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一类Markov跳跃非线性系统的鲁棒自适应镇定

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摘 要: 研究一类具有Markov跳跃参数的随机非线性系统的鲁棒自适应镇定问题. 利用随机控制的Lyapunov设计方法, 对受Wiener噪声干扰的参数严格反馈形式的跳跃系统, 利用backstepping方法设计参数自适应律和控制律, 使得闭环系统状态在4阶矩意义下全局一致有界, 并能收敛到平衡点的任意小邻域内. 仿真结果表明了该设计方法的有效性.

关键词: 随机稳定性; Markov跳跃系统; 鲁棒自适应控制;
中图分类号: O232　　**文献标识码**: A

Robust Adaptive Stabilization for a Class of Markovian Jumping Nonlinear Systems

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Abstract: Robust adaptive control problems for a class of Markovian jumping nonlinear systems are investigated. The stochastic Lyapunov design method is applied for the jumping systems in the form of parametric-strict-feedback driven by Wiener noise. By means of the backstepping method, a parameter adaptive law and a control law are designed to ensure that the states of the closed-loop systems could be globally uniformly bounded in the sense of the 4th moment, and could be within the neighborhood of the equilibrium point as small as possible. The simulation result shows the validity of the method.

Key words: Stochastic stability; Markovian jumping systems; Robust adaptive control

1 引 言

Markov跳跃系统(简称跳跃系统)的概念最先由Krasovskii和Lidskii于20世纪60年代提出[1]. 由于这类模型在社会经济系统和制造系统等方面有着广泛应用, 引起了国内外学者的普遍关注, 但到目前为止, 有关Markov跳跃系统的研究大多局限于跳跃线性系统领域.

随着对跳跃系统研究的深入, 很多专家已着手对跳跃非线性系统领域的探索; 文献[2]探讨了跳跃非线性系统的可控性问题; 文献[3～5]研究了一般形式的跳跃非线性系统方程解的存在唯一性问题和系统的随机稳定性问题. 需要指出的是, 现有对跳跃非线性系统的研究大多局限于系统随机稳定性分析和基本概念的定义, 几乎未涉及控制问题, 特别是受噪声干扰的跳跃系统的控制问题, 国内外尚未有相关文献的报道.

本文针对具有Markov跳跃参数的一类随机非线性系统, 研究其在Wiener噪声干扰下的鲁棒自适应镇定问题, 采用噪声扰动抑制的控制机制, 实现了系统的镇定. 需要注意的是, 由于不确定随机扰动的存在, 难以实现系统渐近稳定, 但可以确保闭环系统状态在4阶矩意义下全局一致有界, 且可以在有限时间内达到平衡点的任意小邻域内.

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2 问题描述

考察参数严格反馈形式的Markov 跳跃系统

$$\begin{cases} dx_i = x_{i+1}dt + \varphi_i(\bar{x}_i, t, r(t))^T\theta^* dt + \\ \qquad g_i(\bar{x}_i, t, r(t))^T dw, \\ dx_n = udt + \varphi_n(x, t, r(t))^T\theta^* dt + \\ \qquad g_n(x, t, r(t))^T dw, \\ y = x_1, i = 1, 2, \cdots, n-1. \end{cases} \tag{1}$$

其中: $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ 是系统状态, 并记 $\bar{x}_i = (x_1, x_2, \cdots, x_i)^T$; $u \in R$ 为输入, $\theta^* \in R^p$ 是未知常参数; $\varphi_i(\bar{x}_i, t, r(t))$ 为 P 维光滑向量值函数, 满足 $\varphi_i(0, t, r(t)) = 0$; $g_i(\bar{x}_i, t, r(t))$ 是 h 维光滑向量值函数, 满足 $g_i(0, t, r(t)) = 0, i = 1, 2, \cdots, n$; y 为系统输出. 噪声干扰 w 为定义于概率空间 (Ω, F, P) 上的 h 维独立Wiener 过程, 记增量 dw 的协方差为 $\Delta\Delta^T dt$, 即均值

$$E\{dw \cdot dw^T\} = \Delta(t)\Delta(t)^T dt,$$

其中函数矩阵 $\Delta(t)$ 有界但不确定. $r(t)$ 为定义于概率空间 (Ω, F, P) 上的一个连续时间Markov 链, 在有限离散模态集 $S = \{1, 2, \cdots, N\}$ 中取值. Markov 过程 $r(t)$ 和噪声 ω 是相互独立的. 定义模态示性函数 $\Psi_{r(t)}$ 为: 对于 $\forall j \in S$ 有

$$\Psi_{r(t)j} = \begin{cases} 1, r(t) = j; \\ 0, r(t) \neq j. \end{cases} \tag{2}$$

