文章编号: 1001-0920(2008)01-0060-05

一类高阶次随机非线性系统的状态反馈镇定

段 纳[1,2], 解学军[1,2], 张嗣瀛[3]
(1. 曲阜师范大学 自动化研究所, 山东 曲阜 273165; 2. 徐州师范大学 电气工程及自动化学院, 江苏 徐州 221116; 3. 东北大学 信息科学与工程学院, 沈阳 110004)

摘 要: 针对一类高阶次随机非线性系统,研究其状态反馈镇定问题. 基于最近发展起来的增加幂次积分器技术,通过适当地选取 Lyapunov 函数和设计参数,给出了一个光滑的状态反馈反推控制器的设计过程,所设计的控制器保证了闭环系统在原点处的平衡点是概率意义下全局渐近稳定的.
关键词: 高阶次随机非线性系统; 状态反馈; 镇定; 反推; 概率意义下全局渐近稳定
中图分类号: TP273 **文献标识码**: A

State-feedback stabilization for a class of high-order stochastic nonlinear system

DUAN Na[1,2], *XIE Xue-jun*[1,2], *ZHANG Si-ying*[3]
(1. Institute of Automation, Qufu Normal University, Qufu 273165, China; 2. School of Electrical Engineering & Automation, Xuzhou Normal University, Xuzhou 221116, China; 3. College of Information Science and Engineering, Northeastern University, Shenyang 110004, China. Correspondent: XIE Xue-jun, E-mail: xxj@mail.qfnu.edu.cn)

Abstract: For a class of high-order stochastic nonlinear systems, the problem of state-feedback stabilization is studied. Based on the adding a power integrator technique developed recently, by appropriately choosing Lyapunov functions and design parameters, a design procedure of smooth state-feedback backstepping controller is presented to guarantee that the equilibrium at the origin of the closed-loop system is globally asymptotically stable in probability.
Key words: High-order stochastic nonlinear systems; State-feedback; Stabilization; Backstepping; Globally asymptotically stable in probability

1 引 言

考虑如下高阶次非线性系统:

$$\dot{x}_i = x_{i+1}^{p_i} + \phi_i^{\mathrm{T}}(\bar{x}_i)\theta,\ i = 1, 2, \cdots, n-1;$$
$$\dot{x}_n = u^{p_n} + \phi_n^{\mathrm{T}}(\bar{x}_n)\theta. \quad (1)$$

其中: p_i 是正奇数, $i = 1, 2, \cdots, n$, 当 $p_i = 1$ 时,系统(1)退化为众所周知的严格反馈形式. 近年来,反推设计方法已成为这类系统的鲁棒和自适应控制研究的有力工具[1,2]. 在此基础上, Lin 和 Qian[3-6] 针对系统(1),考虑了一系列控制问题.

受上述思想的启发,本文考虑如下高阶次随机非线性系统:

$$\mathrm{d}x_i = x_{i+1}^{p}\mathrm{d}t + \phi_i^{\mathrm{T}}(\bar{x}_i)\mathrm{d}\omega,\ i = 1, 2, \cdots, n-1;$$
$$\mathrm{d}x_n = u^{p}\mathrm{d}t + \phi_n^{\mathrm{T}}(\bar{x}_n)\mathrm{d}\omega. \quad (2)$$

其中: $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in R^n$ 和 $u \in R$ 分别是系统的可测状态和控制输入; $\phi_i(\bar{x}_i) \in R^r$ 是满足 $\phi_i(0) = 0$ 的光滑函数,且 $\bar{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in R^i$; $\omega \in R^r$ 是定义在概率空间 (Ω, F, P) 上的独立标准 Wiener 过程向量, Ω, F 和 P 分别是样本空间、σ 代数域和概率测度; $p \geq 1$ 是奇数.

