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不确定系统鲁棒容错 H 控制的 LMI 设计方法

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摘要: 针对不确定线性系统, 研究了执行器失效情况下鲁棒容错 H 控制问题. 基于连续增益故障模式, 利用线性矩阵不等式 LMI 推导了系统 H 指标约束下鲁棒容错镇定的充要条件, 分别给出了输出反馈和状态反馈 H 控制器的设计方法. 通过引入变量代换, 将求解输出反馈 H 指标约束的鲁棒容错控制器的可解条件转化为标准的 LMI. 所获得的控制器不仅能使故障系统鲁棒稳定, 并且能达到给定的 H 性能指标. 仿真实例验证了所提出设计方法的有效性.

关键词: 线性矩阵不等式; 鲁棒容错控制; 输出反馈; 状态反馈; H 控制

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LMI approach to robust fault-tolerant H control for uncertain systems

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Abstract: For uncertain linear systems with actuator faults, the design of robust fault-tolerant H control is studied. Based on an actuator failure model with continuous gain, the necessary and sufficient conditions for robust fault-tolerant stabilization with H index constraints are obtained by using linear matrix inequality (LMI). Then the corresponding design methods of H output feedback and state feedback controllers are given. The solvable condition of output feedback robust fault-tolerant controller is changed into the standard LMI by means of the variable substitution. The proposed controllers in the paper not only enable the systems to keep robust stabilization but also achieve the given H performance. A simulation experiment shows the effectiveness of the proposed approach.

Key words: Linear matrix inequality; Robust fault-tolerant; Output feedback; State feedback; H control

1 引言

近年来,鲁棒容错控制成为容错控制理论中的一大热点.鲁棒容错 H 控制就是将容错控制与 H 控制相结合,使系统在不确定性、外部扰动及故障模式下闭环系统鲁棒稳定,并满足一定的指标约束.近年来针对这方面的研究成果不是很多^[1,2].

文献[3]给出了不确定系统对执行器失效具有完整性的充分条件,将控制器的设计归结为求解 Riccati 方程和不等式.文献[4]研究了不确定关联系统 H 输出反馈控制问题,通过选取适当的同伦函数表示非线性矩阵不等式,通过迭代算法进行求解.文献[5,6]提出了连续增益故障模型,该模型既包含离散故障模型,又表示出离散故障模型不能表

示的故障情况.基于此,本文利用 LMI 对具有连续增益故障系统,给出系统输出反馈和状态反馈 H 指标约束下鲁棒容错镇定的充要条件,通过引入变量代换,将输出反馈 H 指标约束的鲁棒容错控制器非线性矩阵不等式的求解条件转化为标准的 LMI.文中所设计的控制器,当出现执行器故障时,不仅能保证闭环系统渐近稳定,并能满足给定的 H 干扰衰减指标.

2 问题描述

考虑如下形式线性不确定系统:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + B_1 w(t) + \\ &\quad (B_2 + \Delta B_2)u^F(t), \\ z(t) &= C_1 x(t) + D_{12} u^F(t), \end{aligned}$$

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则称系统(4)是输出反馈 H 指标约束鲁棒容错镇定;当故障阵 $M = I$ 时,则称系统(1)是输出反馈 H 指标约束下的鲁棒镇定.其中

$$\begin{aligned} 11 &= 11 + LL^T + \dots^{-1}(LL^T + \dots^T), \quad 21 = 21, \\ 22 &= 22 + (YLL^TY + E_1^TE_1) + \dots^{-1}E_1^TE_1, \\ 11 &= AX + XA^T + B_2MC_cU^T + (B_2MC_cU^T)^T, \\ 21 &= A^T + YAX + VB_cC_2X + \\ &\quad YB_2MC_cU^T + VA_cU^T, \\ 22 &= YA + A^TY + VB_cC_2 + (VB_cC_2)^T, \\ &= E_1X + E_2MC_cU^T. \end{aligned}$$