$\Psi_{r(t)}$ 的微分表达式[1] 为

$$d\Psi_{r(t)} = \Pi\Psi_{r(t)}dt + dM_t. \tag{3}$$

其中: M_t 为定义在概率空间 (Ω, F, P) 上均方可积的连续时间鞅过程, $\Pi = [\pi_{kj}]$ 为Markov 过程的转移率矩阵. 模态间的转移概率为

$$P(r(t+dt) = j \mid r(t) = k) = \begin{cases} \pi_{kj}dt + o(dt), k \neq j; \\ 1 + \pi_{jj}dt + o(dt), k = j, \end{cases}$$

其中 $o(dt)$ 为 dt 的高阶无穷小. 文献[3] 给出了具有Markov 跳跃参数的一般形式随机非线性系统方程存在唯一解的充分条件, 因此在本文中, 若不作特别声明, 上述系统(1) 存在唯一的解.

为研究系统(1), 先对如下随机微分方程

$$dx = f(x, t, r(t))dt + g(x, t, r(t))dw$$

给出有关引理和记号, 式中 $x \in R^n$, Wiener 噪声 w 如在系统(1) 中所述, 函数 $f: R^n \times R_+ \times S \to R^n$ 和 $g: R^n \times R_+ \times S \to R^{n\times h}$ 为局部Lipschitz 连续, 且有 $f(0, t, r(t)) = 0, g(0, t, r(t)) = 0, \forall t \geq 0, \forall r(t) \in S$.

考察函数 $V(x, t, r(t)) \in C^2(R^n \times R_+ \times S)$, 则由广义 Ito 积分公式可得

$$V(x_{t_p}, t_p, r(t_p) = j) = V(x, t, r(t) = k) + \int^p \frac{\partial V(x, s, r(s))}{\partial s}ds + \int^p \frac{\partial V(x, s, r(s))}{\partial x}f(x, s, r(s))ds +$$
$$\int^p \frac{1}{2}\mathrm{Tr}\Big[\Delta^T g^T(x, s, r(s)) \frac{\partial^2 V(x, s, r(s))}{\partial x^2} \times g(x, s, r(s))\Delta\Big]ds + \int^p \frac{\partial V(x, s, r(s))}{\partial x}g(x, s, r(s))d\omega +$$
$$\int^p \sum_{j=1}^N [V(x, s, j) - V(x, s, k)]d\Psi_{r(s)j}, \tag{4}$$

其中 Tr 表示矩阵的迹.

将式(2), (3) 代入(4), 并注意到有

$$\sum_{j=1}^N \pi_{kj}V(x, t, k) = 0, \forall k \in S \tag{5}$$

成立, 从而得到函数 $V(x, t, k)$ 的微分表达式为

$$dV(x, t, k) = \frac{\partial V(x, t, k)}{\partial t}dt + \frac{\partial V(x, t, k)}{\partial x}f(x, t, k)dt + \frac{1}{2}\mathrm{Tr}\Big[\Delta^T g^T(x, t, k) \frac{\partial^2 V(x, t, k)}{\partial x^2} \times g(x, t, k)\Delta\Big]dt + \sum_{j=1}^N \pi_{kj}V(x, t, j)dt +$$
$$\frac{\partial V(x, t, k)}{\partial x}g(x, t, k)d\omega + \sum_{j=1}^N [V(x, t, j) - V(x, t, k)]dM_j. \tag{6}$$

在式(6) 两边取期望, 并利用Wiener 过程和鞅过程的性质, 可以定义关于 $V(x, t, k)$ 的无穷小算子 LV[3] 为

$$LV(x, t, k) = \frac{\partial V(x, t, k)}{\partial t} + \frac{\partial V(x, t, k)}{\partial x}f(x, t, k) + \frac{1}{2}\mathrm{Tr}\Big[\Delta^T g^T(x, t, k) \frac{\partial^2 V(x, t, k)}{\partial x^2} \times g(x, t, k)\Delta\Big] + \sum_{j=1}^N \pi_{kj}V(x, t, j). \tag{7}$$

进一步要求 $V(x, t, r(t) = k)$ 为 Lyapunov 函数 $V(x, t, r(t)) \in C^2(R^n \times R_+ \times S; R_+)$, 从式(7) 可以看出: 对于任给模态 $\forall j \in S$, 即使有 N 个不等式成立

$$\frac{\partial V(x, t, j)}{\partial t} + \frac{\partial V(x, t, j)}{\partial x}f(x, t, j) + \frac{1}{2}\mathrm{Tr}\Big[\Delta^T g^T(x, t, j) \frac{\partial^2 V(x, t, j)}{\partial x^2} \times g(x, t, j)\Delta\Big] < 0, j = 1, 2, \cdots, N,$$

并不能推导出 $LV(x, t, k) < 0$ 成立.