当 $p = 1$ 时, 这类随机系统的全局稳定控制器的设计已经取得了很大进展. 文献[7,8]分别给出了随机系统的稳定性理论. 在此数学基础上的一大突破是 Pan 和 Basar 的工作, 他们在文献[9]中使用二次 Lyapunov 函数和增大反馈权值法设计控制器; 另一主要工作是 Krstic 和 Deng[10,11] 通过引入 4 次 Lyapunov 函数,在闭环系统的平衡点干扰为零的假设下,得到了全局渐近稳定控制. 在最新的一些工作

收稿日期: 2006-08-30; **修回日期**: 2006-11-20.
基金项目: 教育部新世纪优秀人才支持计划项目(NCET-05-0607); 国家自然科学基金项目(60774010); 江苏省高校自然科学基金项目(07KJB510114).
作者简介: 段纳(1981—),女,山东枣庄人,博士生,从事非线性系统的自适应控制研究; 解学军(1968—),男,山东青岛人,教授,博士生导师,从事复杂系统的自适应控制等研究.

© 1994-2010 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

中,文献[12] 通过引入随机小增益定理,文献[13] 通过动态信号和改变供能函数方法, 分别研究了自适应输出反馈控制.

本文针对系统(2), 研究其状态反馈镇定问题. 通过适当选取Lyapunov函数,设计了一个光滑的状态反馈反推控制器,保证了闭环系统在原点处的平衡点是概率意义下全局渐近稳定的.

2　预备知识

用 R_+ 表示 $[0, +\infty)$, X^{T} 表示 X 的转置, $\mathrm{Tr}(X)$ 表示方阵 X 的迹, $|\cdot|$ 表示欧氏空间中向量的 2 范数, C^i 表示相应定义域上的 i 阶连续可微函数. K 表示连续、严格单调、零点等于 0 的 R_+ 到 R_+ 的函数全体; K_∞ 表示 K 中无界函数全体; $R_+ \times R_+$ 到 R_+ 的函数 $\beta(s,t) \in KL$ 表示对给定的 t, $\beta(\cdot,t) \in K$, 而给定 s, $\beta(s,\cdot)$ 是单调减少的且 $\lim\limits_{t\to\infty}\beta(s,t) = 0$.

下面给出一些引理:

引理 1　对任意的实变量 x_i, y 和 z_j $(i = 1,2,\cdots,\bar{l}, j = 1,2,\cdots,l)$, $\bar{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}}$, 给定光滑映射 $g_{1i}: R^i \to R$ 和 $g_2: R^{l+1} \to R$, 则对任意正奇数 p, 存在实数 $M_i > 0$ 和非负光滑函数 $h_{1i}: R^i \to R$, $h_2: R^{l+1} \to R$, 使得

$$|x_i^{p+2}[(y + x_i g_{1i}(\bar{x}_i))^p - (x_i g_{1i}(\bar{x}_i))^p]| \le M_i x_i^{2p+2} + |y|^{2p+2} h_{1i}(\bar{x}_i),$$

$$|y^{p+2}(z_1^p + \cdots + z_l^p + y^p) g_2(\bar{z}_l, y)| \le \frac{p}{2p+2}(|z_1|^{2p+2} + \cdots + |z_l|^{2p+2}) + |y|^{2p+2} h_2(\bar{z}_l, y).$$

其中

$$M_i = \frac{p+2}{2p+2} + C_p^1 \frac{p+3}{2p+2} + \cdots + C_p^{p-1}\frac{2p+1}{2p+2},$$

$$h_{1i}(\bar{x}_i) = \frac{p}{2p+2} + C_p^1 \frac{p-1}{2p+2}|g_{1i}(\bar{x}_i)|^{\frac{2p+2}{p-1}} + \cdots + C_p^{p-1}\frac{1}{2p+2}|g_{1i}(\bar{x}_i)|^{(2p+2)(p-1)},$$

$$h_2(\bar{z}_l, y) = \frac{p+2}{2p+2}|g_2(\bar{z}_l, y)|^{\frac{2p+2}{p+2}} + |g_2(\bar{z}_l, y)|.$$

显然当 $g_2(\bar{z}_l, y) = g_2(\bar{z}_l)$ 时, $h_2(\bar{z}_l, y) = h_2(\bar{z}_l)$.

证明过程类似于文献[5],此略.

