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一类具有未建模动态的非线性系统模糊自适应鲁棒控制

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摘 要: 针对一类单输入单输出未建模动态不确定非线性系统,提出一种模糊自适应 backstepping 控制方法. 设计中利用模糊逻辑系统逼近系统的未知函数,应用非线性阻尼项抵消系统的非线性不确定项,通过引入一个动态信号克服未建模动态. 该模糊自适应控制方法保证了整个闭环系统的有界性,输出信号可调节到零的小邻域内. 仿真结果进一步验证了该方法的有效性.

关键词: 非线性系统; 模糊自适应控制; Backstepping 技术; 稳定性分析
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Fuzzy adaptive robust control for a class of nonlinear system with dynamic uncertainties

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Abstract: A fuzzy adaptive backstepping design procedure is proposed for a class of nonlinear systems with unmodeled dynamics. The fuzzy logic systems are used to approximate the unknown nonlinear functions. Nonlinear damping terms are used to counteract the uncertain nonlinearities and a dynamic signal is introduced to dominate the dynamic disturbance. The derived fuzzy adaptive control approach guarantees not only the global boundedness property for all the signals and the states, but also the output to a small neighbordhood of the origin. Simulation rsults illustrate the effectiveness of the proposed approachs.

Key words: Nonlinear systems; Fuzzy adaptive control; Backstepping technique; Stability

1 引 言

作为处理不确定非线性系统的有效方法之一,模糊自适应控制已取得了一些理论成果和实际效果[1-6]. 一般而言,模糊自适应控制方法和理论促进了模糊控制领域和学科的完善和发展. 然而,这些方法和理论都局限于满足匹配条件的非线性系统,对于那些不满足匹配条件的非线性系统,现有的模糊自适应控制方法和理论则难以应用.

随着非线性 backstepping 自适应设计技术的发展,许多学者把模糊自适应控制与 backstepping 自适应设计相结合,提出了一些非线性模糊自适应 backstepping 控制方法[7,8]. 该控制方法的显著特点是:不但保持了一般模糊自适应控制方法所具有的特性,即不要求非线性函数与不确定参数具有线性关系,而且解决了一般模糊自适应控制方法要求非线性系统必须满足匹配条件的限制,因此它更适用于一般的非线性不确定系统. 由于模糊自适应 backstepping 控制方法所具有的这些特性,近几年来引起了人们的广泛关注,并成为模糊控制领域中一个新的研究方向.

本文针对一类未建模动态不确定非线性系统,提出一种模糊自适应 backstepping 控制方法. 设计中利用模糊逻辑系统逼近系统的未知函数,应用非线性阻尼项抵消系统的非线性不确定项,通过引入一个动态信号克服未建模动态,基于李亚普诺夫函数方法证明了闭环系统的有界性.

2 控制问题描述

考虑具有如下形式的一类非线性未建模动态不

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确定系统:

$$\begin{cases} \dot{z} = q(z,x), \\ \dot{x}_1 = x_2 + f_1(x_1) + \Delta_1(x,z,t), \\ \dot{x}_2 = x_3 + f_2(x_1,x_2) + \Delta_2(x,z,t), \\ \quad \cdots \\ \dot{x}_n = u + f_n(x_1,x_2,\cdots,x_n) + \Delta_n(x,z,t), \\ y = x_1. \end{cases} \tag{1}$$

其中:$(x_1,x_2,\cdots,x_n) \in R^n$ 是系统的状态向量,u 和 y 分别是系统的输入和输出,z 是未知状态,q 是未建模动态,$f_i(x_1,x_2,\cdots,x_i)\ (i=1,2,\cdots,n)$ 是未知的连续光滑函数,$\Delta_i\ (i=1,2,\cdots,n)$ 是非线性动态不确定项. 假设 Δ_i 和 q 是满足 Lipschitz 条件的连续函数.

对式(1) 作如下假设:

假设 1[9] 对于任意的 $i\ (1 \le i \le n)$,存在一个未知的参数 υ_i^*,满足

$$|\Delta_i(x,z,t)| \le \upsilon_i^* \phi_{i1}(|(x_1,x_2,\cdots,x_i)|) + \upsilon_i^* \phi_{i2}(|z|), \tag{2}$$

其中 ϕ_{i1} 和 ϕ_{i2} 是已知的光滑函数.

假设 2[9] 未建模动态 q 满足 exp-ISPS 稳定条件,即 $\dot{z} = q(z,x)$ 是 exp-ISPS 稳定的. 则存在 Lyapunov 函数 $V(z)$,满足

$$\alpha_1(\|z\|) \le V(z) \le \alpha_2(\|z\|), \tag{3}$$

$$\frac{\partial V(z)}{\partial z} q(z,x) \le -c_0 V(z) + \gamma(|x|) + d_0. \tag{4}$$

其中:α_1,α_2 和 γ 是 K_∞ 类函数;$c_0 > 0$ 和 $d_0 > 0$ 是已知常数. 不失一般性,假设 $\gamma(s) = s^2 \gamma_0(s^2)$.

本文引入一个动态信号 r,其定义如下:

$$\dot{r} = -\bar{c} r + \gamma(|x|) + d_0,\ r(t_0) = r_0 > 0, \tag{5}$$

其中 $\bar{c} \in (0, c_0)$.

