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陈明，安思宇

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基于反演算法的严格反馈非线性系统固定时间跟踪控制

陈 明†, 安思宇

(辽宁科技大学 电子与信息工程学院，辽宁 鞍山 114000)

摘 要: 针对一类严格反馈非线性系统,研究固定时间跟踪控制问题. 基于反演控制策略及Lyapunov稳定性理论,给出使系统全局固定时间稳定的充分条件和设计步骤. 所提出的反演控制方案可以消除控制器存在的奇点问题,保证系统的跟踪误差在固定时间内收敛于原点的一个小邻域内,且收敛时间与系统的初始状态无关. 最后,通过一个数值仿真示例验证了所提出设计方案的有效性.

关键词: 有限时间稳定；固定时间控制；跟踪控制；反演控制；虚拟控制律；严格反馈非线性系统

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Fixed-time tracking control for strict-feedback nonlinear systems based on backstepping algorithm

CHEN Ming†, *AN Si-yu*

(School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan 114000, China)

Abstract: The problem of fixed-time tracking control is investigated for a class of strict-feedback nonlinear systems. Based on the backstepping control strategy and Lyapunov stability theory, the sufficient conditions and design steps for globally fixed-time stable of the system are given. The proposed backstepping control scheme eliminates the singularity problem that may occur in the controller design, and ensures that the tracking error converges to a small neighborhood of the origin within fixed-time interval, meanwhile, the convergence time is independent of the initial states of the system. Finally, the effectiveness of the proposed scheme is verified by a numerical simulation example.

Keywords: finite-time stability; fixed-time control; tracking control; backstepping control; virtual control laws; strict-feedback nonlinear systems

0 引 言

在许多实际系统中(如运载器姿态跟踪控制系统等),人们除了对其稳定性感兴趣外,更关心的是系统的瞬态性能. 有限时间稳定,是人们为了改善系统的瞬态性能而提出的、与通常意义下稳定性迥然不同的概念. 近年来,非线性系统有限时间稳定问题已成为研究热点之一,并取得了很多相关成果[1-5]. 然而,传统的有限时间控制虽然能保证系统在有限时间内收敛,但其收敛时间通常与系统的初始状态有关. 系统的初始状态一旦远离系统的平衡点,往往使得系统的收敛时间较长. 目前,固定时间控制为解决对初始状态依赖的快速收敛问题提供了一条新的有效途径[6-9].

固定时间控制是改善系统瞬态性能的有效工具,旨在保证系统具有快速的收敛性,收敛时间存在确定上界且与系统的初始状态无关. 基于有限时间稳定理论,Polyakov等[8]首次提出了固定时间收敛控制问题,并给出了固定时间稳定的定义. 近年来,非线性系统固定时间控制受到了广泛关注,多种研究方案被相继提出[10-16]. 如:文献[11]和文献[12]分别针对动量轮系统和二阶多智能体系统,提出了非奇异固定时间控制策略,实现了固定时间收敛控制器设计;文献[13]提出了一种快速固定时间非奇异终端滑模控制方法,并将其应用到电力系统中以抑制混沌震荡

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†通讯作者. E-mail: cm8061@sina.com.

问题. 另外,运动控制是实现各种控制任务的前提,其中跟踪控制已成为最重要的技术之一. 近年来,文献[14-16]先后提出了很多不同的先进跟踪控制算法,并已广泛应用于机器人跟踪控制、飞机轨迹跟踪、车辆路径和速度跟踪控制等领域.

受上述文献启发,本文基于反演控制策略,将固定时间控制与跟踪控制相结合,针对一类严格反馈非线性系统,提出一种使系统全局固定时间稳定的控制方案. 该方法不仅可以消除控制器设计中存在的奇点问题,而且也能保证系统的跟踪误差在固定的时间内收敛于原点的一个小邻域内,且收敛时间与系统的初始状态无关,同时使得闭环系统的所有信号都是有界的.

1 问题描述及预备知识

1.1 问题描述

考虑如下一类严格反馈非线性系统:

$$\begin{cases} \dot{x}_i(t) = f_i(\bar{x}_i(t)) + \psi_i(\bar{x}_i(t))x_{i+1}(t), \\ \quad 1 \leqslant i \leqslant n-1; \\ \dot{x}_n(t) = f_n(\bar{x}_n(t)) + \psi_n(\bar{x}_n(t))u(t); \\ y(t) = x_1(t). \end{cases} \tag{1}$$

其中: $\bar{x}_i(t) = [x_1(t), \dots, x_i(t)]^{\mathrm{T}} \in R^i\ (i = 1, 2, \dots, n)$, $u(t) \in R$, $y(t) \in R$ 分别是系统的状态变量、输入变量和输出变量; $f_i(\cdot)$、$\psi_i(\cdot)$ 表示系统原点邻域内的光滑函数.

本文的主要控制目标: 在固定时间内,跟踪误差收敛于原点的一个小邻域内,且保证闭环系统内所有信号有界. 下面给出系统全局固定时间稳定的定义及其他重要的引理和假设等.