考虑如下随机非线性系统:

$$\mathrm{d}x = f(x,u)\mathrm{d}t + g(x)\mathrm{d}\omega. \tag{3}$$

其中: $x \in R^n$ 是可测的状态, $u \in R^m$ 是输入, $\omega \in R^r$ 是独立标准 Wiener 过程向量. 对任意 $t \ge 0$, 当 $x \in R^n$ 时, $f: R^{m+n} \to R^n$ 和 $g: R^n \to R^{n\times r}$ 是局部Lipschitz 函数且局部有界, 且 $f(0) = 0$, $g(0) = 0$.

定义 1[10]　如果对于任意的 $\varepsilon > 0$, 存在一类 KL 函数 $\beta(\cdot,\cdot)$, 满足 $P\{|x(t)| < \beta(|x_0|, t)\} \ge 1 - \varepsilon$, $\forall t \ge 0$, $\forall x_0 \in R^n \backslash \{0\}$, 则称系统(3) 在平衡点 $x = 0$ 是概率意义下全局渐近稳定的.

引理 2[10]　对于系统(3), 若存在一个 C^2 函数 $V(x)$ 及 K_∞ 函数 α_1, α_2, K 函数 α_3, 使得

$$\alpha_1(|x|) \le V(x) \le \alpha_2(|x|),$$

$$LV(x) = \frac{\partial V(x)}{\partial x} f + \frac{1}{2}\mathrm{Tr}\left[g^{\mathrm{T}}\frac{\partial^2 V(x)}{\partial x^2} g\right] \le -\alpha_3(|x|),$$

则系统(3) 在 $[0, +\infty)$ 上存在唯一解, 且在平衡点 $x = 0$ 是概率意义下全局渐近稳定的. 其中 $LV(x)$ 称为 $V(x)$ 沿系统(3) 的无穷小算子.

3　反推控制器设计

对随机非线性系统(2), 采用如下假设:

假设 1　对于 $\phi_i(\bar{x}_i)$, 存在一个已知的非负光滑函数 $\rho_i(\cdot)$, 使

$$|\phi_i(\bar{x}_i)| \le \Big(\sum_{j=1}^{i}|x_j|^p\Big)\rho_i(\bar{x}_i), \quad i = 1,2,\cdots,n.$$

控制目标是为系统(2) 设计一个光滑的状态反馈控制器, 使得闭环系统在概率意义下全局渐近稳定.

下面利用反推方法设计控制器. 引进误差变量

$$z_1 = x_1,$$

$$z_i = x_i - \alpha_{i-1}(\bar{x}_{i-1}), \quad i = 2,3,\cdots,n, \tag{4}$$

其中 $\alpha_{i-1}(\bar{x}_{i-1})$ 为待设计的光滑虚拟控制器. 由 Itô 公式, 式(2) 和(4) 得

$$\mathrm{d}z_i = \mathrm{d}(x_i - \alpha_{i-1}(\bar{x}_{i-1})) = \Big(x_{i+1}^p - \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_l}x_{l+1}^p - \frac{1}{2}\sum_{k,m=1}^{i=1}\frac{\partial^2\alpha_{i-1}}{\partial x_k\partial x_m}\cdot\phi_k^{\mathrm{T}}(\bar{x}_k)\phi_m(\bar{x}_m)\Big)\mathrm{d}t + \eta_i^{\mathrm{T}}(\bar{x}_i)\mathrm{d}\omega, \quad i = 1,2,\cdots,n. \tag{5}$$

其中

$$x_{n+1} = u,$$

$$\eta_i(\bar{x}_i) = \phi_i(\bar{x}_i) - \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_l}\phi_l(\bar{x}_l).$$

Step 1　由假设 1 和式(4) 得

$$|\phi_1(x_1)| \le |z_1|^p\tilde{\rho}_1(z_1),$$

$$\tilde{\rho}_1(z_1) = \rho_1(x_1). \tag{6}$$

选取第 1 个 Lyapunov 函数

$$V_1(z_1) = \frac{1}{p+3}z_1^{p+3}, \tag{7}$$

© 1994-2010 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

选择一个光滑的虚拟控制器

$$\alpha_1(x_1) = -z_1\beta_1(z_1),$$
$$\beta_1(z_1) = \left(c_1 + d_1 z_1^{2p-2} + \frac{p+2}{2} z_1^{p-1}\tilde{\rho}_1^2(z_1)\right)^{1/p}, \qquad (8)$$

