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模糊奇异摄动系统 H_∞ 滤波

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摘 要: 针对一类非线性奇异摄动系统,建立基于 T-S 模糊模型的模糊奇异摄动系统模型,通过 Lyapunov 方法和 Schur 补定理,研究其 H_∞ 滤波问题. 将系统 H_∞ 滤波器设计归结为求解一组与摄动参数 ε 无关的线性矩阵不等式,从而避免由 ε 引起数值求解的病态问题. 所获得的滤波器使闭环系统渐近稳定并能达到给定的 H_∞ 性能指标. 该方法适用于标准和非标准非线性奇异摄动系统,仿真实例表明了所提出方法的有效性.

关键词: 模糊奇异摄动; H_∞ 滤波; 线性矩阵不等式; T-S 模糊模型
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H_∞ filtering for fuzzy singularly perturbed systems

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Abstract: To a class of nonlinear singularly perturbed systems, a model of fuzzy singularly perturbed systems based on T-S model is built. The problem of H_∞ filtering for the system is investigated in by Lyapunov approach and Schur complement theorem. The design of the H_∞ filter is solved by a set of the ε independent linear matrix inequalities, so the ill-conditioned problem is avoided effectively. The filter obtained enables the closed-loop systems to be stable and to achieve the given H_∞ performance. The proposed approach can be applied to both standard and nonstandard nonlinear singularly perturbed systems. A simulation example is provided to illustrate the approaches.

Key words: Fuzzy singularly perturbation; H_∞ filtering; Linear matrix inequalities (LMIs); T-S model

1 引 言

非线性奇异摄动系统是多时标非线性系统,由于具有病态动力学特征,其控制问题的研究较一般的常规非线性系统更为复杂[1]. 然而,实际工业工程中存在着大量多时标系统,使得这类问题的研究非常具有实际意义. 近年来,随着机器人控制以及制造业、流程工业等朝着高效、高质量方向发展,线性奇异摄动技术得到了深入研究. 文献[2-5]分别应用奇异摄动技术,研究了移动机器人非理想的非完整运动学约束、受限机械臂建模以及多连杆柔性臂的建模与输出反馈控制;文献[6]通过建立蒸汽锅炉的奇异摄动模型,近似消除了蒸汽产生系统与蒸汽加热系统的耦合,降低了系统维数,但非线性奇异摄动系统的研究[7-10]对系统结构的假设还较多.

目前,随着模糊控制理论的长足发展,模糊控制理论与非线性奇异摄动系统相结合的研究有了一些进展. 文献[11]初步研究了基于非线性奇异摄动系统设计 T-S 控制器的方法,但缺乏理论分析. [12-14]提出了模糊奇异摄动系统的概念,建立了连续模糊奇异摄动系统模型,应用线性矩阵不等式(LMI)方法研究了一类模糊奇异摄动系统的稳定性与状态反馈控制问题. 然而,应用模糊控制理论研究非线性奇异摄动系统的 H_∞ 控制问题的文献尚不多见. [1]提出采用同伦算法求解双线性矩阵不等式(BMI)的方法,研究了模糊奇异摄动系统静态输出反馈 H_∞ 控制问题,但由于同论算法不是一种全局收敛算法,只能获得局部最优解. [15,16]尽管用 LMI 方法研究了非线性奇异摄动系统模糊 H_∞ 滤波器和模糊 H_∞ 动态输出反馈控制,但只研究了矩阵 P 为对角阵的情况,所得结论具有一定的保守性.

本文基于一类非线性奇异摄动系统,建立基于 T-S 模糊模型的连续模糊奇异摄动系统模型,借鉴

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文献[17]中的方法,采用LMI方法研究一类模糊奇异摄动系统 H_∞ 滤波器的设计问题.

2 模糊奇异摄动系统模型

模糊奇异摄动系统第 i 条规则为如下形式:

Plant rule i:

If $\xi_1(t)$ is Φ_{i1} and $\cdots$ and $\xi_g(t)$ is Φ_{ig};

Then

$$E(\varepsilon)\dot{x}(t) = A_i x(t) + B_{1i} w(t), x(0) = 0;$$
$$z(t) = C_{1i} x(t);$$
$$y(t) = C_{2i} x(t) + D_{21i} w(t);$$
$$i = 1, 2, \cdots, r. \tag{1}$$

其中: $E(\varepsilon) = \begin{bmatrix} I_{n\times n} & 0 \\ 0 & \varepsilon I_{m\times m} \end{bmatrix}$, $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, 且 $x_1(t) \in R^n$ 和 $x_2(t) \in R^m$ 为慢、快状态向量; $w(t) \in R^p$ 为噪声信号(包括过程和测量噪声); $y(t) \in R^h$ 为测量输出; $z(t) \in R^l$ 为待估计的信号向量, Φ_{ij} $(j = 1, 2, \cdots, g)$ 为模糊集合; r 为规则数目; A_i, B_{1i}, C_{1i}, C_{2i}, D_{21i} 为适当维数的矩阵; $\xi_1(t), \cdots, \xi_g(t)$ 为可测变量; $0 < \varepsilon \ll 1$ 为摄动参数.