1.2 预备知识

定义1 考虑系统

$$\dot{x}(t) = f(x(t)). \tag{2}$$

其中: $x(t) \in R^n$ 是系统的状态变量, $f(x(t))$ 表示光滑的非线性函数, $x(0) = 0, f(0) = 0$. 假设系统(2)为Lyapunov意义下稳定,若存在有限收敛时间 $T_s(x_0)$,对于所有的 $t \geqslant T_s$ 满足 $x(t) = 0$ 恒成立,则系统(2)是有限时间稳定的.

定义2 若系统(2)为有限时间稳定,收敛时间存在确定上界且上界值与状态变量无关,则称系统(2)是固定时间稳定的.

引理1 [17] 考虑系统(2),若存在连续可微正定函数 $V(x)$,正实数 μ_1、μ_2, $\alpha \in (1, +\infty)$ 及 $\beta \in (0, 1)$,有

$$\dot{V}(x) \leqslant -\mu_1 V^{\alpha}(x) - \mu_2 V^{\beta}(x), \tag{3}$$

则系统(2)是全局固定时间稳定的,且收敛时间满足

$$T_s \leqslant \frac{1}{\mu_1(\alpha - 1)} + \frac{1}{\mu_2(1 - \beta)}. \tag{4}$$

引理2 [18] 对于 $x \in R$ 以及任意正常数 ε,满足

$$0 \leqslant |x| < \varepsilon + \frac{x^2}{\sqrt{x^2 + \varepsilon^2}}. \tag{5}$$

引理3 [19] 对于 $x_i \in R, i = 1, 2, \dots, n$ 及 $\iota \in [0, 1]$,如下关系成立:

$$(|x_1| + \dots + |x_n|)^{\iota} \leqslant |x_1|^{\iota} + \dots + |x_n|^{\iota}. \tag{6}$$

引理4 对于 $x_i \geqslant 0, i = 1, 2, \dots, n$,满足

$$(x_1 + \dots + x_n)^2 \leqslant n(x_1^2 + \dots + x_n^2). \tag{7}$$

假设1 存在正常数 $\overline{\psi}_i \geqslant \underline{\psi}_i > 0$,满足 $\underline{\psi}_i \leqslant |\psi_i(\cdot)| \leqslant \overline{\psi}_i (1 \leqslant i \leqslant n-1), |\psi_n(\cdot)| \geqslant \underline{\psi}_n$. 不失一般性,假设 $\psi_i(\cdot)$ 为严格正的.

假设2 期望输出信号 y_d 连续、n 阶可导且有界.

2 主要成果

2.1 固定时间控制器设计

基于反演算法,本节给出系统(1)的固定时间跟踪控制器设计方法. 首先进行如下坐标变化:

$$z_1 = x_1 - y_d,\ z_i = x_i - \alpha_{i-1},\ i = 2, \dots, n, \tag{8}$$

其中 α_i 为第 i 个子系统的虚拟控制函数. 整个设计过程分 n 步,前 $n-1$ 步设计虚拟控制律,最后一步设计实际控制律 u. 具体递推设计过程如下.

step 1: 根据式(8)及(1)的第1个子系统状态方程,有

$$\dot{z}_1 = \psi_1 z_2 + \psi_1 \alpha_1 + f_1 - \dot{y}_d. \tag{9}$$

构造如下Lyapunov函数:

$$V_1 = \frac{1}{2} z_1^2, \tag{10}$$

对 V_1 求导数,得

$$\dot{V}_1 = z_1(\psi_1 z_2 + \psi_1 \alpha_1 + f_1 - \dot{y}_d). \tag{11}$$

根据引理2, $z_1 f_1$ 可以表示成

$$z_1 f_1 \leqslant |z_1 f_1| < \varepsilon_1 + \frac{(z_1 f_1)^2}{\sqrt{(z_1 f_1)^2 + \varepsilon_1^2}}, \tag{12}$$

其中 $\varepsilon_1 > 0$. 将式(12)代入(11),得

$$\dot{V}_1 \leqslant z_1 \psi_1 z_2 + z_1 \psi_1 \alpha_1 + \varepsilon_1 + \frac{(z_1 f_1)^2}{\sqrt{(z_1 f_1)^2 + \varepsilon_1^2}} - z_1 \dot{y}_d. \tag{13}$$

定义 $\xi_1 = \Big[\frac{z_1 (f_1)^2}{\sqrt{(z_1 f_1)^2 + \varepsilon_1^2}}, 0, \dots, 0\Big]^{\mathrm{T}}_{2n \times 1}$,这样,可将 $\frac{(z_1 f_1)^2}{\sqrt{(z_1 f_1)^2 + \varepsilon_1^2}}$ 表示成

$$\frac{(z_1 f_1)^2}{\sqrt{(z_1 f_1)^2 + \varepsilon_1^2}} = z_1 E \xi_1, \tag{14}$$

