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具有时滞和的连续系统的时滞相关稳定性

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摘 要: 针对一类状态向量中含有时滞和的连续系统,研究其时滞相关稳定性问题. 通过构造新的 Lyapunov 函数, 获得了保证系统稳定、基于线性矩阵不等式的时滞相关充分条件,该条件不需要对原系统进行模型变换. 数值算例表明,所得到的结论较已往文献具有较小的保守性.

关键词: 时滞和; 时滞相关; 线性矩阵不等式
中图分类号: TP13 **文献标识码**: A

Delay-dependent stability for systems with two additive time-varying delay components

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Abstract: The delay-dependent stability for continuous systems with two additive time-varying delay components is studied. By introducing a new Lyapunov function, an LMI approach is used to derive a delay-dependent sufficient condition which can guarantee the system asymptotically stable. The model transformation is unnecessary. An example shows that the conclusion obtained has less conservativeness than the existing ones.

Key words: Additive delay components; Delay-dependent; LMI

1 引 言

时滞现象经常出现在各种工程、生物和经济系统中,常常导致系统不稳定,影响系统的性能,因而引起了众多学者的广泛关注. 常见的时滞系统为

$$\dot{x}(t) = Ax(t) + Bx(t - d(t)). \quad (1)$$

其中 $d(t)$ 为系统的状态时滞,满足

$$0 \le d(t) \le \bar{d} < \infty, \quad \dot{d}(t) \le \tau < \infty.$$

许多关于时滞系统的研究成果大多是基于系统(1)提出的[1-4]. 在这种系统中,状态时滞是一种单一或简单的形式. 然而在实际应用中,有时信号从一点传送到另一点可能要经过一些网络节点,而这些节点在网络传播过程中可能导致一些具有不同特性的时滞出现,例如图 1 所示的网络化控制系统.

图 1 中存在 2 个时滞:传感器与控制器之间的通信时滞 $d_s(t)$; 控制器与执行器之间的通信时滞 $d_a(t)$. 在网络传播过程中 2 个时滞的特性并不完全相同,因而不能将其合并一起进行研究. 当被控对象

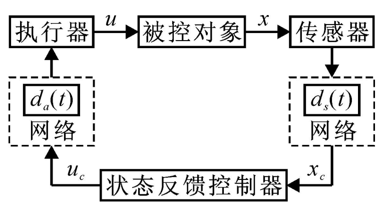

图 1 网络化控制系统

和状态反馈控制器分别以 $\dot{x}(t) = Ax(t) + Bu(t)$ 和 $u_c(t) = Kx_c(t)$ 的形式给出时,闭环系统为

$$\dot{x}(t) = Ax(t) + BKx(t - d_s(t) - d_a(t)). \quad (2)$$

系统(2)和系统(1)的不同之处是在状态变量 $x(t)$ 中存在 2 个时滞,而且这 2 个时滞满足如下关系:

$$0 \le d_s(t) \le \bar{d}_s < \infty, \quad \dot{d}_s(t) \le \tau_s < \infty,$$
$$0 \le d_a(t) \le \bar{d}_a < \infty, \quad \dot{d}_a(t) \le \tau_a < \infty. \quad (3)$$

另一方面,在研究系统(1)的时滞相关稳定性问题时,经常采用的 Lyapunov 函数为

$$V(x(t), t) =$$

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$$x^{\mathrm T}(t)Px(t)+\int_{t-d(t)}^{t}x^{\mathrm T}(s)Qx(s)\mathrm ds+\int_{-\bar d}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds\mathrm d\theta. \quad (4)$$

式中:$P>0,Z>0,Q>0$. 但在 $V(x(t),t)$ 的求导过程中, $-\int_{t-\bar d}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds$ 经常放大为 $-\int_{t-d(t)}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds$,见文献[5-8]. 事实上

$$-\int_{t-\bar d}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds=-\int_{t-d(t)}^{t}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds-\int_{t-\bar d}^{t-d(t)}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds. \quad (5)$$

而在文献[5-8]中, $\dot V(x(t),t)$ 中的积分项 $-\int_{t-\bar d}^{t-d(t)}\dot x^{\mathrm T}(s)Z\dot x(s)\mathrm ds$ 被省略了,这显然会带来一定的保守性.