引理 1[9] 对于 $\dot{z} = q(z,x)$,如果存在一个 exp-ISPS Lyapunov 函数 V,即满足式(3) 和(4),则对于任意常量 $\bar{c} \in (0,c_0)$,初始条件 $x_0 = x_0(t_0)$,$r_0 > 0$,以及任意满足 $\gamma_0(x) \ge \gamma(|x|)$ 的函数 γ_0,都存在有限的时间 $T_0 = T_0(\bar{c}, r_0, z_0) \ge 0$,以及定义在 $t \ge t_0$ 上的非负函数 $D(t_0,t)$,使得当 $t \ge t_0 + T_0$ 时,有 $D(t_0,t) = 0$;当 $t \ge t_0$ 时,有

$$V(x(t)) \le r(t) + D(t_0,t). \tag{6}$$

3 模糊自适应鲁棒控制器的设计

在应用 backstepping 设计方法设计模糊自适应鲁棒控制器之前,首先假设模糊逻辑系统具有如下形式:

$$\hat{f}_i((x_1,x_2,\cdots,x_i) \mid \theta_i) = \theta_i^{\mathrm{T}} \phi_i(x_1,x_2,\cdots,x_i),\quad 1 \le i \le n. \tag{7}$$

假设参数向量 θ_i 的最优向量为

$$\theta_i^* = \arg\min_{\theta_i \in \Omega_i}\Big[\sup_{x \in U_i} |\hat{f}_i((x_1,\cdots,x_i) \mid \theta_i) - f_i(x_1,\cdots,x_i)|\Big],$$

其中 Ω_i 和 U_i 分别是包含 θ_i 和 x 的适当闭集.

定义最小模糊逼近误差

$$w_i = f_i(x_1,\cdots,x_i) - \hat{f}_i((x_1,\cdots,x_i) \mid \theta_i^*). \tag{8}$$

由式(7) 和(8) 得

$$f_i(x_1,\cdots,x_i) = \theta_i^{\mathrm{T}} \phi_i(x_1,\cdots,x_i) + \tilde{\theta}_i^{\mathrm{T}} \phi_i(x_1,\cdots,x_i) + w_i, \tag{9}$$

其中 $\tilde{\theta}_i = \theta_i^* - \theta_i$.

假设 3 最小模糊逼近误差 w_i 满足 $|w_i| \le \delta_i^*$,其中 $\delta_i^* > 0$ 是已知常数.

下面应用 backstepping 设计方法,给出模糊自适应鲁棒控制器的详细设计步骤.

第 1 步 由式(1) 和(9) 得

$$\begin{cases} \dot{z} = q(z,x), \\ \dot{x}_1 = x_2 + \theta_1^{\mathrm{T}} \phi_1(x_1) + \tilde{\theta}_1^{\mathrm{T}} \phi_1(x_1) + \\ \qquad w_1 + \Delta_1(x,z,t). \end{cases} \tag{10}$$

令 $\bar{x}_2 = x_2 - \pi_1(x_1,x_2,t,\theta_1,\hat{p})$,考虑如下 Lyapunov 函数:

$$V_1 = \frac{1}{2} x_1^2 + \frac{1}{\lambda_0} r + \frac{1}{2} \tilde{\theta}_1^{\mathrm{T}} \Gamma_1^{-1} \tilde{\theta}_1 + \frac{1}{2\lambda} (\hat{p} - p^*)^2. \tag{11}$$

其中:$\bar{\gamma}(x_1) = x_1^2 \gamma_0(x_1^2)$,$p^* \ge \max\{\upsilon_1^*,\cdots,\upsilon_n^*\}$,$\hat{p}$ 为 p^* 的估计,$\Gamma_1 = \Gamma_1^{\mathrm{T}} > 0$,$\lambda_0$,$\lambda > 0$ 为设计参数.

求 V_1 对时间的导数,由式(10) 得

$$\begin{aligned} \dot{V}_1 \le\ & x_1(x_2 + \theta_1^{\mathrm{T}} \phi_1(x_1) + \tilde{\theta}_1^{\mathrm{T}} \phi_1(x_1) + \\ & w_1) + |x_1|\,|\Delta_1| - \frac{\bar{c}_0}{\lambda_0} r + \frac{1}{\lambda_0} x_1^2 \gamma_0(x_1^2) + \\ & \frac{1}{\lambda_0} d_0 - \tilde{\theta}_1^{\mathrm{T}} \Gamma_1^{-1} \dot{\theta}_1 + \frac{1}{\lambda} \tilde{p} \dot{\hat{p}}, \end{aligned} \tag{12}$$

其中 $\tilde{p} = \hat{p} - p^*$. 令

$$\tau_1 = \Gamma_1(x_1 \phi_1(x_1) - \sigma_\theta(\theta_1 - \theta_{10})), \tag{13}$$

其中 $\sigma_\theta > 0$ 和 $\theta_{10} \in R^{N_1}$ 是设计参数. 式(13) 代入式(12),得

$$\begin{aligned} \dot{V}_1 \le\ & x_1\Big(x_2 + \theta_1^{\mathrm{T}} \phi_1(x_1) + w_1 + \frac{1}{\lambda_0} x_1 \gamma_0\Big) - \\ & \frac{\bar{c}_0}{\lambda_0} r + \frac{d_0}{\lambda_0} - \tilde{\theta}_1^{\mathrm{T}} \Gamma_1^{-1} (\dot{\theta}_1 - \tau_1) - \\ & \sigma_\theta \tilde{\theta}_1^{\mathrm{T}} (\theta_{10} - \theta_1) + p^* |x_1| \phi_{11}(|(x_1)|) + \\ & p^* |x_1| \phi_{12}(|z|) + \frac{1}{\lambda} \tilde{p} \dot{\hat{p}}. \end{aligned} \tag{14}$$

