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一类离散非线性不确定互联系统的模糊分散控制

佟绍成[1], 曲连江[2], 曹希宁[3]
(1. 辽宁工学院 数理系, 辽宁 锦州 121001; 2. 空军第三飞行学院 训保处, 辽宁 锦州 121001; 3. 沈阳职业技术学院, 辽宁 沈阳 110013)

摘　要: 利用模糊控制方法研究一类离散非线性互联系统的分散控制问题. 首先采用模糊(T-S)模型对离散非线性不确定互联系统进行模糊建模, 应用并行分布补偿算法(PDC)给出状态反馈分散模糊控制方案, 并基于李亚普诺夫函数方法证明了闭环系统的稳定性. 然后当系统的状态不完全可测时, 设计模糊分散观测器来估计各子系统的状态, 从而给出基于观测器的状态反馈分散模糊控制设计的方法. 因为该分散模糊控制设计问题是以线性矩阵不等式的形式给出, 所以很容易用凸优化方法求解. 仿真结果验证了所提出控制方法的有效性.

关键词: 模糊分散控制; 模糊T-S模型; 线性矩阵不等式; 稳定性分析
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Decentralized fuzzy robust control for nonlinear uncertain interconnected systems

TONG Shao-cheng[1], *QU Lian-jiang*[2], *CAO Xi-ning*[3]
(1. Department of Mathematics and Physics, Liaoning Institute of Technology, Jinzhou 121001, China; 2. Training and Logistics Office, The Third Aeronautical Engineering Institute of The Air Force, Jinzhou 121001, China; 3. Shenyang Vocational and Technical College, Shenyang 110013, China. Correspondent: TONG Shao-cheng, E-mail: jztsc@sohu.com)

Abstract: The decentralized control problem is studied for a class of discrete nonlinear interconnected systems via fuzzy control method. The nonlinear interconnected system is represented by an equivalent Takagi-Sugeno type mode. A state feedback decentralized fuzzy control scheme is developed by the parallel distributed compensation algorithm. The stability of nonlinear interconnected systems is proved. In the case that states are not all available, a decentralized fuzzy observer is proposed to estimate the states of each subsystem for decentralized control. Consequently, a fuzzy observer-based state feedback decentralized fuzzy controller is proposed. The problem of decentralized fuzzy control design for nonlinear interconnected systems is formulated in the format of linear matrix inequalities and solved very efficiently using convex optimization techniques. Finally, simulation examples show the effectiveness of the proposed methods.

Key words: decentralized fuzzy control; fuzzy T-S model; linear matrix inequality; stability analysis

1 引　　言

模糊T-S模型是一种非线性模型, 易于表示复杂系统的动态特征. 对于非线性系统不同区域的动态, 可以利用模糊T-S模型建立局部线性模型, 然后将各个局部线性模型用模糊隶属函数连接起来, 得到逼近的非线性系统的模糊模型, 进而进行系统的控制设计及其稳定性分析. 近年来, 基于模糊T-S模型的非线性不确定系统的控制器设计及其理论研究已取得了很多研究成果[1,2], 初步建立了与现代控制理论相平行的设计方法和理论分析体系[3]. 然而, 目前应用模糊T-S模型研究非线性不确定互联系统的分散控制器设计及其稳定性分析的结果却很少[4,5].

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作者简介: 佟绍成(1960—), 男(满族), 辽宁锦州人, 教授, 博士, 从事自适应控制等研究.

本文在文献[4, 5]的研究基础上, 对一类离散非线性不确定互联系统分别给出了模糊状态反馈和基于观测器的输出反馈分散控制设计的方法. 首先, 采用模糊T-S 模型对不确定非线性互联系统进行模糊建模; 然后, 应用并行分布补偿算法(PDC)给出了模糊状态反馈和基于观测器的输出反馈分散控制设计. 同时基于线性矩阵不等式和李亚普诺夫方法, 给出了模糊闭环系统的稳定性分析.

2 非线性离散互联系统的模糊建模及其控制问题

考虑由如下N 个子系统S_i所构成的非线性互联系统S:

$$S_i: x_i(t+1) = f_i(x_i(t)) + g_i(x_i(t))u_i + \sum_{j=1, j\neq i}^{N} f_{ij}(x_j(t)). \qquad (1)$$

其中: $x_i(t) \in R^{n_i}$ 是子系统S_i 的状态向量; $u_i(t) \in R^{m_i}$ 是控制输入; $f_i(x_i(t))$, $g_i(x_i(t))$ 和$f_{ij}(x_i(t))$ 是光滑函数, $f_{ij}(x_i(t))$ 是第 i 个子系统和第 j 个子系统的联接项.

应用模糊 T-S 模型对上述离散非线性系统进行模糊建模. 设模糊系统规则如下:

Rule k:

If $z_{i_1}(t)$ is F_{k1} and $z_{i_2}(t)$ is F_{k2}

and $\cdots$ and $Z_{i_n}(t)$ is F_{kn_i},

Then $x_i(t+1) = A_{ik}x_i(t) + B_{ik}u_i(t) + \sum_{j=1, j\neq i}^{N} A_{ijk}x_j(t)$,

$k = 1, 2, \cdots, L$.

