F,Q_{qF}(q=1,2,3),R_{jF}(j=1,2)$,满足

$$\begin{bmatrix}\varGamma_{11} & 0 & R_{1F} & 0 & \varLambda_{A+BK_1}^{\mathrm{T}}P_F & \tau_1(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{1F} & \tau_{12}(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{2F}\\ & \varGamma_{22} & 2R_{2F} & R_{2F} & (\gamma_{\overline{\lambda}_i}\otimes BK_2)^{\mathrm{T}}P_F & \tau_1(\gamma_{\overline{\lambda}_i}\otimes BK_2)^{\mathrm{T}}R_{1F} & \tau_{12}(\gamma_{\overline{\lambda}_i}\otimes BK_2)^{\mathrm{T}}R_{2F}\\ & & \varGamma_{33} & 0 & 0 & 0 & 0\\ & & & \varGamma_{44} & 0 & 0 & 0\\ & & & & -P_F & 0 & 0\\ & & & & & -R_{1F} & 0\\ & & & & & & -R_{2F}\end{bmatrix}<0,\tag{12}$$

$$\begin{bmatrix}\varGamma_{11} & 0 & R_{1F} & 0 & \varLambda_{A+BK_1}^{\mathrm{T}}P_F & \tau_1(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{1F} & \tau_{12}(\varLambda_{A+BK_1}-I)^{\mathrm{T}}R_{2F}\\ & \varGamma_{22} & R_{2F} & 2R_{2F} & (\gamma_{\overline{\lambda}_i}\otimes BK_2)^{\mathrm{T}}P_F & \tau_1(\gamma_{\overline{\lambda}_i}\otimes BK_2)^{\mathrm{T}}R_{1F} & \tau_{12}(\gamma_{\overline{\lambda}_i}\otimes BK_2)^{\mathrm{T}}R_{2F}\\ & & \varLambda_{33} & 0 & 0 & 0 & 0\\ & & & \varLambda_{44} & 0 & 0 & 0\\ & & & & -P_F & 0 & 0\\ & & & & & -R_{1F} & 0\\ & & & & & & -R_{2F}\end{bmatrix}<0.\tag{13}$$

其中: $\Gamma_{11} = -P_F + Q_{1F} + Q_{2F} + (\tau_{12}+1)Q_{3F} - R_{1F}, \Gamma_{22} = -Q_{3F} - 3R_{2F}, \Gamma_{33} = -Q_{1F} - R_{1F} - 2R_{2F}, \Gamma_{44} = -Q_{2F} - R_{2F}, \Lambda_{33} = -Q_{1F} - R_{1F} - R_{2F}, \Lambda_{44} = -Q_{2F} - 2R_{2F}$.

若上述1)~3)同时成立,则离散异构多自主体系统(1)在协议(4)和协议(5)的作用下可以实现编队-合围控制.

**证明** 若式(9)成立,则有

$$\eta_E(k+1) = (I\otimes(A+BK_1))\eta_E(k) + (\overline{J}_E\otimes BK_3)\eta_E(k-\tau_k). \tag{14}$$

令$\overline{\eta}_i(k) = [\mathrm{Re}(\tilde{\eta}_i(k)); \mathrm{Im}(\tilde{\eta}_i(k))], \tilde{\eta}_i(k+1) = (A+BK_1)\tilde{\eta}_i(k) + \lambda_i BK_3\tilde{\eta}_i(k-\tau_k), i = M+2, M+3, \cdots, N$,则有

$$\overline{\eta}_i(k+1) = \Lambda_{A+BK_1}\overline{\eta}_i(k) + (\gamma_{\lambda_i}\otimes BK_3)\overline{\eta}_i(k-\tau_k). \tag{15}$$

