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线性不确定中立型时滞系统的鲁棒二次性能

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摘 要: 研究具有非匹配条件的范数有界线性不确定中立型时滞系统的稳定和二次性能控制问题. 基于 Lyapunov 方法, 提出了系统鲁棒渐近稳定并满足给定二次性能指标的时滞相关型条件, 该条件等价于线性矩阵不等式(LMI)可解性问题, 并根据LMI的可行解, 构造了状态反馈控制器设计方法.

关键词: 线性矩阵不等式(LMI); 时滞系统; Lyapunov 方法; 二次性能
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Robust quadratic performance for linear neutral systems with uncertainties

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Abstract: The problem of stability and quadratic performance for linear neutral systems with non-match norm-bounded uncertainties is discussed. Based on Lyapunov method, a delay-dependent sufficient criterion for stability and quadratic performance of the systems is derived in the form of linear matrix inequality (LMI). The design method of robust state-feedback controller is constructed in terms of the feasible solutions to the LMI.

Key words: linear matrix inequality(LMI); time-delay system; Lyapunov method; quadratic performance

1 引 言

由于传输距离、计算时间等各种因素, 时滞经常出现在电力系统、化学过程等各种系统中. 对于时滞系统的稳定性分析和综合已经进行了深入的研究[1~5], 其中包括不确定时滞系统的保成本控制[6], H_∞控制[7~9]和无源控制[10,11]等. 这些成果中, 有些成果并不能直接扩展到时变时滞情形, 有些结果对不确定性要求满足一定的匹配条件, 或要求对参数线性搜索以得到最优性能.

本文研究了带时变时滞的线性不确定中立型时滞系统的二次性能控制问题, 采用文献[5]中提出的中立型时滞系统的描述系统模型来研究这类系统时滞相关鲁棒性能. 文中考虑非匹配条件的范数有界不确定性, 采用一种带参数的Lyapunov 泛函, 提出时滞相关型的充分条件, 使这类不确定系统鲁棒渐近稳定且满足给定的性能指标, 进一步, 提出了基于LMI的状态反馈控制器的存在条件, 以及由LMI的可行解构成的控制器增益矩阵的表达式.

2 问题的描述

考虑如下中立型不确定时滞系统:

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$$
\begin{aligned}
&\mathring{x}(t) - F_1\mathring{x}(t-g_1) = \\
&A_0(t)x(t) + A_1(t)x_1(t)x(t- \\
&\tau_1(t)) + B_0w(t) + B_1u(t) = \\
&[A_0 + D_0\Sigma_0(t)E_0]x(t) + [A_1 + \\
&D_1\Sigma_1(t)E_1]x(t-\tau_1(t)) + B_0w(t) + B_1u(t), \\
&z(t) = C_0x(t) + Dw(t), \\
&x(t) = \Psi(t),\ t \in [-h, 0].
\end{aligned} \tag{1}
$$

式中: $x(t) \in R^n$ 为系统状态; $w(t) \in R^r$ 为外部输入, 对于任何 $T > 0$, 有 $w(t) \in L_2[0, T]$; $z(t) \in R^p$ 为被控输出; $F_1, A_0, D_0, E_0, A_1, D_1, E_1, B_0, B_1, C_0, D$ 为适当维数的常数矩阵; $\Psi(t)$ 为连续的状态初值函数; 不确定性矩阵满足 $\Sigma_0^{\mathrm{T}}(t)\Sigma_0(t) \leq I$, $\Sigma_1^{\mathrm{T}}(t)\Sigma_1(t) \leq I$; 时滞 $\tau_1(t)$ 满足:

A 1: $0 \leq \tau_1(t) \leq h_1$,

A 2: $\mathring{\tau}_1(t) \leq d_1 \leq 1$.

取 $h = \max\{g_1, h_1\}$. 下面研究该系统与 h_1, d_1 相关, 与 g_1 无关的性能分析.