利用标准的模糊推理方法,即单点模糊化、乘积推理和加权平均清晰化,可得全局模糊模型为

$$E(\varepsilon)\dot{x}(t) =$$
$$A(\mu) x(t) + B_1(\mu) w(t), \quad x(0) = 0;$$
$$z(t) = C_1(\mu) x(t);$$
$$y(t) = C_2(\mu) x(t) + D_{21}(\mu) w(t). \tag{2}$$

其中

$$\mu_i(\xi(t)) = \frac{w_i(\xi(t))}{\sum_{i=1}^{r} w_i(\xi(t))},$$
$$w_i(\xi(t)) = \prod_{j=1}^{g} \Phi_{ij}(\xi_j(t)),$$
$$A(\mu) = \sum_{i=1}^{r} \mu_i A_i, B_1(\mu) = \sum_{i=1}^{r} \mu_i B_{1i},$$
$$C_1(\mu) = \sum_{i=1}^{r} \mu_i C_{1i}, C_2(\mu) = \sum_{i=1}^{r} \mu_i C_{2i},$$
$$D_{21}(\mu) = \sum_{i=1}^{r} \mu_i D_{21i}, E(\varepsilon) = \begin{bmatrix} I_{n\times n} & 0 \\ 0 & \varepsilon I_{m\times m} \end{bmatrix},$$
$$x(t) \in R^{(n+m)}, \mu_i = \mu_i(\xi(t)).$$

对于系统(2),如果应用常规模糊系统理论和Lyapunov方法,则可得出一组依赖于小参数 ε 的稳定性条件. 但由于 $\varepsilon \ll 1$,系统矩阵会存在严重的病态特性. 目前通用的一些LMI求解工具对矩阵的条件数都较为敏感[18],故这类病态LMIs不适于用常规LMI工具求解. 对于稳定性分析问题如此,对于综合问题亦如此.

下面提出系统(2)不依赖于小参数 ε 的稳定性分析与 H_∞ 滤波方法.

3 H_∞ 滤波器设计

假设1 系统(2)是渐近稳定的.

对于系统(2),采用如下的控制规律设计 H_∞ 滤波器:

$$E(\varepsilon)\dot{\hat{x}}(t) = A_f(\mu)\hat{x}(t) + B_f(\mu) y(t),$$
$$\hat{z}(t) = C_f(\mu)\hat{x}(t). \tag{3}$$

其中

$$\mu_i(\xi(t)) = \frac{w_i(\xi(t))}{\sum_{i=1}^{r} w_i(\xi(t))},$$
$$w_i(\xi(t)) = \prod_{j=1}^{g} \Phi_{ij}(\xi_j(t)),$$
$$\mu_i = \mu_i(\xi(t)), A_f(\mu) = \sum_{i=1}^{r} \mu_i A_{fi},$$
$$B_f(\mu) = \sum_{i=1}^{r} \mu_i B_{fi}, C_f(\mu) = \sum_{i=1}^{r} \mu_i C_{fi},$$
$$\hat{x}(t) = \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \end{bmatrix},$$

$\hat{x}_1(t) \in R^n$ 和 $\hat{x}_2(t) \in R^m$ 分别为滤波器的慢、快状态向量.

滤波器规则 i:

If $\xi_1(t)$ is Φ_{i1} and $\cdots$ and $\xi_g(t)$ is Φ_{ig};

Then

$$E(\varepsilon)\dot{\hat{x}}(t) = A_{fi}\hat{x}(t) + B_{fi} y(t),$$
$$\hat{z}(t) = C_{fi}\hat{x}(t).$$

将滤波器(3)应用于系统(2),可得闭环系统

$$E_{cl}(\varepsilon)\dot{\tilde{x}}(t) =$$
$$A_{cl}(\mu, \mu)\tilde{x}(t) + B_{cl}(\mu, \mu) w(t), \tilde{x}(0) = 0;$$
$$\tilde{z}(t) = C_{cl}(\mu, \mu)\tilde{x}(t). \tag{4}$$