这个关系反映了跳跃系统的特性: 由于 $r(t)$ 的

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引入导致了系统不同模态间的随机切换, 使得即使在各个模态下子系统是稳定的, 当组合在一起构成跳跃系统时, 整个系统仍有可能是不稳定的. 在设计控制器时, 不能选择各个子系统分别设计控制器, 而是要通过 π_{kj} 耦合项进行设计. 当 $S = \{1\}, k \equiv j = 1$ 时, 由式(5) 结果易见, $\mathcal{L}V$ 就退化为一般非跳跃随机系统的无穷小算子, 因此一般非跳跃随机系统可看作跳跃系统的一个特例.

下面给出证明过程中将会用到的一个引理.

引理 1(Young 不等式)　对于任意两个向量 x, $y \in R^n$, 如下不等式成立:

$$x^{\mathrm{T}}y \le \frac{\epsilon^p}{p}|x|^p + \frac{1}{q\epsilon^q}|y|^q,$$

其中: $\epsilon > 0$; 常数 $p > 1, q > 1$, 且满足 $(p-1)(q-1) = 1$; $|\cdot|$ 表示向量的 Euclidean 范数.

3　控制设计

下面采用 backstepping 递归设计思想, 来设计系统(1) 的鲁棒自适应控制方案. 记参数 θ^* 的估计值为 $\hat{\theta}$. 引入虚拟控制量 $\alpha_{i-1}(\bar{x}_{i-1}, \hat{\theta}, t, r(t))$; 从而

$$z_i = x_i - \alpha_{i-1}(\bar{x}_{i-1}, \hat{\theta}, t, r(t)),\ i = 1, 2, \cdots, n.$$

式中取 $\alpha_0 = 0, z_{n+1} = 0$, 则 $\alpha_n = u$. 为叙述方便, 将 $\alpha_{i-1}(\bar{x}_{i-1}, \hat{\theta}, t, k), \varphi_i(\bar{x}_i, t, k), g_i(\bar{x}_i, t, k)$ 简记为 $\alpha_{i-1}(\cdot,\cdot,\cdot,k), \varphi_i, g_i$. 利用式(6) 得到

$$\begin{aligned}
&\mathrm{d}z_i = \mathrm{d}x_i - \mathrm{d}\alpha_{i-1}(\cdot,\cdot,\cdot,k) = \\
&\Big[z_{i+1} + \alpha_i(\cdot,\cdot,\cdot,k) + \varphi_i^{\mathrm{T}}\theta^* - \\
&\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat{\theta}}\dot{\hat{\theta}} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}(x_{j+1} + \\
&\varphi_j^{\mathrm{T}}\theta^*) - \frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial t} - \\
&\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}g_p^{\mathrm{T}}\Delta\Delta^{\mathrm{T}}g_q - \\
&\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(\cdot,\cdot,\cdot,j)\Big]\mathrm{d}t + \\
&\Big(g_i^{\mathrm{T}} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j^{\mathrm{T}}\Big)\mathrm{d}w + \\
&\sum_{j=1}^{N}[\alpha_{i-1}(\cdot,\cdot,\cdot,k) - \alpha_{i-1}(\cdot,\cdot,\cdot,j)]\mathrm{d}M_j,
\end{aligned}\tag{8}$$

其中 $i = 1, 2, \cdots, n$. 定义

$$\begin{aligned}
&\Gamma_i(k) = \\
&[\alpha_{i-1}(\cdot,\cdot,\cdot,k) - \alpha_{i-1}(\cdot,\cdot,\cdot,1), \\
&\alpha_{i-1}(\cdot,\cdot,\cdot,k) - \alpha_{i-1}(\cdot,\cdot,\cdot,2), \\
&\cdots, \alpha_{i-1}(\cdot,\cdot,\cdot,k) - \alpha_{i-1}(\cdot,\cdot,\cdot,N)],
\end{aligned}$$

鞅过程 $M = [M_1, M_2, \cdots, M_N]^{\mathrm{T}}$, 则

$$\sum_{j=1}^{N}[\alpha_{i-1}(\cdot,\cdot,\cdot,k) - \alpha_{i-1}(\cdot,\cdot,\cdot,j)]\mathrm{d}M_j = \Gamma_i(k)\mathrm{d}M.$$

由文献[6] 中鞅表示定理可知, 对于定义在概率空间(Ω, F, P) 上的均方可积鞅过程 M, 存在独立的 Wiener 过程 ω 和 Lebesgue 均方可积的过程 Φ, 使得

$$\mathrm{d}M = \Phi\cdot\mathrm{d}\omega,$$

其中 ω 与 w 独立. 记增量 $\mathrm{d}\omega$ 的协方差为 $\Xi\Xi^{\mathrm{T}}\mathrm{d}t$, 即

$$E\{\mathrm{d}\omega\mathrm{d}\omega^{\mathrm{T}}\} = \Xi(t)\Xi^{\mathrm{T}}(t)\mathrm{d}t,$$

函数矩阵 $\Xi(t)$ 有界但不确定.