考虑二次性能指标为

$$
J(w) = \int [w^{\mathrm{T}}(t)\quad z^{\mathrm{T}}(t)] \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^{\mathrm{T}} & M_{22} \end{bmatrix} \begin{bmatrix} w(t) \\ z(t) \end{bmatrix} \mathrm{d}t \tag{2}
$$

采用文献[5]中引入的描述系统模型将系统(1)重写为

$$
\begin{aligned}
&\mathring{x}(t) = y(t), \\
&0 = -y(t) + F_1\mathring{x}(t-g_1) + [A_0(t) + \\
&\quad A_1(t)]x(t) - A_1(t)\int_{\tau_1(t)} y(s)\mathrm{d}s + \\
&\quad B_0w(t) + B_1u(t), \\
&z(t) = C_0x(t) + Dw(t), \\
&x(t) = \Psi(t),\ t \in [-h, 0].
\end{aligned} \tag{3}
$$

采用无记忆状态反馈控制器 $u(t) = K_0x(t)$, 寻求控制器增益 K_0, 使不确定时滞系统(1)和控制器构成的闭环系统鲁棒渐近稳定且满足 $J(w) < 0$.

3 结 果

在证明本文主要结果前, 先给出如下引理.

引理 1[11] 设 $F(t) \in \Omega$, $\Omega = \{F(t): F^{\mathrm{T}}(t) \times F(t) \leq I$, $F(t)$ 是元素为Lebesgue可测未知函数矩阵$\}$, A, D, E, X 为适当维数的常数矩阵, 则下列各式成立:

1) 对任何标量 $\epsilon > 0$, 有

$$DF(t)E + [DF(t)E]^{\mathrm{T}} \leq \epsilon^{-1}DD^{\mathrm{T}} + \epsilon E^{\mathrm{T}}E.$$

2) 对满足 $(X - \mu E^{\mathrm{T}}E)^{-1} > 0$ 的标量 $\mu > 0$ 和 $X > 0$, 有

$$
\begin{aligned}
&[A + DF(t)E]^{\mathrm{T}}X^{-1}[A + DF(t)E] \leq \\
&A(X - \mu E^{\mathrm{T}}E)^{-1}A^{\mathrm{T}} + 1/\mu DD^{\mathrm{T}}.
\end{aligned}
$$

3) 对满足 $(\mu I - D^{\mathrm{T}}XD)^{-1} > 0$ 的标量 $\mu > 0$ 和 $X > 0$, 有

$$
\begin{aligned}
&[A + DF(t)E]^{\mathrm{T}}X[A + DF(t)E] \leq \\
&A^{\mathrm{T}}XA + A^{\mathrm{T}}XD(\mu I - D^{\mathrm{T}}XD)^{-1}D^{\mathrm{T}}XA + \mu E^{\mathrm{T}}E.
\end{aligned}
$$

定理 1 给定不确定时滞系统(1)和性能指标(2)(设 M_{22} 为非奇异), 如果存在矩阵 $Q_1 > 0, Q_2, Q_3, M_0$, $\bar{S}_1 > 0$, $\bar{R}_1 > 0$, $\bar{U}_1 > 0$, $\bar{Z}_1 = \bar{Z}_1^{\mathrm{T}} = \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} \\ \bar{Z}_{12}^{\mathrm{T}} & \bar{Z}_{22} \end{bmatrix} > 0$ 和标量 $\lambda_1 > 0$, $\lambda_2 > 0$, 使下列LMI有可行解:

$$
\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & 0 & 0 & Q_1\Psi_1 & Q_1E_0^{\mathrm{T}} & 0 & Q_1 & Q_2^{\mathrm{T}} & h_1Q_2^{\mathrm{T}}A_1 & h_1Q_2^{\mathrm{T}}D_1 & Q_1C_0^{\mathrm{T}} \\
* & \Gamma_{22} & A_1\bar{S}_1 & F_1\bar{U}_1 & B_0 & 0 & 0 & 0 & Q_3^{\mathrm{T}} & h_1Q_3^{\mathrm{T}}A_1 & h_1Q_3^{\mathrm{T}}D_1 & 0 \\
* & * & -(1-d_1)\bar{S}_1 & 0 & 0 & 0 & \bar{S}_1E_1^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\bar{U}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \Psi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\lambda_1I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\lambda_2I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\bar{S}_1 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\bar{U}_1 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\Psi_3 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -h_1\mu I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\bar{M}_{22}
\end{bmatrix} < 0 \tag{4}
$$