为了处理式(14) 中的后 3 项,利用文献[9] 中的引理 3.3 和引理 3.4,对于任意的 $\varepsilon_{11} > 0$,存在光滑函

数 ψ_{11}(其中 $\psi_{11}(0)=0$),使得

$$p^*|x_1|\psi_{11}(|(x_1)|)\le p^*x_1\psi_{11}(x_1)+\varepsilon_{11}. \tag{15}$$

由式(6),假设 2 以及不等式

$$x^{\mathrm T}y\le\frac{1}{4\varepsilon}|x|^2+\varepsilon|y|^2,$$

式(15)可进一步表示成

$$\begin{aligned}
&p^*|x_1|\psi_{12}(|z|)\le\\
&p^*|x_1|\psi_{12}\circ\alpha_1^{-1}(r+D(t_0,t))\le\\
&p^*|x_1|\psi_{12}\circ\alpha_1^{-1}(2r)+\\
&p^*|x_1|\psi_{12}\circ\alpha_1^{-1}(2D(t_0,t))\le\\
&p^*|x_1|\psi_{12}\circ\alpha_1^{-1}(2r)+\frac14x_1^2+d_1(t_0,t),
\end{aligned}\tag{16}$$

其中 $d(t_0,t)=[p^*\psi_{12}\circ\alpha_1^{-1}(2D(t_0,t))]^2$. 对于任意 $t\ge t_0+T_0$,有 $d_1(t_0,t)=0$.

根据文献[9]引理 3.4 可知,存在光滑函数 ψ_{12}(其中 $\psi_{12}(0)=0$),使得

$$\psi_{12}\circ\alpha_1^{-1}(2r)\le\psi_{12}(r)+1. \tag{17}$$

因此,对于任意的 $\varepsilon_{12}>0$,重复应用文献[9]引理 3.3,可得

$$\begin{aligned}
&p^*|x_1|\psi_{12}(|z|)\circ\alpha_1^{-1}(2r)\le\\
&p^*|x_1|\psi_{12}(r)+p^*|x_1|\le\\
&p^*x_1\psi_{12}(r)\psi_{13}(x_1,r)+\\
&p^*x_1\psi_{14}(x_1)+2p^*\varepsilon_{12},
\end{aligned}\tag{18}$$

其中 ψ_{13} 和 ψ_{14} 是两个足够光滑的函数,且在零点等于零.

将式(15)～(18)代入式(14),得

$$\begin{aligned}
\dot V_1\le{}&x_1\Big(x_2+\hat\theta_1^{\mathrm T}\varphi_1(x_1)+\frac{1}{\lambda_0}x_1\gamma_0+\\
&\hat p\psi_{11}(x_1)+\hat p\psi_{12}(r)\psi_{13}(x_1,r)+\\
&\hat p\psi_{14}(x_1)+\frac14x_1+w_1\Big)-\frac{\bar c_0}{\lambda_0}r+\\
&\frac{d_0}{\lambda_0}+p^*(\varepsilon_{11}+2\varepsilon_{12})+d_1(t_0,t)-\\
&\tilde\theta_1^{\mathrm T}\Gamma_1^{-1}(\dot{\hat\theta}_1-\tau_1)-\sigma_0\tilde\theta_1^{\mathrm T}(\theta_{10}-\\
&\theta_1^*)-\sigma_0\tilde\theta_1^{\mathrm T}\tilde\theta_1+\frac{1}{\lambda}\tilde p(\dot{\hat p}-\tilde\omega_1)-\\
&\sigma_p\tilde p(\hat p-p_0),
\end{aligned}\tag{19}$$

其中

$$\begin{aligned}
\tilde\omega_1={}&\lambda[x_1\psi_{11}(x_1)+x_1\psi_{12}(r)\psi_{13}(x_1,r)+\\
&x_1\psi_{14}(x_1)-\sigma_p(\hat p-p_0)].
\end{aligned}\tag{20}$$

因此,设计虚拟的控制器

$$\begin{aligned}
\pi_1={}&\\
&-k_1x_1-\hat\theta_1^{\mathrm T}\varphi_1(x_1)-\frac{1}{\lambda_0}x_1\gamma_0-\frac14x_1-\\
&x_1\delta_1\tanh(x_1\delta_1/\varepsilon_0)-\hat p[\psi_{11}(x_1)+\\
&\psi_{12}(r)\psi_{13}(x_1,r)+\psi_{14}(x_1)].
\end{aligned}\tag{21}$$

其中:$\varepsilon_0>0$ 和 $k_1>0$ 是设计参数,$\delta_1>\sup\limits_{t\ge0}[|w_1|]$. 利用式(19)和 $\bar x_2=x_2-\pi_1$,可得

$$\begin{aligned}
\dot V_1\le{}&-k_1x_1^2+x_1\bar x_2+\mu_1(t_0,t)-\\
&\frac12\sigma_0\tilde\theta_1^{\mathrm T}\tilde\theta_1-\tilde\theta_1^{\mathrm T}\Gamma_1^{-1}(\dot{\hat\theta}_1-\tau_1)+\\
&\frac{1}{\lambda}\tilde p(\dot{\hat p}-\tilde\omega_1),
\end{aligned}\tag{22}$$