证明 先证充分性.根据引理1及式(6),设 X_d 具有如下形式:

$$X_d = \begin{bmatrix} Y & V \\ V^T & * \end{bmatrix}, \quad X_d^{-1} = \begin{bmatrix} X & U \\ U^T & * \end{bmatrix},$$

由 $X_d X_d^{-1} = I$,可以得到

$$UV^T = I - XY, \quad * = U^{-1}(XYX - X)U^{-T}. \quad (8)$$

由矩阵 Schur 补性质,将式(6)改写为 $X - Y^{-1} > 0$,再根据式(8),有 $* - V^T Y^{-1} V = U(X - Y^{-1})U^{-T} > 0$,即 $X_d > 0$ 是正定对称矩阵.令

$$\begin{aligned} 11 &= AX + XA^T + B_2MC_cU^T + \\ &\quad (B_2MC_cU^T)^T, \\ 21 &= A^T + YAX + YB_2MC_cU^T, \\ 22 &= YA + A^TY, \\ 1 &= \begin{bmatrix} AX + B_2MC_cU^T & 0 \\ 0 & 0 \end{bmatrix}, \\ 2 &= \begin{bmatrix} 0 & 0 \\ 0 & YA \end{bmatrix}, \quad 3 = \begin{bmatrix} 0 & 0 \\ A^T & 0 \end{bmatrix}, \\ 4 &= \begin{bmatrix} 0 & 0 \\ Y(AAX + B_2MC_cU^T) & 0 \end{bmatrix}, \end{aligned}$$

所以 $\begin{bmatrix} 11 & 21 \\ 21 & 22 \end{bmatrix} = \sum_{i=1}^4 (i + i^T)$.根据引理2及式

(2) 可得

$$\begin{aligned} 1 + 1^T &= \begin{bmatrix} L \\ 0 \end{bmatrix} (tI \quad 0J) + \left(\begin{bmatrix} L \\ 0 \end{bmatrix} (tI \quad 0J) \right)^T \\ &= \begin{bmatrix} LL^T & 0 \\ 0 & 0 \end{bmatrix} + \dots^{-1} \begin{bmatrix} \dots^T & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

同理可得

$$\begin{aligned} 2 + 2^T &= \begin{bmatrix} 0 & 0 \\ 0 & YLL^TY \end{bmatrix} + \dots^{-1} \begin{bmatrix} 0 & 0 \\ 0 & E_1^TE_1 \end{bmatrix}, \\ 3 + 3^T &= \begin{bmatrix} 0 & 0 \\ 0 & E_1^TE_1 \end{bmatrix} + \dots^{-1} \begin{bmatrix} LL^T & 0 \\ 0 & 0 \end{bmatrix}, \\ 4 + 4^T &= \begin{bmatrix} 0 & 0 \\ 0 & YLL^TY \end{bmatrix} + \dots^{-1} \begin{bmatrix} \dots^T & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

由

$$\begin{bmatrix} 11 & 21 \\ 21 & 22 \end{bmatrix} = \begin{bmatrix} LL^T + \dots^{-1}(LL^T + \dots^T) & 0 \\ 0 & (YLL^TY + E_1^TE_1) + \dots^{-1}E_1^TE_1 \end{bmatrix}, \quad (9)$$

有

$$\begin{aligned} &= \begin{bmatrix} 11 + 11 & * & * & 0 \\ 21 + 21 & 22 + 22 & * & 0 \\ B_1^T & (YB_1 + VB_cD_{21})^T & -I & * \\ C_1X + D_{12}MC_cU^T & C_1 & 0 & -I \end{bmatrix} \\ & \quad (X, Y, A_c, B_c, C_c, \dots) < 0. \end{aligned} \quad (10)$$

设

$$T_1 = \begin{bmatrix} X & I \\ U^T & 0 \end{bmatrix}, \quad T_2 = X_d T_1 = \begin{bmatrix} I & Y \\ 0 & V^T \end{bmatrix}. \quad (11)$$

可将式(10)整理得

$$\begin{aligned} &= \begin{bmatrix} T_1^T(A_d^T X_d + X_d A_d) T_1 & T_1^T X_d B_d & T_1^T C_d^T \\ B_d^T X_d T_1 & -I & 0 \\ C_d T_1 & 0 & -I \end{bmatrix} < 0. \end{aligned} \quad (12)$$