由于 Φ 是 Lebesgue 均方可积的, 即 $E\{|\Phi\Phi^{\mathrm{T}}|\} = \Phi(t)\Phi^{\mathrm{T}}(t) < \infty$, 从而存在常数 $Q > 0$, 使得 $\Phi\Phi^{\mathrm{T}} \le Q$. 则式(8) 可改写为

$$\begin{aligned}
&\mathrm{d}z_i = \\
&(x_{i+1} + \varphi_i^{\mathrm{T}}\theta^*)\mathrm{d}t + g_i^{\mathrm{T}}\mathrm{d}w - \mathrm{d}\alpha_{i-1}(\cdot,\cdot,\cdot,k) = \\
&\Big[z_{i+1} + \alpha_i(\cdot,\cdot,\cdot,k) + \\
&\varphi_i^{\mathrm{T}}\theta^* - \frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat{\theta}}\dot{\hat{\theta}} - \\
&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}(x_{j+1} + \varphi_j^{\mathrm{T}}\theta^*) - \\
&\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}g_p^{\mathrm{T}}\Delta\Delta^{\mathrm{T}}g_q - \\
&\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(\cdot,\cdot,\cdot,j) - \frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial t}\Big]\mathrm{d}t + \\
&\Big(g_i^{\mathrm{T}} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j^{\mathrm{T}}\Big)\mathrm{d}w + \\
&\Gamma_i(k)\Phi\mathrm{d}\omega.
\end{aligned}\tag{9}$$

选用 4 次 Lyapunov 函数

$$V = \frac{1}{4}\sum_{i=1}^{n}z_i^4 + \frac{1}{2\gamma}\tilde{\theta}^{\mathrm{T}}\tilde{\theta},$$

其中: 参数估计误差为 $\tilde{\theta} = \theta^* - \hat{\theta}$, 常数 $\gamma > 0$.

下面计算 V 沿系统(9) 的时间变化率

$$\begin{aligned}
&\mathcal{L}V = \\
&\sum_{i=1}^{n}z_i^3\Big[z_{i+1} + \alpha_i(\cdot,\cdot,\cdot,k) + \varphi_i^{\mathrm{T}}\theta^* - \\
&\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}(x_{j+1} + \varphi_j^{\mathrm{T}}\theta^*) - \\
&\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial t} - \frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat{\theta}}\dot{\hat{\theta}} - \\
&\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}g_p^{\mathrm{T}}\Delta\Delta^{\mathrm{T}}g_q - \\
&\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(\cdot,\cdot,\cdot,j)\Big] - \frac{1}{\gamma}\tilde{\theta}^{\mathrm{T}}\dot{\hat{\theta}} + \\
&\frac{3}{2}\sum_{i=1}^{n}z_i^2\Big[g_i^{\mathrm{T}} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j^{\mathrm{T}}\Big]\Delta\Delta^{\mathrm{T}}\times \\
&\Big[g_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j\Big] + \\
&\frac{3}{2}\sum_{i=1}^{n}z_i^2\Gamma_i(k)\Phi\Xi\Xi^{\mathrm{T}}\Phi^{\mathrm{T}}\Gamma_i^{\mathrm{T}}(k) \le
\end{aligned}$$

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$$\sum_{i=1}^{n} z_i^3\Big[\Big(\frac{3}{4}\delta_i^{\frac{4}{3}}+\frac{1}{4\delta_{i-1}^4}\Big)z_i+\alpha_i(\cdot,\cdot,\cdot,k)+\tau_i^{\mathrm T}(k)\hat\theta-\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\dot{\hat\theta}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}x_{j+1}-\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial t}+\lambda z_i^3\sum_{p,q=1}^{i-1}\Big(\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}\Big)^2 g_p^{\mathrm T}g_pg_q^{\mathrm T}g_q+\mu_1 z_i[\sigma_i^{\mathrm T}(k)\sigma_i(k)]^2+\mu_2 z_i[\Gamma_i(k)\Gamma_i^{\mathrm T}(k)]^2-\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(\cdot,\cdot,\cdot,j)\Big]+\iota|\Delta|^4+\frac{9n}{16\mu_2}|\Xi|^4Q^2-\tilde\theta^{\mathrm T}\Big(\frac{1}{\gamma}\dot{\hat\theta}-\sum_{i=1}^{n}z_i^3\tau_i(k)\Big). \tag{10}$$