其中

$$\begin{aligned}
\mu_1(t_0,t)={}&\frac{d_0}{\lambda_0}+p^*(\varepsilon_{11}+2\varepsilon_{12})+d_1(t_0,t)+\\
&0.2785\varepsilon_0+\frac{\sigma_0}{2}|\theta_1^*-\theta_{10}|^2+\\
&\frac{\sigma_p}{2}|p^*-p_0|^2.
\end{aligned}\tag{23}$$

第 2 步　令 $\bar x_3=x_3-\pi_2(x_1,x_2,x_3,r,\hat\theta_1,\hat\theta_2,\hat p)$,由式(1)和(9)可得

$$\begin{aligned}
\dot{\bar x}_2={}&\bar x_3+\pi_2+\hat\theta_2^{\mathrm T}\varphi_2+w_2+\tilde\theta_2^{\mathrm T}\varphi_2-\\
&\frac{\partial\pi_1}{\partial x_1}\hat\theta_1^{\mathrm T}\varphi_1-\frac{\partial\pi_1}{\partial x_1}x_2-\frac{\partial\pi_1}{\partial x_1}w_1-\\
&\frac{\partial\pi_1}{\partial\hat\theta_1}\dot{\hat\theta}_1-\frac{\partial\pi_1}{\partial\hat p}\dot{\hat p}-\frac{\partial\pi_1}{\partial x_1}\tilde\theta_1^{\mathrm T}\varphi_1+\\
&\Delta_2-\frac{\partial\pi_1}{\partial x_1}\Delta_1.
\end{aligned}\tag{24}$$

选择 Lyapunov 函数

$$V_2=V_1+\frac12\bar x_2^2+\frac12\tilde\theta_2^{\mathrm T}\Gamma_2^{-1}\tilde\theta_2, \tag{25}$$

由式(24)和(25)得

$$\begin{aligned}
\dot V_2\le{}&-k_1x_1^2+\mu_1(t_0,t)+\bar x_2\Big[x_1+\bar x_3+\pi_2+\\
&\hat\theta_2^{\mathrm T}\varphi_2+w_2-\frac{\partial\pi_1}{\partial x_1}\hat\theta_1^{\mathrm T}\varphi_1-\frac{\partial\pi_1}{\partial x_1}x_2-\\
&\frac{\partial\pi_1}{\partial x_1}w_1-\frac{\partial\pi_1}{\partial\hat\theta_1}\dot{\hat\theta}_1-\frac{\partial\pi_1}{\partial\hat p}\dot{\hat p}\Big]+|\bar x_2\bar\Delta_2|-\\
&\frac12\sigma_0\tilde\theta_1^{\mathrm T}\tilde\theta_1-\tilde\theta_1^{\mathrm T}\Gamma_1^{-1}\Big[\dot{\hat\theta}_1-\tau_1+\\
&\Gamma_1\bar x_2\frac{\partial\pi_1}{\partial x_1}\varphi_1\Big]+\frac{1}{\lambda}(\hat p-p^*)(\dot{\hat p}-\tilde\omega_1)+\\
&\tilde\theta_2^{\mathrm T}\Gamma_2^{-1}(\dot{\hat\theta}_2+\Gamma_2\varphi_2\bar x_2),
\end{aligned}\tag{26}$$

其中 $\bar\Delta_2=\Delta_2-\frac{\partial\pi_1}{\partial x_1}\Delta_1$.

类似于第 1 步的推导,利用假设 1 和 $p^*\ge\max\{\vartheta_1^*,\cdots,\vartheta_n^*\}$,对于任意的 $\varepsilon_{21},\varepsilon_{22}>0$,存在光滑函数 ψ_{21} 和 ψ_{22},使得

$$\begin{aligned}
&|\bar x_2\bar\Delta_2|\le\\
&p^*\bar x_2\psi_{21}+p^*\varepsilon_{21}+p^*\bar x_2\psi_{22}+\frac{\bar x_2^2}{4}\Big(1+\\
&\Big(\frac{\partial\pi_1}{\partial x_1}\Big)^2\Big)+2\times2p^*\varepsilon_{22}+d_2(t_0,t),
\end{aligned}\tag{27}$$

其中

$$d_2(t_0,t)=\sum_{j=1}^{2}[p^*\phi_{j2}\circ\alpha_1^{-1}(2D(t_0,t))]^2.$$

则

$$\begin{aligned}\dot V_2\le\ & -k_1x_1^2+\mu_1(t_0,t)+\bar x_2\Big[x_1+\bar x_3+\\ &\pi_2+\theta_2^T\phi_2+w_2-\frac{\partial\pi_1}{\partial x_1}\theta_1^T\phi_1+\\ &\hat p(\phi_{21}+\phi_{22})+\frac{\bar x_2}{4}\Big(1+\Big(\frac{\partial\pi_1}{\partial x_1}\Big)^2\Big)-\\ &\frac{\partial\pi_1}{\partial x_1}x_2-\frac{\partial\pi_1}{\partial x_1}w_1-\frac{\partial\pi_1}{\partial\hat\theta_1}\dot{\hat\theta}_1-\\ &\frac{\partial\pi_1}{\partial\hat p}\dot{\hat p}\Big]+p^*(\varepsilon_{21}+2\times 2p^*\varepsilon_{22})+\\ &d_2(t_0,t)-\tilde\theta_1^T\Gamma_1^{-1}\Big(\dot{\hat\theta}_1-\tau_1-\\ &\Gamma_1\bar x_2\frac{\partial\pi_1}{\partial x_1}\phi_1\Big)-\frac{1}{2}\sigma_\theta\tilde\theta_1^T\hat\theta_1+\\ &\frac{1}{\lambda}(\hat p-p^*)[\dot{\hat p}-\tilde\omega_1-\lambda(\phi_{21}+\\ &\phi_{22})\bar x_2]+\tilde\theta_2^T\Gamma_2^{-1}(\dot{\hat\theta}_2-\Gamma_2\phi_2\bar x_2).\end{aligned}\tag{28}$$

