《流量处理能力均衡的无模型自适应迭代学习交通信号控制》附录

定理 1 证明过程

证明： 1、证明 $\hat{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i)$ 的有界性

若 $\left\|\tilde{\boldsymbol{\Phi}}_{n,L}(k,i)\right\| \le \varepsilon$ 或 $\left\|\Delta\bar{\mathbf{U}}_n(k,i)\right\| \le \varepsilon$ 或 $\mathrm{sign}(\hat{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i)) \ne \mathrm{sign}(\hat{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1))$，$\hat{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i)$ 显然有界. PPG 估计误差定义为：$\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i) = \hat{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i) - \bar{\boldsymbol{\Phi}}_{n,L}(k,i)$,将(18)式两边同时减去 $\bar{\boldsymbol{\Phi}}_{n,L}(k,i)$ 得：

$$\tilde{\boldsymbol{\Phi}}_{n,L}(k,h) = \tilde{\boldsymbol{\Phi}}_{n,L}(k,h-1)\left(\mathbf{I} - \frac{\eta\Delta\mathbf{U}_n(k,h) * \Delta\mathbf{U}^T{}_n(k,h)}{\mu + \left\|\Delta\mathbf{U}_n(k,h-1)\right\|^2}\right) - \Delta\boldsymbol{\Phi}_{n,L}(k,h-1). \tag{23}$$

由引理 1 可知 $\left\|\bar{\boldsymbol{\Phi}}_{n,L}(k,i)\right\|$ 有上界，即存在一个正常数 $\bar{b}$，使得 $\left\|\bar{\boldsymbol{\Phi}}_{n,L}(k,i)\right\| \le \bar{b}$，因此 $\left\|\Delta\bar{\boldsymbol{\Phi}}_{n,L}(k,i-1)\right\| \le 2\bar{b}$. (23)式两边取范数得：

$$\begin{aligned}\left\|\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i)\right\| &\le \left\|(\mathbf{I} - \frac{\eta\Delta\bar{\mathbf{U}}_n(k,i) * \Delta\bar{\mathbf{U}}_n^T(k,i)}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2})\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\right\| + \left\|\Delta\hat{\bar{\boldsymbol{\Phi}}}^T_{n,L}(k,i-1)\right\| \\ &\le \left\|(\mathbf{I} - \frac{\eta\Delta\bar{\mathbf{U}}_n(k,h) * \Delta\bar{\mathbf{U}}_n^T(k,i)}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2})\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\right\| + 2\bar{b}.\end{aligned} \tag{24}$$

(24)式右边第一项平方得：

$$\begin{aligned}&\left\|(\mathbf{I} - \frac{\eta\Delta\bar{\mathbf{U}}_n(k,i) * \Delta\bar{\mathbf{U}}_n^T(k,i)}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2})\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\right\|^2 = \left\|\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\right\|^2 + \\ &\left[-2 + \frac{\eta\left\|\Delta\bar{\mathbf{U}}_n(k,i)\right\|^2}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2}\right]\frac{\eta\left\|\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\Delta\bar{\mathbf{U}}_n(k,i)\right\|^2}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2}.\end{aligned} \tag{25}$$

对于 $0 < \eta \le 2$ 和 $\mu > 0$, 下式成立:

$$\left[-2 + \frac{\eta\left\|\Delta\bar{\mathbf{U}}_n(k,i)\right\|^2}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2}\right] < 0. \tag{26}$$

不等式(24)与不等式(25)意味着存在 $0 < d < 1$, 使得下式成立:

$$\left\|(\mathbf{I} - \frac{\eta\Delta\bar{\mathbf{U}}_n(k,i)\Delta\bar{\mathbf{U}}_n^T(k,i)}{\mu + \left\|\Delta\bar{\mathbf{U}}_n(k,i-1)\right\|^2})\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\right\|^2 < d\left\|\Delta\tilde{\bar{\boldsymbol{\Phi}}}_{n,L}(k,i-1)\right\|. \tag{27}$$

