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扩展风险性滤波算法的修正方法

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摘要: 针对非线性系统扩展风险性滤波(ERSF)估计精度较低的缺点,提出一种基于二阶 Taylor 展开对其进行高阶修正的算法.该算法利用高阶项对一阶扩展卡尔曼滤波(EKF)的状态估计向量及协方差阵进行适当修正,并由新息滤波方法得到修正的扩展风险性滤波(MERSF).高度非线性的仿真研究表明,所提出的算法在计算量增加不多的情况下,滤波精度有明显的提高.

关键词: 风险性滤波; 扩展卡尔曼滤波; 泰勒级数展开

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Modified method for extension risk-sensitive filter

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Abstract: In order to overcome the flaw that the extended risk-sensitive filter (ERSF) has poor filtering precision, an appropriate modified principle based on the second order Taylor series extension theory is presented. The modified extended risk-sensitive filter (MERSF) is obtained by using innovation filter and making the higher order terms derived from the nonlinear dynamic system to modify the state estimation and its covariance matrix of the first order extended Kalman filter (EKF). Simulation results of a severely nonlinear plant show that the filtering precision of the proposed algorithm is improved notably with appropriately increasing computing load.

Key words: Risk-sensitive filter; Extended Kalman filter; Taylor series extension

1 引言

对于模型不确定性系统,由于风险敏感性滤波(RSF)比风险中立滤波具有更强的鲁棒性,已受到众多学者的关注^[1-11].风险敏感性滤波以最小化指数型二次代价函数的期望为准则,其难点在于高维积分的计算.对于线性高斯系统,有类似卡尔曼滤波的闭环形式的解;对于非线性随机高斯模型,一种数值有效且积分可及的方法是扩展风险敏感性滤波(ERSF)^[4].ERSF将扩展卡尔曼滤波的线性化技术用于风险敏感性滤波,所求得的状态估计有较大的截断误差,导致滤波精度降低,但计算量较小.而基于栅格方法^[5,6]和粒子滤波算法^[7,8]的风险敏感性滤波,虽然不要求模型有高斯性的假定,且估计精度较高,但计算量太大.

本文利用二阶 Taylor 展开,对扩展风险敏感性滤波(ERSF)的状态估计及其协方差阵进行适当修正,目的在于减小截断误差.同时利用矩阵的 QR 分解和 Cholesky 分解技术,对协方差阵提前进行分解,以保证其正定性,并由信息滤波方法,得到一种新的风险敏感性滤波方法——修正的扩展风险敏感性滤波(MERSF).

2 问题描述

非线性离散系统状态方程和量测方程可表示为

$$x_{k+1} = f_k(x_k) + w_k, \quad (1)$$

$$z_k = h_k(x_k) + v_k. \quad (2)$$

其中: x_k 和 w_k 为 k 时刻系统的状态和相互独立的过程噪声向量; $f_k; R^n \rightarrow R^n$ 为非线性系统状态演化映射; w_k 为噪声转移矩阵; z_k 和 v_k 为 k 时刻的量测和

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量测噪声; $h_k \ R^n \ R^m$ 为非线性量测映射; w_k 和 v_k 为相互统计独立的噪声随机向量, 而且与状态的初值 x_0 不相关.

本文的目的是估计状态变量的函数 $\phi = \phi(X)$. 具有一个参数的风险灵敏度代价函数定义为

$$J_{RS}(\Phi_1^*, \Phi_2^*, \dots, \Phi_{k-1}^*, \Phi_k^*) = E[\exp(\mu \sum_{i=1}^{k-1} (\phi(x_i) - \Phi_i^*)) + \mu_2 (\phi(x_k) - \Phi_k^*)]. \quad (3)$$

其中: $\Phi_i^*, i \in \{0, 1, \dots, k-1\}$, 表示 $\phi(X)$ 在过去直到 $k-1$ 时刻的最优值; $\mu > 0$ 为风险参数; $\mu_1(\cdot)$ 和 $\mu_2(\cdot)$ 为 2 个严格凸的、连续有下界的函数. 当前的最优估计 Φ_k^* 可由最小化风险函数(3) 得到, 即

$$\Phi_k^* = \arg \min J_{RS}(\Phi_1^*, \Phi_2^*, \dots, \Phi_{k-1}^*, \Phi_k^*). \quad (4)$$