定义$\mu_i(k) = \overline{\eta}_i(k+1) - \overline{\eta}_i(k)$,对于系统(15),构造如下Lyapunov函数:

$$V_i(k) = V_{i1}(k) + V_{i2}(k). \tag{16}$$

其中

$$V_{i1}(k) = \overline{\eta}_i(k)^{\mathrm{T}}P_E\overline{\eta}_i(k) + \sum_{j=k-\tau_1}^{k-1}\overline{\eta}_i(j)^{\mathrm{T}}Q_{1E}\overline{\eta}_i(j) + \sum_{j=k-\tau_2}^{k-1}\overline{\eta}_i(j)^{\mathrm{T}}Q_{2E}\overline{\eta}_i(j) + \sum_{j=-\tau_2}^{-\tau_1}\sum_{l=k+j}^{k-1}\overline{\eta}_i(l)^{\mathrm{T}}Q_{3E}\overline{\eta}_i(l),$$

$$V_{i2}(k) = \tau_{12}\sum_{j=-\tau_2}^{-\tau_1-1}\sum_{l=k+j}^{k-1}\mu_i^{\mathrm{T}}(l)R_{2E}\mu_i(l) + \tau_1\sum_{j=-\tau_2}^{-\tau_1}\sum_{l=k+j}^{k-1}\mu_i^{\mathrm{T}}(l)R_{1E}\mu_i(l).$$

由文献[14-15]中的相关引理和S-procedure[16]可知,当式(10)和(11)成立时,$\Delta V_i(k) < 0$,则系统(14)渐近稳定,即$\lim\limits_{k\to\infty}\eta_E(k) = 0$.

对于follower自主体,采用相同分析方法可以得到相应结论. 由引理3可知,当1)~3)同时成立时,异构多自主体系统(1)在协议(4)和协议(5)的作用下可以实现编队-合围控制. □

**引理4** 对于给定的$\tau_1, \tau_2$和$\tilde{\lambda}_i (i = 1,2,3,4)$,若存在正定对称矩阵$P, Q_q(q = 1,2,3), R, S$和矩阵$T_1, T_2, M_1, M_2, N_1, N_2$满足

$$\begin{bmatrix} \phi_{11} & \phi_{12} & M_1 & -T_1 & \sqrt{\tau_{12}}T_1 & \sqrt{\tau_{12}}M_1 & \sqrt{\tau_2}N_1 \\ & \phi_{22} & M_2 & -T_2 & \sqrt{\tau_{12}}T_2 & \sqrt{\tau_{12}}M_2 & \sqrt{\tau_2}N_2 \\ & & -Q_1 & 0 & 0 & 0 & 0 \\ & & & -Q_2 & 0 & 0 & 0 \\ & & & & -R-S & 0 & 0 \\ & & & & & -R & 0 \\ & & & & & & -S \end{bmatrix} < 0, \tag{17}$$

则系统(15)渐近稳定,其中$\phi_{11}, \phi_{12}, \phi_{22}$如文献[15]所示.

**定理2** 若存在正定对称矩阵$P, Q_q(q = 1,2,3), R, S$和$T_1, T_2, M_1, M_2, N_1, N_2$使式(17)成立,则$P, Q_q(q = 1,2,3), R_1 = \tau_1^{-1}S, R_2 = \tau_{12}^{-1}(R+S)$为LMIs(10)和(11)的一组可行解.

**证明** 若式(17)成立,则根据Schur's补,有

$$\Psi = \begin{bmatrix} \overline{\Psi}_{11} & 0 & \tau_1^{-1}S & 0 & \Lambda_{A+BK_1}^{\mathrm{T}}P & (\Lambda_{A+BK_1}-I)^{\mathrm{T}}S & (\Lambda_{A+BK_1}-I)^{\mathrm{T}}\Xi \\ & \overline{\Psi}_{22} & \tau_{12}^{-1}\Xi & \tau_{12}^{-1}\Xi & (\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}P & (\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}S & (\gamma_{\tilde{\lambda}_i}\otimes BK_3)^{\mathrm{T}}\Xi \\ & & \overline{\Psi}_{33} & 0 & 0 & 0 & 0 \\ & & & \overline{\Psi}_{44} & 0 & 0 & 0 \\ & & & & -P & 0 & 0 \\ & & & & & -\tau_1^{-1}S & 0 \\ & & & & & & -\tau_{12}^{-1}\Xi \end{bmatrix} < 0,$$

