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基于LMI的一类中立型延迟神经网络的全局渐近稳定判据

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摘 要: 针对一类能够由中立型变延迟非线性微分方程描述的神经网络模型, 给出了全局渐近稳定的不依赖于时间延迟的充分条件. 所得到的稳定判据不仅考虑了神经元的激励和抑制对网络的影响, 而且易于验证. 仿真示例验证了所得结论的有效性.

关键词: 延迟神经网络; 中立型; 全局渐近稳定; 线性矩阵不等式; 时变延迟
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Global Asymptotical Stability Criteria for a Class of Delayed Neural Network Model of Neutral Type Based on LMI

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Abstract: A sufficient condition guaranteeing the global asymptotical stability of the equilibrium point is derived for a class of neural network models with variable delay and neutral type delay. The stability criterion not only eliminates the differences between excitatory and inhibitory effects on the neural networks, but also can be conveniently checked. Numerical example shows the effectiveness of the obtained results.

Key words: Delayed neural networks; Neutral type; Global asymptotical stability; Linear matrix inequality (LMI); Time varying delay

1 引 言

在神经网络的电子线路实现中, 由于运算放大器的有限切换速度, 时间延迟的存在是不可避免的. 时间延迟会影响神经网络的动态特性, 如产生震荡或不稳定等[1]. 于是, 人们对不同延迟类型的神经网络的动态特性进行了广泛研究. 文献[2~4]针对定常延迟的神经网络给出了渐近稳定和指数稳定的充分条件; 文献[4~6]针对时变延时神经网络分别给出了全局指数稳定和渐近稳定的充分条件; 文献[7]针对分布延迟神经网络给出了全局指数稳定条件. 此外, 由于神经网络是一个复杂的非线性动力系统, 利用延迟可产生某些动态行为, 如混沌和周期解等[8].

在神经网络VLSI实现中, 用来产生延迟的元件主要有延迟传输线和部分元件等效电路(PEEC)两类, 而由PEEC构成的电路将会产生中立型延迟[9]. 文献[9]针对一类中立型线性定常延迟微分方程模型, 给出了零解渐近稳定的条件. 本文则针对一类能够由中立型非线性变延迟微分方程描述的神经网络, 基于LMI技术, 对其全局渐近稳定性进行了分析, 所得结果消除了神经元的激励和抑制对网络的影响, 且易于验证.

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2 问题描述

考虑如下延迟神经网络:

$$\begin{aligned}\dot{u}_i(t) = &-a_i u_i(t) + \sum_{j=1}^{n} w_{0ij} g_j(u_j(t)) + \\ &\sum_{j=1}^{n} w_{1ij} g_j(u_j(t-\tau_j(t))) + \\ &\sum_{j=1}^{n} d_{ij}\dot{u}_j(t-\tau_j(t)) + U_i,\\ u_i(t) = &\ \phi_i(t),\\ &-\tau_i(t) \le t \le 0,\ i = 1,\cdots,n,\end{aligned} \tag{1}$$

其中: $u_i(t)$ 表示第 i 个神经元的状态; $a_i > 0$ 表示自反馈系数; w_{0ij}, w_{1ij} 和 d_{ij} 表示已知的连接矩阵系数; $\tau_j(t) \ge 0$ 表示时变延迟; $g_j(\cdot)$ 表示神经元激励函数; U_i 表示外界常值输入, $\phi_i(t)$ 表示初始条件, $i, j = 1,\cdots,n$.