其中$E=[1\ \ 1\ \ \dots\ \ 1]_{1\times 2n}$. 根据引理2,得

$$z_1E\xi_1<\varPhi\varepsilon_1+\frac{\varPhi z_1^2\xi_1^{\mathrm T}\xi_1}{\sqrt{z_1^2\xi_1^{\mathrm T}\xi_1+\varepsilon_1^2}}.\tag{15}$$

其中: $\|E\|=\varPhi>0$, $\|\cdot\|$表示欧几里得范数. 将式(15)代入(14),有

$$\begin{aligned}\dot V_1\leqslant{}& z_1\psi_1z_2+z_1\psi_1\alpha_1-z_1\dot y_d+\\&(1+\varPhi)\varepsilon_1+\frac{\varPhi z_1^2\xi_1^{\mathrm T}\xi_1}{\sqrt{z_1^2\xi_1^{\mathrm T}\xi_1+\varepsilon_1^2}}.\end{aligned}\tag{16}$$

这里,设

$$\begin{aligned}\tilde\alpha_1={}&-\dot y_d+K_{11}\Big(\frac12\Big)^{\frac34}\frac{S_{z_1}}{z_1}+\\&K_{12}^3\Big(\frac12\Big)^2z_1^3+\frac{\varPhi z_1\xi_1^{\mathrm T}\xi_1}{\sqrt{z_1^2\xi_1^{\mathrm T}\xi_1+\varepsilon_1^2}}.\end{aligned}\tag{17}$$

其中: K_{11}、K_{12}为大于零的设计参数; S_{z_1}设计如下:

$$S_{z_1}=\begin{cases}(z_1^2)^{\frac34},\ |z_1|\geqslant\varepsilon_{10}>0;\\ \displaystyle\sum_{j=1}^n a_j(z_1^2)^j(\varepsilon_{10}^2)^{-j+\frac34}.\end{cases}\tag{18}$$

$a_j(j=1,2,\dots,n)$的各系数由如下方程计算求得:

$$\begin{bmatrix}1&1&\dots&1&1\\1&2&\dots&n-1&n\\0&2\times1&\dots&(n-1)(n-2)&n(n-1)\\\vdots&\vdots&\ddots&\vdots&\vdots\\0&0&\dots&\displaystyle\prod_{j=0}^{n-2}(n-1-j)&\displaystyle\prod_{j=0}^{n-2}(n-j)\end{bmatrix}\times$$

$$\begin{bmatrix}a_1\\a_2\\a_3\\\vdots\\a_n\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\\\vdots\\b_n\end{bmatrix}.$$

其中: $b_1=1, b_2=\dfrac34, b_3=\dfrac34\Big(\dfrac34-1\Big), b_n=\displaystyle\prod_{j=0}^{n-2}\Big(\frac34-j\Big)$.

根据式(16)和(17),得到

$$\begin{aligned}\dot V_1\leqslant{}&-K_{11}\Big(\frac12\Big)^{\frac34}S_{z_1}-K_{12}\Big(\frac12\Big)^2z_1^4+\\&(1+\varPhi)\varepsilon_1+z_1\psi_1z_2+z_1\psi_1\alpha_1+z_1\tilde\alpha_1.\end{aligned}\tag{19}$$

设计第1个子系统的虚拟控制律α_1为

$$\alpha_1=-\frac{z_1\tilde\alpha_1^2}{\underline{\psi}_1\sqrt{z_1^2\tilde\alpha_1^2+\varepsilon_1^2}}.\tag{20}$$

利用式(20)、假设1及引理2, $z_1\psi_1\alpha_1$可以变换成

$$z_1\psi_1\alpha_1\leqslant\varepsilon_1-z_1\omega\tilde\alpha_1.\tag{21}$$

将式(21)代入(19),易得

$$\dot V_1\leqslant-K_{11}\Big(\frac12\Big)^{\frac34}S_{z_1}-K_{12}\Big(\frac{z_2^2}{2}\Big)^2+z_1\psi_1z_2+\sigma_1,\tag{22}$$

其中$\sigma_1=(2+\varPhi)\varepsilon_1>0$.

当$|z_1|\geqslant\varepsilon_{10}$时,将$S_{z_1}$代入式(22),得到

$$\dot V_1\leqslant-K_{11}\Big(\frac{z_1^2}{2}\Big)^{\frac34}-K_{12}\Big(\frac{z_1^2}{2}\Big)^2+z_1\psi_1z_2+\sigma_1;\tag{23}$$

当$|z_1|<\varepsilon_{10}$时,可得

$$\begin{aligned}\dot V_1\leqslant{}&-K_{11}\Big(\frac{z_1^2}{2}\Big)^{\frac34}-K_{12}\Big(\frac{z_2^2}{2}\Big)^2+\\&z_1\psi_1z_2+\sigma_1+K_{11}\Big(\frac{z_1^2}{2}\Big)^{\frac34}-\\&K_{11}\Big(\frac12\Big)^{\frac34}\Big(\sum_{j=1}^n a_j(z_1^2)^j(\varepsilon_{10}^2)^{-j+\frac34}\Big).\end{aligned}\tag{24}$$

注1　当$|z_1|<\varepsilon_{10}$时,与式(23)相比较,式(24)中存在一个附加项,可以将其视为常数项σ_1发生小的增量变化. 为了便于讨论,在如下分析过程中仅讨论$|z_i|\geqslant\varepsilon_{i0}$的情况;同时为了书写及表示方便,各式中所有时间变量t均省略.

step i $(2\leqslant i\leqslant n-1)$: 计算$z_i$的导数,有

$$\begin{aligned}\dot z_i={}&\psi_iz_{i+1}+\psi_i\alpha_i+f_i+\chi_i-\\&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\psi_jx_{j+1}+f_j),\end{aligned}\tag{25}$$

其中$\chi_i=-\displaystyle\sum_{j=1}^{i}\frac{\partial^j\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j)}$.