其中: $\Xi = R+S, \overline{\Psi}_{11} = -P + (\tau_{12}+1)Q_3 + Q_1 + Q_2 - \tau_1^{-1}S, \overline{\Psi}_{22} = -Q_3 - 2\tau_{12}^{-1}\Xi, \overline{\Psi}_{33} = -Q_1 - \tau_1^{-1}S - \tau_{12}^{-1}\Xi, \overline{\Psi}_{44} = -Q_1 - \tau_{12}^{-1}\Xi$.

令$\Omega_1 = [0,\ -I,\ I,\ 0,\ 0,\ 0,\ 0], \Omega_2 = [0,\ I,\ 0,\ -I,\ 0,\ 0,\ 0]$,且

$$\Psi - \Omega_1^{\mathrm{T}}R_2\Omega_1 < 0, \tag{18}$$

$$\Psi - \Omega_2^{\mathrm{T}}R_2\Omega_2 < 0. \tag{19}$$

分别比较式(10)和(18),式(11)和(19),可以得到$P,\ Q_q(q = 1,2,3), R_1 = \tau_1^{-1}S,\ R_2 = \tau_{12}^{-1}(R+S)$是式(10)和(11)的一组可行解. □

**推论1**　当系统(1)在协议(4)和协议(5)的作用下实现编队-合围控制时,编队参考函数$r(k)$满足

$$\lim_{k\to\infty} r(k) = -(1_{N-M}/\sqrt{N-M})\otimes(\tilde{u}_{M+1}^H\otimes I)h_E(k)+(1_{N-M}/\sqrt{N-M})\otimes(\tilde{u}_{M+1}^H\otimes(A+BK_1)^k)x_E(0). \tag{20}$$

**证明**　当系统实现编队-合围控制时,有

$$z_E(k)=z_{\rm EC}(k)=(1_{N-M}/\sqrt{N-M})\otimes\beta_E(k). \tag{21}$$

根据$\beta_E(k)$的定义,有

$$\beta_E(k)=(\tilde{u}_{M+1}^H\otimes(A+BK_1)^k)x_E(0)-(\tilde{u}_{M+1}^H\otimes I)h_E(k). \tag{22}$$

当leader实现时变编队时,存在$\lim\limits_{k\to\infty} r(k) = \lim\limits_{k\to\infty}(x_E(k)-h_E(k)) = \lim\limits_{k\to\infty} z_E(k)$,由式(21)和(22)即可得到式(20). □

**注2**　推论1表明,时变时延对系统的编队参考函数没有影响.

## 4　仿真验证

### 4.1　固定编队

考虑由3个follower自主体(标号1～3)和4个leader自主体(标号4～7)组成的多自主体系统,其网络拓扑如图1所示,满足假设1和假设2. 期望编队$h_E(k)=[-1,\ 1,\ 1,\ 1,\ 1,\ -1,\ -1,\ -1]^{\rm T}$.

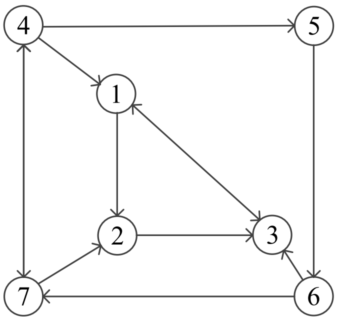


**图1**　多自主体系统网络拓扑1

图2和图3是多自主体系统的状态轨迹图. 4个leader最终形成了正方形编队,并且follower运行在leader形成的区域内,验证了所设计的协议有效.