将(27)式代入(24)式, 得:

$$\left\|\tilde{\bar{\mathbf{\Phi}}}_{n,L}(k,i)\right\| \le d\left\|\tilde{\bar{\mathbf{\Phi}}}_{n,L}(k,i)\right\| + 2\bar{b} \le \cdots \le d^{i-1}\tilde{\bar{\mathbf{\Phi}}}_{n,L}(k,1) + \frac{2\bar{b}(1-d^{i-1})}{1-d}. \tag{28}$$

不等式(28)证明了 $\tilde{\bar{\mathbf{\Phi}}}_{n,L}(k,i)$ 是有界的. 由引理 1 可知 $\bar{\mathbf{\Phi}}_{n,L}(k,i)$ 是有界的, 因此 $\hat{\bar{\mathbf{\Phi}}}_{n,L}(k,i)$ 是有界的.

2、证明输出误差收敛性

定义系统的跟踪误差为:

$$e_n(k+1,i) = y_{n,d}(k+1) - y(k+1,i). \tag{29}$$

令:

$$A(k,i) = \begin{bmatrix} \frac{\rho_2 \hat{\mathbf{\Phi}}^T_{n,1}(k,i)\hat{\mathbf{\Phi}}_{n,2}(k,i)}{\lambda + \left\|\hat{\mathbf{\Phi}}^T_{n,1}(k,i)\right\|^2} & \cdots & \frac{\rho_L \hat{\mathbf{\Phi}}^T_{n,1}(k,i)\hat{\mathbf{\Phi}}_{n,L}(k,i)}{\lambda + \left\|\hat{\mathbf{\Phi}}^T_{n,1}(k,i)\right\|^2} & \mathbf{0} \\ \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{C}(k,i) = \begin{bmatrix} \frac{\hat{\mathbf{\Phi}}^T_{n,1}(k,i)}{\lambda + \left\|\hat{\mathbf{\Phi}}^T_{n,1}(k,i)\right\|^2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

则控制算法(21)式重新写为:

$$\Delta\mathbf{U}_n(k,i) = \mathbf{A}(k,i)\Delta\mathbf{U}_n(k,i-1) - \rho_1\mathbf{C}_1(k,i)e(k,i). \tag{30}$$

由矩阵谱半径结论可知, $\left\|\mathbf{A}(k,i)\right\|_v < \rho(\mathbf{A}(k,i)) + \varepsilon_1 < \max_{1\le j\le i}\left\{\rho(\mathbf{A}(k,j))\right\} + \varepsilon_1 = d_1$. 其中, $\left\|\mathbf{A}(k,i)\right\|_v$, $\rho(\mathbf{A}(k,i))$ 分别为 $\mathbf{A}(k,j)$ 的相容范数和谱半径, ε_1 是任意小的正数. 由 $\Delta\mathbf{U}_n(k,0) = 0$ 可知 $\left\|\Delta\mathbf{U}_n(k,0)\right\|_v = 0$.

令 $r = \left\|\mathbf{C}(k,i)\right\|_v$, 在(30)式两边去相容范数得:

$$\begin{aligned} \left\|\Delta\mathbf{U}_n(k,i)\right\|_v &\le \left\|\mathbf{A}(k,i)\right\|_v \left\|\Delta\mathbf{U}_n(k-1,i)\right\|_v + r\left|e(k+1,i-1)\right| < d_1\left\|\Delta\mathbf{U}_n(k-1,i)\right\|_v + r\left|e(k+1,i-1)\right| < \\ \cdots &< d_1^i\left\|\Delta\mathbf{U}_n(k-1,0)\right\|_v + d_1^{i-1}r\left|e(k+1,0)\right| + \cdots + r\left|e(k+1,i-1)\right| = r\sum_{j=1}^{i} d_1^{i-j}\left|e(k+1,j-1)\right|. \end{aligned} \tag{31}$$