其中

$$E_{cl}(\varepsilon) = \begin{bmatrix} E(\varepsilon) & 0 \\ 0 & E(\varepsilon) \end{bmatrix},$$
$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}, \tilde{z}(t) = z(t) - \hat{z}(t),$$
$$A_{cl}(\mu, \mu) = \begin{bmatrix} A(\mu) & 0 \\ B_f(\mu) C_2(\mu) & A_f(\mu) \end{bmatrix},$$
$$B_{cl}(\mu, \mu) = \begin{bmatrix} B_1(\mu) \\ B_f(\mu) D_{21}(\mu) \end{bmatrix},$$
$$C_{cl}(\mu, \mu) = [C_1(\mu) - C_f(\mu)].$$

定理1 对于任意给定常数 $\gamma > 0$,若存在矩阵 $X = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}$ 和 $Y = \begin{bmatrix} Y_{11} & 0 \\ Y_{21} & Y_{22} \end{bmatrix}$,其中 $X_{11} \in$

$R^{n\times n}$, $X_{22}\in R^{m\times m}$, $Y_{11}\in R^{n\times n}$, $Y_{22}\in R^{m\times m}$ 为对称正定矩阵,以及适当维数矩阵 M_i, N_i, J_i 满足如下不等式:

$$\Pi_{ii}<0,\ i=1,2,\cdots,r; \tag{5a}$$

$$\Pi_{ij}<0,\ i\neq j,\ i=1,2,\cdots,r,\ j=1,2,\cdots,r; \tag{5b}$$

$$X_{11}-Y_{11}>0; \tag{5c}$$

$$X_{22}-Y_{22}>0. \tag{5d}$$

其中

$$\Pi_{ij}=\begin{bmatrix} YA_i+A_i^{\mathrm T}Y^{\mathrm T} & * & * & * \\ A_i^{\mathrm T}Y+X^{\mathrm T}A_i+J_iC_{2j}+M_j & \Delta_{22} & * & * \\ B_{1i}^{\mathrm T}Y & B_{1i}^{\mathrm T}X+D_{21i}^{\mathrm T}J_j^{\mathrm T} & -I & * \\ C_{1i}-N_j & C_{1i} & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Delta_{22}=X^{\mathrm T}A_i+A_i^{\mathrm T}X+J_iC_{2j}+(J_iC_{2j})^{\mathrm T}.$$

则存在 ε^*, $0<\varepsilon^*\ll 1$, 对于 $\forall\varepsilon\in(0,\varepsilon^*]$, 模糊奇异摄动系统(2),存在 γ-次优 H_∞ 滤波器(3),使构成的闭环系统(4)渐近稳定,并具有给定的 H_∞ 性能指标 γ. 此时,控制器参数取为

$$A_{fi}=(Y-X)^{-1}M_i,$$
$$B_{fi}=(Y-X)^{-1}J_i,$$
$$C_{fi}=N_i. \tag{6}$$

证明　对于充分小的 $0<\varepsilon\ll 1$, 定义正定矩阵

$$P_\varepsilon=\begin{bmatrix} X_\varepsilon & P_{12} \\ P_{12} & L_\varepsilon \end{bmatrix},$$

其中

$$X_\varepsilon=\begin{bmatrix} X_{11} & \varepsilon X_{21}^{\mathrm T} \\ X_{21} & X_{22} \end{bmatrix},$$

$$P_{12}=\begin{bmatrix} P_{121} & 0 \\ 0 & P_{122} \end{bmatrix},\quad L_\varepsilon=\begin{bmatrix} L_{11} & \varepsilon L_{21}^{\mathrm T} \\ L_{21} & L_{22} \end{bmatrix},$$

且 $X_{11}\in R^{n\times n}$, $X_{22}\in R^{m\times m}$, $L_{11}\in R^{n\times n}$, $L_{22}\in R^{m\times m}$ 分别为对称正定矩阵, $X_{21}\in R^{m\times n}$ 和 $L_{21}\in R^{m\times n}$ 为任意矩阵, $P_{121}\in R^{n\times n}$ 和 $P_{122}\in R^{m\times m}$ 为对称矩阵. 则 $\exists\varepsilon_0^*>0$, 对于 $\forall\varepsilon\in(0,\varepsilon_0^*]$, 矩阵 P_ε 满足

$$E_{cl}(\varepsilon)P_\varepsilon=P_\varepsilon^{\mathrm T}E_{cl}(\varepsilon)>0.$$