选择如下Lyapunov函数:

$$V_i=V_{i-1}+\frac12z_i^2,\tag{26}$$

对V_i求导,可得

$$\begin{aligned}\dot V_i\leqslant{}&-\sum_{j=1}^{i-1}K_{j1}\Big(\frac{z_j^2}{2}\Big)^{\frac34}-\sum_{j=1}^{i-1}K_{j2}\Big(\frac{z_j^2}{2}\Big)^2+z_if_i+\\&z_{i-1}\psi_{i-1}z_i+\sigma_{i-1}+z_i\psi_iz_{i+1}+z_i\psi_i\alpha_i+\\&z_i\chi_i-z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\psi_jx_{j+1}+f_j).\end{aligned}\tag{27}$$

设计相应的虚拟控制律为

$$\alpha_i=-\frac{z_i\tilde\alpha_i^2}{\underline{\psi}_i\sqrt{z_i^2\tilde\alpha_i^2+\varepsilon_i^2}},\tag{28}$$

$$\begin{aligned}\tilde\alpha_i={}&\Big(\frac12\Big)^{\frac34}K_{i1}\frac{S_{z_i}}{z_i}+\Big(\frac12\Big)^2K_{i2}z_i^3+\\&\chi_i+\frac{\varPhi z_i\xi_i^{\mathrm T}\xi_i}{\sqrt{z_i^2\xi_i^{\mathrm T}\xi_i+\varepsilon_i^2}},\end{aligned}\tag{29}$$

以及

$$S_{z_i}=\begin{cases}(z_i^2)^{\frac34},\ \|z_i\|\geqslant\varepsilon_{i0}>0;\\ \displaystyle\sum_{j=1}^n a_j(z_i^2)^j(\varepsilon_{i0}^2)^{-j+\frac34}.\end{cases}\tag{30}$$

其中: $K_{j1}>0, K_{j2}>0(j=2,\ldots,i)$ 为设计参数. 利用式(28)和(29),可有

$$z_i\psi_i\alpha_i \leqslant \varepsilon_i - z_i\bar{\alpha}_i. \tag{31}$$

将式(30)和(31)代入(27),得

$$\begin{aligned}\dot{V}_i \leqslant & -\sum_{j=1}^{i}K_{j1}\Big(\frac{z_j^2}{2}\Big)^{\frac{3}{4}} - \sum_{j=1}^{i}K_{j2}\Big(\frac{z_j^2}{2}\Big)^2 - \\ & \frac{\varPhi z_i^2\xi_i^{\rm T}\xi_i}{\sqrt{z_i^2\xi_i^{\rm T}\xi_i+\varepsilon_i^2}} + z_i\psi_i z_{i+1} + \varepsilon_i + \\ & z_{i-1}\psi_{i-1}z_i + z_if_i - z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j x_{j+1} - \\ & z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j + \sigma_{i-1}.\end{aligned} \tag{32}$$

与step 1相类似,下面不等式成立:

$$\begin{aligned}& -z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j x_{j+1} + z_{i-1}\psi_{i-1}z_i < \\ & \sum_{j=1}^{i-1}\frac{\bar{\psi}_j z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 x_{j+1}^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 x_{j+1}^2+\varepsilon_i^2}} + \\ & \frac{\bar{\psi}_{i-1}z_{i-1}^2z_i^2}{\sqrt{z_{i-1}^2z_i^2+\varepsilon_i^2}} + \varepsilon_i\sum_{j=1}^{i-2}\bar{\psi}_j + 2\varepsilon_i\bar{\psi}_{i-1}.\end{aligned} \tag{33}$$

对于式(32)中的 z_if_i 及 $-z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j$,有

$$z_if_i \leqslant |z_if_i| < \varepsilon_i + \frac{(z_if_i)^2}{\sqrt{(z_if_i)^2+\varepsilon_i^2}}, \tag{34}$$

$$\begin{aligned}& -z_i\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j < \\ & \sum_{j=1}^{i-1}\frac{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 f_j^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 f_j^2+\varepsilon_i^2}} + (i-1)\varepsilon_i.\end{aligned} \tag{35}$$