将等价 PFDL 数据模型(16)式以及(30)式代入误差方程(29)式得:

$$e_n(k+1,i)=y_{n,d}(k+1)-y_n(k+1,i)=e_n(k+1,i-1)-\bar{\mathbf{\Phi}}_{n,L}^T(k,i)\Delta\bar{\mathbf{U}}_n(k,i)$$
$$=\left[1-\frac{\rho_1\mathbf{\Phi}_{n,1}^T(k,i)\hat{\mathbf{\Phi}}_{n,1}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right]e_n(k+1,i-1)-\bar{\mathbf{\Phi}}_{n,L}^T(k,i)\mathbf{A}(k,i-1)\Delta\bar{\mathbf{U}}_n(k-1,i). \tag{32}$$

令 $d_2=1-\min\limits_{1\le j\le i}\left\{\frac{\rho_1\sum\limits_{n=1}^{N}\mathbf{\Phi}_{n,1}^T(k,j)\hat{\mathbf{\Phi}}_{n,1}(k,j)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2}\right\}$，(32)式取范数，由 $\left\|\mathbf{A}(k,i)\right\|_v<d_1$ 可知:

$$\left|e_n(k+1,i)\right|\le d_2\left|e_n(k+1,i-1)\right|+\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,i)\right\|_v\left\|\mathbf{A}(k,i)\right\|_v\left\|\Delta\mathbf{U}_n(k-1,i)\right\|_v<$$
$$d_2\left|e_n(k+1,i-1)\right|+d_1\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,i)\right\|_v\left\|\Delta\mathbf{U}_n(k-1,i)\right\|_v<\cdots<d_2^{i-1}\left|e_n(k+1,1)\right|+$$
$$d_2^{i-2}d_1\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,2)\right\|_v\left\|\Delta\mathbf{U}_n(k,2)\right\|_v+\cdots+d_1\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,i)\right\|_v\left\|\Delta\mathbf{U}_n(k-1,i)\right\|_v. \tag{33}$$

将(31)代入(33)式得:

$$\left|e_n(k+1,i)\right|<d_2^{i-1}\left|e_n(k+1,1)\right|+d_2^{i-2}d_1\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,2)\right\|_v\left\|\Delta\mathbf{U}_n(k-1,2)\right\|_v+$$
$$\cdots+d_1\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,i)\right\|_v\left\|\Delta\mathbf{U}_n(k-1,i)\right\|_v\le d_2^{i-1}\left|e_n(k+1,1)\right|+$$
$$\max_{1\le j\le i}\left\{\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,j)\right\|_v\right\}\sum_{j=2}^{i}d_2^{i-j}d_1\left\|\Delta\mathbf{U}_n(k-1,j)\right\|_v\le d_2^{i-1}\left|e_n(k+1,1)\right|+$$
$$\max_{1\le j\le i}\left\{\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,j)\right\|_v\right\}\sum_{j=2}^{i}d_2^{i-j}d_1r\sum_{i=1}^{j-1}d_1^{i-1-j}\left|e(k,i-1)\right|$$
$$=d_2^{i-1}\left|e_n(k+1,1)\right|+d_3\sum_{j=2}^{i}d_2^{i-j}\sum_{j=1}^{j-1}d_1^{j-i}\left|e(k,i-1)\right|. \tag{34}$$

其中，$d_3=\max\limits_{1\le j\le i}\left\{\left\|\bar{\mathbf{\Phi}}_{n,L}^T(k,j)\right\|_v\right\}r$.整理(34)式得:

$$\left|e_n(k+1,i)\right|<d_2^{i-1}\left|e_n(k+1,1)\right|+d_3\sum_{j=2}^{i}d_2^{i-j}\sum_{l=1}^{j-1}d_1^{j-l}\left|e(k,l-1)\right|=d_2^{i-1}\left|e_n(k+1,1)\right|+$$
$$d_3\sum_{l=1}^{i-1}d_1^ld_2^{i-1-l}\left|e_n(k,0)\right|+\cdots+d_3\sum_{l=1}^{2}d_1^ld_2^{2-l}\left|e_n(k,i-3)\right|+d_3d_1\left|e_n(k,i-2)\right|=g_1(k+1,i-1). \tag{35}$$