并考虑式(6),则

$$LV_1 = z_1^{p+2}x_2^p + \frac{p+2}{2}\mathrm{Tr}[z_1^{p+1}\phi_1(x_1)\phi_1^{\mathrm{T}}(x_1)] \le -c_1 z_1^{2p+2} - d_1 z_1^{4p} + z_1^{p+2}(x_2^p - \alpha_1^p), \qquad (9)$$

其中 $c_1, d_1 > 0$ 是设计参数.

Step i(递归过程) 假设在第 $i-1$ 步已经得到光滑虚拟控制器 $\alpha_1(x_1), \cdots, \alpha_{i-1}(\bar{x}_{i-1})$ 形如

$$\alpha_1(x_1) = -z_1\beta_1(z_1), \cdots, \alpha_{i-1}(\bar{x}_{i-1}) = -z_{i-1}\beta_{i-1}(\bar{z}_{i-1}), \qquad (10)$$

$\beta_1(z_1), \cdots, \beta_{i-1}(\bar{z}_{i-1}) > 0$ 是光滑函数,使得第 $i-1$ 个 Lyapunov 函数

$$V_{i-1}(z_1, \cdots, z_{i-1}) = \frac{1}{p+3}\sum_{j=1}^{i-1} z_j^{p+3}$$

满足

$$LV_{i-1} \le -\sum_{j=1}^{i-2}(c_{j,i-1} z_j^{2p+2} + d_{j,i-1} z_j^{4p}) - c_{i-1} z_{i-1}^{2p+2} - d_{i-1} z_{i-1}^{4p} + z_{i-1}^{p+2}(x_i^p - \alpha_{i-1}^p). \qquad (11)$$

其中

$$c_{j,i-1} = c_{j,i-2} - \frac{p}{2p+2},$$
$$d_{j,i-1} = d_{j,i-2} - \frac{i-2}{2} - \frac{(p+2)(i-1)}{4},$$
$$j = 1, 2, \cdots, i-3,$$
$$c_{i-2,i-1} = c_{i-2} - M_{i-2} - \frac{p}{2p+2},$$
$$d_{i-2,i-1} = d_{i-2} - \frac{i-2}{2} - \frac{(p+2)(i-1)}{4},$$
$$M_{i-2} = \cdots = M_1 = \frac{p+2}{2p+2} + C_p^1\frac{p+3}{2p+2} + \cdots + C_p^{p-1}\frac{2p+1}{2p+2},$$

$c_{i-1} > 0, d_{i-1} > 0$ 是设计参数. 下面证明对于第 i 个 Lyapunov 函数

$$V_i(\bar{z}_i) = V_{i-1}(\bar{z}_{i-1}) + \frac{1}{p+3} z_i^{p+3}, \qquad (12)$$

式(11)仍同样成立.

由式(5)和(11)得

$$LV_i \le -\sum_{j=1}^{i-2}(c_{j,i-1} z_j^{2p+2} + d_{j,i-1} z_j^{4p}) - c_{i-1} z_{i-1}^{2p+2} - d_{i-1} z_{i-1}^{4p} + z_{i-1}^{p+2}(x_i^p - \alpha_{i-1}^p) + z_i^{p+2}(x_{i+1}^p - \alpha_i^p) + z_i^{p+2}\alpha_i^p + z_i^{p+2}\Big[-\sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_l}x_{l+1}^p - \frac{1}{2}\sum_{k,m=1}^{i-1}\frac{\partial^2\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_k\partial x_m}\phi_k^{\mathrm{T}}(\bar{x}_k)\phi_m(\bar{x}_m)\Big] + \frac{p+2}{2}\mathrm{Tr}[z_i^{p+1}\eta_i(\bar{x}_i)\eta_i^{\mathrm{T}}(\bar{x}_i)]. \qquad (13)$$