其中

$$
\begin{aligned}
&\Gamma_{11} = Q_2 + Q_2^{\mathrm{T}} + h_1\bar{Z}_{11}, \\
&\Gamma_{12} = Q_3^{\mathrm{T}} - Q_2 + Q_1A_0^{\mathrm{T}} + h_1\bar{Z}_{12} + M_0^{\mathrm{T}}B_1^{\mathrm{T}}, \\
&\Gamma_{22} = -Q_3 - Q_3^{\mathrm{T}} + h_1\bar{Z}_{22} + \lambda_1D_0D_0^{\mathrm{T}} + \lambda_2D_1D_1^{\mathrm{T}},
\end{aligned}
$$

$$
\begin{aligned}
&\Psi_2 = M_{11} + D^{\mathrm{T}}M_{22}D + 2M_{12}D, \\
&\bar{M}_{22} = -M_{22}^{-1}, \\
&\Psi_1 = C_0^{\mathrm{T}}M_{12}^{\mathrm{T}} + C_0^{\mathrm{T}}M_{22}C_0, \\
&\Psi_3 = h_1(R_1 - \mu E_1^{\mathrm{T}}E_1),
\end{aligned}
$$

则对于所有允许的不确定性和满足A1,A2条件的滞后, 状态反馈控制器 $u(t) = K_0 x(t)$, K_0 存在, 而且闭环系统鲁棒渐近稳定且二次性能指标 $J(w) < 0$. 这时控制器增益 $K_0 = M_0 Q_1^{-1}$.

证明　设 $u(t) = 0$, 由引理1, 存在 $R_1 > 0$, $\mu > 0$, 有

$$A_1^T(t)R_1^{-1}A_1(t) = [A_1 + D_1\Sigma_1(t)E_1]^T R_1^{-1}[A_1 + D_1\Sigma_1(t)E_1] \le A_1\left(R_1 - \frac{1}{\mu}E_1^T E_1\right)^{-1}A_1^T + \mu D_1 D_1^T.$$

定义

$$\Omega(R_1,\mu) := A_1\left(R_1 - \frac{1}{\mu}E_1^T E_1\right)^{-1}A_1^T + \mu D_1 D_1^T, \tag{5}$$

引入Lyapunov-Krasvokii泛函[3]

$$V(t) = [x^T(t)\quad y^T(t)]EP\begin{bmatrix} x(t) \\ y(t)\end{bmatrix} + V_2(t) + V_3(t) + V_4(t),$$

$$V_2(t) = \int_{-h_1}^{0}\int_{t+\theta}^{t} y^T(s)\Omega(R_1,\mu)y(s)\,\mathrm{d}s\,\mathrm{d}\theta,$$

$$V_3(t) = \int_{t-g_1}^{t} y^T(s)U_1 y(s)\,\mathrm{d}s,$$

$$V_4(t) = \int_{t-\tau_1(t)}^{t} x^T(s)S_1 x(s)\,\mathrm{d}s. \tag{6}$$

这里

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, P_1 = P_1^T > 0, E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},$$

$U_1 > 0, S_1 > 0, R_1 > 0$.

设

$$\bar{x}^T(t) = [x^T(t)\quad y^T(t)],$$

$$\xi^T(t) = [\bar{x}^T(t)\quad x^T(t-\tau_1(t))\quad y^T(t-g_1)\quad w^T(t)].$$

沿系统(3) 的轨迹求 $\mathrm{d}V(t)/\mathrm{d}t$, 得到

$$\dot{V}(t) \le \xi^T(t)H_1(t)\xi(t) - \int_{t-h_1}^{t} y^T(s)\Omega(R_1,\mu)y(s)\,\mathrm{d}s - 2\int_{t-\tau_1(t)}^{t}\bar{x}^T(t)P^T\begin{bmatrix} 0 \\ A_1(t)\end{bmatrix}y(s)\,\mathrm{d}s$$

其中

$$H_1(t) = \begin{bmatrix} \Phi & 0 & P^T\begin{bmatrix} 0 \\ F_1 \end{bmatrix} & P^T\begin{bmatrix} 0 \\ B_0 \end{bmatrix} \\ * & -(1-d_1)S_1 & 0 & 0 \\ * & * & -U_1 & 0 \\ * & * & * & 0 \end{bmatrix},$$

$$\Phi = P^T\begin{bmatrix} 0 & I \\ \sum_{i=0}^{1}A_i(t) & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ \sum_{i=0}^{1}A_i(t) & -I \end{bmatrix}^T P + \begin{bmatrix} S_1 & 0 \\ 0 & U_1 + h_1\Omega(R_1,\mu) \end{bmatrix},$$