其中: F_{kj} 是模糊集; $z_i(t) = [z_{i_1}(t), \cdots, z_{i_k}(t)]^{\mathrm{T}}$ 是可测系统变量, 即前件变量; k 是模糊推理的规则数; A_{ik}, B_{ik} 和A_{ijk} 是适当维数的矩阵.

采用与文献[1] 相同的模糊推理方法, 则可得到第 i 个子系统S_i 的模糊互联系统

$$x_i(t+1) = \sum_{k=1}^{L} \mu_k(z_i(t))\Big[A_{ik}x_i(t) + B_{ik}u_i(t) + \sum_{j=1, j\neq i}^{N} A_{ijk}x_j(t)\Big]. \qquad (2)$$

其中

$$\alpha_k(z_i(t)) = \prod_{j=1}^{n_i} F_{kj}(z_j(t)),$$

$$\mu_k(z_i(t)) = \frac{\alpha_k(z_i(t))}{\sum_{k=1}^{N} \alpha_k(z_i(t))}.$$

控制目标为: 对模糊互联系统(2), 应用并行补偿算法(PDC), 设计模糊状态反馈和基于观测器的模糊输出反馈分散控制, 使得模糊互联系统(2) 是全局渐近稳定的.

3 模糊状态反馈分散控制及其稳定性分析

如果非线性系统的状态是完全可测的, 则根据 PDC 设计第 i 个子系统S_i 的模糊分散控制如下:

Rule k: If $z_{i_1}(t)$ is F_{k1} and $z_{i_2}(t)$ is F_{k2}

and $\cdots$ and $z_{i_n}(t)$ is F_{kn_i},

Then $u_i(t) = -K_{ik}x_i(t)$,

$k = 1, 2, \cdots, L$, (3)

其中K_{ik} 为反馈增益矩阵. 于是第 i 个子系统S_i 的模糊分散控制器为

$$u_i(t) = -\sum_{k=1}^{L} \mu_k(z_i(t))K_{ik}x_i(t),\ i = 1, 2, \cdots, N. \qquad (4)$$

将式(4) 代入(2) 可得模糊闭环系统为

$$\begin{aligned} x_i(t+1) =& \\ &\sum_{k=1}^{L}\sum_{m=1}^{L} \mu_k\mu_m(A_{ik} - B_{ik}K_{im})x_i(t) + \\ &\sum_{k=1}^{L}\sum_{j=1, j\neq i}^{N} \mu_k A_{ijk}x_j(t) = \\ &\sum_{k=1}^{L}\sum_{m=1}^{L} \mu_k\mu_m G_{ikm}x_i(t) + \\ &\sum_{k=1}^{L}\sum_{j=1, j\neq i}^{N} \mu_k A_{ijk}x_j(t), \end{aligned} \qquad (5)$$

其中$G_{ikm} = A_{ik} - B_{ik}K_{im}$.

定理 1 如果存在一个正定矩阵 P_i 和矩阵 K_{ik}, 对所有的 $i = 1, 2, \cdots, N$, 满足下列线性矩阵不等式(LMI):

$$\begin{bmatrix} -\frac{1}{N}X_i & * & * \\ A_{ik}X_i - B_{ik}M_{ik} & -X_i & 0 \\ \underline{A}_{ik}X_i & 0 & -\underline{X}_i \end{bmatrix} < 0, \qquad (6)$$

$$\begin{bmatrix} -\frac{1}{N}X_i & * & * & * \\ A_{ik}X_i - B_{ik}M_{ik} & -\frac{1}{\sqrt{2}}X_i & 0 & 0 \\ A_{ik}X_i - B_{ik}M_{im} & 0 & -\frac{1}{\sqrt{2}}X_i & 0 \\ \underline{A}_{ik}X_i & 0 & 0 & -\underline{X}_i \end{bmatrix} < 0,$$

$$k < m, \qquad (7)$$

其中

$M_{ik} = K_{ik}X_i$, $X_i = P_i^{-1}$,

$\underline{A}_{ik} = [A_{1ik}^{\mathrm{T}}, \cdots, A_{(i-1)ik}^{\mathrm{T}}, A_{(i+1)ik}^{\mathrm{T}}, \cdots A_{Nik}^{\mathrm{T}}]^{\mathrm{T}}$,

$\underline{X}_i = \mathrm{diag}[X_i^{-1}, \cdots, X_{i-1}^{-1}, X_{i-1}^{-1}, \cdots, -X_N^{-1}]$.

则模糊状态反馈分散控制器(4), 使得模糊不确定分散系统(5) 全局渐近稳定.