由式(4),(10)和假设1知,存在非负光滑函数 $\tilde{\rho}_i(\cdot)$ 和 $\rho_{i1}(\cdot)$ 使得

$$|\phi_i(\bar{x}_i)| \le \sum_{j=1}^{i}|x_j|^p\rho_i(\bar{x}_i) \le |z_1|^p\rho_i(\bar{x}_i) + 2^{p-1}\sum_{j=2}^{i}(|z_j|^p + |z_{j-1}|^p\beta_{j-1}^p(\bar{z}_{j-1}))\rho_i(\bar{x}_i) \le \sum_{j=1}^{i}|z_j|^p\tilde{\rho}_i(\bar{z}_i),$$
$$|\eta_i(\bar{x}_i)| \le \sum_{j=1}^{i}|z_j|^p\tilde{\rho}_i(\bar{z}_i) + \sum_{l=1}^{i-1}\left|\frac{\partial\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_l}\right|\cdot\Big(\sum_{j=1}^{l}|z_j|^p\tilde{\rho}_l(\bar{z}_l)|\Big) \le \sum_{j=1}^{i}|z_j|^p\rho_{i1}(\bar{z}_i). \qquad (14)$$

由式(4),(10)和引理1易得

$$z_{i-1}^{p+2}(x_i^p - \alpha_{i-1}^p) = z_{i-1}^{p+2}((z_i + \alpha_{i-1})^p - \alpha_{i-1}^p) \le M_{i-1}z_{i-1}^{2p+2} + z_i^{2p+2}\gamma_{i1}(\bar{z}_{i-1}). \qquad (15)$$

其中

$$M_{i-1} = \frac{p+2}{2p+2} + C_p^1\frac{p+3}{2p+2} + \cdots + C_p^{p-1}\frac{2p+1}{2p+2},$$
$$\gamma_{i1}(\bar{z}_{i-1}) = \frac{p}{2p+2} + C_p^1\frac{p-1}{2p+2}\beta_{i-1}^{\frac{2p+2}{p-1}}(\bar{z}_{i-1}) + \cdots + C_p^{p-1}\frac{1}{2p+2}\beta_{i-1}^{(2p+2)(p-1)}(\bar{z}_{i-1}).$$

利用引理1和 Young 不等式得

$$|z_i|^{p+2}\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_j}\right|\|z_{j+1} + \alpha_j|^p \le \sum_{j=1}^{i}|z_i|^{p+2}|z_j|^p\gamma_{i2}(\bar{z}_{i-1}) \le \frac{p}{2p+2}\sum_{j=1}^{i-1}z_j^{2p+2} + z_i^{2p+2}\tilde{\gamma}_{i2}(\bar{z}_{i-1}). \qquad (16)$$

其中

$$\tilde{\gamma}_{i2}(\bar{z}_{i-1}) = \frac{p+2}{2p+2}\gamma_{i2}^{\frac{2p+2}{p+2}}(\bar{z}_{i-1}) + \gamma_{i2}(\bar{z}_{i-1}).$$

由式(14)得

© 1994-2010 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net

$$|z_i|^{p+2}\frac{1}{2}\sum_{k,m=1}^{i-1}\left|\frac{\partial^2\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_k\partial x_m}\right|\times$$
$$|\phi_k(\bar{x}_k)|\,|\phi_m(\bar{x}_m)|\le$$
$$|z_i|^{p+2}\frac{1}{2}\sum_{k,m=1}^{i-1}\gamma_{i3}(\bar{z}_{i-1})(|\phi_k(\bar{x}_k)|^2+$$
$$|\phi_m(\bar{x}_m)|^2)=$$
$$(i-1)\sum_{j=1}^{i-1}|z_i|^{p+2}\gamma_{i3}(\bar{z}_{i-1})|\phi_j(\bar{x}_j)|^2\le$$
$$(i-1)\sum_{j=1}^{i-1}|z_i|^{p+2}\gamma_{i3}(\bar{z}_{i-1})\,j\Big(\sum_{m=1}^{j}z_m^{2p}\tilde{\rho}_j^2(\bar{z}_j)\Big)=$$
$$(i-1)\sum_{j=1}^{i-1}z_j^{2p}\gamma_{i3}(\bar{z}_{i-1})|z_i|^{p+2}\Big(\sum_{m=j}^{i-1}m\tilde{\rho}_m^2(\bar{z}_m)\Big)\le$$
$$\frac{i-1}{2}\sum_{j=1}^{i-1}z_j^{4p}+$$
$$\frac{i-1}{2}z_i^{2p+4}\gamma_{i3}^2(\bar{z}_{i-1})\sum_{j=1}^{i-1}\Big(\sum_{m=j}^{i-1}m\tilde{\rho}_m^2(\bar{z}_m)\Big)^2. \qquad (17)$$