将式(12)两边分别左乘 $\text{diag}\{ (T_1^T)^{-1}, 0, 0 \}$ 和右乘 $\text{diag}\{ T_1^{-1}, 0, 0 \}$,有

$$\begin{bmatrix} A_d^T X_d + X_d A_d & X_d B_d & C_d^T \\ B_d^T X_d & -I & 0 \\ C_d & 0 & -I \end{bmatrix} < 0, \quad (13)$$

即 $A_d^T X_d + X_d A_d < -X_d B_d B_d^T X_d - C_d^T C_d < 0$.所以 $A_d^T X_d + X_d A_d$ 是负定的,即 A_d 矩阵为稳定矩阵.根据引理3,有 $G_{zw}(s) < \dots$,所以系统(4)是输出反馈 H 指标约束的鲁棒容错镇定;当故障阵 $M = I$ 时,系统是输出反馈 H 指标约束下的鲁棒镇定.

必要性证明.假设系统(4)是输出反馈 H 指标约束的鲁棒容错镇定,则存在一个正定对称解 X_d ,使得式(13)成立.将式(13)两边分别左乘 $\text{diag}\{ T_1^T, 0, 0 \}$ 和右乘 $\text{diag}\{ T_1, 0, 0 \}$,有

$$\begin{bmatrix} T_1^T(A_d^T X_d + X_d A_d) T_1 & T_1^T X_d B_d & T_1^T C_d^T \\ B_d^T X_d T_1 & -I & 0 \\ C_d T_1 & 0 & -I \end{bmatrix} < 0, \quad \text{即}$$

将式(18)左乘和右乘矩阵 $\text{diag}\{P^{-1}, I, I, I, I\}$, 并令 $X = P^{-1}$, 可求得

$$\begin{aligned} & \lambda_2(P, K, \dots) = \\ & \begin{bmatrix} E_1^T & K^T M^T E_2^T & C_1^T & K^T M^T \\ E_1 & -I & 0 & 0 \\ E_2 M K & 0 & -I & 0 \\ C_1 & 0 & 0 & -I \\ M K & 0 & 0 & -I \end{bmatrix} < 0, \end{aligned} \tag{19}$$

其中

$$\begin{aligned} & = A^T P + P A + K^T M^T B_2^T P + P B_2 M K + \\ & \quad -I P P + -I P L L^T P + -I P B_1 B_1^T P. \end{aligned}$$

由引理 1, 式(19)可以等价改写成如下形式:

$$\begin{aligned} & \lambda_2(P, K, \dots) = \\ & A^T P + P A + K^T M^T B_2^T P + P B_2 M K + E_1^T E_1 + \\ & \quad -I P P + K^T E_2^T E_2 K + -I P L L^T P + \\ & \quad -I P B_1 B_1^T P + C_1^T C_1 + K^T K < 0. \end{aligned} \tag{20}$$

因为

$$\begin{aligned} & A^T P + P A + E_1^T E_1 + -I P P, \\ & K^T M^T B_2^T P + P B_2 M K \\ & K^T M^T E_2^T E_2 K + -I P L L^T P, \end{aligned} \tag{21}$$

所以

$$\begin{aligned} & A^T P + P A + A^T P + P A + K^T M^T B_2^T P + \\ & P B_2 M K + K^T M^T B_2^T P + P B_2 M K + \\ & -I P B_2 D^T P + C_1^T C_1 + K^T K \\ & (P, K, \dots) < 0. \end{aligned} \tag{22}$$

又由假设 1 得, $D_{12}^T C_1 = 0, D_{12}^T D_{12} = I$, 所以

$$\begin{aligned} & C_1^T C_1 + K^T K = \\ & C_1^T C_1 + K^T D_{12}^T C_1 + C_1^T D_{12} K + K^T D_{12}^T D_{12} K = \end{aligned}$$

$$\begin{aligned} & C_1^T C_1 + (D_{12} K)^T C_1 + C_1^T (D_{12} K) + \\ & (D_{12} K)^T D_{12} K = \\ & C_1^T (C_1 + D_{12} K) + (D_{12} K)^T (C_1 + D_{12} K) = \\ & (C_1 + D_{12} K)^T (C_1 + D_{12} K). \end{aligned}$$