可将系统(1) 写成如下矩阵形式:

$$\begin{aligned}\dot{u}(t) = &-Au(t) + W_0 g(u(t)) + \\ &W_1 g(u(t-\tau(t))) + \\ &D\dot{u}(t-\tau(t)) + U,\\ u(\theta) = &\ \phi(\theta),\ -\max(\tau_i(t)) \le \theta \le 0,\end{aligned} \tag{2}$$

其中

$$\begin{aligned}&u(t) = [u_1(t)\ \cdots\ u_n(t)]^{\mathrm{T}},\\ &A = \mathrm{diag}(a_1\ \cdots\ a_n),\ W_0 = [w_{0ij}]_{n\times n},\\ &g(u(t)) = [g_1(u_1(t))\ \cdots\ g_n(u_n(t))]^{\mathrm{T}},\\ &W_i = [w_{1ij}]_{n\times n},\ \tau(t) = [\tau_1(t)\ \cdots\ \tau_n(t)]^{\mathrm{T}},\\ &g(u(t-\tau(t))) = [g_1(u_1(t-\tau_1(t)))\cdots,\\ &\qquad g_n(u_n(t-\tau_n(t)))]^{\mathrm{T}},\\ &U = [U_1\ \cdots\ U_n]^{\mathrm{T}},\end{aligned}$$

$\phi(\theta)$ 表示连续向量值函数.

假设 1 激励函数满足如下条件:

$$0 \le \frac{g_i(s) - g_i(\omega)}{s - \omega} \le \sigma_i, \tag{3}$$

其中: $s, \omega \in \mathbf{R}$, $s \ne \omega$, $\sigma_i > 0$, $i = 1,\cdots,n$. 令 $\Delta = \mathrm{diag}(\sigma_1\ \cdots\ \sigma_n)$.

令 $u^* = [u_1^*\ \cdots\ u_n^*]^{\mathrm{T}}$ 为系统(1) 的一个平衡点, 则通过坐标变换 $x(t) = u(t) - u^*$ 后, 式(2) 变为

$$\begin{aligned}\dot{x}(t) = &-Ax(t) + W_0 f(x(t)) + \\ &W_1 f(x(t-\tau(t))) + D\dot{x}(t-\tau(t)),\end{aligned} \tag{4}$$

其中

$$\begin{aligned}&f(x(t)) = [f_1(x_1(t))\ \cdots\ f_n(x_n(t))]^{\mathrm{T}},\\ &f_i(x_i(t)) = g_i(x_i(t) + u_i^*) - g_i(u_i^*),\end{aligned}$$

显然, $f_i(0) = 0$, $i = 1,\cdots,n$, 且 $f_i(x_i(t))$ 满足假设 1. 若原点 $x = 0$ 是系统(4) 的唯一平衡点, 则 u^* 也是系统(1) 或(2) 的唯一平衡点.

3 全局渐近稳定分析

定理 1 考虑延迟系统(1) 或(2), $\tau_i(t) > 0$, $\dot{\tau}_i(t) < 0$, $1 < \eta = \min(1 - \dot{\tau}_i(t))$, $i = 1,\cdots,n$, 如果 LMI

$$\begin{bmatrix}\Omega & K_{12} & PW_0 & PW_1 & X\\ K_{12}^{\mathrm{T}} & K_{22} & 0 & 0 & 0\\ W_0^{\mathrm{T}}P & 0 & -M_0 & 0 & 0\\ W_1^{\mathrm{T}}P & 0 & 0 & -M_1 & 0\\ X^{\mathrm{T}} & 0 & 0 & 0 & Y\end{bmatrix} < 0 \tag{5}$$

存在对称正定矩阵 P, 正定对角矩阵 M_0, M_1 和 Q, 则延迟系统(1) 或(2) 的平衡点是全局渐近稳定的, 其中

$$\begin{aligned}&X = PW_1(1 - \sqrt{\eta}),\ Y = -(\eta - 1)M_1,\\ &\Omega = -PA - AP + (PW_0\Delta + \Delta W_0^{\mathrm{T}}P)/2 + \\ &\quad \Delta M_0\Delta/4 + Q,\\ &K_{12} = -PAD + PW_0\Delta D/2 + \Delta M_0\Delta D/4 + \\ &\quad \sqrt{\eta}PW_1\Delta/2 + QD,\\ &K_{22} = -\eta Q + D^{\mathrm{T}}\Delta M_0\Delta D/4 + D^{\mathrm{T}}QD + \\ &\quad \eta\Delta M_1\Delta/4.\end{aligned}$$