可构造 Lyapunov 函数

$$V(\tilde{x}(t))=\tilde{x}^{\mathrm T}(t)E_{cl}(\varepsilon)P_\varepsilon\tilde{x}(t)>0,$$

$$\begin{aligned}
&\dot V(\tilde{x}(t))=\\
&\dot{\tilde{x}}^{\mathrm T}(t)E_{cl}(\varepsilon)P_\varepsilon\tilde{x}(t)+\tilde{x}^{\mathrm T}(t)E_{cl}(\varepsilon)P_\varepsilon\dot{\tilde{x}}(t)=\\
&\dot{\tilde{x}}^{\mathrm T}(t)[A_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon+P_\varepsilon^{\mathrm T}A_{cl}(\mu,\hat\mu)]\tilde{x}(t)+\\
&w^{\mathrm T}(t)B_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon\tilde{x}(t)+\tilde{x}^{\mathrm T}(t)P_\varepsilon^{\mathrm T}B_{cl}(\mu,\hat\mu)w(t)=\\
&\tilde{x}^{\mathrm T}(t)[A_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon+P_\varepsilon^{\mathrm T}A_{cl}(\mu,\hat\mu)]\tilde{x}(t)+\\
&w^{\mathrm T}(t)B_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon\tilde{x}(t)+\tilde{x}^{\mathrm T}(t)P_\varepsilon^{\mathrm T}B_{cl}(\mu,\hat\mu)w(t)-\\
&w^{\mathrm T}(t)w(t)+w^{\mathrm T}(t)w(t).
\end{aligned}$$

在等式两边加 $\gamma^2\tilde{z}^{\mathrm T}(t)\tilde{z}(t)$, $\gamma>0$ 为任意常数,然后对于任意 $T>0$, 等式两边积分得

$$\begin{aligned}
&V(\tilde{x}(T))+\gamma^2\int_0^T\tilde{z}^{\mathrm T}(t)\tilde{z}(t)\mathrm{d}t=\\
&\int_0^T\left\{\begin{bmatrix}\tilde{x}(t)\\ w(t)\end{bmatrix}^{\mathrm T}\begin{bmatrix} W & P_\varepsilon^{\mathrm T}B_{cl}(\mu,\hat\mu) \\ B_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon & -I\end{bmatrix}\begin{bmatrix}\tilde{x}(t)\\ w(t)\end{bmatrix}\right\}\mathrm{d}t+\\
&\int_0^T w^{\mathrm T}(t)w(t)\mathrm{d}t,
\end{aligned}$$

其中 $W=A_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon+P_\varepsilon^{\mathrm T}A_{cl}(\mu,\hat\mu)+\gamma^2C_{cl}^{\mathrm T}(\mu,\hat\mu)C_{cl}(\mu,\hat\mu)$.

$V(\tilde{x}(T))>0$, 因此当且仅当矩阵 P_ε 满足如下不等式时:

$$\begin{bmatrix} W & * \\ B_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon & -I\end{bmatrix}<0, \tag{7}$$

有

$$\int_0^T\tilde{z}^{\mathrm T}(t)\tilde{z}(t)\mathrm{d}t\le\gamma^2\int_0^T w^{\mathrm T}(t)w(t)\mathrm{d}t\le\gamma^2\int_0^\infty w^{\mathrm T}(t)w(t)\mathrm{d}t$$

成立. 显然,可将不等式(7)改写为

$$\begin{aligned}
&\begin{bmatrix} W & * \\ B_{cl}^{\mathrm T}(\mu,\hat\mu)P_\varepsilon & -I\end{bmatrix}=\\
&\begin{bmatrix} A_{cl}^{\mathrm T}(\mu,\hat\mu)P+P^{\mathrm T}A_{cl}(\mu,\hat\mu)+\gamma^2C_{cl}^{\mathrm T}(\mu,\hat\mu)C_{cl}(\mu,\hat\mu) & * \\ B_{cl}^{\mathrm T}(\mu,\hat\mu)P & -I\end{bmatrix}+\\
&O(\varepsilon)<0.
\end{aligned} \tag{8}$$

其中

$$P=\begin{bmatrix} X & P_{12} \\ P_{12} & L\end{bmatrix},\ X=\begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22}\end{bmatrix},$$

$$L=\begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22}\end{bmatrix},\ P_{12}=\begin{bmatrix} P_{121} & 0 \\ 0 & P_{122}\end{bmatrix};$$

$X_{11}\in R^{n\times n}$, $X_{22}\in R^{m\times m}$, $L_{11}\in R^{n\times n}$, $L_{22}\in R^{m\times m}$ 为对称正定矩阵; $P_{121}\in R^{n\times n}$ 和 $P_{122}\in R^{m\times m}$ 为对称矩阵.