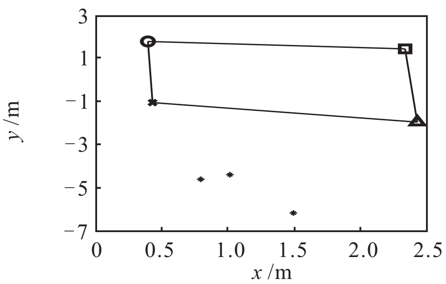


**图2**　多自主体系统状态($k=200$)

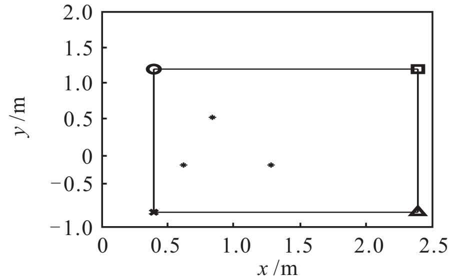


**图3**　多自主体系统状态($k=1\,000$)

### 4.2　时变编队

考虑如图4所示的多自主体网络拓扑,其中1～3是follower, 4～6是leader, 网络拓扑满足假设1和假设2. $h_E(k)=[\sin(kT)\ \ \cos(kT)\ \ \sin(kT+2\pi/3)\ \ \cos(kT+2\pi/3)\ \ \sin(kT+4\pi/3)\ \ \cos(kT+4\pi/3)]^{\rm T}$.

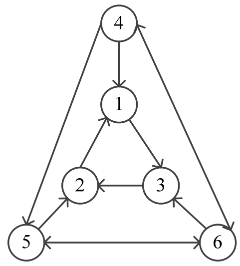


**图4**　多自主体系统网络拓扑2

图5和图6是多自主体系统时变编队-合围控制的状态图. 图5和图6中leader形成了所要求的编队,实时变化时,follower始终运行在leader形成的区域内,验证了所设计的协议有效.

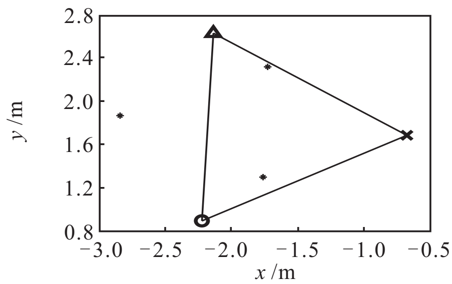


**图5**　多自主体系统时变编队-合围控制状态($k=100$)

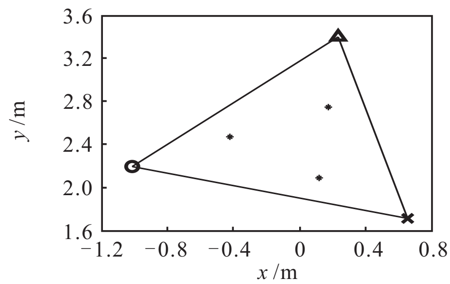


**图6**　多自主体系统时变编队-合围控制状态($k=1\,050$)

## 5　结　论

针对异构离散高阶多自主体系统,设计了时变编队-合围控制协议,通过合理的数学假设和模型转换

将编队-合围问题转换为子系统的稳定性问题,给出了系统实现时变编队-合围控制的充分条件. 未来研究方向主要是通信受限时多自主体系统的协调控制.

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## 作者简介


韩娜妮 (1986−), 女, 讲师, 从事多自主体系统协调控制的研究, E-mail: hannani@yeah.net;

罗小元 (1976−), 男, 教授, 博士, 从事多移动机器人、移动传感器网络等研究, E-mail: xyluo@ysu.edu.cn;

朱亚锟 (1984−), 男, 讲师, 博士, 从事多智能体系统协调控制等研究, E-mail: zyk@neuq.edu.cn.


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