2　问题描述

考虑如下时滞系统:

$$\begin{cases}\dot x(t)=Ax(t)+Bx(t-d_1(t)-d_2(t)),\\ x(t)=\phi(t),\ t\in[-\bar d,0].\end{cases} \quad (6)$$

其中:$x(t)\in R^n$ 是系统的状态向量;$d_1(t)$ 和 $d_2(t)$ 是系统状态的 2 个时滞部分,满足

$$0\le d_1(t)\le\bar d_1<\infty,\ \dot d_1(t)\le\tau_1<\infty,$$
$$0\le d_2(t)\le\bar d_2<\infty,\ \dot d_2(t)\le\tau_2<\infty, \quad (7)$$

其中 $\bar d\triangleq\bar d_1+\bar d_2$.

针对系统(6),本文构造新的 Lyapunov 函数. 保留式(5)中被省略的积分项,设计时滞相关稳定条件,使得系统(6)对于满足式(7)的时滞 $d_1(t)$ 和 $d_2(t)$ 是渐近稳定的.

3　主要结果及证明

定理 1　如果存在矩阵 $P>0,R>0,Q_1>Q_2>0,Q_3>Q_4>0,Z_i>0,i=1,\cdots,6$,且 $Z_1>Z_2+Z_3,Z_4>Z_5+Z_6$ 以及适当维数的矩阵

$$N=\begin{bmatrix}N_1\\N_2\\N_3\\N_4\\N_5\end{bmatrix},\ S=\begin{bmatrix}S_1\\S_2\\S_3\\S_4\\S_5\end{bmatrix},\ M=\begin{bmatrix}M_1\\M_2\\M_3\\M_4\\M_5\end{bmatrix},\ W=\begin{bmatrix}W_1\\W_2\\W_3\\W_4\\W_5\end{bmatrix},$$

$$V=\begin{bmatrix}V_1\\V_2\\V_3\\V_4\\V_5\end{bmatrix},\ Y=\begin{bmatrix}Y_1\\Y_2\\Y_3\\Y_4\\Y_5\end{bmatrix},\ T=\begin{bmatrix}T_1\\T_2\\T_3\\T_4\\T_5\end{bmatrix},$$

使得下面线性矩阵不等式成立:

$$\Omega:=\begin{bmatrix}\Phi & \bar d_1N & \bar d_2S & (\bar d_1+\bar d_2)M & (\bar d_1+\bar d_2)W & \bar d_2V & \bar d_1Y & (\bar d_1+\bar d_2)T & A_c^{\mathrm T}\Pi_{99}\\ * & -\bar d_1Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & -\bar d_2Z_2 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -(\bar d_1+\bar d_2)(Z_2+Z_5) & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -(\bar d_1+\bar d_2)Z_3 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -\bar d_2Z_4 & 0 & 0 & 0\\ * & * & * & * & * & * & -\bar d_1Z_5 & 0 & 0\\ * & * & * & * & * & * & * & -(\bar d_1+\bar d_2)Z_6 & 0\\ * & * & * & * & * & * & * & * & -\Pi_{99}\end{bmatrix}<0. \quad (8)$$

其中

$$\Phi=\Phi_1+\Phi_2+\Phi_2^{\mathrm T},$$

$$\Phi_1=\begin{bmatrix}\Sigma & 0 & 0 & PB & 0\\ * & -(1-\tau_1)(Q_1-Q_2) & 0 & 0 & 0\\ * & * & -(1-\tau_2)(Q_3-Q_4) & 0 & 0\\ * & * & * & -(1-\tau_1-\tau_2)(Q_2+Q_4) & 0\\ * & * & * & * & -R\end{bmatrix}<0,$$

$$\Phi_2=[N+V\quad -N+S+W\quad Y+T-V\quad -Y+M-S\quad -W-T-M],$$

$$\Sigma=PA+A^{\mathrm T}P+Q_1+Q_3+R,$$

$$A_c=[A\quad 0\quad 0\quad B\quad 0],$$

$$\Pi_{99}=\bar d_1Z_1+\bar d_2(Z_2+Z_3)+\bar d_2Z_4+\bar d_1(Z_5+Z_6),$$

则系统(6)是渐近稳定的.