* 表示矩阵中的对称项

若存在矩阵 $Z_1 > 0$, 显然下式成立:

$$-2\int_{t-\tau_1(t)}^{t}\bar{x}^T(t)P^T\begin{bmatrix} 0 \\ A_1(t)\end{bmatrix}y(s)\,\mathrm{d}s \le \int_{t-\tau_1(t)}^{t}[y^T(s)A_1^T(t)\quad \bar{x}^T(t)] \times \begin{bmatrix} R_1^{-1} & -[0\quad I]P \\ * & Z_1 \end{bmatrix}\begin{bmatrix} A_1(t)y(s) \\ \bar{x}(t)\end{bmatrix}\mathrm{d}s \le \int_{t-h_1}^{t} y^T(s)\Omega(R_1,\mu)y(s)\,\mathrm{d}s + h_1\bar{x}^T(t)Z_1\bar{x}(t) - 2[x^T(t) - x^T(t-\tau_1)]A_1^T(t)[0\quad I]P\bar{x}(t).$$

计算下式:

$$\dot{V}(t) + w^T(t)M_{11}w(t) + 2w^T(t)M_{12}z(t) + z^T(t)M_{22}z(t) \le \xi^T(t)H_2(t)\xi(t). \tag{7}$$

其中

$$H_2(t) = \begin{bmatrix} \bar{\Psi} & P^T\begin{bmatrix} 0 \\ A_1(t)\end{bmatrix} & P^T\begin{bmatrix} 0 \\ F_1 \end{bmatrix} & P^T\begin{bmatrix} 0 \\ B_0 \end{bmatrix} + \begin{bmatrix} \Psi_1 \\ 0 \end{bmatrix} \\ * & -(1-d_1)S_1 & 0 & 0 \\ * & * & -U_1 & 0 \\ * & * & * & \Psi_2 \end{bmatrix},$$

$$\Psi_1 = C_0^T M_{12}^T + C_0^T M_{22}D,$$

$$\Psi_2 = M_{11} + D^T M_{22}D + 2M_{12}D,$$

$$\bar{\Psi} = P^T\begin{bmatrix} 0 & I \\ A_0(t) & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A_0(t) & -I \end{bmatrix}^T P + \begin{bmatrix} S_1 & 0 \\ 0 & U_1 + h_1\Omega(R_1,\mu) \end{bmatrix} + h_1 Z_1 + \begin{bmatrix} C_0^T M_{22}C_0 & 0 \\ 0 & 0 \end{bmatrix}.$$

这样, $H_2(t) < 0$ 为系统(3) 渐近稳定且 $J(w) < 0$ 的充分条件. 分析 $\bar{\Psi}$ 的(2,2) 块可知, $-P_3 - P_3^T < 0$, 则 P_3, P 非奇异, 设

$$P^{-1} = Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix},$$

$$Q_1 = Q_1^T > 0.$$

将 $H_2(t) < 0$ 左右两边分别乘 $\mathrm{diag}\{Q^T, S_1^{-1}, U_1^{-1}, I\}$ 和它的转置, 得到

$$H_3(t)=\begin{bmatrix}\Gamma & \begin{bmatrix}0\\ A_1(t)\end{bmatrix}S_1^{-1} & \begin{bmatrix}0\\ F_1\end{bmatrix}U_1^{-1} & \begin{bmatrix}0\\ B_0\end{bmatrix}+Q^{\mathrm T}\begin{bmatrix}\Psi_1\\ 0\end{bmatrix}\\ * & -(1-d_1)S_1^{-1} & 0 & 0\\ * & * & -U_1^{-1} & 0\\ * & * & * & \Psi_2\end{bmatrix}<0 \qquad (8)$$