证明 考虑李亚普诺夫函数

$$V(t) = \sum_{i=1}^{N} x_i^{\mathrm{T}}(t) P_i x_i(t), \qquad (8)$$

求 $V(t)$ 对时间的差分, 并由式(5) 得

$$\Delta V(t) = V(t+1) - V(t) = \sum_{i=1}^{N} x_i^{\mathrm{T}}(t+1) P_i x_i(t+1) - \sum_{i=1}^{N} x_i^{\mathrm{T}}(t) P_i x_i(t) =$$

$$\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{m=1}^{L}\sum_{l=1}^{L}\sum_{h=1}^{L}\mu_k\mu_m\mu_l\mu_h \times x_i^{\mathrm{T}}(t)(G_{ikm}^{\mathrm{T}} P_i G_{ilh} - P_i) x_i(t) +$$

$$\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{m=1}^{L}\sum_{l=1}^{L}\sum_{j=1, j\neq i}^{N}\mu_k\mu_m\mu_l \times x_i^{\mathrm{T}}(t) G_{ikm}^{\mathrm{T}} P_i A_{jil} x_j(t) +$$

$$\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{l=1}^{L}\sum_{h=1}^{L}\sum_{f=1, j\neq i}^{N}\mu_k\mu_l\mu_h \times x_j^{\mathrm{T}}(t) A_{ijk}^{\mathrm{T}} P_i G_{ilh} x_i(t) +$$

$$\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{l=1}^{L}\sum_{j=1, j\neq i}^{N}\sum_{f=1, f\neq j}^{N}\mu_k\mu_l \times x_j^{\mathrm{T}}(t) A_{ijk}^{\mathrm{T}} P A_{ifl} x_f(t) \leq$$

$$\frac{1}{4}\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{m=1}^{L}\sum_{l=1}^{L}\sum_{h=1}^{L}\mu_k\mu_m\mu_l\mu_h \times x_i^{\mathrm{T}}(t)[(G_{ikm} + G_{ilh})^{\mathrm{T}} P_i (G_{ikm} + G_{ilh}) - 4P_i] x_i(t) +$$

$$\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{m=1}^{L}\sum_{l=1}^{L}\sum_{j=1, j\neq i}^{N}\mu_k\mu_m\mu_l \times [x_i^{\mathrm{T}}(t) G_{ikm}^{\mathrm{T}} P_i G_{ikm} x_i(t) + x_j^{\mathrm{T}}(t) A_{ijl}^{\mathrm{T}} P A_{ijl} x_j(t)] +$$

$$\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{L}\sum_{l=1}^{L}\sum_{j=1, i\neq j}^{N}\sum_{f=1, f\neq i}^{N}\mu_k\mu_l \times [x_j^{\mathrm{T}}(t) A_{ijk}^{\mathrm{T}} P A_{ijk} x_j(t) + x_f^{\mathrm{T}}(t) A_{ifl}^{\mathrm{T}} P A_{ifl} x_f(t)] \leq$$

$$N\sum_{i=1}^{N}\sum_{k=1}^{L}\mu_k^2 x_i^{\mathrm{T}}(t)\Big(G_{ikk}^{\mathrm{T}} P_i G_{ikk} + \sum_{j=1, j\neq i}^{N} A_{jik}^{\mathrm{T}} P_j A_{jik} - \frac{1}{N} P_i\Big) x_i(t) +$$

$$2N\sum_{i=1}^{N}\sum_{k<m}^{L}\mu_k\mu_m x_i^{\mathrm{T}}(t) \times \Big[(G_{ikm}^{\mathrm{T}} P_i G_{ikm} + G_{imk}^{\mathrm{T}} P_i G_{imk})/2 + \sum_{j=1, j\neq i}^{N} A_{jik}^{\mathrm{T}} P_j A_{jik} - \frac{1}{N} P_i\Big] x_i(t). \qquad (9)$$

为保证除 $x_i(t) = 0$ 之外, 式(8) 的差分对任意 $x_i(t)$ 都是负定的, 因此, 假设式(9) 中每个和式都是负定的, 那么模糊分散控制系统在其平衡点 0 处是渐近稳定的

假设第 1 个和式是负定的, 即

$$G_{ikk}^{\mathrm{T}} P_i G_{ikk} + \sum_{j=1, j\neq i}^{N} A_{ijk}^{\mathrm{T}} P_j A_{ijk} - \frac{1}{N} P_i = (A_{ik} - B_{ik} K_{ik})^{\mathrm{T}} P_i (A_{ik} - B_{ik} K_{ik}) + \sum_{j=1, j\neq i}^{N} A_{ijk}^{\mathrm{T}} P_j A_{ijk} - \frac{1}{N} P_i < 0. \qquad (10)$$

对式(10) 应用 Schur 分解方法得

$$\begin{bmatrix} -\frac{1}{N} P_i & * & * \\ A_{ik} - B_{ik} K_{ik} & -X_i & 0 \\ \underline{A}_{ik} & 0 & -X_i \end{bmatrix} < 0. \qquad (11)$$

在不等式(11) 左右同时乘以 $\mathrm{diag}[P_i^{-1}\ I\ \cdots\ I]$, 可得 LMI(6).