根据文献[19,20]中的结论,可知 $\exists\varepsilon_1^*>0$, 对于 $\forall\varepsilon\in(0,\varepsilon_1^*]$, 当如下不等式成立时:

$$\begin{bmatrix} A_{cl}^{\mathrm T}(\mu,\hat\mu)P+P^{\mathrm T}A_{cl}(\mu,\hat\mu)+\gamma^2C_{cl}^{\mathrm T}(\mu,\hat\mu)C_{cl}(\mu,\hat\mu) & * \\ B_{cl}^{\mathrm T}(\mu,\hat\mu)P & -I\end{bmatrix}<0, \tag{9}$$

不等式(8)成立.

对于不等式(9),应用 Schur 补原理,可得

$$\begin{bmatrix} A_{cl}^{\mathrm T}(\mu,\hat\mu)P+P^{\mathrm T}A_{cl}(\mu,\hat\mu) & * & * \\ B_{cl}^{\mathrm T}(\mu,\hat\mu)P & -I & * \\ C_{cl}(\mu,\hat\mu) & 0 & -\gamma^2 I\end{bmatrix}<0, \tag{10}$$

其中 $P=\begin{bmatrix} X & P_{12} \\ P_{12} & L\end{bmatrix}$. 令 $P^{-1}=\begin{bmatrix} Z & S_{21} \\ S_{12} & S_{22}\end{bmatrix}$, 则

$$P\begin{bmatrix} Z & I \\ S_{12} & 0 \end{bmatrix} = \begin{bmatrix} I & X \\ 0 & P_{12} \end{bmatrix}.$$

其中: $Z = \begin{bmatrix} Z_{11} & 0 \\ Z_{21} & Z_{22} \end{bmatrix}$, $Z_{11} \in R^{n\times n}$ 和 $Z_{22} \in R^{m\times m}$ 为对称正定矩阵; $S_{12} = \begin{bmatrix} S_{121} & 0 \\ S_{122} & S_{123} \end{bmatrix}$, $S_{121} \in R^{n\times n}$ 和 $S_{123} \in R^{m\times m}$ 为任意阵.

令 $E_1 = \begin{bmatrix} Z & I \\ S_{12} & 0 \end{bmatrix}$, $E_2 = \begin{bmatrix} I & X \\ 0 & P_{12} \end{bmatrix}$, 则对于不等式(10) 左边的矩阵, 分别左乘 $\mathrm{diag}\{E_1^{\mathrm T}, I, I\}$, 右乘 $\mathrm{diag}\{E_1, I, I\}$, 并结合式(4), 可得矩阵不等式

$$\begin{bmatrix} A(\mu)Z + Z^{\mathrm T}A^{\mathrm T}(\mu) & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * \\ B_1^{\mathrm T}(\mu) & \begin{matrix} B_1^{\mathrm T}(\mu)X + \\ D_{21}^{\mathrm T}(\mu)B_f^{\mathrm T}(\mu)P_{12} \end{matrix} & -I & * \\ C_1(\mu)Z - C_f(\mu)S_{12} & C_1(\mu) & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{11}$$

其中

$$\begin{aligned} \Gamma_{21} = {} & A^{\mathrm T}(\mu) + X^{\mathrm T}A(\mu)Z + \\ & P_{12}^{\mathrm T}B_f(\mu)C_2(\mu)Z + P_{12}^{\mathrm T}A_f(\mu)S_{12}, \\ \Gamma_{22} = {} & X^{\mathrm T}A(\mu) + A^{\mathrm T}(\mu)X + \\ & P_{12}^{\mathrm T}B_f(\mu)C_2(\mu) + C_2^{\mathrm T}(\mu)B_f^{\mathrm T}(\mu)P_{12}. \end{aligned}$$

令

$$\begin{aligned} J(\mu) &= P_{12}^{\mathrm T}B_f(\mu), \\ \tilde J(\mu) &= C_f(\mu)S_{12}, \\ \hat J(\mu) &= P_{12}^{\mathrm T}A_f(\mu)S_{12}, \end{aligned} \tag{12}$$

则矩阵不等式(11) 可改写为

$$\begin{bmatrix} A(\mu)Z + Z^{\mathrm T}A^{\mathrm T}(\mu) & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * \\ B_1^{\mathrm T}(\mu) & \begin{matrix} B_1^{\mathrm T}(\mu)X + \\ D_{21}^{\mathrm T}(\mu)J^{\mathrm T}(\mu) \end{matrix} & -I & * \\ C_1(\mu)Z - \tilde J(\mu) & C_1(\mu) & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{13}$$

其中

$$\begin{aligned} \Psi_{21} = {} & A^{\mathrm T}(\mu) + X^{\mathrm T}A(\mu)Z + \\ & J(\mu)C_2(\mu)Z + \hat J(\mu), \\ \Psi_{22} = {} & X^{\mathrm T}A(\mu) + A^{\mathrm T}(\mu)X + \\ & J(\mu)C_2(\mu) + C_2^{\mathrm T}(\mu)J^{\mathrm T}(\mu). \end{aligned}$$