证明 首先证明, 在条件(5) 成立的前提下, 系统(4) 具有唯一平衡点. 这里采用反证法. 考虑系统(4) 的平衡点方程

$$-Av + W_0 f(v) + W_1 f(v) = 0, \tag{6}$$

为简洁起见, 这里略去了对时间 t 的表示. 假设 $v = [v_1\ \cdots\ v_n]^{\mathrm{T}}$ 是延迟系统(4) 的一个平衡点, 显然, 若 $f(v) = 0$, 则 $v = 0$. 现令 $v \ne 0$. 从式(5) 可知, $D^{\mathrm{T}}QD - \eta Q < 0$, 即 $\|D\| < \sqrt{\eta}$. 这样 $I - D/\sqrt{\eta}$ 可逆. 令 $z = y/\sqrt{\eta} = (I - D/\sqrt{\eta})v$, 则 $z \ne 0$, $y \ne 0$. 在式(6) 两侧同乘 $2z^{\mathrm{T}}P$, 则

$$-2z^{\mathrm{T}}PAv + 2z^{\mathrm{T}}PW_0 f(v) + 2z^{\mathrm{T}}PW_1 f(v) = 0. \tag{7}$$

因为激励函数 $0 \le f_i(v_i)/v_i \le \sigma_i$, $\forall v_i \ne 0$, 令 $F_i(v_i) = f_i(v_i)/v_i$, 则

$$-\frac{1}{2}\sigma_i \le F_i(v_i) - \frac{1}{2}\sigma_i \le \frac{1}{2}\sigma_i.$$

当 $v_i = 0$ 时, 必有 $f_i(v_i) = 0$, 则令 $F_i(v_i) = 0$, 这样, $-\frac{1}{2}\Delta \le F(v) - \frac{1}{2}\Delta \le \frac{1}{2}\Delta$, 其中 $F(v) = \mathrm{diag}(F_1(v_1)\ \cdots\ F_n(v_n))$. 因此, 式(7) 等价于

$$\begin{aligned}&-2z^{\mathrm{T}}PA\left(z + \frac{D}{\sqrt{\eta}}v\right) + 2z^{\mathrm{T}}PW_0\left(F(v) - \frac{1}{2}\Delta\right)v + \\ &z^{\mathrm{T}}PW_0\Delta v + 2z^{\mathrm{T}}PW_1\left(F(v) - \frac{1}{2}\Delta\right)v + \\ &z^{\mathrm{T}}PW_1\Delta v = 0.\end{aligned} \tag{8}$$

因为对于任意适维向量 ζ 和 ξ, 适维矩阵 S 和

N, 以及对于任意正定矩阵 H, 下式成立[10]:

$$2\zeta^T SN\xi \le \zeta^T SH^{-1}S^T\zeta + \xi^T N^T HN\xi, \tag{9}$$

则

$$2z^T PW_0\Big(F(v) - \frac{1}{2}\Delta\Big)v \le z^T PW_0M_0^{-1}W_0^T Pz + \frac{1}{4}v^T\Delta M_0\Delta v, \tag{10}$$

$$2z^T PW_1\Big(F(v) - \frac{1}{2}\Delta\Big)v \le z^T PW_1M_1^{-1}W_1^T Pz + \frac{1}{4}v^T\Delta M_1\Delta v. \tag{11}$$

将式(10)和(11)代入式(8), 得

$$0 \le \frac{1}{\eta}\Big\{y^T\Big(PW_0M_0^{-1}W_0^TP + PW_1M_1^{-1}W_1^TP - 2PA + Q + \frac{1}{4}\Delta M_0\Delta + PW_0\Delta\Big)y + 2y^T\Big(-PAD + \frac{1}{2}PW_0\Delta D + \frac{1}{4}\Delta M_0\Delta D + \frac{1}{2}\sqrt{\eta}PW_1\Delta + QD\Big)v + v^T\Big(\frac{1}{4}\eta\Delta M_1\Delta - \eta Q + D^TQD + \frac{1}{4}D^T\Delta M_0\Delta D\Big)v + (1-\sqrt{\eta})^2y^TPW_1((\eta-1)M_1)^{-1}W_1^TPy\Big\}. \tag{12}$$