重写(35)式得:

$$\left|e_n(k+1,i)\right|<g_1(k+1,i-1). \tag{36}$$

用 d_2 乘以 $g_1(k+1,i-1)$ 得:

$$d_2 g_1(k+1,i-1)=d_2^i|e_n(k+1,1)|+d_3\sum_{i=1}^{i-1} d_1^l d_2^{i-i}|e_n(k,0)|+$$
$$\cdots+d_3\sum_{l=1}^{2} d_1^l d_2^{3-l}|e_n(k,i-3)|+d_3 d_2 d_1|e_n(k,i-2)|. \tag{37}$$

由(36)式得:

$$|e_n(k+1,i+1)|<g_1(k+1,i)=d_2^i|e_n(k+1,1)|+d_3\sum_{l=1}^{i} d_1^l d_2^{i-l}|e_n(k,1)|+\cdots+d_3\sum_{l=1}^{2} d_1^l d_2^{3-l}|e_n(k,i-2)|$$
$$+d_3 d_1|e_n(k,i-1)|=d_2 g_1(k+1,i-1)+d_3 d_1^i|e_n(k,0)|+\cdots+d_3 d_1^2|e_n(k,i-2)|+d_3 d_1|e_n(k,i-1)|. \tag{38}$$

由(36)式可知: $|e_n(k+1,i-1)|<g_1(k+1,i-2)$, 把其代入(38)式可得:

$$|e_n(k+1,i+1)|<d_2 g_1(k+1,i-1)+d_3 d_1^i|e_n(k,0)|+\cdots+d_3 d_1|e_n(k,i-1)|<d_2 g_1(k+1,i-1)+$$
$$d_3 d_1^i|e_n(k,0)|+\cdots+d_3 d_1^2|e_n(k,i-2)|+d_3 d_1 g_1(k+1,i-2)=d_2 g_1(k+1,i-1)+d_3 d_1^i|e_n(k,0)|+$$
$$\cdots+d_3 d_1^2|e_n(k,i-2)|+d_3 d_1 d_2^{i-2}|e_n(k+1,1)|+d_3^2\sum_{i=1}^{i-2} d_1^{l+1} d_2^{i-2+l}|e_n(k,0)|+\cdots+ \tag{39}$$
$$d_3^2\sum_{l=1}^{2} d_1^{l+1} d_2^{2-l}|e_n(k,i-4)|+d_3^2 d_1^2|e_n(k,i-3)|=d_2 g_1(k+1,i-1)+g_2(k+1,i-1).$$

其中

$$g_2(k+1,i-1)=d_3 d_1 d_2^{i-2}|e_n(k+1,1)|+d_3^2\sum_{l=1}^{i-2} d_1^{l+1} d_2^{i-2-l}|e_n(k,0)|+d_3 d_1^{i-1}|e_n(k,0)|+$$
$$\cdots+d_3^2 d_1^2|e_n(k,i-3)|+d_3 d_1^3|e_n(k,i-3)|+d_3 d_1^2|e_n(k,i-2)|. \tag{40}$$

用 d_1 乘以 $g_1(k+1,i-1)$ 得:

$$d_1 g_1(k+1,i-1)=d_2^{i-1} d_1|e_n(k+1,1)|+d_3\sum_{l=1}^{i-1} d_1^{l+1} d_2^{i-1-l}|e_n(k,0)|+$$
$$\cdots+d_3\sum_{l=1}^{2} d_1^{l+1} d_2^{2-l}|e_n(k,i-3)|+d_3 d_1^2|e_n(k,i-2)|. \tag{41}$$