利用式(16)和(17)得

$$z_i^{p+2}\Big[-\sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_l}x_{l+1}^p-$$
$$\frac{1}{2}\sum_{k,m=1}^{i-1}\frac{\partial^2\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_k\partial x_m}\cdot\phi_k^{\mathrm{T}}(\bar{x}_k)\phi_m(\bar{x}_m)\Big]\le$$
$$|z_i|^{p+2}\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_j}\right||z_{j+1}+\alpha_j|^p+$$
$$|z_i|^{p+2}\frac{1}{2}\sum_{k,m=1}^{i-1}\left|\frac{\partial\alpha_{i-1}(\bar{x}_{i-1})}{\partial x_k\partial x_m}\right|\times$$
$$|\phi_k(\bar{x}_k)|\,|\phi_m(\bar{x}_m)|\le$$
$$\frac{p}{2p+2}\sum_{j=1}^{i-1}z_j^{2p+2}+z_i^{2p+2}\tilde{\gamma}_{i2}(\bar{z}_{i-1})+\frac{i-1}{2}\sum_{j=1}^{i-1}z_j^{4p}+$$
$$\frac{i-1}{2}z_i^{2p+4}\gamma_{i3}^2(\bar{z}_{i-1})\sum_{j=1}^{i-1}\Big(\sum_{m=j}^{i-1}\tilde{\rho}_m^2(\bar{z}_m)\Big)^2. \qquad (18)$$

最后考虑式(13)中的最后一项. 首先由 Young 不等式得

$$(p+2)\sum_{j=1}^{i-1}|z_i|^{2p+1}|z_j|^p\rho_{i1}^2(\bar{z}_i)\le$$
$$\frac{p+2}{2}\sum_{j=1}^{i-1}z_i^{2p}z_j^{2p}+\frac{p+2}{2}\sum_{j=1}^{i-1}z_i^{2p+2}\rho_{i1}^4(\bar{z}_i)\le$$
$$\frac{(p+2)(i-1)}{4}z_i^{4p}+\frac{p+2}{4}\sum_{j=1}^{i-1}z_j^{4p}+$$
$$\frac{(p+2)(i-1)}{2}z_i^{2p+2}\rho_{i1}^4(\bar{z}_i),$$
$$\frac{p+2}{2}\sum_{k,m=1}^{i-1}z_i^{p+1}|z_k|^p|z_m|^p\rho_{i1}^2(\bar{z}_i)\le$$
$$\frac{(p+2)(i-1)}{2}\sum_{j=1}^{i-1}z_i^{p+1}z_j^{2p}\rho_{i1}^2(\bar{z}_i)\le$$
$$\frac{(p+2)(i-1)}{4}\sum_{j=1}^{i-1}z_j^{4p}+$$
$$\frac{(p+2)(i-1)^2}{4}z_i^{2p+2}\rho_{i1}^4(\bar{z}_i). \qquad (19)$$