因为从式(5)可知,

$$\frac{1}{4}\eta\Delta M_1\Delta - \eta Q + D^TQD + \frac{1}{4}D^T\Delta M_0\Delta D < 0,$$

将式(12)中的第2项和第3项应用不等式(9), 再整理(12)式得

$$0 \le \frac{1}{\eta}\Big\{y^T\Big(PW_0M_0^{-1}W_0^TP + PW_1M_1^{-1}W_1^TP - 2PA + Q + \frac{1}{4}\Delta M_0\Delta + PW_0\Delta + \Big(-PAD + \frac{1}{2}PW_0\Delta D + \frac{1}{4}\Delta M_0\Delta D + \frac{1}{2}PW_1\Delta + QD\Big)\Big(\eta Q - \frac{1}{4}\Delta M_1\Delta - D^TQD - \frac{1}{4}D^T\Delta M_0\Delta D\Big)^{-1}\Big(-PAD + \frac{1}{2}PW_0\Delta D + \frac{1}{4}\Delta M_0\Delta D + \frac{1}{2}PW_1\Delta + QD\Big)^T + (1-\sqrt{\eta})^2PW_1((\eta-1)M_1)^{-1}W_1^TP\Big)\Big\}y. \tag{13}$$

根据 Schur 补引理, 从式(5)可知, 如果 $y \ne 0$, 则

$$y^T\Big\{PW_0M_0^{-1}W_0^TP + PW_1M_1^{-1}W_1^TP - 2PA + Q + \frac{1}{4}\Delta M_0\Delta + PW_0\Delta + \Big(-PAD + \frac{1}{2}PW_0\Delta D + \frac{1}{4}\Delta M_0\Delta D + \frac{1}{2}PW_1\Delta + QD\Big)\Big(\eta Q - \frac{1}{4}\Delta M_1\Delta - D^TQD - \frac{1}{4}D^T\Delta M_0\Delta D\Big)^{-1}\Big(-PAD + \frac{1}{2}PW_0\Delta D + \frac{1}{4}\Delta M_0\Delta D + \frac{1}{2}PW_1\Delta + QD\Big)^T + (1-\sqrt{\eta})^2PW_1((\eta-1)M_1)^{-1}W_1^TP\Big\}y < 0. \tag{14}$$

显然, 式(13)与(14)矛盾, 这意味着 $v = 0$, 即原点是延迟系统(4)的唯一平衡点, 进而延迟系统(1)或(2)具有唯一平衡点.

其次, 证明延迟系统(4)的唯一平衡点 $x = 0$ 是全局渐近稳定的. 为此, 定义如下 Lyapunov-Krasovskii 泛函:

$$V(x(t)) = (x - Dx(t-\tau(t)))^TP(x - Dx(t-\tau(t))) + \sum_{i=1}^{n}\int_{\tau_i(t)} q_ix_i^2(s)\,\mathrm{d}s, \tag{15}$$

其中 $q_i > 0$. 则沿着式(4)的轨迹求导得

$$\dot V(x(t)) \le 2(x - Dx(t-\tau(t)))^T \times P(\dot x - D\dot x(t-\tau(t))) + x^T(t)Qx(t) - \min(1-\dot\tau_i(t))x^T(t-\tau(t))Qx(t-\tau(t)) = -2(x(t) - Dx(t-\tau(t)))^TPA(x(t) - Dx(t-\tau(t))) - 2(x(t) - Dx(t-\tau(t)))^TPADx(t-\tau(t)) + 2(x(t) - Dx(t-\tau(t)))^TP(W_0f(x(t)) + W_1f(x(t-\tau(t)))) + x^T(t)Qx(t) - \eta x^T(t-\tau(t))Qx(t-\tau(t)), \tag{16}$$