证明　构造新的 Lyapunov 函数

$$V(x(t),t)=V_1(x(t),t)+V_2(x(t),t)+V_3(x(t),t)+V_4(x(t),t)+V_5(x(t),t)+V_6(x(t),t).$$

其中

$$V_1(x(t),t)=x^{\mathrm T}(t)Px(t),$$

$$V_2(x(t),t)=\int_{t-\bar d_1-\bar d_2}^{t}x^{\mathrm T}(s)Rx(s)\mathrm ds,$$

$$V_3(x(t),t)=$$

$$\int_{t-d_1(t)}^{t} x^{\mathrm T}(s)Q_1x(s)\mathrm{d}s+\int_{t-d_1(t)-d_2(t)}^{t-d_1(t)} x^{\mathrm T}(s)Q_2x(s)\mathrm{d}s,$$

$$V_4(x(t),t)=\int_{-\bar d_1}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)Z_1\dot x(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\bar d_1-\bar d_2}^{-\bar d_1}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)(Z_2+Z_3)\dot x(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_5(x(t),t)=\int_{t-d_2(t)}^{t} x^{\mathrm T}(s)Q_3x(s)\mathrm{d}s+\int_{t-d_1(t)-d_2(t)}^{t-d_2(t)} x^{\mathrm T}(s)Q_4x(s)\mathrm{d}s,$$

$$V_6(x(t),t)=\int_{-\bar d_2}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)Z_4\dot x(s)\mathrm{d}s\mathrm{d}\theta+\int_{-\bar d_1-\bar d_2}^{-\bar d_2}\int_{t+\theta}^{t}\dot x^{\mathrm T}(s)(Z_5+Z_6)\dot x(s)\mathrm{d}s\mathrm{d}\theta.$$

式中:$P>0,Q_i>0(i=1,\cdots,4)$ 和 $Z_i>0(i=1,\cdots,6)$ 为满足式(8) 的矩阵.

对 $V(x(t),t)$ 沿系统(6) 的轨线求导,可得

$$\dot V_1(x(t),t)=x^{\mathrm T}(t)(PA+A^{\mathrm T}P)x(t)+2x^{\mathrm T}(t)PBx(t-d_1(t)-d_2(t));$$

$$\dot V_2(x(t),t)=x^{\mathrm T}(t)Rx(t)-x^{\mathrm T}(t-\bar d_1-\bar d_2)Rx(t-\bar d_1-\bar d_2);$$

$$\begin{aligned}\dot V_3(x(t),t)={}&x^{\mathrm T}(t)Q_1x(t)-x^{\mathrm T}(t-d_1(t))(Q_1-Q_2)x(t-d_1(t))+\dot d_1(t)x^{\mathrm T}(t-d_1(t))(Q_1-Q_2)x(t-d_1(t))-{}\\&(1-\dot d_1(t)-\dot d_2(t))x^{\mathrm T}(t-d_1(t)-d_2(t))Q_2x(t-d_1(t)-d_2(t))\leqslant\\&x^{\mathrm T}(t)Q_1x(t)-(1-\tau_1)x^{\mathrm T}(t-d_1(t))(Q_1-Q_2)x(t-d_1(t))-(1-\tau_1-\tau_2)x^{\mathrm T}(t-d_1(t)-d_2(t))Q_2x(t-d_1(t)-d_2(t)),\\&Q_1>Q_2>0;\end{aligned}$$

$$\begin{aligned}\dot V_4(x(t),t)={}&\bar d_1\dot x^{\mathrm T}(t)Z_1\dot x(t)-\int_{t-\bar d_1}^{t}\dot x^{\mathrm T}(s)Z_1\dot x(s)\mathrm{d}s+\bar d_2\dot x^{\mathrm T}(t)(Z_2+Z_3)\dot x(t)-\int_{t-\bar d_1-\bar d_2}^{t-\bar d_1}\dot x^{\mathrm T}(s)(Z_2+Z_3)\dot x(t)\mathrm{d}s\leqslant\\&\dot x^{\mathrm T}(t)[\bar d_1Z_1+\bar d_2(Z_2+Z_3)]\dot x(t)-\int_{t-\bar d_1}^{t}\dot x^{\mathrm T}(s)Z_1\dot x(s)\mathrm{d}s+\int_{t-\bar d_1}^{t-d_1(t)}\dot x^{\mathrm T}(s)Z_1\dot x(s)\mathrm{d}s-{}\\&\int_{t-\bar d_1-\bar d_2}^{t-\bar d_1}\dot x^{\mathrm T}(s)(Z_2+Z_3)\dot x(s)\mathrm{d}s-\int_{t-\bar d_1}^{t-d_1(t)}\dot x^{\mathrm T}(s)(Z_2+Z_3)\dot x(s)\mathrm{d}s={}\\&\dot x^{\mathrm T}(t)[\bar d_1Z_1+\bar d_2(Z_2+Z_3)]\dot x(t)-\int_{t-d_1(t)}^{t}\dot x^{\mathrm T}(s)Z_1\dot x(s)\mathrm{d}s-\int_{t-\bar d_1-\bar d_2}^{t-d_1(t)}\dot x^{\mathrm T}(s)(Z_2+Z_3)\dot x(s)\mathrm{d}s={}\\&\dot x^{\mathrm T}(t)[\bar d_1Z_1+\bar d_2(Z_2+Z_3)]\dot x(t)-\int_{t-d_1(t)}^{t}\dot x^{\mathrm T}(s)Z_1\dot x(s)\mathrm{d}s-\int_{t-d_1(t)-d_2(t)}^{t-d_1(t)}\dot x^{\mathrm T}(s)Z_2\dot x(s)\mathrm{d}s-{}\\&\int_{t-\bar d_1-\bar d_2}^{t-d_1(t)-d_2(t)}\dot x^{\mathrm T}(s)Z_2\dot x(s)\mathrm{d}s-\int_{t-\bar d_1-\bar d_2}^{t-d_1(t)}\dot x^{\mathrm T}(s)Z_3\dot x(s)\mathrm{d}s,\ Z_1>Z_2+Z_3>0;\end{aligned}$$