假设第 2 个和式是负定的, 即

$$(G_{ikm}^{\mathrm{T}} P_i G_{ikm} + G_{imk}^{\mathrm{T}} P_i G_{imk})/2 + \sum_{j=1, j\neq i}^{N} A_{ijk}^{\mathrm{T}} P_j A_{ijk} - \frac{1}{N} P_i < 0. \qquad (12)$$

对式(12) 应用 Schur 分解方法, 类似于 LMI(6) 的推导过程, 可将式(12) 化成 LMI(7). □

4　模糊输出分散反馈控制及其稳定性分析

在上节中, 假设所有的系统变量是可以利用的, 然而实际中, 这种假设往往难以成立 因此需要首先估计系统的状态, 然后设计模糊输出分散反馈控制

设第 i 个子系统 S_i 的模糊观测器设计为

Rule: k

If $z_{i_1}(t)$ is F_{k1} and $z_{i_2}(t)$ is F_{k2} and $\cdots$ and $z_{i_n}(t)$ is F_{kn_i},

Then $\hat{x}_i(t+1) = A_{ik}\hat{x}_i(t) + B_{ki} u_i(t) + \sum_{j=1, j\neq i}^{N} A_{ijk}\hat{x}_j(t) + L_{ik}(y_i(t) - \hat{y}_i(t))$,

$\hat{y}(t) = C_{ik}\hat{x}_i(t),\ k = 1, 2, \cdots, L.$

则第 i 个子系统 S_i 的模糊观测器为

$$\hat{x}_i(t+1) = \sum_{k=1}^{L}\mu_k\Big[A_{ik}\hat{x}_i(t) + B_{ik} u_i(t) + \sum_{j=1, j\neq i}^{N} A_{ijk}\hat{x}_j(t) + L_{ik}(y_i(t) - \hat{y}_i(t))\Big],$$

$$\hat{y}(t) = \sum_{k=1}^{L}\mu_k C_{ik}\hat{x}_i(t). \qquad (13)$$

于是, 基于观测器的模糊分散控制变成

$$u_i(t) = -\sum_{k=1}^{L}\mu_k K_{ik}\hat{x}_i(t),\ i = 1, 2, \cdots, N. \qquad (14)$$

将式(14) 代入(2) 得

$$x_i(t+1) = \sum_{k=1}^{L}\sum_{m=1}^{L}\mu_k\mu_m\Big[G_{ikm} x_i(t) + \sum_{j=1, j\neq i}^{N} A_{ijk} x_j(t)\Big] +$$

$$\sum_{k=1}^{L}\sum_{m=1}^{L}\mu_k\mu_m B_{ik}K_{im}e_i(t). \tag{15}$$

定义观测误差为

$$e_i(t) = x_i(t) - \hat{x}_i(t). \tag{16}$$

由式(2)和式(13),得误差的动态方程

$$e_i(t+1) = \sum_{k=1}^{L}\sum_{m=1}^{L}\mu_k\mu_m\Big[H_{ikm}e_i(t) + \sum_{j=1,j\neq i}^{N}A_{ijk}e_j(t)\Big], \tag{17}$$

其中 $H_{ikm} = A_{ik} - L_{ik}C_{im}$.

引入合成变量 $x_{\alpha i}(t) = [x_i(t)\ \ e_i(t)]^{\mathrm{T}}$, 根据式(15)和(17)得

$$x_{\alpha i}(t+1) = \sum_{k=1}^{L}\sum_{m=1}^{L}\mu_k\mu_m\Big[\hat{G}_{ikm}x_{\alpha i}(t) + \sum_{j=1,j\neq i}^{N}\hat{A}_{ijk}x_{\alpha j}(t)\Big], \tag{18}$$

其中

$$\hat{G}_{ikm} = \begin{bmatrix} G_{ikm} & B_{ik}K_{im} \\ 0 & H_{ikm}\end{bmatrix}, \hat{A}_{ijk} = \begin{bmatrix} A_{ijk} & 0 \\ 0 & A_{ijk}\end{bmatrix}.$$

定理 2 如果存在正定矩阵 $\hat{P}_i = \mathrm{diag}[P_{1i}, P_{2i}]$, 矩阵 K_{im} 和 L_{im}, 对所有的 $i = 1, 2, \cdots, N$, 满足下面的矩阵不等式:

$$\hat{G}_{ikk}^{\mathrm{T}}\hat{P}_i\hat{G}_{ikk} + \sum_{j=1,j\neq i}^{N}\hat{A}_{jik}^{\mathrm{T}}\hat{P}_j\hat{A}_{jik} - \frac{1}{N}\hat{P}_i < 0, \tag{19}$$

$$(\hat{G}_{ikm}^{\mathrm{T}}\hat{P}_i\hat{G}_{ikm} + \hat{G}_{imk}^{\mathrm{T}}\hat{P}_i\hat{G}_{imk})/2 + \sum_{j=1,j\neq i}^{N}\hat{A}_{jik}^{\mathrm{T}}\hat{P}_j\hat{A}_{jik} - \frac{1}{N}\hat{P}_i < 0, k < m, \tag{20}$$

则基于模糊观测器的分散控制系统是全局渐近稳定的.