令

$$\tau_{12}=\tau_{11}-\Gamma_1\bar x_2\frac{\partial\pi_1}{\partial x_1}\phi_1,\tag{29}$$

$$\tau_{22}=\Gamma_2(\phi_2\bar x_2-\sigma_\theta(\hat\theta_2-\theta_{20})),\tag{30}$$

$$\tilde\omega_2=\tilde\omega_1+\lambda(\phi_{21}+\phi_{22})\bar x_2.\tag{31}$$

设计虚拟控制

$$\begin{aligned}\pi_2=\ & -x_1-k_2\bar x_2-\hat\theta_2^T\phi_2+\frac{\partial\pi_1}{\partial x_1}\hat\theta_1^T\phi_1-\\ &\bar x_2\delta_2\tanh(\bar x_2\delta_2/\varepsilon_0)-\hat p(\phi_{21}+\phi_{22})-\\ &\frac{\bar x_2}{4}\Big(1+\Big(\frac{\partial\pi_1}{\partial x_1}\Big)^2\Big)+\frac{\partial\pi_1}{\partial x_1}x_2-\\ &\bar x_2\frac{\partial\pi_1}{\partial x_1}\delta_1\tanh\Big(\bar x_2\frac{\partial\pi_1}{\partial x_1}\delta_1/\varepsilon_0\Big)+\\ &\frac{\partial\pi_1}{\partial\hat\theta_1}\tau_{12}+\frac{\partial\pi_1}{\partial\hat p}\tilde\omega_2.\end{aligned}\tag{32}$$

式(39)～(32)代入式(28),得

$$\begin{aligned}\dot V_2\le\ & -k_1\bar x_1^2-k_2\bar x_2^2+\bar x_2\bar x_3+\mu_2(t_0,t)-\\ &\Big(\tilde\theta_1^T\Gamma_1^{-1}+\bar x_2\frac{\partial\pi_1}{\partial\hat\theta_1}\Big)(\dot{\hat\theta}_1-\tau_{12})+\\ &\Big(\frac{1}{\lambda}(\hat p-p^*)-\bar x_2\frac{\partial\pi_1}{\partial\hat p}\Big)(\dot{\hat p}-\tilde\omega_2)-\\ &\tilde\theta_2^T\Gamma_2^{-1}(\dot{\hat\theta}_2-\tau_{22})-\frac{\sigma_\theta}{2}\sum_{j=1}^{2}\tilde\theta_j^T\tilde\theta_j,\end{aligned}\tag{33}$$

其中

$$\begin{aligned}\mu_2(t_0,t)=\ &\\ &\mu_1(t_0,t)+p^*(\varepsilon_{21}+2\times 2p^*\varepsilon_{22})+\\ &2\times 0.2785\varepsilon_0+d_2(t_0,t)+\frac{1}{2}\sigma_\theta|\theta_2^*-\theta_{20}|^2.\end{aligned}$$

第 i 步$(2\le i\le n-1)$ 令

$$\bar x_{i+1}=x_{i+1}-\pi_i(x_1,\cdots,x_i,r,\hat\theta_1,\cdots,\hat\theta_i,\hat p),$$
$$2\le i\le n.$$

反复使用类似于第 2 步的推导方法,利用式(1)和(13),$\bar x_i$-子系统可表示为

$$\begin{aligned}\dot{\bar x}_i=\ &\bar x_{i+1}+\pi_i-\kappa_i+w_i+\hat\theta_i^T\phi_i-\\ &\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\hat\theta_j^T\phi_j-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}w_j-\\ &\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial\hat\theta_j}\dot{\hat\theta}_j+\tilde\theta_i^T\phi_i-\\ &\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\tilde\theta_j^T\phi_j-\frac{\partial\pi_{i-1}}{\partial\hat p}\dot{\hat p}+\bar\Delta_i.\end{aligned}\tag{34}$$

其中

$$\begin{aligned}\kappa_i=\ &\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}x_{j+1}+\frac{\partial\pi_{i-1}}{\partial r}\times\\ &(-cr+x_1^2\gamma_0+d_0),\end{aligned}\tag{35}$$

$$\bar\Delta_i=\Delta_i-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\Delta_j.\tag{36}$$

考虑 Lyapunov 函数

$$V_i=V_{i-1}+\frac{1}{2}\bar x_i^2+\frac{1}{2}\tilde\theta_i^T\Gamma_i^{-1}\tilde\theta_i,\tag{37}$$

利用假设 1,类似于第 2 步的设计方法,对于 $\varepsilon_{i1},\varepsilon_{i2}>0$,存在光滑函数 ϕ_{i1} 和 ϕ_{i2},使得

$$\begin{aligned}|\bar x_i\bar\Delta_i|\le\ & p^*\bar x_i\phi_{i1}+p^*\varepsilon_{i1}+p^*\bar x_i\phi_{i2}+\\ &\frac{\bar x_i^2}{4}\Big(1+\sum_{j=1}^{i-1}\Big(\frac{\partial\pi_{i-1}}{\partial x_j}\Big)^2\Big)+\\ &2ip^*\varepsilon_{i2}+d_i(t_0,t),\end{aligned}\tag{38}$$