其中

$$\Gamma=\Pi_1+Q^{\mathrm T}h_1Z_1Q+Q^{\mathrm T}\begin{bmatrix}S_1 & 0\\ 0 & U_1+h_1\Omega(R_1,\mu)\end{bmatrix}Q+Q^{\mathrm T}\begin{bmatrix}C_0^{\mathrm T}M_{22}C_0 & 0\\ 0 & 0\end{bmatrix}Q=$$

$$\begin{bmatrix}\Pi_{11} & \Pi_{12}\\ \Pi_{12}^{\mathrm T} & \Pi_{22}\end{bmatrix}+Q^{\mathrm T}\begin{bmatrix}S_1 & 0\\ 0 & U_1+h_1\Omega(R_1,\mu)\end{bmatrix}Q+Q^{\mathrm T}\begin{bmatrix}C_0^{\mathrm T}M_{22}C_0 & 0\\ 0 & 0\end{bmatrix}Q,$$

$$\Pi_1=\begin{bmatrix}0 & I\\ A_0(t) & -I\end{bmatrix}Q+Q^{\mathrm T}\begin{bmatrix}0 & I\\ A_0(t) & -I\end{bmatrix}^{\mathrm T}.$$

设

$$Q^{\mathrm T}Z_1Q=\bar Z=\begin{bmatrix}\bar Z_{11} & \bar Z_{12}\\ \bar Z_{12}^{\mathrm T} & \bar Z_{22}\end{bmatrix},$$

$$S_1^{-1}=\bar S_1,\ U_1^{-1}=\bar U_1,$$

$$\Pi_{11}=Q_2+Q_2^{\mathrm T}+h_1\bar Z_{11},$$

$$\Pi_{12}=Q_3-Q_2^{\mathrm T}+Q_1A_0^{\mathrm T}(t)+h_1\bar Z_{12},$$

$$\Pi_{22}=-Q_3-Q_3^{\mathrm T}+h_1\bar Z_{22}.$$

再设

$$\xi_1(t)^{\mathrm T}=[x(t)\quad y(t)\quad x(t-\tau_1(t))\quad y(t-g_1)\quad w(t)]^{\mathrm T}$$

将式 $\xi_1^{\mathrm T}(t)H_3(t)\xi(t)<0$ 展开, 应用引理 1, 可知对任意标量 $\lambda_1>0,\lambda_2>0$, 有

$$2x^{\mathrm T}(t)Q_1A_0^{\mathrm T}(t)y(t)\le 2x^{\mathrm T}(t)Q_1A_0^{\mathrm T}y(t)+\frac{1}{\lambda_1}x^{\mathrm T}(t)Q_1E_0^{\mathrm T}E_0Q_1x(t)+\lambda_1y^{\mathrm T}(t)D_0D_0^{\mathrm T}y(t),$$

$$2y^{\mathrm T}(t)A_1(t)\bar S_1x(t-\tau_1(t))\le 2y^{\mathrm T}(t)A_1\bar S_1x(t-\tau_1(t))+\lambda_2y^{\mathrm T}(t)D_1D_1^{\mathrm T}y(t)+\frac{1}{\lambda_2}x^{\mathrm T}(t-\tau_1(t))\bar S_1E_1^{\mathrm T}E_1\bar S_1x(t-\tau_1(t)). \qquad (9)$$

一般情况下, 性能指标中 M_{11} 和 M_{22} 要么等于零阵, 要么为对角阵且非奇异. 下面分别考虑这两种情况.

1) 当 M_{22} 为非奇异时, 将

$$\Omega(R_1,\mu):=A_1\left(R_1-\frac{1}{\mu}E_1^{\mathrm T}E_1\right)^{-1}A_1^{\mathrm T}+\mu D_1D_1^{\mathrm T}$$

和式(9) 代入 $\xi_1^{\mathrm T}(t)H_3(t)\xi_1(t)<0$, 得到

$$\xi_1^{\mathrm T}(t)H_3(t)\xi_1(t)<\xi_1^{\mathrm T}(t)H_4\xi_1(t)<0,$$

若式

$$H_4=\begin{bmatrix}\begin{bmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{12}^{\mathrm T} & \Gamma_{22}\end{bmatrix}+Q^{\mathrm T}\begin{bmatrix}S_1 & 0\\ 0 & U_1\end{bmatrix}Q & \begin{bmatrix}0\\ A_1\bar S_1\end{bmatrix} & \begin{bmatrix}0\\ F_1\bar U_1\end{bmatrix} & \begin{bmatrix}Q_1^{\mathrm T}\Psi_1\\ B_0\end{bmatrix}\\ * & \Gamma_{33} & 0 & 0\\ * & * & -\bar U_1 & 0\\ * & * & * & \Psi_2\end{bmatrix}<0 \qquad (10)$$