证明 考虑李亚普诺夫函数

$$V(t) = \sum_{i=1}^{N}x_{\alpha i}^{\mathrm{T}}(t)\hat{P}_i x_{\alpha i}(t). \tag{21}$$

类似于定理 1 的证明过程, 如果满足矩阵不等式(19)和(20), 则 $\Delta V(t) < 0$, 因此基于模糊观测器的分散控制系统是全局渐近稳定的. □

从定理 2 可知, 基于观测器的模糊分散控制的最重要问题是: 如何求出公共正定矩阵 $\hat{P}_i$ 以及反馈控制和观测器增益矩阵 K_{im} 和 L_{im}. 下面应用矩阵不等式理论给出一种解耦设计方法.

由不等式(19)得

$$\begin{bmatrix} (A_{ik} - B_{ik}K_{ik})^{\mathrm{T}} & 0 \\ K_{ik}^{\mathrm{T}}B_{ik}^{\mathrm{T}} & (A_{ik} - L_{ik}C_{ik})^{\mathrm{T}}\end{bmatrix}\begin{bmatrix} P_{1i} & \\ & P_{2i}\end{bmatrix} \times \begin{bmatrix} (A_{ik} - B_{ik}K_{ik}) & B_{ik}K_{ik} \\ 0 & (A_{ik} - L_{ik}C_{ik})\end{bmatrix} + \sum_{j=1,j\neq i}^{N}\begin{bmatrix} A_{ijk}^{\mathrm{T}} & \\ & A_{ijk}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix} P_{1i} & \\ & P_{2i}\end{bmatrix}\begin{bmatrix} A_{ijk} & \\ & A_{ijk}\end{bmatrix} - \frac{1}{N}\begin{bmatrix} P_{1i} & \\ & P_{2i}\end{bmatrix} = \begin{bmatrix} S_{11ik} & S_{12ik} \\ S_{12ik}^{\mathrm{T}} & S_{22ik}\end{bmatrix} < 0. \tag{22}$$

其中

$$S_{11ik} = (A_{ik} - B_{ik}K_{ik})^{\mathrm{T}}P_{1i}(A_{ik} - B_{ik}K_{ik}) + \sum_{j=1,j\neq i}^{N}A_{ijk}^{\mathrm{T}}P_{1j}A_{ijk} - \frac{1}{N}P_{1i},$$

$$S_{12ik} = (A_{ik} - B_{ik}K_{ik})^{\mathrm{T}}P_{1i}B_{ik}K_{ik},$$

$$S_{22ik} = K_{ik}^{\mathrm{T}}B_{ik}^{\mathrm{T}}P_{1i}B_{ik}K_{ik} + (A_{ik} - L_{ik}C_{ik})^{\mathrm{T}}P_{2i}(A_{ik} - L_{ik}C_{ik}) + \sum_{j=1,j\neq i}^{N}A_{ijk}^{\mathrm{T}}P_{2j}A_{jik} - \frac{1}{N}P_{2i}.$$

记 $Y_{2i} = P_{2i}^{-1}$, $\underline{Y}_{2i} = \mathrm{diag}[X_{2i}^{-1}, \cdots, X_{2i-1}^{-1}, X_{2i-1}^{-1}, \cdots, -X_{2N}^{-1}]$, 则矩阵不等式(22)等价于

$$\begin{bmatrix} S_{11ik} & * & * & * \\ S_{12ik} & -\frac{1}{N}P_{2i} & 0 & 0 \\ 0 & K_{ik}^{\mathrm{T}}B_{ik}^{\mathrm{T}} & -P_{1i}^{-1} & 0 \\ 0 & \underline{A}_{ki} & 0 & \underline{Y}_{2i}\end{bmatrix} < 0. \tag{23}$$

令

$$S_{11ik} = (A_{ik} - B_{ik}K_{ik})^{\mathrm{T}}P_{1i}(A_{ik} - B_{ik}K_{ik}) + \sum_{j=1,j\neq i}^{N}A_{ijk}^{\mathrm{T}}P_{1j}A_{ijk} - \frac{1}{N}P_{1i} < 0, \tag{24}$$

记 $X_{1i} = P_{1i}^{-1}$, $\underline{X}_{1i} = \mathrm{diag}[X_{1i}^{-1}, \cdots, X_{1i-1}^{-1}, X_{1i-1}^{-1}, \cdots, -X_{1N}^{-1}]$, 则不等式(24)等价于

$$\begin{bmatrix} -\frac{1}{N}P_{1i} & * & * \\ A_{ik} - B_{ik}K_{ik} & -X_{1i} & 0 \\ \underline{A}_{ik} & 0 & -\underline{X}_{1i}\end{bmatrix} < 0. \tag{25}$$

在不等式(25)左右同时乘以 $\mathrm{diag}[P_{1i}^{-1}\ \ I \cdots I]$, 可得如下LMI:

$$\begin{bmatrix} -\frac{1}{N}X_{1i} & * & * \\ A_{ik} - B_{ik}M_{ik} & -X_{1i} & 0 \\ \underline{A}_{ik} & 0 & -\underline{X}_{1i}\end{bmatrix} < 0. \tag{26}$$