式中定义

$$\tau_i(k)=\varphi_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}\varphi_j,$$

$$\sigma_i(k)=g_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j,$$

在式(10)的推导过程中,使用了以下的不等式,它们是从 Young 不等式得到的[7]:

$$\sum_{i=1}^{n}z_i^3z_{i+1}\leqslant\frac{3}{4}\sum_{i=1}^{n-1}\delta_i^{\frac{4}{3}}z_i^4+\frac{1}{4}\sum_{i=1}^{n-1}\frac{1}{\delta_i^4}z_{i+1}^4=\sum_{i=1}^{n}\Big(\frac{3}{4}\delta_i^{\frac{4}{3}}+\frac{1}{4\delta_{i-1}^4}\Big)z_i^4,$$

其中:$\delta_0=\infty$, $\delta_n=0$, 而 $\delta_i>0$, $i=1,2,\cdots,n-1$.

$$\frac{1}{2}\sum_{i=1}^{n}z_i^3\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}g_p^{\mathrm T}\Delta\Delta^{\mathrm T}g_q\leqslant\sum_{i=1}^{n}\lambda z_i^6\sum_{p,q=1}^{i-1}\Big(\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}\Big)^2g_p^{\mathrm T}g_pg_q^{\mathrm T}g_q+\frac{|\Delta\Delta^{\mathrm T}|^2}{96\lambda}(n-1)n(2n-1).$$

$$\frac{3}{2}\sum_{i=1}^{n}z_i^2\Big(g_i^{\mathrm T}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j^{\mathrm T}\Big)\times\Delta\Delta^{\mathrm T}\Big(g_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j\Big)\leqslant\sum_{i=1}^{n}\mu_1z_i^4\Big[\Big(g_i-\sum_{j=1}^{i}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j\Big)^{\mathrm T}\times\Big(g_i-\sum_{j=1}^{i}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}g_j\Big)\Big]^2+\sum_{i=1}^{n}\frac{9}{16\mu_1}|\Delta\Delta^{\mathrm T}|^2.$$

$$\frac{3}{2}\sum_{i=1}^{n}z_i^2\Gamma_i(k)\Phi\Xi\Xi^{\mathrm T}\Phi\Gamma_i^{\mathrm T}(k)\leqslant\sum_{i=1}^{n}\mu_2z_i^4[\Gamma_i(k)\Gamma_i^{\mathrm T}(k)]^2+\sum_{i=1}^{n}\frac{9}{16\mu_2}|\Xi\Xi^{\mathrm T}|^2Q^2,$$

其中:$\lambda>0$, $\mu_1>0$, $\mu_2>0$, 并记

$$\iota=\frac{(n-1)n(2n-1)}{96\lambda}+\frac{9n}{16\mu_1}.$$

根据式(10),可以选定计算虚拟控制如下:

$$\alpha_i(\cdot,\cdot,\cdot,k)=-c_iz_i-\Big(\frac{3}{4}\delta_i^{\frac{4}{3}}+\frac{1}{4\delta_{i-1}^4}\Big)z_i-\tau_i(k)\hat\theta+\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\dot{\hat\theta}+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}x_{j+1}+\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial t}-\lambda z_i^3\sum_{p,q=1}^{i-1}\Big(\frac{\partial^2\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_p\partial x_q}\Big)^2g_p^{\mathrm T}g_pg_q^{\mathrm T}g_q-\mu_1z_i[\sigma_i^{\mathrm T}(k)\sigma_i(k)]^2-\mu_2z_i[\Gamma_i(k)\Gamma_i^{\mathrm T}(k)]^2+\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(\cdot,\cdot,\cdot,j). \tag{11}$$

自适应控制律取为[8]

$$\dot{\hat\theta}=\gamma\Big[\sum_{i=1}^{n}z_i^3\tau_i(k)-m(\hat\theta-\theta^0)\Big], \tag{12}$$

其中:$m>0$, $\theta^0\in R^p$ 为给定常数.