进一步,将式(34)和(35)代入(32),得

$$\begin{aligned}& \dot{V}_i \leqslant \\ & -\sum_{j=1}^{i}K_{j1}\Big(\frac{z_j^2}{2}\Big)^{\frac{3}{4}} - \sum_{j=1}^{i}K_{j2}\Big(\frac{z_j^2}{2}\Big)^2 - \\ & \frac{\varPhi z_i^2\xi_i^{\rm T}\xi_i}{\sqrt{z_i^2\xi_i^{\rm T}\xi_i+\varepsilon_i^2}} + z_i\psi_i z_{i+1} + \\ & \frac{(z_if_i)^2}{\sqrt{(z_if_i)^2+\varepsilon_i^2}} + \sum_{j=1}^{i-1}\frac{\bar{\psi}_j z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 x_{j+1}^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 x_{j+1}^2+\varepsilon_i^2}} + \\ & \frac{\bar{\psi}_{i-1}z_{i-1}^2z_i^2}{\sqrt{z_{i-1}^2z_i^2+\varepsilon_i^2}} + \sum_{j=1}^{i-1}\frac{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 f_j^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 f_j^2+\varepsilon_i^2}} + \\ & \sigma_{i-1} + \varepsilon_i\Big(2\bar{\psi}_{i-1} + 1 + i + \sum_{j=1}^{i-2}\bar{\psi}_j\Big).\end{aligned} \tag{36}$$

定义

$$\begin{aligned}& \xi_i = \\ & \Bigg[\frac{z_i\Big(\frac{\partial\alpha_{i-1}}{\partial x_1}\Big)^2 f_1^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_1}\Big)^2 f_1^2+\varepsilon_i^2}}, \frac{\bar{\psi}_j z_i\Big(\frac{\partial\alpha_{i-1}}{\partial x_1}\Big)^2 x_2^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_1}\Big)^2 x_2^2+\varepsilon_i^2}}, \\ & \frac{z_i\Big(\frac{\partial\alpha_{i-1}}{\partial x_2}\Big)^2 f_2^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_2}\Big)^2 f_2^2+\varepsilon_i^2}}, \ldots, \\ & \frac{\bar{\psi}_j z_i\Big(\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\Big)^2 x_i^2}{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\Big)^2 x_i^2+\varepsilon_i^2}, \frac{\bar{\psi}_{i-1}z_iz_{i-1}^2}{\sqrt{z_{i-1}^2z_i^2+\varepsilon_i^2}}, \frac{z_if_i^2}{\sqrt{z_i^2f_i^2+\varepsilon_i^2}}\Bigg]^{\rm T},\end{aligned}$$

可得

$$\begin{aligned}& \frac{(z_if_i)^2}{\sqrt{(z_if_i)^2+\varepsilon_i^2}} + \sum_{j=1}^{i-1}\frac{\bar{\psi}_j z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 x_{j+1}^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 x_{j+1}^2+\varepsilon_i^2}} + \\ & \frac{\bar{\psi}_j z_{i-1}^2z_i^2}{\sqrt{z_{i-1}^2z_i^2+\varepsilon_i^2}} + \sum_{j=1}^{i-1}\frac{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 f_j^2}{\sqrt{z_i^2\Big(\frac{\partial\alpha_{i-1}}{\partial x_j}\Big)^2 f_j^2+\varepsilon_i^2}} = \\ & z_iE\xi_i \leqslant \varPhi|z_i|\|\xi_i\| < \\ & \varPhi\varepsilon_i + \frac{\varPhi z_i^2\xi_i^{\rm T}\xi_i}{\sqrt{z_i^2\xi_i^{\rm T}\xi_i+\varepsilon_i^2}}.\end{aligned} \tag{37}$$

最后,式(36)可以重新写成

$$\begin{aligned}\dot{V}_i \leqslant & -\sum_{j=1}^{i}K_{j1}\Big(\frac{z_j^2}{2}\Big)^{\frac{3}{4}} - \sum_{j=1}^{i}K_{j2}\Big(\frac{z_j^2}{2}\Big)^2 + \\ & z_i\psi_i z_{i+1} + \sigma_i,\end{aligned}$$

其中 $\sigma_i = \sigma_{i-1} + \varepsilon_i\Big(2\bar{\psi}_{i-1} + 1 + \varPhi + i + \sum_{j=1}^{i-2}\bar{\psi}_j\Big)$.

step n: 考虑最后一个子系统,引入 $z_n = x_n - \alpha_{n-1}$,有

$$\dot{z}_n = \psi_n u + f_n + \chi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\psi_j x_{j+1} + f_j). \tag{38}$$

构造Lyapunov函数 $V_n = V_{n-1} + \frac{1}{2}z_n^2$,采用与前面类

似的方法,计算V_n的导数,有

$$\dot{V}_n \leqslant -\sum_{j=1}^{n-1} K_{j1}\left(\frac{z_j^2}{2}\right)^{\frac{3}{4}} - \sum_{j=1}^{n-1} K_{j2}\left(\frac{z_j^2}{2}\right)^2 + z_{n-1}\psi_{n-1}z_n + z_n\psi_n u + z_n f_n + z_n\chi_n - z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(\psi_j x_{j+1} + f_j) + \sigma_{n-1}. \tag{39}$$