因为式(26)为标准的LMI, 所以可求出 P_{1i} 和 K_{ik}. 记 $N_{ik} = L_{ik}P_{2i}^{-1}$, 将 P_{1i} 和 K_{ik} 代入式(23), 并在该式左右同时乘以矩阵 $\mathrm{diag}[I\ \ P_{2i}^{-1}\ \ I \cdots I]$, 得

$$\begin{bmatrix} S_{11ik} & * & * & * \\ S_{12ik} & -\frac{1}{N}P_{2i} & 0 & 0 \\ 0 & K_{ik}^{\mathrm{T}}B_{ik}^{\mathrm{T}} & -P_{1i}^{-1} & 0 \\ 0 & \underline{A}_{ki} & 0 & \underline{Y}_{2i}\end{bmatrix} < 0. \tag{27}$$

解LMI(27), 可得到 P_{2i} 和 L_{ik}. 由

$$(\hat{G}_{ikm}^{\mathrm{T}}\hat{P}_i\hat{G}_{ikm} + \hat{G}_{imk}^{\mathrm{T}}\hat{P}_i\hat{G}_{imk})/2 + \sum_{j=1,j\neq i}^{N}\hat{A}_{ijk}^{\mathrm{T}}\hat{P}_j\hat{A}_{ijk} - \frac{1}{N}\hat{P}_i < 0,$$

可得

$$\begin{bmatrix} S_{11ikm} & S_{12ikm} \\ S_{12ikm}^{\mathrm{T}} & S_{22ikm} \end{bmatrix} < 0 \tag{28}$$

其中

$$\begin{aligned} S_{11ikm} = {} & \frac{1}{2}(A_{ik} - B_{ik}K_{ik})^{\mathrm{T}} P_{1i}(A_{ik} - B_{ik}K_{ik}) + \\ & \frac{1}{2}(A_{ik} - B_{ik}K_{im})^{\mathrm{T}} P_{1i}(A_{ik} - B_{ik}K_{im}) + \\ & \sum_{j=1, j\neq i}^{N} A_{ijk}^{\mathrm{T}} P_{1j} A_{ijk} - \frac{1}{N} P_{1i}, \\ S_{12ikm} = {} & \frac{1}{2}(A_{ik} - B_{ik}K_{ik})^{\mathrm{T}} P_{1i} B_{ik} K_{ik} + \\ & \frac{1}{2}(A_{ik} - B_{ik}K_{im})^{\mathrm{T}} P_{1i} B_{ik} K_{im}, \\ S_{22ikm} = {} & K_{ik}^{\mathrm{T}} B_{ik}^{\mathrm{T}} P_{1i} B_{ik} K_{ik} + K_{im}^{\mathrm{T}} B_{ik}^{\mathrm{T}} P_{1i} B_{ik} K_{mk} + \\ & \frac{1}{2}(A_{ik} - L_{ik}C_{ik})^{\mathrm{T}} P_{2i}(A_{ik} - L_{ik}C_{ik}) + \\ & \frac{1}{2}(A_{ik} - L_{im}C_{ik})^{\mathrm{T}} P_{2i}(A_{ik} - L_{im}C_{ik}) + \\ & \sum_{j=1, j\neq i}^{N} A_{ijk}^{\mathrm{T}} P_{2j} A_{ijk} - \frac{1}{N} P_{i2}. \end{aligned}$$

式(28) 等价于

$$\begin{bmatrix} S_{11ikm} & S_{12ikm} & * & * & * & * & * \\ S_{12ikm}^{\mathrm{T}} & -\frac{1}{N}P_{2i} & 0 & 0 & 0 & 0 & 0 \\ 0 & B_{ik}K_{ik} & -X_{1i} & 0 & 0 & 0 & 0 \\ 0 & B_{ik}K_{im} & 0 & -X_{1i} & 0 & 0 & 0 \\ 0 & A_{ik} - L_{ik}C_{ik} & 0 & 0 & -Y_{2i} & 0 & 0 \\ 0 & A_{ik} - L_{im}C_{ik} & 0 & 0 & 0 & -Y_{2i} & 0 \\ 0 & \underline{A}_{ki} & 0 & 0 & 0 & 0 & -Y_{2i} \end{bmatrix} < 0 \tag{29}$$

令 $S_{11ikm} < 0$, 经过适当的变换可得

$$\begin{bmatrix} -\frac{1}{N}Y_{2i} & * & * & * \\ A_{ik}X_i - B_{ik}M_{ik} & -\frac{1}{\sqrt{2}}X_i & 0 & 0 \\ A_{ik}X_i - B_{ik}M_{im} & 0 & -\frac{1}{\sqrt{2}}X_i & 0 \\ \underline{A}_{ik} & 0 & 0 & -Y_{2i} \end{bmatrix} < 0 \tag{30}$$

由式(30) 求出 Y_{2i} 和 K_{im}. 再将 Y_{2i} 和 K_{im} 代入式(29) 中, 并作变换 $\mathrm{diag}[I \quad P_{2i}^{-1} \quad I \cdots I]$, 得

$$\begin{bmatrix} S_{11ikm} & S_{12ikm} & * & * & * & * & * \\ S_{12ikm}^{\mathrm{T}} & -\frac{1}{N}Y_{2i} & 0 & 0 & 0 & 0 & 0 \\ 0 & B_{ik}K_{ik} & -X_{1i} & 0 & 0 & 0 & 0 \\ 0 & B_{ik}K_{im} & 0 & -X_{1i} & 0 & 0 & 0 \\ 0 & A_{ik}Y_{2i} - N_{ik}C_{ik} & 0 & 0 & -Y_{2i} & 0 & 0 \\ 0 & A_{ik}Y_{2i} - N_{im}C_{ik} & 0 & 0 & 0 & -Y_{2i} & 0 \\ 0 & \underline{A}_{ki}Y_{2i} & 0 & 0 & 0 & 0 & -Y_{2i} \end{bmatrix} < 0 \tag{31}$$

通过解LMI(31), 得到正定矩阵 P_{2i} 和观测增益矩阵 L_{im}.