同理

$$\begin{aligned}\dot V_5(x(t),t)\leqslant{}&x^{\mathrm T}(t)Q_3x(t)-(1-\tau_2)x^{\mathrm T}(t-d_2(t))(Q_3-Q_4)x(t-d_2(t))-{}\\&(1-\tau_1-\tau_2)x^{\mathrm T}(t-d_1(t)-d_2(t))Q_4x(t-d_1(t)-d_2(t));\end{aligned}$$

$$\begin{aligned}\dot V_6(x(t),t)\leqslant{}&\dot x^{\mathrm T}(t)[\bar d_2Z_4+\bar d_1(Z_5+Z_6)]\dot x(t)-\int_{t-d_2(t)}^{t}\dot x^{\mathrm T}(s)Z_4\dot x(s)\mathrm{d}s-\int_{t-d_1(t)-d_2(t)}^{t-d_2(t)}\dot x^{\mathrm T}(s)Z_5\dot x(s)\mathrm{d}s-{}\\&\int_{t-\bar d_1-\bar d_2}^{t-d_1(t)-d_2(t)}\dot x^{\mathrm T}(s)Z_5\dot x(s)\mathrm{d}s-\int_{t-\bar d_1-\bar d_2}^{t-d_2(t)}\dot x^{\mathrm T}(s)Z_6\dot x(s)\mathrm{d}s.\end{aligned}$$

利用牛顿-莱布尼兹公式,以下方程对于适当维数的 N,S,M,W,V,Y,T 成立:

$$\alpha_1(t):=2\xi^{\mathrm T}(t)N\Big[x(t)-x(t-d_1(t))-\int_{t-d_1(t)}^{t}\dot x(s)\mathrm{d}s\Big]=0,$$

$$\alpha_2(t):=2\xi^{\mathrm T}(t)S\Big[x(t-d_1(t))-x(t-d_1(t)-d_2(t))-\int_{t-d_1(t)-d_2(t)}^{t-d_1(t)}\dot x(s)\mathrm{d}s\Big]=0,$$

$$\alpha_3(t):=2\xi^{\mathrm T}(t)M\Big[x(t-d_1(t)-d_2(t))-x(t-\bar d_1-\bar d_2)-\int_{t-\bar d_1-\bar d_2}^{t-d_1(t)-d_2(t)}\dot x(s)\mathrm{d}s\Big]=0,$$

$$\alpha_4(t):=2\xi^{\mathrm T}(t)W\Big[x(t-d_1(t))-x(t-\bar d_1-\bar d_2)-\int_{t-\bar d_1-\bar d_2}^{t-d_1(t)}\dot x(s)\mathrm{d}s\Big]=0,$$

$\alpha_5(t) :=$

$2\xi^T(t)V[x(t) - x(t - d_2(t)) - \int_{t-d_2(t)}^{t} \dot{x}(s)\mathrm{d}s] = 0,$

$\alpha_6(t) :=$

$2\xi^T(t)Y[x(t - d_2(t)) - x(t - d_1(t) - d_2(t)) - \int_{t-d_1(t)-d_2(t)}^{t-d_2(t)} \dot{x}(s)\mathrm{d}s] = 0,$