将式(23)代入(22), 整理得

$$A_{ck}^T P + P A_{ck} < -I P B_{ck} B_{ck}^T P - C_{ck}^T C_{ck} < 0,$$

所以 A_{ck} 是稳定矩阵. 根据引理 3, 系统(5) 是状态反馈 H 指标约束下的鲁棒容错镇定.

4 仿真示例

考虑如下不确定系统:

$$\dot{x}(t) = (A + \Delta A) x(t) + B_1 w(t) + (B_2 + \Delta B_2) u^F(t),$$

$$z(t) = C_1 x(t) + D_{12} u^F(t),$$

$$y(t) = C_2 x(t) + D_{21} w(t),$$

其可控输出为 $z(t) = [x^T(t) \quad u^T(t)]^T$. 其中

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.47 \\ 0.45 \end{bmatrix}, L = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, D_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C_2 = [1 \quad 1], D_{21} = 1, E_1 = [1 \quad 0.1],$$

$$E_2 = 0.1, \Delta A = \sin(t), \Delta B_2 = 2,$$

$$\Delta C_1 = 10, \Delta C_2 = 10, \Delta D_{12} = 10.$$

根据文中设计的方法, 求得该系统输出反馈 H 指标约束的鲁棒容错控制武器为

$$A_c = \begin{bmatrix} -1.5598 & -0.5252 \\ 0.0541 & -2.1347 \end{bmatrix},$$

表 1 闭环系统在不同执行器故障和不确定性下的极点分布及 H 指标

执行器故障	不确定性 $(t) = \sin(t)$	输出反馈		状态反馈	
		极点分布	H 性能指标	极点分布	H 性能指标
$M = 1$	$t = 0$	- 1.1050	0.1400	- 3.4404	0.1556
		- 1.9581 + 0.3728i			
		- 1.9581 - 0.3728i			
		- 1.6733			
$M = 2$	$t = 0.5$	- 1.0540	0.0935	- 3.4173	0.1568
		- 1.9593 + 0.3708i			
		- 1.9593 - 0.3708i			
		- 1.6739			
$M = 0.5$	$t = 1$	- 1.0156	0.2543	- 3.3999	0.1578
		- 1.9602 + 0.3694i			
		- 1.9602 - 0.3694i			
		- 1.6733			
$M = 0.1$	$t = 3$	- 1.0899	1.2202	- 3.4336	0.1560
		- 1.9585 + 0.3722i			
		- 1.9585 - 0.3722i			
		- 1.6735			

$$B_c = \begin{bmatrix} -1.7328 \\ 0.8957 \end{bmatrix},$$

$$C_c = [0.0748 \quad -0.3327]^T.$$

状态反馈 H 指标约束的鲁棒容错器律为 $K = [-4.7202 \quad -0.4918]$.

分别采用上述控制器得到闭环系统在不同故障模式及不确定性情况下的极点分布和 H 指标如表 1 所示. 从表 1 中可以看出,对于不同的执行器故障和系统的不确定性,闭环系统的极点均位于复平面的左半平面,即闭环系统是渐近稳定的;同时闭环系统还满足预先给定的 衰减指标.

5 结 论

本文研究了不确定系统鲁棒容错 H 输出反馈和状态反馈控制器设计问题. 基于有界实引理给出了两种控制器 H 指标约束鲁棒容错镇定的充分必要条件. 引入变量代换,将求解输出反馈指标约束的鲁棒容错控制器的可解条件化为标准的 LMI. 文中所设计的控制器不仅能使系统对不确定具有鲁棒性,而且当系统出现故障时,系统仍保持渐近稳定并满足给定的 H 干扰衰减指标. 仿真结果表明所求控制器是有效的.

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