其中

$$d_i(t_0,t)=\sum_{j=1}^{i}[p_1^*\phi_{j2}\circ\alpha_1^{-1}(2D(t_0,t))]^2.$$

由式(34)和(38)得

$$\begin{aligned}\dot V_i\le\ &\\ &-\sum_{j=1}^{i-1}k_{j-1}\bar x_{j-1}^2+\bar x_i\bar x_{i-1}+\mu_{i-1}(t_0,t)+\\ &p^*(\varepsilon_{i1}+2ip^*\varepsilon_{i2})+d_i(t_0,t)-\\ &\sum_{j=1}^{i-1}\Big[\tilde\theta_j^T\Gamma_j^{-1}+\sum_{l=j}^{i-2}\Big(\bar x_{l+1}\frac{\partial\pi_l}{\partial\hat\theta_j}\Big)\Big](\dot{\hat\theta}_j-\\ &\tau_{j(i-1)})+\tilde\theta_{i-1}^T\Gamma_{i-1}^{-1}(\dot{\hat\theta}_{i-1}-\tau_{2(i-1)})-\\ &\frac{\sigma_\theta}{2}\sum_{j=1}^{i-1}\tilde\theta_j^T\tilde\theta_j+\tilde\theta_i^T\Gamma_i^{-1}\dot{\hat\theta}_i+\Big(\frac{1}{\lambda}(\hat p-p^*)-\\ &\sum_{j=1}^{i-2}\bar x_{j+1}\frac{\partial\pi_j}{\partial\hat p}\Big](\dot{\hat p}-\tilde\omega_{i-1})+\bar x_i\Big[\bar x_{i-1}+\bar x_{i+1}+\\ &\pi_i-\kappa_i+\hat\theta_i^T\phi_i+w_i+\hat p(\phi_{i1}+\phi_{i2})+\end{aligned}$$

$$\frac{\bar{x}_i}{4}\Big(1+\sum_{j=1}^{i-1}\Big(\frac{\partial \pi_{i-1}}{\partial x_j}\Big)^2\Big)-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\theta_j^{\mathrm T}\phi_j-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}w_j-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial\theta_j}\theta_j-\frac{\partial\pi_{i-1}}{\partial \hat p}\dot{\hat p}\Big]+\bar x_i\theta_i^{\mathrm T}\phi_i-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\theta_j^{\mathrm T}\phi_j\bar x_i+(p^*-\hat p)(\phi_{i1}+\phi_{i2})\bar x_i. \tag{39}$$

基于式(39)，设计虚拟控制器

$$\pi_i = -k_i\bar x_i^2-\bar x_{i-1}+\kappa_i-\bar x_i\delta_i\tanh(\bar x_i\delta_i/\varepsilon_0)-\theta_i^{\mathrm T}\phi_i-\frac{\bar x_i}{4}\Big(1+\sum_{j=1}^{i-1}\Big(\frac{\partial\pi_{i-1}}{\partial x_j}\Big)^2\Big)-\Big(\hat p-\lambda\Big(\sum_{j=1}^{i-2}\bar x_{j+1}\frac{\partial\pi_j}{\partial\hat p}\Big)\Big)(\phi_{i1}+\phi_{i2})+\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial\theta_j}\tau_{ji}+\frac{\partial\pi_{i-1}}{\partial\hat p}\tilde\omega_i+\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\theta_j^{\mathrm T}\phi_j-\sum_{j=1}^{i-1}\bar x_j\frac{\partial\pi_{i-1}}{\partial x_j}\delta_j\tanh\Big(\bar x_j\frac{\partial\pi_{i-1}}{\partial x_j}\delta_j/\varepsilon_0\Big)-\sum_{j=1}^{i-1}\Big(\frac{\partial\pi_{i-1}}{\partial x_j}\phi_j\Gamma_j\sum_{l=j}^{i-2}\Big(\Big(\frac{\partial\pi_l}{\partial\theta_j}\Big)^{\mathrm T}\bar x_{l+1}\Big)\Big). \tag{40}$$

令

$$\tau_{ji} = \tau_{j(i-1)}-\Gamma_j\bar x_i\frac{\partial\pi_{i-1}}{\partial x_j}\phi_j^{\mathrm T}, \tag{41}$$

$$\tau_{ii} = \Gamma_i[\bar x_i\phi_i^{\mathrm T}-\sigma_\theta(\theta_i-\theta_{i0})], \tag{42}$$

$$\tilde\omega_i = \tilde\omega_{i-1}+\lambda\bar x_i(\phi_{i1}+\phi_{i2}). \tag{43}$$

其中：$\Gamma_i = \Gamma_i^{\mathrm T} > 0$，$\lambda > 0$，$\theta_{i0}$ 为 θ_i 的一个给定的初值.