其中 $Q = \mathrm{diag}(q_1 \cdots q_n)$. 定义

$$F(x(t)) = \mathrm{diag}(F_1(x_1(t)) \quad \cdots \quad F_n(x_n(t))),$$

其中, $F_i(x_i(t)) = f_i(x_i(t))/x_i(t)$, 与平衡点唯一性证明部分类似, 有 $-\frac{1}{2}\Delta \le F(x(t)) - \frac{1}{2}\Delta \le \frac{1}{2}\Delta$. 这样,

$$2(x(t) - Dx(t-\tau(t)))^TPW_0f(x(t)) = 2(x(t) - Dx(t-\tau(t)))^TPW_0\Big(F(x(t)) - \frac{1}{2}\Delta\Big)x(t) + (x(t) - Dx(t-\tau(t)))^TPW_0\Delta x(t), \tag{17}$$

$$2(x(t) - Dx(t-\tau(t)))^TPW_1f(x(t-\tau(t))) = 2(x(t) - Dx(t-\tau(t)))^TPW_1\Big(F(x(t-\tau(t))) - \frac{1}{2}\Delta\Big)x(t-\tau(t)) + (x(t) - Dx(t-\tau(t)))^TPW_1\Delta x(t-\tau(t)). \tag{18}$$

将式(17) 和(18) 代入(16) 后, 有

$$\begin{aligned}&\dot V(x(t)) \le \\&-2(x(t)-Dx(t-\tau(t)))^{\mathrm T}PA(x(t)-Dx(t-\tau(t)))-2(x(t)-\\&Dx(t-\tau(t)))^{\mathrm T}PADx(t-\tau(t))+2(x(t)-\\&Dx(t-\tau(t)))^{\mathrm T}PW_0\Big(F(x(t))-\frac{1}{2}\Delta\Big)x(t)+\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_0\Delta x(t)+2(x(t)-\\&Dx(t-\tau(t)))^{\mathrm T}PW_1\Big(F(x(t-\tau(t)))-\\&\frac{1}{2}\Delta\Big)x(t-\tau(t))+\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_1\Delta x(t-\tau(t))+\\&x^{\mathrm T}(t)Qx(t)-\eta x^{\mathrm T}(t-\tau(t))Qx(t-\tau(t)). \qquad (19)\end{aligned}$$

考虑不等式(9) 成立, 则

$$\begin{aligned}&2(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_0\times\\&\Big(F(x(t))-\frac{1}{2}\Delta\Big)x(t)\le\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_0M_0^{-1}W_0^{\mathrm T}P(x(t)-\\&Dx(t-\tau(t)))+x^{\mathrm T}(t)\Big(F(x(t))-\\&\frac{1}{2}\Delta\Big)^{\mathrm T}M_0\Big(F(x(t))-\frac{1}{2}\Delta\Big)x(t)\le\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_0M_0^{-1}W_0^{\mathrm T}P(x(t)-\\&Dx(t-\tau(t)))+x^{\mathrm T}(t)\frac{1}{2}\Delta M_0\frac{1}{2}\Delta x(t). \qquad (20)\end{aligned}$$

同理

$$\begin{aligned}&2(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_1\Big(F(x(t-\\&\tau(t)))-\frac{1}{2}\Delta\Big)x(t-\tau(t))\le\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_1\times\\&M_1^{-1}W_1^{\mathrm T}P(x(t)-Dx(t-\tau(t)))+\\&x^{\mathrm T}(t-\tau(t))\frac{1}{2}\Delta M_1\frac{1}{2}\Delta x(t-\tau(t)). \qquad (21)\end{aligned}$$