选定参数自适应律(12)后,由于在 $\alpha_i(\cdot,\cdot,\cdot,k)$ 求解式(11)中包含了 $\dot{\hat\theta}$, 它与 $z_1,\cdots,z_n$ 有关,于是式(11)不能直接递归求解 α_i, 为此本文对 LV 表达式中含 $\dot{\hat\theta}$ 的项进行如下处理:

$$\sum_{i=1}^{n}z_i^3\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\dot{\hat\theta}=\sum_{i=1}^{n}z_i^3\Big[\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\gamma\sum_{j=1}^{i}z_j^3\tau_j(k)+\Big(\sum_{j=1}^{i-1}z_j^3\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\Big)\gamma\tau_i(k)-\gamma m(\hat\theta-\theta^0)\Big]. \tag{13}$$

将式(13)代入 LV 表达式(10)中,设计可递归计算的虚拟控制

$$\alpha_i(\cdot,\cdot,\cdot,k)=-c_iz_i-\Big(\frac{3}{4}\delta_i^{\frac{4}{3}}+\frac{1}{4\delta_{i-1}^4}\Big)z_i-\tau_i(k)\hat\theta+\gamma\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\sum_{j=1}^{i}z_j^3\tau_j(k)+\gamma\sum_{j=1}^{i-1}z_j^3\frac{\partial\alpha_{j-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}\tau_i(k)-\gamma m\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial\hat\theta}(\hat\theta-\theta^0)+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial x_j}x_{j+1}+\frac{\partial\alpha_{i-1}(\cdot,\cdot,\cdot,k)}{\partial t}-$$

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$$\lambda z_i^3\sum_{p,q=1}^{i-1}\Big(\frac{\partial^2\alpha_{i-1}(\bullet,\bullet,\bullet,k)}{\partial\hat{\alpha}_p\partial\hat{\alpha}_q}\Big)^2 g_p^{\mathrm T}g_pg_q^{\mathrm T}g_q-$$
$$\mu_1z_i[\sigma_i^{\mathrm T}(k)\sigma_i(k)]^2-\mu_2z_i[\Gamma_i(k)\Gamma_i^{\mathrm T}(k)]^2+$$
$$\sum_{j=1}^{N}\pi_{kj}\alpha_{i-1}(\bullet,\bullet,\bullet,j),\tag{14}$$

其中: 常数 $c_i>0$, $i=1,2,\cdots,n$; 实际控制输入为

$$u(k)=\alpha_n(\bullet,\bullet,\bullet,k).\tag{15}$$

将式(12), (14) 和(15) 带入 LV 表达式(10), 并考虑式(13), 可得

$$LV\le-\sum_{i=1}^{n}c_iz_i^4+m\tilde{\theta}^{\mathrm T}(\theta-\theta^0)+\iota|\Delta|^2+\frac{9n}{16\mu_2}|\Xi|^4Q^2.$$

又有

$$m\tilde{\theta}^{\mathrm T}(\theta-\theta^0)=-\frac{1}{2}m\tilde{\theta}^{\mathrm T}\tilde{\theta}-\frac{1}{2}m(\theta-\theta^0)^{\mathrm T}(\theta-\theta^0)+\frac{1}{2}m(\theta^*-\theta^0)^{\mathrm T}(\theta^*-\theta^0),$$

从而

$$LV\le-\sum_{i=1}^{n}c_iz_i^4-\frac{1}{2}m\tilde{\theta}^{\mathrm T}\tilde{\theta}+\frac{1}{2}m(\theta^*-\theta^0)^{\mathrm T}(\theta^*-\theta^0)+\iota|\Delta|^4+\frac{9n}{16\mu_2}|\Xi|^4Q^2.$$

取

$$c=\min(4c_i,m\gamma),$$
$$\lambda=\frac{1}{2}m(\theta^*-\theta^0)^{\mathrm T}(\theta^*-\theta^0)+\iota|\Delta|^4+\frac{9n}{16\mu_2}|\Xi|^4Q^2,$$

则得到

$$LV\le-cV+\lambda\tag{16}$$

根据文献[5] 中引理4 1:

$$EV(x(t),t,r(t))\le e^{-ct}\Big[V(x_0,t_0,r_0)-\frac{\lambda}{c}\Big]+\frac{\lambda}{c},$$

其中 $V(x_0,t_0,r_0)$ 表示 $V(x(t),t,r(t))$ 在系统初始点 (x_0,t_0,r_0) 的取值 由于

$$V=\frac{1}{4}\sum_{i=1}^{n}z_i^4+\frac{1}{2\gamma}\tilde{\theta}^{\mathrm T}\tilde{\theta}\ge\frac{1}{4}\sum_{i=1}^{n}z_i^4,$$

令向量 $Z=[z_1,z_2,\cdots,z_n]$, 在上式不等号两边取期望, 得

$$E(|Z|_4^4)\le 4EV(x,t,r(t))\le 4e^{-ct}\Big[V(x_0,t_0,r_0)-\frac{\lambda}{c}\Big]+\frac{4\lambda}{c},\tag{17}$$