由式(41)～(43)，式(39)可变为

$$\dot V_i \le -\sum_{j=1}^{i}k_j\bar x_j^2+\bar x_i\bar x_{i+1}+\mu_i(t_0,t)+\theta_i^{\mathrm T}\Gamma_i^{-1}(\dot\theta_i-\tau_{ii})-\sum_{j=1}^{i-1}\Big(\theta_j^{\mathrm T}\Gamma_j^{-1}+\sum_{l=j}^{i-2}\Big(\bar x_{l+1}\frac{\partial\pi_l}{\partial\theta_j}\Big)(\theta_j-\tau_{j(i-1)})\Big)+\theta_{i-1}^{\mathrm T}\Gamma_{i-1}^{-1}(\dot\theta_{i-1}-\tau_{2(i-1)})+\Big(\frac{1}{\lambda}(\hat p-p^*)-\sum_{j=1}^{i-2}\bar x_{j+1}\frac{\partial\pi_j}{\partial\hat p}\Big)(\dot{\hat p}-\tilde\omega_{i-1})+\bar x_i\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial\theta_j}\tau_{ji}+\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial\theta_j}\theta_j\bar x_i+\bar x_i\frac{\partial\pi_{i-1}}{\partial\hat p}(\tilde\omega_i-\dot{\hat p})-\sum_{j=1}^{i-1}\frac{\partial\pi_{i-1}}{\partial x_j}\theta_j^{\mathrm T}\phi_j\bar x_i+(p^*-\hat p)(\phi_{i1}+\phi_{i2})\bar x_i-\frac{1}{2}\sum_{j=1}^{i}\sigma_\theta\theta_j^{\mathrm T}\theta_j, \tag{44}$$

其中

$$\mu_i(t_0,t) = \mu_{i-1}(t_0,t)+p^*(\varepsilon_{i1}+2ip^*\varepsilon_{i2})i\times 0.2785\varepsilon_0+d_i(t_0,t)+\frac{1}{2}\sigma_\theta|\theta_i^*-\theta_{i0}|^2.$$

则式(44)可进一步写成

$$\dot V_i \le -\sum_{j=1}^{i}\Big(k_j\bar x_j^2+\frac{1}{2}\sigma_\theta\theta_j^{\mathrm T}\theta_j\Big)+\bar x_i\bar x_{i+1}-\sum_{j=1}^{i}\Big[\Big(\theta_j^{\mathrm T}\Gamma_j^{-1}+\sum_{l=j}^{i-1}\Big(\bar x_{l+1}\frac{\partial\pi_l}{\partial\theta_j}\Big)\Big)(\theta_j-\tau_{ji})\Big]+\frac{1}{\lambda}\Big[(\hat p-p^*)+\mu_i(t_0,t)-\Big(\sum_{j=1}^{i-1}\bar x_{j+1}\frac{\partial\pi_j}{\partial\hat p}\Big)\Big](\dot{\hat p}-\tilde\omega_i). \tag{45}$$

第 n 步　在最后一步中，将得到实际的控制器 u. 考虑整个 Lyapunov 函数

$$V_n = V_{n-1}+\frac{1}{2}\bar x_n^2+\frac{1}{2}\theta_n^{\mathrm T}\Gamma_n^{-1}\theta_n, \tag{46}$$

在式(40)～(43)中，对于中间的自适应函数 τ_{ji} 和 $\tilde\omega_i$，令其中的 $i = n$. 对式(46)两边求导，得

$$\dot V_n \le -\sum_{i=1}^{n}\sum_{j=1}^{i}\Big(k_j\bar x_j^2+\frac{1}{2}\sigma_\theta\theta_j^{\mathrm T}\theta_j\Big)+\mu_n(t_0,t)-\sum_{j=1}^{n}\Big[\Big(\theta_j^{\mathrm T}\Gamma_j^{-1}+\sum_{l=j}^{n-1}\Big(\bar x_{l+1}\frac{\partial\pi_l}{\partial\theta_j}\Big)\Big)(\theta_j-\tau_{jn})\Big]+\frac{1}{\lambda}\Big[(\hat p-p^*)+\frac{n(n+1)}{2}0.2785\varepsilon_0-\Big(\sum_{j=1}^{n-1}\bar x_{j+1}\frac{\partial\pi_j}{\partial\hat p}\Big)\Big](\dot{\hat p}-\tilde\omega_n). \tag{47}$$

根据以上递归设计方法，最后一步得到 $u = \pi_n$(因 $\bar x_{n+1} = 0$) 和自适应律 $\dot\theta_n = \tau_{jn}$，$\dot{\hat p} = \tilde\omega_n$. 则得

$$\dot V_n \le -c_nV_n+\mu_n(t_0,t). \tag{48}$$

其中 c_n 和非负函数 μ_n 定义如下：

$$c_n = \min\{2k_i, \bar c_0, \sigma_p\lambda, \sigma_\theta/\lambda_{\max}(\Gamma_i^{-1})\}, \tag{49}$$

$$\mu_n(t_0,t) = \frac{d_0}{\lambda_0}+\sum_{i=1}^{n}p^*(\varepsilon_{i1}+2\varepsilon_{i2}+d_i(t_0,t))+\frac{\sigma_\theta}{2}\sum_{i=1}^{n}|\theta_i^*-\theta_{i0}|^2+\frac{\sigma_p}{2}|p^*-p_0|^2+\frac{n(n+1)}{2}0.2785\varepsilon_0. \tag{50}$$

以上设计和分析步骤可总结成如下定理：

定理 1　在假设 1～假设 3 存在的条件下，控制方案(40)～(43)能使闭环系统(1)一致终结有界，而且对于任意的 $\mu > 0$，存在 T 使得 $|y(t)| \le \mu$.