将式(20) 和(21) 代入(19), 得

$$\begin{aligned}&\dot V(x(t))\le\\&-2(x(t)-Dx(t-\tau(t)))^{\mathrm T}PA(x(t)-\\&Dx(t-\tau(t)))-2(x(t)-\\&Dx(t-\tau(t)))^{\mathrm T}PADx(t-\tau(t))+\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_0M_0^{-1}W_0^{\mathrm T}P(x(t)-\\&Dx(t-\tau(t)))+x^{\mathrm T}(t)\frac{1}{4}\Delta M_0\Delta x(t)+\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_0\Delta x(t)+\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_1M_1^{-1}W_1^{\mathrm T}P(\Delta x(t)-\\&Dx(t-\tau(t)))+x^{\mathrm T}(t-\tau(t))\frac{1}{4}\Delta M_1\Delta x(t-\\&\tau(t))+(x(t)-Dx(t-\tau(t)))^{\mathrm T}PW_1\Delta x(t-\\&\tau(t))+(x(t)-Dx(t-\tau(t))+Dx(t-\\&\tau(t)))Q(x(t)-Dx(t-\tau(t))+\\&Dx(t-\tau(t)))-\eta x^{\mathrm T}(t-\tau(t))Qx(t-\tau(t))=\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}\{-2PA+\\&PW_0M_0^{-1}W_0^{\mathrm T}P+PW_1M_1^{-1}W_1^{\mathrm T}P+\\&\frac{1}{4}\Delta M_0\Delta+PW_0\Delta+Q\}(x(t)-\\&Dx(t-\tau(t)))+2(x(t)-Dx(t-\\&\tau(t)))^{\mathrm T}\{-PAD+QD+\frac{1}{2}PW_0\Delta D+\\&\frac{1}{4}\Delta M_0\Delta D+\frac{1}{2}\sqrt{\eta}PW_1\Delta\}x(t-\\&\tau(t))+x^{\mathrm T}(t-\tau(t))\Big\{\frac{1}{4}\eta\Delta M_1\Delta+\\&\frac{1}{4}D^{\mathrm T}\Delta M_0\Delta D+D^{\mathrm T}QD-\eta Q\Big\}x(t-\tau(t))+\\&2(x(t)-Dx(t-\tau(t)))^{\mathrm T}\Big(\frac{1}{2}PW_1\Delta-\\&\frac{1}{2}\sqrt{\eta}PW_1\Delta\Big)x(t-\tau(t))+x^{\mathrm T}(t-\tau(t))\times\\&\Big(\frac{1}{4}\Delta M_1\Delta-\frac{1}{4}\eta\Delta M_1\Delta\Big)x(t-\tau(t)). \qquad (22)\end{aligned}$$

因为 $\eta > 1$, 则 $\frac{1}{4}\eta\Delta M_1\Delta - \frac{1}{4}\Delta M_1\Delta$ 为正定矩阵. 再考虑不等式(9), 则

$$\begin{aligned}&2(x(t)-Dx(t-\tau(t)))^{\mathrm T}\Big(\frac{1}{2}PW_1\Delta-\\&\frac{1}{2}\sqrt{\eta}PW_1\Delta\Big)x(t-\tau(t))-x^{\mathrm T}(t-\tau(t))\times\\&\Big(\frac{1}{4}\eta\Delta M_1\Delta-\frac{1}{4}\Delta M_1\Delta\Big)x(t-\tau(t))\le\\&(x(t)-Dx(t-\tau(t)))^{\mathrm T}(1-\sqrt{\eta})^2PW_1((\eta-\\&1)M_1)^{-1}(PW_1)^{\mathrm T}(x(t)-Dx(t-\tau(t))). \qquad (23)\end{aligned}$$

将式(23) 代入(22) 中, 则

$$\begin{aligned}&\dot V(x(t))\le\\&[(x(t)-Dx(t-\tau(t)))^{\mathrm T}\quad x^{\mathrm T}(t-\tau(t))]M\times\\&[(x(t)-Dx(t-\tau(t)))^{\mathrm T}\quad x^{\mathrm T}(t-\tau(t))]^{\mathrm T}, \qquad (24)\end{aligned}$$