对于不等式(13) 左边的矩阵, 左乘 $\mathrm{diag}\{Z^{-T}, I, I, I\}$, 右乘 $\mathrm{diag}\{Z^{-1}, I, I, I\}$, 并对所得不等式, 令 $Z^{-1} = Y$, $M(\mu) = \hat J(\mu)Y$, $N(\mu) = \hat J(\mu)Y$, 可得如下不等式:

$$\begin{bmatrix} YA(\mu) + A^{\mathrm T}(\mu)Y & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ B_1^{\mathrm T}(\mu)Y & \begin{matrix} B_1^{\mathrm T}(\mu)X + \\ D_{21}^{\mathrm T}(\mu)J^{\mathrm T}(\mu) \end{matrix} & -I & * \\ C_1(\mu) - N(\mu) & C_1(\mu) & 0 & -\gamma^2 I \end{bmatrix} < 0. \tag{14}$$

其中

$$\begin{aligned} \Omega_{21} = {} & A^{\mathrm T}(\mu)Y + X^{\mathrm T}A(\mu) + \\ & J(\mu)C_2(\mu) + M(\mu), \\ \Omega_{22} = {} & X^{\mathrm T}A(\mu) + A^{\mathrm T}(\mu)X + \\ & J(\mu)C_2(\mu) + C_2^{\mathrm T}(\mu)J^{\mathrm T}(\mu). \end{aligned}$$

由式(12) 得

$$\begin{aligned} A_f(\mu) &= P_{12}^{-\mathrm T}\hat J(\mu)S_{12}^{-1}, \\ B_f(\mu) &= P_{12}^{-T}J(\mu), \\ C_f(\mu) &= \tilde J(\mu)S_{12}^{-1}. \end{aligned}$$

因此, 滤波器(3) 的传函矩阵为

$$\begin{aligned} & H_{\hat z y}(s) = \\ & \tilde J(\mu)S_{12}^{-1}[sI - P_{12}^{-\mathrm T}\hat J(\mu)S_{12}^{-1}]^{-1}P_{12}^{-\mathrm T}J(\mu) = \\ & \tilde J(\mu)[s(I - XZ) - \hat J(\mu)]^{-1}J(\mu) = \\ & N(\mu)[sI - (Y - X)^{-1}M(\mu)]^{-1}(Y - X)^{-1}J(\mu). \end{aligned}$$

由此可得

$$\begin{aligned} A_f(\mu) &= (Y - X)^{-1}M(\mu), \\ B_f(\mu) &= (Y - X)^{-1}J(\mu), \\ C_f(\mu) &= N(\mu). \end{aligned}$$

根据式(2) 和(3), 可将不等式(14) 改写为

$$\sum_{i=1}^{r}\mu_i\mu_i\Pi_{ii} + \sum_{i,j=1,i\neq j}^{r}\mu_i\mu_j\Pi_{ij} < 0.$$

因此, $\exists\varepsilon^* > 0$, $\varepsilon^* = \min\{\varepsilon_0^*, \varepsilon_1^*\}$, 对于 $\forall\varepsilon \in (0, \varepsilon^*]$, 线性矩阵不等式(5a) 和(5b) 成立时, 矩阵不等式(8) 成立.

另外, 由 $E_d(\varepsilon)P_\varepsilon > 0$, 可得

$$\begin{aligned} & E_1^{\mathrm T}E_d(\varepsilon)P_\varepsilon E_1 = \\ & \begin{bmatrix} \begin{matrix} Z^{\mathrm T}E(\varepsilon)(X_\varepsilon Z + P_{12}S_{12}) + \\ S_{12}^{\mathrm T}E(\varepsilon)(P_{12}Z + L_\varepsilon S_{12}) \end{matrix} & (Z^{\mathrm T}X_\varepsilon^{\mathrm T} + S_{12}^{\mathrm T}P_{12})E(\varepsilon) \\ E(\varepsilon)(X_\varepsilon Z + P_{12}S_{12}) & X_\varepsilon^{\mathrm T}E(\varepsilon) \end{bmatrix} = \\ & \begin{bmatrix} \begin{matrix} Z^{\mathrm T}E(\varepsilon)(XZ + P_{12}S_{12}) + \\ S_{12}^{\mathrm T}E(\varepsilon)(P_{12}Z + LS_{12}) \end{matrix} & (Z^{\mathrm T}X^{\mathrm T} + S_{12}^{\mathrm T}P_{12})E(\varepsilon) \\ E(\varepsilon)(XZ + P_{12}S_{12}) & X^{\mathrm T}E(\varepsilon) \end{bmatrix} + \\ & O(\varepsilon) > 0. \end{aligned} \tag{15}$$