5　仿真研究

考虑由如下两个子系统构成的非线性系统:

$$S_1: \begin{cases} x_{11}(t+1) = 0.06x_{11}(t) + 0.24x_{21}(t) + \\ \qquad u_1 + 0.03x_{12}^2 + 0.01x_{11}, \\ x_{21}(x+1) = 0.02x_{11}^2(t) + 0.29x_{21}(t) + \\ \qquad 7x_{11}(t) + 0.22x_{22}(t), \\ y_1(t) = x_{11}(t); \end{cases} \tag{32}$$

$$S_2: \begin{cases} x_{12}(t+1) = 0.23x_{12}^2(t) + 0.2x_{22}(t) + \\ \qquad u_2 + 7x_{12}(t) + 0.2x_{22}, \\ x_{22}(t+1) = 0.39\sin(x_{12}(t)) + u_2 + \\ \qquad 0.01x_{11}^2(t) + 0.05x_{21}(t), \\ y_1(t) = x_{12}(t). \end{cases} \tag{33}$$

这里假设 $x_{11}(t)$ 和 $x_{12}(t)$ 是可测变量, $x_{21}(t)$ 和 $x_{22}(t)$ 是不可测变量, 于是建立如下模糊 T-S 模型来逼近非线性互联系统(32) 和(33).

模糊规则如下:

Rule 1: If $x_{11}(t)$ is about $\pi/3$ and $x_{12}(t)$ is about $\pi/3$,

Then $x_i(t+1) = A_{i1}x_i(t) + B_{i1}u(t) + \sum_{j=1, j\neq 1}^{2} A_{ji1}x_j(t)$,

$y_i(t+1) = C_{i1}x_i(t)$;

Rule 2: If $x_{11}(t)$ is about $\pi/3$ and $x_{12}(t)$ is about 0,

Then $x_i(t+1) = A_{i2}x_i(t) + B_{i2}u(t) +$

$$\sum_{j=1,j\neq i}^{2} A_{ji2}x_j(t),$$
$$y_i(t+1) = C_{i2}x_i(t);$$

Rule 3: If $x_{11}(t)$ is about $\pi/3$ and
$x_{12}(t)$ is about $-\pi/3$,
Then $x_i(t+1) = A_{i3}x_i(t) + B_{i3}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji3}x_j(t)$,
$y_i(t+1) = C_{i3}x_i(t)$;

Rule 4: If $x_{11}(t)$ is about 0 and
$x_{12}(t)$ is about $-\pi/3$,
Then $x_i(t+1) = A_{i4}x_i(t) + B_{i4}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji4}x_j(t)$,
$y_i(t+1) = C_{i4}x_i(t)$;

Rule 5: If $x_{11}(t)$ is about 0 and
$x_{12}(t)$ is about 0,
Then $x_i(t+1) = A_{i5}x_i(t) + B_{i5}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji5}x_j(t)$,
$y_i(t+1) = C_{i5}x_i(t)$;

Rule 6: If $x_{11}(t)$ is about 0 and
$x_{12}(t)$ is about $-\pi/3$,
Then $x_i(t+1) = A_{i6}x_i(t) + B_{i6}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji6}x_j(t)$,
$y_i(t+1) = C_{i6}x_i(t)$;

Rule 7: If $x_{11}(t)$ is about $-\pi/3$ and
$x_{12}(t)$ is about $\pi/3$,
Then $x_i(t+1) = A_{i7}x_i(t) + B_{i7}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji7}x_j(t)$,
$y_i(t+1) = C_{i7}x_i(t)$;

Rule 8: If $x_{11}(t)$ is about 0 and
$x_{12}(t)$ is about $-\pi/3$,
Then $x_i(t+1) = A_{i8}x_i(t) + B_{i8}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji8}x_j(t)$,
$y_i(t+1) = C_{i8}x_i(t)$;

Rule 9: If $x_{11}(t)$ is about $-\pi/3$ and
$x_{12}(t)$ is about $-\pi/3$,
Then $x_i(t+1) = A_{i9}x_i(t) + B_{i9}u(t) + \sum_{j=1,j\neq i}^{2} A_{ji9}x_j(t)$,
$y_i(t+1) = C_{i9}x_i(t)$.