最终,设计如下的实际控制律:

$$u = \alpha_n = -\frac{z_n\tilde{\alpha}_n^2}{\underline{\psi}_n\sqrt{z_n^2\tilde{\alpha}_i^2 + \varepsilon_n^2}}, \tag{40}$$

$$\tilde{\alpha}_n = \left(\frac{1}{2}\right)^{\frac{3}{4}} K_{n1}\frac{S_{z_n}}{z_n} + \left(\frac{1}{2}\right)^2 K_{n2} z_n^3 + \chi_n + \frac{\varPhi z_n\xi_n^{\mathrm{T}}\xi_n}{\sqrt{z_n^2\xi_n^{\mathrm{T}}\xi_n + \varepsilon_n^2}}, \tag{41}$$

以及

$$S_{z_n} = \begin{cases} (z_n^2)^{\frac{3}{4}},\ \|z_n\| \geqslant \varepsilon_{n0} > 0; \\ \sum_{j=1}^{n} a_j(z_n^2)^j(\varepsilon_{n0}^2)^{-j+\frac{3}{4}}. \end{cases} \tag{42}$$

根据式(40)和(41),得到

$$z_n\psi_n u \leqslant -\frac{z_n^2\tilde{\alpha}_n^2}{\sqrt{z_n^2\tilde{\alpha}_n^2 + \varepsilon_n^2}} \leqslant \varepsilon_n - z_n\tilde{\alpha}_n. \tag{43}$$

把式(43)代入(39),整理得到

$$\dot{V}_n \leqslant -\sum_{j=1}^{n} K_{j1}\left(\frac{z_j^2}{2}\right)^{\frac{3}{4}} - \sum_{j=1}^{n} K_{j2}\left(\frac{z_j^2}{2}\right)^2 + \sigma_{n-1} - z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j x_{j+1} + z_{n-1}\psi_{n-1}z_n + \varepsilon_n + z_n f_n - z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j - \frac{\varPhi z_n^2\xi_n^{\mathrm{T}}\xi_n}{\sqrt{z_n^2\xi_n^{\mathrm{T}}\xi_n + \varepsilon_n^2}}. \tag{44}$$

同样地,对于式(44)中的$-z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j x_{j+1} + z_{n-1}\psi_{n-1}z_n$,可推导出

$$-z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j x_{j+1} + z_{n-1}\psi_{n-1}z_n < \sum_{j=1}^{n-1}\bar{\psi}_j\frac{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 x_{j+1}^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 x_{j+1}^2 + \varepsilon_n^2}} + \frac{\bar{\psi}_{n-1}z_{n-1}^2 z_n^2}{\sqrt{z_{n-1}^2 z_n^2 + \varepsilon_n^2}} + \varepsilon_n\sum_{j=1}^{n-2}\bar{\psi}_j + 2\varepsilon_n\bar{\psi}_{n-1}. \tag{45}$$

进一步,可以得到

$$z_n f_n \leqslant \varepsilon_n + \frac{(z_n f_n)^2}{\sqrt{(z_n f_n)^2 + \varepsilon_n^2}}, \tag{46}$$

$$-z_n\sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j \leqslant |z_n|\sum_{j=1}^{n-1}\frac{\partial\alpha_{i-1}}{\partial x_j}f_j < \sum_{j=1}^{n-1}\frac{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 f_j^2}{\sqrt{z_i^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 f_j^2 + \varepsilon_n^2}} + (n-1)\varepsilon_n. \tag{47}$$

将式(45)、(46)及(47)代入(44),得

$$\dot{V}_n \leqslant -\sum_{j=1}^{n} K_{j1}\left(\frac{z_j^2}{2}\right)^{\frac{3}{4}} - \sum_{j=1}^{n} K_{j2}\left(\frac{z_j^2}{2}\right)^2 + \sum_{j=1}^{n-1}\frac{\bar{\psi}_j z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 x_{j+1}^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 x_{j+1}^2 + \varepsilon_n^2}} + \frac{\bar{\psi}_{n-1}z_n^2 z_{n-1}^2}{\sqrt{z_{n-1}^2 z_n^2 + \varepsilon_n^2}} + \frac{z_n^2 f_n^2}{\sqrt{(z_n f_n)^2 + \varepsilon_n^2}} + \sum_{j=1}^{n-1}\frac{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 f_j^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 f_j^2 + \varepsilon_n^2}} - \frac{z_n\varPhi z_n\xi_n^{\mathrm{T}}\xi_n}{\sqrt{z_n^2\xi_n^{\mathrm{T}}\xi_n + \varepsilon_n^2}} + \sigma_{n-1} + (n-1)\varepsilon_n + 2\varepsilon_n + \varepsilon_n\sum_{j=1}^{n-2}\bar{\psi}_j + 2\varepsilon_n\bar{\psi}_{n-1}. \tag{48}$$