根据 $PP^{-1} = I$ 和文献[19,20] 的结论, 可知 $\exists\varepsilon_2^* > 0$, 对于 $\forall\varepsilon \in (0, \varepsilon_2^*]$, 当不等式

$$\begin{bmatrix} Z^{\mathrm T}E(\varepsilon) & E(\varepsilon) \\ E(\varepsilon) & X^{\mathrm T}E(\varepsilon) \end{bmatrix} > 0 \tag{16}$$

成立时, 不等式(15) 成立.

应用 Schur 补定理, 并将 $Y = Z^{-1}$ 代入, 得

$$Y_{11} = Y_{11}^{\mathrm T} > 0, \tag{17a}$$

$$Y_{22} = Y_{22}^{\mathrm T} > 0, \tag{17b}$$

$$X_{11} - Y_{11} > 0, \tag{17c}$$

$$X_{22} - Y_{22} > 0. \tag{17d}$$

因此, $\forall\varepsilon^* > 0$, $\varepsilon^* = \min\{\varepsilon_0^*, \varepsilon_1^*, \varepsilon_2^*\}$, 对于 $\forall\varepsilon \in (0, \varepsilon^*]$, 闭环系统(4) 满足给定 H_∞ 性能指标 γ.

当 $w(t) \equiv 0$ 时, 有

$$\dot V(\tilde x(t)) =$$

$$\tilde{x}^{\mathrm{T}}(t)[A_d^{\mathrm{T}}(\mu,\mu)P_\varepsilon + P_\varepsilon^{\mathrm{T}}A_d(\mu,\mu)]\tilde{x}(t).$$

根据文献[19,20]的结论和不等式(10),得 $\dot{V}(\tilde{x}(t)) < 0$,闭环系统(4) 渐近稳定. □

矩阵不等式(5) 是与 ε 无关的线性矩阵不等式组,因此可应用LMI工具箱中的求解器feasp 求解. 若有可行解,则根据定理1可构造系统(2) 的一个 γ-次优 H_∞ 滤波器. 考虑求解如下优化问题:

$$\min\ \gamma.$$
$$\text{s.t}$$
$$\Pi_{ii} < 0,\ i = 1,2,\cdots,r; \tag{18a}$$
$$\Pi_{ij} < 0,\ i \neq j,\ i = 1,2,\cdots,r,\ j = 1,2,\cdots,r; \tag{18b}$$
$$X_{11} - Y_{11} > 0; \tag{18c}$$
$$X_{22} - Y_{22} > 0. \tag{18d}$$

如果该问题有解,则结合定理1,利用其最优解可得到系统(2) 的最优 H_∞ 滤波器. 本文应用LMI中的求解器mincx 求解.

4　仿真实例

采用文献[16] 中的由2条规则描述的模糊奇异摄动系统

R1: If $x_1(t)$ is $\Phi_1(x_1(t))$,

Then

$$E(\varepsilon)\dot{x}(t) = A_1x(t) + B_{11}w(t),$$
$$z(t) = C_{11}x(t),$$
$$y(t) = C_{21}x(t) + D_{211}w(t);$$

R2: If $x_1(t)$ is $\Phi_2(x_1(t))$,

Then

$$E(\varepsilon)\dot{x}(t) = A_2x(t) + B_{12}w(t),$$
$$z(t) = C_{12}x(t),$$
$$y(t) = C_{22}x(t) + D_{212}w(t).$$

其中

$$A_1 = \begin{bmatrix} -0.1 & 10 \\ -1 & -1 \end{bmatrix},\ B_{11} = B_{12} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$
$$C_{11} = C_{12} = [1\quad 0],\ D_{121} = D_{122} = 0.1,$$
$$A_2 = \begin{bmatrix} -4.6 & 10 \\ -1 & -1 \end{bmatrix},\ C_{21} = C_{22} = [0\quad 1],$$
$$D_{211} = D_{212} = 0.1,\ E(\varepsilon) = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}.$$