这里的 $|\bullet|_4$ 表示 4- 阶范数

从式(17) 可以看出, $Z=(z_1,z_2,\cdots,z_n)$ 在 4- 阶矩意义下全局一致有界, 则 $X=(x_1,x_2,\cdots,x_n)$ 在 4- 阶矩意义下全局一致有界. 并且 $\exists\, T>0$, 当 $t>T$ 时,

$$4e^{-ct}\Big[V(x_0,t_0,r_0)-\frac{\lambda}{c}\Big]\le\frac{4\lambda}{c},$$

从而有 $E|Z|_4^4\le 8\lambda/c$ 成立 进一步可知, 对于任意给定 $\epsilon>0$, 可通过选取合适的控制设计参数 $c_i,m,\gamma,\theta^0,\mu_1,\mu_2$, 使得 $8\lambda/c<\epsilon$ 因此有当 $t>T$ 时,

$$E|Z|_4^4\le 8\lambda/c<\epsilon$$

即闭环系统状态在 4 阶矩意义下全局一致有界, 且可在有限时间内达到包含平衡点的任意小邻域内

定理 1　随机跳跃非线性系统(1) 在控制律(11), (14) 和参数自适应律(12) 的作用下, 闭环系统的状态在 4 阶矩意义下全局一致有界, 且能够在有限时间内达到平衡点的任意小邻域内

4　算　　例

考察二阶系统并设系统的模态集为 $S=\{1,2\}$, 模态转移率矩阵为

$$\Pi=\begin{bmatrix}-2 & 2\\ 5 & -5\end{bmatrix}.$$

系统动力学方程为

$$\begin{cases}\mathrm{d}x_1=x_2\mathrm{d}t+\varphi_1(x_1,r(t))\theta^*\mathrm{d}t,\\ \mathrm{d}x_2=u\mathrm{d}t+\varphi_2(x,r(t))\theta^*\mathrm{d}t+\\ \qquad g_2(x,r(t))\mathrm{d}w.\end{cases}$$

其中

$$\varphi_1(x_1,1)=x_1^2,\ \varphi_1(x_1,2)=x_1\sin x_1,$$
$$\varphi_2(x,1)=x_1x_2,\ \varphi_2(x,2)=x_1^2\sin x_2,$$
$$g_2(x,1)=\sin x_2,\ g_2(x,2)=x_2^2.$$

按本文所给的方法, 推出该系统的控制律和参数自适应律如下(取 $\delta_1=1$):

当系统模态 $r(t)=1$ 时,

$$\alpha_1(r(t)=1)=-\Big(c_1+\frac{3}{4}\Big)z_1-\tau_1\hat{\theta},$$
$$\alpha_2(r(t)=1)=$$
$$-\Big(c_2+\frac{1}{4}+\mu_1\sigma_2^4\Big)z_2-\tau_2\hat{\theta}-$$
$$\Big(c_1+\frac{3}{4}+2x_1\hat{\theta}\Big)x_2-\gamma\tau_1(z_1^3\tau_1+$$
$$z_2^3\tau_2)+\gamma m\tau_1(\hat{\theta}-\theta^0)+$$
$$\pi_{11}\alpha_1(r(t)=1)+\pi_{12}\alpha_1(r(t)=2)-$$
$$\mu_2z_2[\alpha_1(r(t)=2)-\alpha_1(r(t)=1)]^4,$$
$$\dot{\hat{\theta}}=\gamma[z_1^3\tau_1+z_2^3\tau_2-m(\hat{\theta}-\theta^0)].$$

其中

$$\sigma_1=0,\ \sigma_2=\sin x_2,\ \tau_1=x_1^2,$$

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$$\tau_2 = x_1x_2 + \left(c_1 + \frac{3}{4} + 2x_1\theta\right)x_1^2,$$

$$z_1 = x_1, z_2 = x_2 - \alpha_1(r(t) = 1).$$

当系统模态 $r(t) = 2$ 时,

$$\alpha_1(r(t) = 2) = -\left(c_1 + \frac{3}{4}\right)z_1 - \tau_1\theta,$$

$$\alpha_2(r(t) = 2) =$$

$$-\left(c_1 + \frac{1}{4} + \mu_1\sigma_2^4\right)z_2 - \tau_2\theta -$$

$$\left[c_1 + \frac{3}{4} + (x_1\cos x_1 + \sin x_1)\theta\right]x_2 -$$

$$\gamma\tau_1(z_1^3\tau_1 + z_2^3\tau_2) + \gamma m\tau_1(\theta - \theta^0) +$$

$$\pi_{21}\alpha_1(r(t) = 1) + \pi_{22}\alpha_1(r(t) = 2) -$$

$$\mu_2 z_2[\alpha_1(r(t) = 1) - \alpha_1(r(t) = 2)]^4,$$

$$\dot{\theta} = \gamma[z_1^3\tau_1 + z_2^3\tau_2 - m(\theta - \theta^0)].$$

其中

$$\sigma_1 = 0, \sigma_2 = x_2^2, \tau_1 = x_1\sin x_1,$$

$$\tau_2 = x_1^2\sin x_2 + \left[c_1 + \frac{3}{4} + (\sin x_1 + x_1\cos x_1)\theta\right]x_1\sin x_1,$$