采用与step i类似的方法,有

$$\sum_{j=1}^{n-1}\frac{\bar{\psi}_j z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 x_{j+1}^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 x_{j+1}^2 + \varepsilon_n^2}} + \frac{z_n^2\bar{\psi}_{n-1}z_{n-1}^2}{\sqrt{z_{n-1}^2 z_n^2 + \varepsilon_n^2}} + \frac{z_n^2 f_n^2}{\sqrt{(z_n f_n)^2 + \varepsilon_n^2}} + \sum_{j=1}^{n-1}\frac{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 f_j^2}{\sqrt{z_i^2\left(\frac{\partial\alpha_{n-1}}{\partial x_j}\right)^2 f_j^2 + \varepsilon_n^2}} \leqslant \varPhi\varepsilon_n + \frac{\varPhi z_n^2\xi_n^{\mathrm{T}}\xi_n}{\sqrt{z_n^2\xi_n^{\mathrm{T}}\xi_n + \varepsilon_n^2}}, \tag{49}$$

其中

$$\xi_n = \left[\frac{z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 f_1^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 f_1^2 + \varepsilon_n^2}}, \frac{\bar{\psi}_1 z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 x_2^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_1}\right)^2 x_2^2 + \varepsilon_n^2}}, \frac{z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_2}\right)^2 f_2^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_2}\right)^2 f_2^2 + \varepsilon_n^2}}, \frac{\bar{\psi}_2 z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_2}\right)^2 x_3^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_2}\right)^2 x_3^2 + \varepsilon_n^2}},\right.$$

$$\ldots,\frac{z_n\left(\frac{\partial\alpha_{n-1}}{\partial x_{n-1}}\right)^2 f_{n-1}^2}{\sqrt{z_n^2\left(\frac{\partial\alpha_{n-1}}{\partial x_{n-1}}\right)^2 f_{n-1}^2+\varepsilon_n^2}},$$
$$\left.\frac{\bar{\psi}_{n-1}z_n z_{n-1}^2}{\sqrt{z_{n-1}^2 z_n^2+\varepsilon_n^2}},\frac{z_n f_n^2}{\sqrt{z_n^2 f_n^2+\varepsilon_n^2}}\right]^{\mathrm{T}}_{2n\times 1}.$$

最后,整理得到

$$\dot{V}_n=\dot{V}_{n-1}+z_n\dot{z}_n\leqslant -\sum_{j=1}^{n}K_{j1}\left(\frac{z_j^2}{2}\right)^{\frac{3}{4}}-\sum_{j=1}^{n}K_{j2}\left(\frac{z_j^2}{2}\right)^2+\sigma_n, \tag{50}$$

其中$\sigma_n=\sigma_{n-1}+\varepsilon_n\Big(2\bar{\psi}_{n-1}+1+\varPhi+n+\sum_{j=1}^{n-2}\bar{\psi}_j\Big)$.

继续作如下处理. 设$\mu_1=\min(K_{11},\ldots,K_{n1})$, $\mu_2=\min(K_{12},\ldots,K_{n2})$,根据引理3和引理4,可得

$$-\sum_{j=1}^{n}K_{j1}\left(\frac{z_j^2}{2}\right)^{\frac{3}{4}}\leqslant-\mu_1\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{\frac{3}{4}}, \tag{51}$$

$$-\sum_{j=1}^{n}K_{j2}\left(\frac{z_j^2}{2}\right)^{2}\leqslant-\frac{\mu_2}{n}\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{2}. \tag{52}$$

将式(51)和(52)代入(50),有

$$\dot{V}_n\leqslant-\mu_1\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{\frac{3}{4}}-\frac{\mu_2}{n}\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{2}+\sigma_n=-\mu_1V_n^{\frac{3}{4}}-\frac{\mu_2}{n}V_n^2+\sigma_n. \tag{53}$$

至此,基于反演算法的控制器设计完成.

2.2 主要成果

定理1 对于系统(1),满足假设1和假设2,选取合适的设计参数$K_{j1}>0,K_{j2}>0,j=1,2,\ldots,n$,采用控制律(28)、(29)、(40)和(41),则闭环系统(1)是全局固定时间稳定的,即在固定时间内,跟踪误差收敛于原点的一个小邻域内,同时保证闭环系统内所有信号都是有界的,且收敛时间存在确定上界.

注2 根据$\dot{V}_n\leqslant-\mu_1V_n^{\frac{3}{4}}-\frac{\mu_2}{n}V_n^2+\sigma_n$,因为$V_n^2\geqslant\frac{n\sigma_n}{\mu_2}$, $\dot{V}_n\leqslant-\mu_1V_n^{\frac{3}{4}}-\frac{\mu_2}{n}V_n^2+\sigma_n<0$,所以$V_n$有界,进而推出$z_i$有界. 进一步,根据$z_i$、$y_d$及其各阶导数的有界性得出$\xi_i$有界,于是$\tilde{\alpha}_i$有界,从而确定$\alpha_i$也是有界的. 因为$z_i=x_i-\alpha_i$,所以闭环系统的所有状态变量及控制变量$u$均有界,易得跟踪误差$z_1=x_1-y_d$收敛于原点的一个小邻域.