根据式(14)和(19),则

$$\frac{p+2}{2}\mathrm{Tr}[z_i^{p+1}\eta_i(\bar{x}_i)\eta_i^{\mathrm{T}}(\bar{x}_i)]\le$$
$$\frac{p+2}{2}z_i^{p+1}\Big(\sum_{j=1}^{i}|z_j|^p\rho_{i1}(\bar{z}_i)\Big)^2=$$
$$\frac{p+2}{2}z_i^{3p+1}\rho_{i1}^2(\bar{z}_i)+(p+2)\sum_{j=1}^{i-1}|z_i|^{2p+1}|z_j|^p\cdot$$
$$\rho_{i1}^2(\bar{z}_i)+\frac{p+2}{2}\sum_{k,m=1}^{i-1}z_i^{p+1}|z_k|^p|z_m|^p\rho_{i1}^2(\bar{z}_i)\le$$
$$\frac{(p+2)(i-1)}{2}z_i^{2p+2}\rho_{i1}^4(\bar{z}_i)+\frac{(p+2)(i-1)^2}{4}\cdot$$
$$z_i^{2p+2}\rho_{i1}^4(\bar{z}_i)+\frac{p+2}{2}z_i^{3p+1}\rho_{i1}^2(\bar{z}_i)+$$
$$\frac{(p+2)(i-1)}{4}z_i^{4p}+\frac{(p+2)i}{4}\sum_{j=1}^{i-1}z_j^{4p}. \qquad (20)$$

这里的函数 $\gamma_{i1}(\bar{z}_{i-1}),\gamma_{i2}(\bar{z}_{i-1}),\tilde{\gamma}_{i2}(\bar{z}_{i-1}),\gamma_{i3}(\bar{z}_{i-1})$ 均为非负光滑函数. 选取第 i 个光滑虚拟控制器

$$\alpha_i(\bar{x}_i)=-z_i\beta_i(\bar{z}_i),$$
$$\beta_i(\bar{z}_i)=$$
$$\Big(c_i+d_iz_i^{2p-2}+\gamma_{i1}(\bar{z}_{i-1})+\tilde{\gamma}_{i2}(\bar{z}_{i-1})+$$
$$\frac{(p+2)(i-1)}{2}\rho_{i1}^4(\bar{z}_i)+$$
$$\frac{(p+2)(i-1)^2}{4}\rho_{i1}^4(\bar{z}_i)+$$
$$\frac{i-1}{2}z_i^2\gamma_{i3}^2(\bar{z}_{i-1})\sum_{j=1}^{i-1}\Big(\sum_{m=j}^{i-1}m\tilde{\rho}_m^2(\bar{z}_m)\Big)^2+$$
$$\frac{p+2}{2}z_i^{p-1}\rho_{i1}^2(\bar{z}_i)+\frac{(p+2)(i-1)}{4}z_i^{2p-2}\Big)^{1/p}. \qquad (21)$$

其中

$$c_{ji}=c_{j,i-1}-\frac{p}{2p+2},$$
$$d_{ji}=d_{j,i-1}-\frac{i-1}{2}-\frac{(p+2)i}{4},$$
$$j=1,2,\cdots,i-2,$$
$$c_{i-1,i}=c_{i-1}-M_{i-1}-\frac{p}{2p+2},$$
$$d_{i-1,i}=d_{i-1}-\frac{i-1}{2}-\frac{(p+2)i}{4}.$$

将式(15),(18),(20)和(21)代入(13)得

$$LV_i(z_1,z_2,\cdots,z_i)\le$$
$$-\sum_{j=1}^{i-1}(c_{j,i}z_j^{2p+2}+d_{j,i}z_j^{4p})-$$
$$c_iz_i^{2p+2}-d_iz_i^{4p}+z_i^{p+2}(x_{i+1}^p-\alpha_i^p),$$

$c_i,d_i>0$ 为设计参数且 $\beta_i(\bar{z}_i)>0$ 是光滑函数.

© 1994-2010 China Academic Journal Electronic Publishing House. All rights reserved.　http://www.cnki.net

Step n 选取控制律

$$u = - z_n \beta_n (\bar{z}_n). \tag{22}$$

从而

$$LV_n \le - \sum_{j=1}^{n-1} (c_{jn} z_j^{2p+2} + d_{jn} z_j^{4p}) - c_n z_n^{2p+2} - d_n z_n^{4p},$$

$$c_{jn} = c_j - M_j - \frac{p(n-j)}{2p+2},$$

$$d_{jn} = d_j - \frac{1}{2}\sum_{k=j}^{n-1} k - \frac{p+2}{4}\sum_{k=j+1}^{n} k,$$

$$M_j = \frac{p+2}{2p+2} + C_p^1 \frac{p+3}{2p+2} + \cdots + C_p^{p-1} \frac{2p+1}{2p+2},\ j = 1,2,\cdots,n-1.$$

其中 :$V_n = \frac{1}{p+3}\sum_{i=1}^{n} z_i^{p+3}$;$c_n, d_n > 0$ 为设计参数. 至此,可得如下结论.