其中

$$M=\begin{bmatrix}K_{11} & K_{12}\\ K_{12}^{\mathrm T} & K_{22}\end{bmatrix}, \qquad (25)$$

$$\begin{aligned}K_{11}=\;&\Omega+\sum_{i=0}^{1}PW_iM_i^{-1}W_i^{\mathrm T}P+\\&(1-\sqrt{\eta})^2PW_1((\eta-1)M_1)^{-1}W_1^{\mathrm T}P,\\K_{12}=\;&-PAD+PW_0\Delta D/2+\\&\Delta M_0\Delta D/4+\sqrt{\eta}PW_1\Delta/2+QD.\\K_{22}=\;&-\eta Q+D^{\mathrm T}\Delta M_0\Delta D/4+\\&D^{\mathrm T}QD+\eta\Delta M_1\Delta/4,\end{aligned}$$

如果 $M < 0$, 则 $\dot V(x(t)) < 0$, $\forall\,[x(t)-D(x(t-\tau(t)))\quad x(t-\tau(t))]\ne 0$.

根据Schur补性质, 可知式(25)等价于式(5). 这样就证明了如果$M < 0$, 则系统(4)是全局渐近稳定的. □

推论 1　考虑延迟系统(1)或(2), 且$\tau_i(t) = \tau_i \geq 0$, $i = 1, \cdots, n$, 如果LMI

$$\begin{bmatrix} \Omega & K_{12} & PW_0 & PW_1 \\ K_{12}^{\mathrm{T}} & K_{22} & 0 & 0 \\ W_0^{\mathrm{T}}P & 0 & -M_0 & 0 \\ W_1^{\mathrm{T}}P & 0 & 0 & -M_1 \end{bmatrix} < 0 \tag{26}$$

存在对称正定矩阵P, 正定对角矩阵M_0, M_1和Q, 则延迟系统(1)或(2)的平衡点是全局渐近稳定的, 其中

$$\Omega = -PA - AP + (PW_0\Delta + \Delta W_0^{\mathrm{T}}P)/2 + \Delta M_0\Delta/4 + Q,$$

$$K_{12} = -PAD + PW_0\Delta D/2 + \Delta M_0\Delta D/4 + PW_1\Delta/2 + QD,$$

$$K_{22} = -Q + D^{\mathrm{T}}\Delta M_0\Delta D/4 + D^{\mathrm{T}}QD + \Delta M_1\Delta/4.$$

注 1　如果$\tau_i(t) = d(t) \geq 0$, 则在定理1中的Q可放宽为对称正定矩阵, 而不必为对角矩阵.

注 2　如果在系统(2)中令$D = 0$, 则这类网络模型在大量文献中已被研究, 如文献[3～7]等. 此时, 只要在定理1的证明中令$D = 0$即可得到与定理1相类似的结果. 由此可见, 本文结果更具有一般性.

4　仿真示例

考虑系统(1)或(2), 其中$A = \mathrm{diag}(4.1\ \ 3.1\ \ 3.1)$, $\tau_j(t) = d > 0$为任意有界延迟, 激励函数

$$g_i(x_i(t)) = 0.5(|x_i(t) + 1| - |x_i(t) - 1|),$$

$$W_0 = \begin{bmatrix} -3 & 1/2 & 1/3 \\ 1/2 & 1 & 1/6 \\ 1 & 2 & -1 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 1 & 1/4 & 1/6 \\ 1/2 & 1 & 1/6 \\ 1/2 & 1/4 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.37 & 0.92 & -1.38 \\ 0.18 & 0.09 & -0.13 \\ 0.50 & 0.7 & -0.5 \end{bmatrix},$$