$\alpha_7(t) :=$

$2\xi^T(t)T[x(t - d_2(t)) - x(t - \bar{d}_1 - \bar{d}_2) - \int_{t-\bar{d}_1-\bar{d}_2}^{t-d_2(t)} \dot{x}(s)\mathrm{d}s] = 0.$

其中

$\xi(t) = [x^T(t) \quad x^T(t - d_1(t)) \quad x^T(t - d_2(t)) \quad x^T(t - d_1(t) - d_2(t)) \quad x^T(t - \bar{d}_1 - \bar{d}_2)]^T.$

所以

$$\dot{V}(x(t),t) = \sum_{i=1}^{6}\dot{V}_i(x(t),t) + \sum_{i=1}^{7}\alpha_i(t) \le \xi^T(t)[\Phi + \bar{d}_1 N Z_1^{-1} N^T + \bar{d}_2 S Z_2^{-1} S^T + (\bar{d}_1 + \bar{d}_2) M(Z_2 + Z_5)^{-1} M^T + (\bar{d}_1 + \bar{d}_2) W Z_3^{-1} W^T + \bar{d}_2 V Z_4^{-1} V^T + \bar{d}_1 Y Z_5^{-1} Y^T + (\bar{d}_1 + \bar{d}_2) T Z_6^{-1} T^T + A_c^T \Pi_{99} A_c]\xi(t) - \int_{t-d_1(t)}^{t} [\xi^T(t) N + \dot{x}^T(s) Z_1] Z_1^{-1} \times [N^T\xi(t) + Z_1\dot{x}(s)]\mathrm{d}s - \int_{t-d_1(t)-d_2(t)}^{t-d_1(t)} [\xi^T(t) S + \dot{x}^T(s) Z_2] Z_2^{-1} \times [S^T\xi(t) + Z_2\dot{x}(s)]\mathrm{d}s - \int_{t-\bar{d}_1-\bar{d}_2}^{t-d_1(t)-d_2(t)} [\xi^T(t) M + \dot{x}^T(s)(Z_2 + Z_5)](Z_2 + Z_5)^{-1} \times [M^T\xi(t) + (Z_2 + Z_5)\dot{x}(s)]\mathrm{d}s - \int_{t-\bar{d}_1-\bar{d}_2}^{t-d_1(t)} [\xi^T(t) W + \dot{x}^T(s) Z_3] Z_3^{-1} \times [W^T\xi(t) + Z_3\dot{x}(s)]\mathrm{d}s - \int_{t-d_2(t)}^{t} [\xi^T(t) V + \dot{x}^T(s) Z_4] Z_4^{-1} \times [V^T\xi(t) + Z_4\dot{x}(s)]\mathrm{d}s - \int_{t-d_1(t)-d_2(t)}^{t-d_2(t)} [\xi^T(t) Y + \dot{x}^T(s) Z_5] Z_5^{-1} \times [Y^T\xi(t) + Z_5\dot{x}(s)]\mathrm{d}s - \int_{t-\bar{d}_1-\bar{d}_2}^{t-d_2(t)} [\xi^T(t) T + \dot{x}^T(s) Z_6] Z_6^{-1} \times [T^T\xi(t) + Z_6\dot{x}(s)]\mathrm{d}s. \tag{9}$$

利用 Schur 补性质,式(8) 等价于

$$\Omega := \Phi + \bar{d}_1 N Z_1^{-1} N^T + \bar{d}_2 S Z_2^{-1} S^T + (\bar{d}_1 + \bar{d}_2) M(Z_2 + Z_5)^{-1} M^T + \bar{d}_2 V Z_4^{-1} V^T + (\bar{d}_1 + \bar{d}_2) W Z_3^{-1} W^T + \bar{d}_1 Y Z_5^{-1} Y^T + (\bar{d}_1 + \bar{d}_2) T Z_6^{-1} T^T + A_c^T \Pi_{99} A_c < 0.$$

又因为式(9) 中 $Z_i > 0, i = 1, \cdots, 6$,所以

$$\dot{V}(x(t),t) \le \lambda_{\max}(\Omega) \|x(t)\|^2.$$

令 $\varepsilon = -\lambda_{\max}(\Omega)$,则 $\varepsilon > 0$,从而有

$$\dot{V}(x(t),t) \le -\varepsilon \|x(t)\|^2,$$

所以系统(6) 是渐近稳定的. □

4　数值算例

考虑时滞系统(6),其中

$$A = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.09 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$\dot{d}_1(t) \le 0.1$, $\dot{d}_2(t) \le 0.8$.