对应模糊集合的隶属度函数为

$$\mu_1(x_1(t)) = 0.5x_1^2(t),$$
$$\mu_2(x_1(t)) = 1 - \mu_1(x_1(t)).$$

应用LMI工具箱及其求解器mincx,求解不等式(18a)～(18d),得到最小扰动参数 $\gamma = 4$ 时,实例系统最优 H_∞ 滤波器参数如下:

$$A_{f1} = \begin{bmatrix} -2.9140 & 14.5957 \\ -0.3043 & -5.4658 \end{bmatrix},$$
$$A_{f2} = \begin{bmatrix} -2.8214 & 14.7454 \\ -0.3313 & -5.5240 \end{bmatrix},$$
$$B_{f1} = \begin{bmatrix} -1.7707 \\ 0.4091 \end{bmatrix},\ B_{f2} = \begin{bmatrix} -1.4588 \\ 0.3738 \end{bmatrix},$$
$$C_{f1} = [0.9874\quad 1.4474],$$
$$C_{f2} = [0.8135\quad 0.8205].$$

则系统最优 H_∞ 滤波器为

$$E(\varepsilon)\dot{\hat{x}}(t) = \sum_{i=1}^{2}\mu_i(\hat{x}_1(t))A_{fi}\hat{x}(t) + \sum_{i=1}^{2}\mu_i(\hat{x}_1(t))B_{fi}y(t),$$
$$\hat{z}(t) = \sum_{i=1}^{2}\mu_i(\hat{x}_1(t))C_{fi}\hat{x}(t).$$

其中

$$\mu_1(\hat{x}_1(t)) = 0.5\hat{x}_1^2(t),$$
$$\mu_2(\hat{x}_1(t)) = 1 - \mu_1(\hat{x}_1(t)).$$

以初始条件 $x(0) = [1,0,0,0]^{\mathrm{T}}$ 和 $w(t) = \sin(\pi t)e^{-0.1t}$ 进行仿真,图1～图4分别为对应 $\varepsilon = 0.0001$ 和 $\varepsilon = 0.2$ 时的闭环系统响应. 仿真结果表明,对于充分小的 ε,系统能够获得满意的性能.

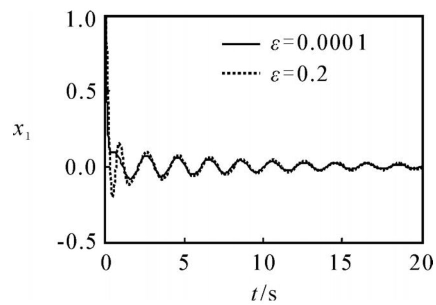

图1　$\varepsilon = 0.0001$,$\varepsilon = 0.2$ 时状态变量 $x_1(t)$

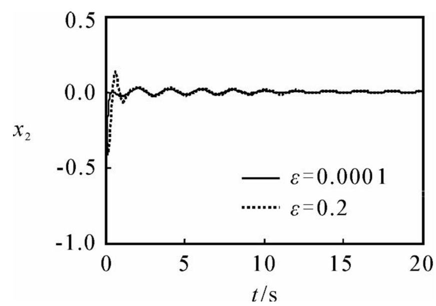

图2　$\varepsilon = 0.0001$,$\varepsilon = 0.2$ 时状态变量 $x_2(t)$

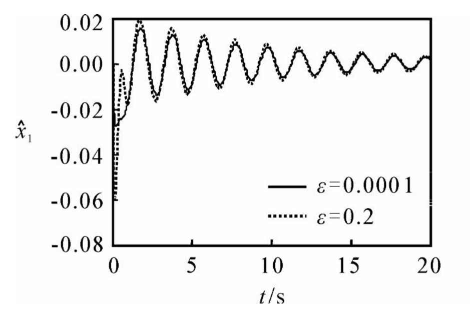

图3　$\varepsilon = 0.0001$,$\varepsilon = 0.2$ 时状态变量 $\hat{x}_1(t)$

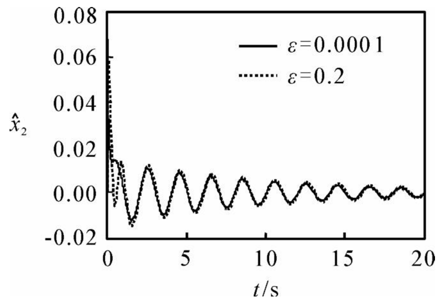

图 4 $\varepsilon = 0.0001, \varepsilon = 0.2$ 时状态变量 $\hat{x}_2(t)$

5 结 论

本文建立了模糊奇异摄动系统模型,用 Lyapunov 方法和 Schur 补定理研究了模糊奇异摄动系统 H_∞滤波问题. 将系统 H_∞滤波器设计转化为求解一组 LMIs,并利用 LMI 工具箱中的求解器求解. 求解过程与摄动参数 ε 无关,从而避免了数值求解过程中的病态问题,而且能获得稳定的数值解. 所得滤波器使闭环模糊奇异摄动系统渐近稳定,而且具有给定的 H_∞性能指标. 设计方法无需实现快慢分解,适用于标准和非标准非线性奇异摄动系统.

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