将(41)式减去(40)的差为 $g(k+1,i-1)$, 整理得:

$$g(k+1,i-1)=(d_2-d_3)d_2^{i-2} d_1|e_n(k+1,1)|+d_3(d_2-d_3)\sum_{l=1}^{i-2} d_1^{l+1} d_2^{i-2-l}|e_n(k,0)|+$$
$$\cdots+d_3(d_2-d_3)d_1^2|e_n(k,i-3)|. \tag{42}$$

若 $d_3<d_2$, 则(42)式有 $g(k+1,i-1)>0$, 把其代入(38)式得:

$$|e_n(k+1,i+1)|<g_1(k+1,i)<d_2 g_1(k+1,i-1)+g_2(k+1,i-1)<$$
$$d_2 g_1(k+1,i-1)+d_1 g_1(k+1,i-1)=d_4 g_1(k+1,i-1). \tag{43}$$

其中, $d_4=d_2+d_1$, 若 $d_4<1$, 则(43)式可得:

$$
\begin{aligned}
&\lim_{i\to\infty}\left|e(k+1,i)\right| < \lim_{i\to\infty}\left|g_1(k+1,i-1)\right| < \lim_{i\to\infty} d_4\left|g_1(k+1,i-2)\right| < \\
&\lim_{i\to\infty} d_4^2\left|g_1(k+1,i-3)\right| < \cdots < \lim_{i\to\infty} d_4^{i-2}\left|g_1(k+1,1)\right| = 0.
\end{aligned} \tag{44}
$$

因此, 只需 $d_3 < d_2$ 和 $d_2 + d_1 = d_4 < 1$ 这两个条件满足即可保证收敛误差.

由前面推导可知 d_2 、 d_3 以及 $\mathbf{C}$ 的表达式.选取 $\rho_1 < 1$, 因为 $\lim_{\lambda\to\infty} d_3 = 0, \lim_{\lambda\to\infty} d_2 = 1$, 所以存在正数 $\lambda_{\min,1}$, 当 $\lambda > \lambda_{\min,1}$ 时有 $d_3 < d_2$ 成立.

由参数估计有界性证明可知 $\left\|\hat{\mathbf{\Phi}}_{n,l}(k,i)\right\| < M_1$, 以及假设 3 可知 $\alpha \le \mathbf{\Phi}_{n,1}(k,i) \le \beta$, 其中 $n = 1,2\cdots N; l = 1,2\cdots L$, 由此存在一个足够大的正数 $\lambda_{\min,2}$, 当 $\lambda > \lambda_{\min,2}$ 时有:

$$
M_2 < \min_{1\le j\le i} \frac{\rho_1 \sum_{n=1}^{N} \mathbf{\Phi}_{n,1}^T(k,j)\hat{\mathbf{\Phi}}_{n,1}(k,j)}{\lambda + \left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2} < 1. \tag{45}
$$

$$
\max_{1\le j\le i}\left(\sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\hat{\mathbf{\Phi}}_{n,1}^T(k,j)\hat{\mathbf{\Phi}}_{n,l}(k,j)}{\lambda + \left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2}\right|\right)^{\frac{1}{L-1}} < M_3 < 1. \tag{46}
$$

选择 $0 < \rho_1 < 1, \cdots, 0 < \rho_L < 1$ 可以使得下面的不等式成立:

$$
\max_{1\le j\le i}\left(\sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\hat{\mathbf{\Phi}}_{n,1}^T(k,j)\hat{\mathbf{\Phi}}_{n,l}(k,j)}{\lambda + \left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2}\right|\right)^{\frac{1}{L-1}} < \left\{\max_{l\in\{2,\cdots L\}} \rho_l\right\}^{\frac{1}{L-1}} M_3 < 1. \tag{47}
$$