定理 1 对满足假设 1 的高阶次随机非线性系统(2),通过适当地选取设计参数 c_i 和 d_i,可使闭环随机系统(2) 和(22) 存在唯一解且在原点处的平衡点是概率意义下全局渐近稳定的.

证明 选取

$$c_j > M_j + \frac{p(n-j)}{2p+2},$$

$$d_j > \frac{1}{2}\sum_{k=j}^{n-1} k + \frac{p+2}{4}\sum_{k=j+1}^{n} k,$$

$$j = 1,2,\cdots,n-1,\ c_n > 0, d_n > 0,$$

$$c = \min\{c_{1n},\cdots,c_{n-1,n},c_n\},$$

$$d = \min\{d_{1n},\cdots,d_{n-1,n},d_n\},$$

则 $LV_n \le - c\sum_{i=1}^{n} z_i^{2p+2} - d\sum_{i=1}^{n} z_i^{4p}$. 由 V_n 的定义,据引理 2 知结论成立. □

4 结 语

本文研究了一类高阶次随机非线性系统. 在一系列增长条件下,通过适当地选取 Lyapunov 函数,设计了一个光滑的状态反馈控制器,使闭环系统在原点处的平衡点是概率意义下全局渐近稳定的. 未来的工作将深入研究高阶次随机非线性系统的输出反馈控制问题.

参考文献(References)

[1] Marino R, Tomei P. Nonlinear control design-geometric, adaptive and robust[M]. London: Prentice-Hall, 1995.

[2] Krstic M, Kanellakopoulos I, Kokotovic P V. Nonlinear and adaptive control design[M]. New York: John Wiley, 1995.

[3] Lin W, Qian C J. Adding one power integrator: A tool for global stabilization of high-order lower-triangular systems[J]. System and Control Letters, 2000, 39(5): 339-351.

[4] Lin W, Qian C J. Adaptive regulation of high-order cascade nonlinear systems: An adding one power integrator approach[J]. System and Control Letters, 2000, 39(5): 353-364.

[5] Qian C J, Lin W. Almost disturbance decoupling for a class of high-order nonlinear systems[J]. IEEE Trans on Automatic Control, 2000, 45(6): 1209-1214.

[6] Qian C J, Lin W. Non-Lipschitz continuous stabilizers for nonlinear systems with uncontrollable unstable linearization[J]. System and Control Letters, 2001, 42(3): 185-200.

[7] Has'minskii R Z. Stochastic stability of differential equations [M]. Rockville, Maryland: S & N International Publisher, 1980.

[8] Kushner H J. Stochastic stability and control[M]. New York: Academic Press, 1967.

[9] Pan Z, Basar T. Backstepping controller design for nonlinear stochastic systems under a risk-sensitive cost criterion[J]. SIAM J of Control and Optimization, 1999, 37(3): 957-995.

[10] Krstic M, Deng H. Stability of nonlinear uncertain systems[M]. New York: Springer, 1998.

[11] Deng H, Krstic M. Output-feedback stochastic nonlinear stabilization[J]. IEEE Trans on Automatic Control, 1999, 44(2): 328-333.

[12] Wu Z J, Xie X J. Adaptive backstepping controller design using stochastic small-gain theorem [J]. Automatica, 2007, 43(4): 608-620.

[13] Wu Z J, Xie X J, Zhang S Y. Stochastic adaptive backstepping controller design by introducing dynamic signal and changing supply function [J]. Int J of Control, 2006, 79(12): 1635-1646.

© 1994-2010 China Academic Journal Electronic Publishing House. All rights reserved. http://www.cnki.net