其中

$$A_{11} = A_{12} = A_{13} = \begin{bmatrix} 0.06 & 0.24 \\ 0.0209 & 0.19 \end{bmatrix},$$
$$A_{14} = A_{15} = A_{16} = \begin{bmatrix} 0.06 & 0.24 \\ 0 & 0.19 \end{bmatrix},$$
$$A_{17} = A_{18} = A_{19} = \begin{bmatrix} 0.06 & 0.24 \\ -0.0209 & 0.19 \end{bmatrix},$$
$$A_{121} = A_{124} = A_{127} = \begin{bmatrix} 0.0314 & 0.01 \\ 0 & 0.02 \end{bmatrix},$$
$$A_{122} = A_{125} = A_{128} = \begin{bmatrix} 0 & 0.01 \\ 0 & 0.02 \end{bmatrix},$$
$$A_{123} = A_{126} = A_{129} = \begin{bmatrix} -0.0314 & 0.01 \\ 0 & 0.02 \end{bmatrix},$$
$$A_{21} = A_{24} = A_{27} = \begin{bmatrix} 0.2409 & 0.2 \\ 0.5706 & 0 \end{bmatrix},$$
$$A_{22} = A_{26} = A_{28} = \begin{bmatrix} 0 & 0.2 \\ 0.39 & 0 \end{bmatrix},$$
$$A_{23} = A_{25} = A_{29} = \begin{bmatrix} -0.2409 & 0.2 \\ 0.5706 & 0 \end{bmatrix},$$
$$A_{211} = A_{212} = A_{213} = \begin{bmatrix} 0.01 & 0 \\ 0.1047 & 0.05 \end{bmatrix},$$
$$A_{214} = A_{215} = A_{216} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.05 \end{bmatrix},$$
$$A_{217} = A_{218} = A_{219} = \begin{bmatrix} 0.01 & 0 \\ -0.1047 & 0.05 \end{bmatrix},$$
$$B_{1i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_{2i} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$C_{ik} = [1 \quad 0], i = 1, 2, \cdots, 9, k = 1, 2$$

通过解LMI(25),(27),(30)和(31),得到反馈控制器和增益矩阵为

$$K_{i1} = [-0.06 \quad -0.2399],$$
$$K_{i2} = [-0.1421 \quad -0.134], i = 1, 2, \cdots, 9;$$
$$L_{11} = L_{21} = L_{31} = \begin{bmatrix} 0.060346 \\ 0.020965 \end{bmatrix},$$
$$L_{41} = L_{51} = L_{61} = \begin{bmatrix} 0.060339 \\ 0 \end{bmatrix},$$
$$L_{71} = L_{81} = L_{91} = \begin{bmatrix} 0.060331 \\ -0.020935 \end{bmatrix},$$
$$L_{12} = L_{42} = L_{72} = \begin{bmatrix} 0.24175 \\ 0.57131 \end{bmatrix},$$
$$L_{22} = L_{52} = L_{82} = \begin{bmatrix} 0.00089 \\ 0.39071 \end{bmatrix},$$
$$L_{32} = L_{62} = L_{92} = \begin{bmatrix} -0.23996 \\ 0.57131 \end{bmatrix}.$$

如果选取初始条件为

$$x_1(0) = [2 \quad -3]^T, x_2(0) = [1.5 \quad -1]^T,$$
$$\hat{x}_1(0) = [1 \quad -2]^T, \hat{x}_2(0) = [1 \quad 0]^T,$$

则各个子系统在控制器用下的闭环状态响应曲线及控制输入曲线分别如图 1 和图 2 所示. 图中的采样周期 $T = 0.0001$ s.

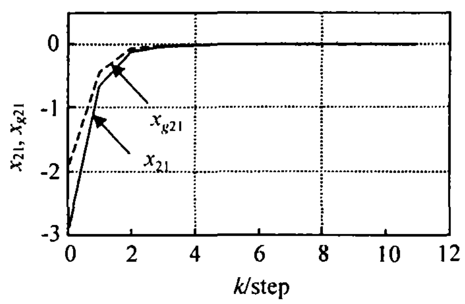

图 1　x_{21} 状态响应及其估计曲线

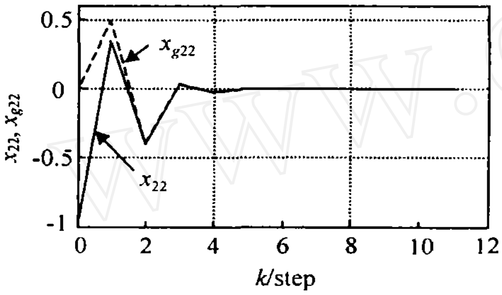

图 2　x_{22} 状态响应及其估计曲线

6　结　　语

本文针对一类状态可测和不可测离散不确定非线性互联系统, 给出了模糊状态反馈和基于观测器的输出反馈分散控制设计的方法. 设计中, 首先采用模糊T-S 模型对不确定非线性互联系统进行模糊建模; 然后, 应用并行分布补偿算法(PDC) 给出模糊状态反馈和基于观测器的输出反馈分散控制设计. 同时, 基于线性矩阵不等式和李亚普诺夫方法, 给出了模糊闭环系统的稳定性分析. 仿真结果验证了该方法的有效性.

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