4　仿真实例

考虑如下二阶非线性系统：

$$\dot{z} = -z + x_1^2,$$
$$\dot{x}_1 = x_2 + x_1 e^{-0.5x_1} + (x_1 e^{\xi_1 x_1} + \xi_2 \sin(t^2)) + \xi_3 z,$$
$$\dot{x}_2 = u + x_1 x_2^2 + \xi_4 z^2 x_1.$$

其中:$f_1(x_1) = x_1 e^{-0.5x_1}$ 和 $f_2(x_1, x_2) = x_1 x_2^2$ 是未知函数,$\Delta_1 = (x_1 e^{\xi_1 x_1} + \xi_2 \sin(t^2)) + \xi_3 z$ 和 $\Delta_2 = \xi_4 z^2 x_1$ 中的 $\xi_i (1 \leq i \leq 4)$ 是未知常值参数,z 是未知动态. 定义动态信号

$$\dot{r} = -1.2r + 1.25s^4 + 1.68.$$

选取模糊隶属函数

$$\mu_{F_1^l}(x_1) = \exp[-(x_1 - 3 + l)^2/16],$$
$$\mu_{F_2^l}(x_1, x_2) = \exp[-(x_1 - 3 + l)^2/4] \times \exp[-(x_2 - 3 + l)^2/4],$$
$$l = 1, 2, \cdots, 5.$$

控制器和参数自适应律中的设计参数选为

$$k_1 = 10,\ k_2 = 2,\ \delta_1 = 0.1,\ \delta_2 = 0.2,$$
$$\lambda_0 = 1,\ \lambda = 4,\ \Gamma_1 = \Gamma_2 = \mathrm{diag}\{10, 10\},$$
$$\sigma_\theta = 0.4,\ \sigma_p = 4,\ p_0 = 0.01,$$
$$\theta_{10} = \theta_{20} = [0.1, 0.1, 0.1, 0.1, 0.1].$$

初始条件选取

$$x_1(0) = 1,\ x_2(0) = -7,$$
$$\hat{p}(0) = 0.1,\ r(0) = 0.6.$$

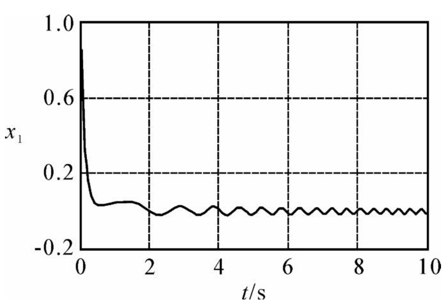

图 1 状态 x_1 的仿真曲线

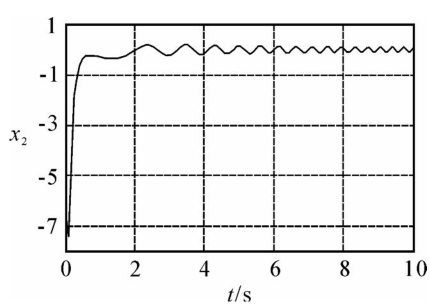

图 2 状态 x_2 的仿真曲线

$$\theta_1(0) = [0.1, 0.6, 0.9, 0.7, 0.9],$$
$$\theta_2(0) = [0.6, 0.9, 0.7, 0.7, 0].$$

图 1 和图 2 分别给出了状态 x_1 和 x_2 的计算机仿真曲线.

5 结 论

本文针对一类具有未建模动态的非线性不确定系统,基于模糊自适应控制和 backstepping 设计技术,提出一种自适应鲁棒模糊控制器的设计方法. 与现有结果相比,本文研究的不确定非线性系统不仅包含未知函数和动态不确定项,而且包含未建模动态,因此本文结果更具普遍性.

参考文献(References)

[1] Wang L X. Adaptive fuzzy systems and control: Design and stability analysis[M]. Englewood Cliffs: Prentice-Hall, 1994.

[2] Chen B S, Lee C H, Chang Y C. H_∞ tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach[J]. IEEE Trans on Fuzzy Systems, 1996, 4(2): 32-43.

[3] Chai T Y, Tong S C. Fuzzy direct adaptive control for a class of nonlinear systems[J]. Fuzzy Sets and Systems, 1999, 103(3): 379-389.

[4] Tong S C, Tang J T, Wang T. Fuzzy adaptive control of multivariable nonlinear systems[J]. Fuzzy Sets and Systems, 2000, 111(2): 153-167.

[5] Yang Y C. Robust tracking control of nonlinear MIMO systems via fuzzy approaches[J]. Automatica, 2000, 36(10): 1535-1545.

[6] Yang Y S, Zhou C J. Adaptive fuzzy H_∞ stabilization for strict-feedback canonical nonlinear systems via backstepping and small-gain approach[J]. IEEE Trans on Fuzzy Systems, 2005, 13(1): 104-114.

[7] Zhou S S, Feng G, Feng C B. Robust control for a class of uncertain nonlinear systems: Adaptive fuzzy approach based on backstepping[J]. Fuzzy Sets and Systems, 2005, 151(1): 1-20.

[8] Chen B, Liu X P. Fuzzy approximate disturbance decoupling of MIMO nonlinear systems by backstepping and application to chemical processes[J]. IEEE Trans on Fuzzy Systems, 2005, 13(6): 832-847.

[9] Jiang Z P, Praly L. Design of robust adaptive controllers for nonlinear systems with dynamic uncertainties[J]. Automatica, 1998, 34(7): 825-840.