求解式(26)得到

$$P = \begin{bmatrix} 0.1672 & 0.0855 & -0.1052 \\ -0.0855 & 0.6727 & -0.1953 \\ -0.1052 & -0.1953 & 0.6738 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.6416 & 0.5779 & -0.5404 \\ 0.5779 & 1.8679 & -1.3182 \\ -0.5404 & -1.3182 & 1.9395 \end{bmatrix},$$

$$M_0 = \begin{bmatrix} 1.0515 & 0 & 0 \\ 0 & 2.3893 & 0 \\ 0 & 0 & 0.7586 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.7183 & 0 & 0 \\ 0 & 1.2052 & 0 \\ 0 & 0 & 0.8013 \end{bmatrix},$$

根据推论1可知, 神经网络是全局渐近稳定的. $[U_1, U_2, U_3] = [1, 2, 2]$时, 状态轨迹如图1所示. 平衡点为

$$[u_1^* \ \ u_2^* \ \ u_3^*] = [0.3056 \ \ 1.4258 \ \ 0.2285].$$

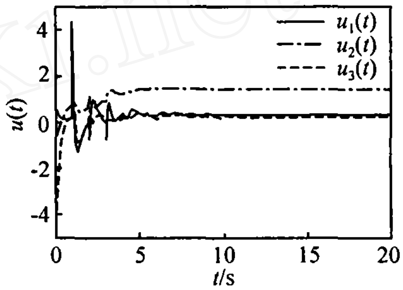

图1　状态响应轨迹

5　结　　论

针对一类能够由中立型变延迟非线性微分方程描述的神经网络模型, 给出了平衡点唯一性和不依赖于延迟的全局渐近稳定的充分条件. 该稳定判据易于验证, 并消除了神经元激励和抑制对网络的影响, 仿真验证了所得结论的有效性.

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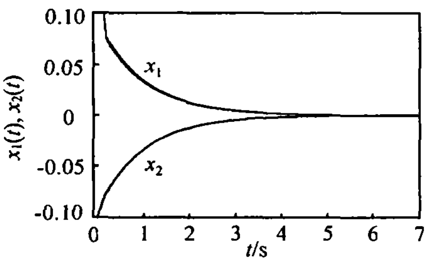

图 1　常规到达条件下 x_1, x_2 的运动

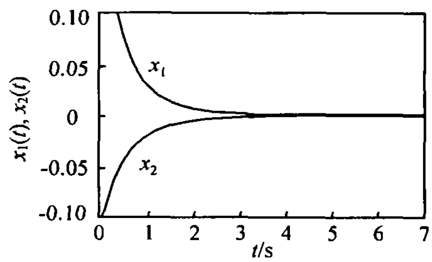

图 2　本文到达条件下 x_1, x_2 的运动

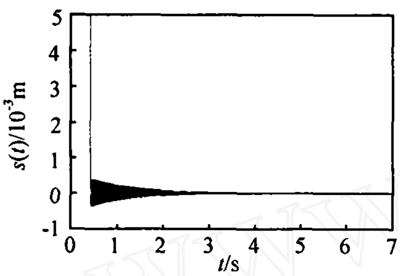

图 3　常规到达条件下 $s(t)$ 的运动轨线

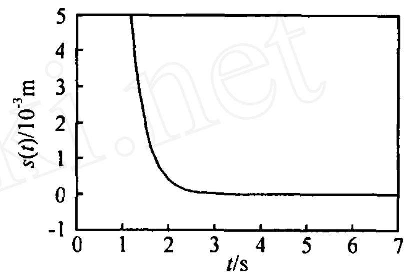

图 4　本文到达条件下 $s(t)$ 的运动轨线

明显的抖振; 而本文到达条件(3) 下, 图 2 和图 4 基本看不到抖振, 可见本文到达条件确实起到了抑制抖振的作用

6　结　　论

本文对多输入系统变结构控制提出了一个新的到达条件, 该条件的特点是, 既保留了常规到达条件的优点, 又能使穿越切换流形的速度为零, 从而能有效地抑制抖振, 改善控制系统的动态品质 另外给出了到达时间的计算公式, 分析了参数对快速性的影响, 并讨论了参数的物理实现, 为到达条件的有效性和可行性提供了理论依据

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