注3 接下来讨论固定时间收敛问题. 当$V_n^2\geqslant\frac{\sigma_n n}{\varpi\mu_2}$, $0<\varpi<1$时,有

$$\varpi\mu_2V_n^2/n\geqslant\sigma_n. \tag{54}$$

根据式(53),有

$$\dot{V}_n\leqslant-\mu_1V_n^{\frac{3}{4}}-\frac{(1-\varpi)\mu_2}{n}V_n^2. \tag{55}$$

由引理1可推导出,在固定时间内V_n收敛于集合$\left\{V_n:V_n<\sqrt{\frac{\sigma_n n}{\varpi\mu_2}}\right\}$,且收敛时间为

$$T_s\leqslant\frac{n}{(1-\varpi)\mu_2}+\frac{4}{\mu_1}. \tag{56}$$

3 仿真分析

考虑如下二阶严格反馈非线性系统[20]:

$$\begin{cases}\dot{x}_1=f_1+\psi_1x_2,\\ \dot{x}_2=f_2+\psi_2u,\\ y=x_1.\end{cases} \tag{57}$$

其中: $f_1=0.1x_1^2,f_2=0.1x_1x_2-0.2x_1,\psi_1=1,\psi_2=1+x_1^2$. 所以, $\overline{\psi}_1=\underline{\psi}_1=1,\underline{\psi}_2=1$. 显见,系统(57)是一个满足假设1的严格反馈非线性系统. 已知系统的初始状态为$x_1(0)=0.5,x_2(0)=-0.3$. 本文的控制目标是,在固定时间内系统的输出信号跟踪期望输出信号$y_d=0.3\sin t$,满足假设2.

根据定理1,设计控制虚拟和实际控制律. 仿真过程中发现,设计参数选择合适与否对系统性能有很大影响. 例如,设计参数K_{11}、K_{21}、K_{12}、K_{22}选择较大时,系统跟踪效果较好,但是会产生很大的控制律u. 通过反复试凑,各设计参数选择为: $K_{11}=K_{21}=30,K_{12}=K_{22}=30,\varepsilon_1=\varepsilon_2=1,\varepsilon_{10}=\varepsilon_{20}=0.001$. 仿真结果如图1～图4所示.

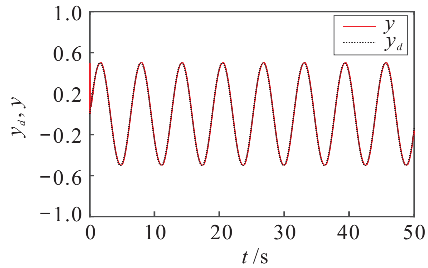

图1 实际输出y和期望输出y_d的响应曲线

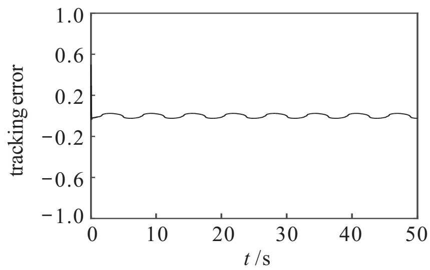

图2 跟踪误差响应曲线

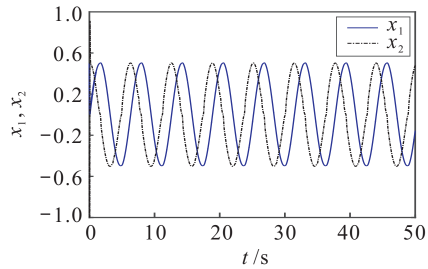

图3 x_1,x_2响应曲线

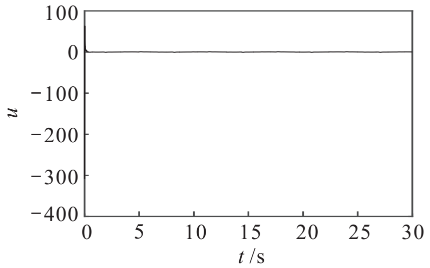

图 4　控制输入 u 响应曲线

仿真结果表明,在固定时间内,闭环系统中的全部信号均是有界的,系统跟踪误差收敛于原点的一个小邻域内,且收敛时间与系统初始状态无关.

4 结　论

本文基于反演控制策略,针对一类严格反馈非线性系统,提出了一种固定时间跟踪控制方法. 该策略避免了控制器设计中存在的奇点问题,同时使得系统的跟踪误差在预先设定的固定时间内收敛于原点的一个小邻域内,且收敛时间与系统的初始状态无关,同时保证闭环系统内所有信号均是有界的. 仿真结果验证了该设计方法的有效性. 下一步,将该成果推广到一般形式的非线性系统,尤其是工程实际系统,这将是未来的主要研究方向.

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作者简介

陈明 (1977−), 女, 教授, 博士, 从事非线性系统理论及其应用等研究, E-mail: cm8061@sina.com;

安思宇 (1996−), 女, 硕士生, 从事非线性系统有限时间控制理论与方法的研究, E-mail: 2780173730@qq.com.

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