利用定理 1 及 Matlab 软件中的 LMI 工具箱,可得到表 1 和表 2.

表 1　给定时滞界 $\bar{d}_1$,最大时滞界 $\bar{d}_2$ 的比较

$\bar{d}_2$ 最大值	$\bar{d}_1$	
	1	1.1
文献[7]	0.180	0.080
文献[8]	0.180	0.080
本文定理 1	0.199	0.095

表 2　给定时滞界 $\bar{d}_2$,最大时滞界 $\bar{d}_1$ 的比较

$\bar{d}_1$ 最大值	$\bar{d}_2$	
	0.2	0.3
文献[7]	0.980	0.080
文献[8]	0.980	0.880
本文定理 1	0.999	0.901

5　结　　论

本文通过构造新的 Lyapunov 函数,讨论了一类状态向量中含有时滞和的连续系统的时滞相关稳定性,得到了基于 LMI 的时滞相关稳定充分条件.数值算例说明了所得到的结论具有较小的保守性.

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因素校正(APFC),其中用得最广的是Boost-PFC.单相Boost-PFC的大信号瞬时值状态方程可写成

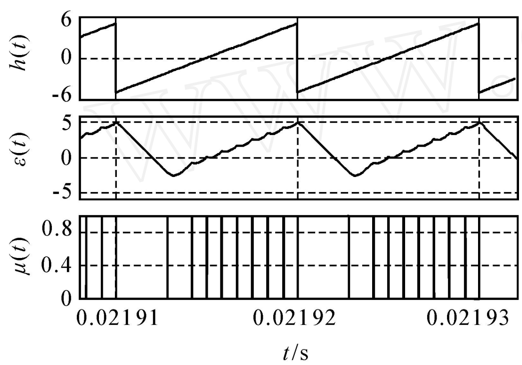
$$\begin{cases} \dfrac{\mathrm{d}i_L}{\mathrm{d}t} = \dfrac{1}{L}u_g - \dfrac{1-\mu}{L}V_c, \\ \dfrac{\mathrm{d}V_C}{\mathrm{d}t} = -\dfrac{1}{RC}V_C + \dfrac{1-\mu}{C}i_L, \end{cases} \tag{15}$$

其中各变量的定义可参见文献[5].定义 $e = Gu_g - R_1 i_L$, $\varepsilon = K_p e$, $s = \varepsilon - h(t)$,其中 $R_1, G, K_p > 0$ 采用控制律(6).利用定理2便可证明,只要系统参数满足

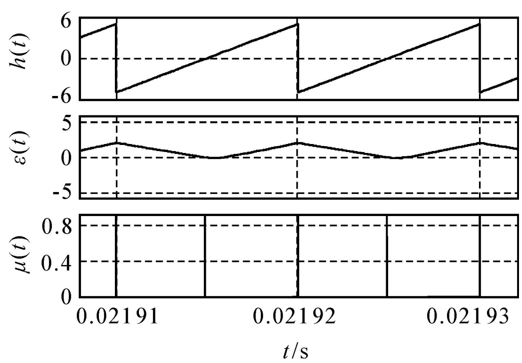
$$\frac{K_p R_1 V_m}{L}\left(M + \frac{\omega L G}{R_1}\right) < \frac{2A_h}{T_h},$$

$$\omega L G / R_1 < M - 1,$$

电流环就一定稳定,且存在 $t_1 > t_0$ 使得 $t \geq t_1$ 时 $|e(t)| \leq A_h / k_p$.其中 $M = V_c / V_m$.



(a) 锯齿波滑动模态



(b) PWM-准滑动模态

图2 两种模态下的 h(t),ε(t) **和** μ(t)

对单相Boost-PFC进行Matlab仿真,取不同的切换函数和参数,可得到不同的运行状态.图2给出了两种运行状态,图中从上到下的波形依次为 $h(t)$, $\varepsilon(t)$ 和 $\mu(t)$.仿真结果验证了上述结论.用PWM-准滑模控制原理设计了一种单/三相(10 A) PFC装置,经测试,其输入电流接近正弦,功率因数 > 0.99,效率 > 0.96,样机已稳定运行1年.

6 结 论

PWM-准滑模是用VSC理论描述恒频PWM控制的新概念,借助这一概念,PWM控制可直接用VSC理论进行分析.分析表明,PWM控制系统存在3种运行状态,满足一定的条件便能进入相应的状态.分析结果为电力电子系统提供了一种非线性、大信号的设计方法.

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