$$z_1 = x_1, z_2 = x_2 - \alpha_1(r(t) = 2).$$

当噪声方差 $\Delta(t) = 1$, 参数真值 $\theta^* = 2$, 有关设计常数取 $c_1 = c_2 = 5, \mu_1 = 100, \mu_2 = 50, \theta^0 = 1, \gamma = 0.5, m = 1$, 计算初始值取 $x_1 = 8.5, x_2 = -1.5$, $\theta(0) = 0$ 时, 系统模态转移曲线如图 1 所示; 闭环系统状态 $x = (x_1, x_2)$ 如图 2 所示, 其中实线是 x_1 的轨迹, 虚线是 x_2 的轨迹; 控制曲线如图 3 所示. 计算时间步长取 1 s, 时间轴以时间步长为单位. 为保证计算的稳定性, 时间步长在计算过程中可作适当的调整.

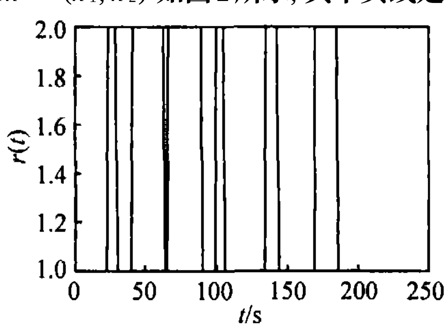

图 1 模态转移曲线

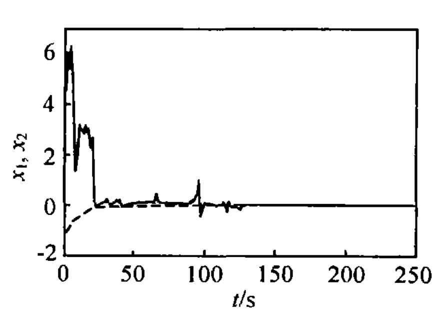

图 2 闭环系统状态曲线

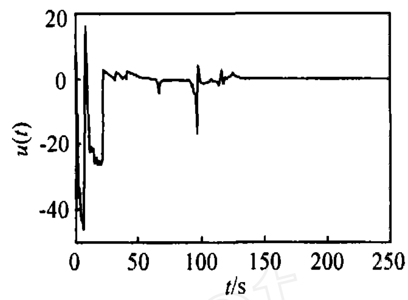

图 3 反馈控制曲线

从图中可以看出, 状态 $x = (x_1, x_2)$ 的 4 阶矩有界, 并且随着模态 $r(t)$ 的转换, x 的向量场和控制量 u 有明显的跳跃. 仿真结果验证了本文控制方案是有效的.

5 结 论

本文将Markov 跳跃过程和随机非线性系统的控制问题结合起来, 研究了具有Markov 跳跃参数的一类随机非线性系统的鲁棒自适应控制器设计问题. 通过构造 4 次随机控制Lyapunov 函数和运用backstepping 的递归设计方法, 使闭环系统状态在 4 阶矩意义下全局一致有界, 且能在有限时间内到达包含平衡点的任意小邻域内. 由于一般非跳跃的随机系统可看作是跳跃系统的一个特例, 因此, 本文控制器设计方案也同样适用于非跳跃随机系统.

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6 结　　语

遮挡给运动目标的跟踪带来很大困难, 本文提出的算法能很好地解决刚性运动目标的遮挡问题. 本文算法的特点有: 1) 该算法基于灰度相关匹配, 不依赖于运动目标分割这个较为困难的环节, 提高了跟踪的速度和精度; 2) 采用自适应的分块算法, 克服了固定分块算法中可能存在的子块特征不明显以及子块相似度高等问题造成的误配; 3) 采用整体相关匹配和各子块表决相结合的算法, 很好地利用了目标的灰度信息和结构信息, 提高了跟踪的可靠性; 4) 不用检测遮挡发生的开始和结束; 5) 采用子块来判定遮挡区域, 并且实时更新. 对遮挡子块的判定是基于位移量的误差大小, 阈值选取简单, 克服了用匹配误差判定算法中阈值难以选择的缺点.

实际中大多数刚性运动目标符合本文提出的算法, 但也有少数例外, 如: 对于体积很小的目标, 不适合分块; 对于灰度单一的目标或者是灰度分布很有规律的目标, 子块的相似度很高, 也不适合分块. 对这些以及非刚性目标遮挡问题的研究是下一步的工作.

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