选择 $\rho_1, \cdots, \rho_L$ 同时满足 $\rho_1 M_2 > \left\{\max_{l\in\{2,\cdots L\}} \rho_l\right\}^{\frac{1}{L-1}} M_3$, 则下面不等式成立:

$$
0 < \min_{1\le j\le i} \frac{\rho_1 \sum_{n=1}^{N} \mathbf{\Phi}_{n,1}^T(k,j)\hat{\mathbf{\Phi}}_{n,1}(k,j)}{\lambda + \left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2} - \max_{1\le j\le i}\left(\left|\sum_{l=2}^{L}\sum_{n=1}^{N}\frac{\hat{\mathbf{\Phi}}_{n,1}^T(k,j)\hat{\mathbf{\Phi}}_{n,l}(k,j)}{\lambda + \left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2}\right|\right)^{\frac{1}{L-1}} < 1. \tag{48}
$$

$\mathrm{A}(k,i)$ 的特征方程为:

$$
v^{L+1}\det\left(v^{L-1} + \sum_{l=2}^{L} v^{L-l}\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^T(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda + \left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right) = 0. \tag{49}
$$

由此可得:

敛.

$$\left|v^{L-1}\right| \leq \sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^{T}(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right|\left|v\right|^{L-l}. \tag{50}$$

由(47)式成立可知, 当$\lambda > \lambda_{\min,2}$, $0<\rho_1<1$,$0<\rho_L<1$时有:

$$\sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^{T}(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right|<1. \tag{51}$$

假设$\left|v\right|\geq 1$, 则$1\leq\left|v\right|^1\leq\left|v\right|^2\leq\cdots\leq\left|v\right|^{L-2}$, 由(51)式得:

$$\left|v\right|^{L-1}\leq\left|v\right|^{L-2}\sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^{T}(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right|<1. \tag{52}$$

由(52)式可知$\left|v\right|<1$这与$\left|v\right|\geq 1$的假设不符, 因此$\left|v\right|<1$, 由(50)式和$\left|v\right|<1$可得:

$$\left|v\right|^{L-1}\leq\sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^{T}(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right|\left|v\right|^{L-l}<\sum_{n=1}^{N}\sum_{l=2}^{L}\left|\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^{T}(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right|. \tag{53}$$

由(53)式可得:

$$\rho(\mathbf{A}(k,i))<\left(\sum_{l=2}^{L}\sum_{n=1}^{N}\left|\frac{\rho_l \hat{\mathbf{\Phi}}_{n,1}^{T}(k,i)\hat{\mathbf{\Phi}}_{n,l}(k,i)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,i)\right\|^2}\right|\right)^{\frac{1}{L-1}}. \tag{54}$$

由(48)式和(54)式可知选择$0<\rho_1<1,\cdots,0<\rho_L<1$，且$0<\rho_1,\cdots,0<\rho_L$满足

$\rho_1>\left\{\max\limits_{l\in\{2,\cdots L\}}\rho_l\right\}^{\frac{1}{L-1}}\dfrac{M_3}{M_2}$可以使得下面不等式成立:

$$0<\min_{1\leq j\leq i}\frac{\rho_1\sum_{n=1}^{N}\mathbf{\Phi}_{n,1}^{T}(k,j)\hat{\mathbf{\Phi}}_{n,1}(k,j)}{\lambda+\left\|\hat{\mathbf{\Phi}}_{n,1}(k,j)\right\|^2}-\max_{1\leq j\leq i}\left\{\rho(\mathbf{A}(k,j))\right\}<1. \tag{55}$$

由(55)式可知不等式$d_2+d_1=d_4<1$成立. 因此，存在一个正数$\lambda_{\min}=\max\left\{\lambda_{\min,1},\lambda_{\min,2}\right\}$，当$\lambda>\lambda_{\min}$时有$d_3<d_2$和$d_2+d_1=d_4<1$成立. 故输出误差收敛.