成立, 则

$$\xi_1^{\mathrm T}(t)H_3(t)\xi_1(t)<0\Rightarrow \dot V(t)+w^{\mathrm T}(t)M_{11}w(t)+2w^{\mathrm T}(t)M_{12}z(t)+z^{\mathrm T}(t)M_{22}z(t)<0.$$

其中

$$\Gamma_{11}=Q_2+Q_2^{\mathrm T}+h_1\bar Z_{11}+\lambda_1^{-1}Q_1E_0^{\mathrm T}E_0Q_1+h_1Q_2^{\mathrm T}A_1(R_1-\mu E_1^{\mathrm T}E_1)^{-1}A_1^{\mathrm T}Q_2+\frac{h_1}{\mu}Q_2^{\mathrm T}D_1D_1^{\mathrm T}Q_2+Q_1C_0^{\mathrm T}M_{22}C_0Q_1,$$

$$\Gamma_{12}=Q_3-Q_2^{\mathrm T}+Q_1^{\mathrm T}A_0^{\mathrm T}+h_1\bar Z_{12}+h_1Q_2^{\mathrm T}A_1(R_1-\mu E_1^{\mathrm T}E_1)^{-1}A_1^{\mathrm T}Q_3+\frac{h_1}{\mu}Q_2^{\mathrm T}D_1D_1^{\mathrm T}Q_3,$$

$$\Gamma_{22}=-Q_3-Q_3^{\mathrm T}+h_1\bar Z_{22}+\lambda_2D_1D_1^{\mathrm T}+\lambda_1D_0^{\mathrm T}D_0+h_1Q_3^{\mathrm T}A_1(R_1-\mu E_1^{\mathrm T}E_1)^{-1}A_1^{\mathrm T}Q_3+\frac{h_1}{\mu}Q_3^{\mathrm T}D_1D_1^{\mathrm T}Q_3,$$

$$\Gamma_{33}=-(1-d_1)\bar S_1+\lambda_2^{-1}\bar S_1E_1^{\mathrm T}E_1\bar S_1.$$

对式(10) 应用 Schur 补展开, 用 $A_0+B_1K_0$ 代替式(10) 中的 A_0 矩阵, 并设 $M_0=K_0Q_1$, 得到式(4).

2) 当 $M_{22}=0$ 时, 得到的综合条件式就是将式(4) 中最后一行和最后一列删去.

4 数值实例

采用文献[3] 中的实例, 设 $F_1=0$, 考虑如下系统:

$$\dot x(t)-\dot x(t-g_1)=\begin{bmatrix}-2+\delta_1\cos t & 0\\ 0 & -1+\delta_2\sin t\end{bmatrix}x(t)+\begin{bmatrix}-1+\gamma_1\cos t & 0\\ -1 & -1+\gamma_2\sin t\end{bmatrix}x(t-\tau_1(t))+$$

$$\begin{bmatrix}1\\1\end{bmatrix} w(t) + \begin{bmatrix}0\\1\end{bmatrix} u(t),$$

$$z(t) = [0 \quad 1] x(t) + 0.1 w(t).$$

系统可写成式(1) 形式, 即

$$A_0 = \begin{bmatrix}-2 & 0\\0 & -1\end{bmatrix}, A_1 = \begin{bmatrix}-1 & 0\\-1 & -1\end{bmatrix},$$

$$D_0 = E_0 = \begin{bmatrix}\sqrt{1.6} & 0\\0 & \sqrt{0.05}\end{bmatrix},$$

$$D_1 = E_1 = \begin{bmatrix}\sqrt{0.1} & 0\\0 & \sqrt{0.3}\end{bmatrix},$$

$$\dot{\tau}_1(t) \leq 0.9, \tau_1(t) < 1.$$

考虑 H_∞ 控制, $M = \begin{bmatrix}-\gamma I & 0\\0 & \frac{1}{\gamma} I\end{bmatrix}$, 取 $\gamma = 0.8$, 利用Matlab 软件中的LMI toolbox 求解式(4), 求得状态反馈控制器增益矩阵 $K_0 = -[0.524 \quad 0.972]$.

5 结　　语

本文基于Lyapunov 方法, 研究了不确定中立型线性时滞系统的二次性能综合问题, 获得系统鲁棒渐近稳定且满足二次性能指标的状态反馈控制器的存在条件, 及控制器的构造方法.

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