记 $(x_{k+1} / x_k) \triangleq p_{x_{k+1}/x_k}(\cdot / x_k), (y_k / x_k) \triangleq p_{y_k/x_k}(\cdot / x_k)$, 则式(4) 的递归形式可由信息状态 \hat{x}_k 和参数 \hat{P}_k 表示为

$$\hat{x}_{k+1} = (x_{k+1} / x_k) (y_k / x_k) \exp(\mu_1(\phi(x_k) - \Phi_k^*)) \hat{x}_k, \quad (5)$$

$$\hat{P}_k = \arg \min_R \exp(\mu_2(\phi(x_k) - \Phi_k^*)) \hat{P}_k. \quad (6)$$

当所估计的函数为状态向量自身, 即 $\phi(x) = x$, $\mu_1(\cdot)$ 和 $\mu_2(\cdot)$ 为二次函数, $\mu_j(x) = x^T x (j = 1, 2)$ 时, 式(5) 和(6) 变为

$$\hat{x}_{k+1} = (x_{k+1} / x_k) (y_k / x_k) \exp(\mu(x_k - \hat{x}_{k/k})^T (x_k - \hat{x}_{k/k})) \hat{x}_k, \quad (7)$$

$$\hat{P}_k = \arg \min_R \exp(\mu(x_k - \hat{x}_k)^T (x_k - \hat{x}_k)) \hat{P}_k. \quad (8)$$

其中: $\hat{x}_{k/k}$ 表示状态 x_k 的最优估计, 且递归依赖于 $\hat{x}_{k-1/k-1}$.

3 修正的扩展风险性滤波

3.1 扩展风险性滤波(ERSF)

对于非线性系统(1) 和(2), 在二次风险灵敏度代价函数意义下, 基于扩展卡尔曼新息滤波的思想, 式(7) 和(8) 的解为

$$\hat{x}_{k/k} = f_{k-1}(\hat{x}_{k-1/k-1}) + (P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} \times C_k^T R_k^{-1} (z_k - h_k(f_{k-1}(\hat{x}_{k-1/k-1}))), \hat{x}_0 = \bar{x}_0. \quad (9)$$

其中: $C_k = \frac{\partial h_k}{\partial x} \Big|_{x=f_{k-1}(\hat{x}_{k-1/k-1})}$, R_k 为量测噪声的协方差矩阵. 先验估计误差的协方差矩阵 P_k 满足下列方程:

$$P_k = P_{k-1} Q_{k-1}^{-1} + A_{k-1} (P_{k-1}^{-1} +$$

$$C_{k-1}^T R_{k-1}^{-1} C_{k-1} - \mu I)^{-1} A_{k-1}^T. \quad (10)$$

其中: $A_{k-1} = \frac{\partial f_{k-1}}{\partial x} \Big|_{x=\hat{x}_{k-1/k-1}}$, Q_k 为系统噪声的协方差矩阵, I 为相应的单位矩阵. 滤波的增益为

$$K_k = (P_k^{-1} + C_k^T R_k^{-1} C_k)^{-1} C_k^T R_k^{-1}, \quad (11)$$

当且仅当

$$P_i^{-1} + C_i^T R_i^{-1} C_i - \mu I > 0, \quad i = 1, 2, \dots, N \quad (12)$$

成立时, 滤波存在.

3.2 修正的扩展风险性滤波(MERSF)

扩展卡尔曼滤波用一阶 Taylor 展开逼近非线性函数, 为了减少截断误差的影响, 本文用二阶 Taylor 展开对扩展风险性滤波进行修正. 假设在 $k-1$ 时刻已经获得系统的最优估计 $\hat{x}_{k-1/k-1}$ 及其协方差 P_{k-1} , 在 k 时刻, 首先对式(1) 中的 $f_{k-1}(x_{k-1})$ 和式(2) 中的 $h_k(x_k)$ 分别基于 $x_{k-1} = \hat{x}_{k-1/k-1}$ 和 $x_k = f_{k-1}(\hat{x}_{k-1/k-1})$ 进行二阶 Taylor 展开, 得

$$x_k = f_{k-1}(\hat{x}_{k-1/k-1}) + A_{k-1} \tilde{x}_{k-1/k-1} + \frac{1}{2} \sum_{i=1}^n e_i \tilde{x}_{k-1/k-1} f_{i,k-1}^{xx} \tilde{x}_{k-1/k-1} + \dots + \mu_{k-1} w_{k-1}, \quad (13)$$

$$y_k = h(x_k) + C_k \tilde{x}_{k/k-1} + \frac{1}{2} \sum_{j=1}^m e_j \tilde{x}_{k/k-1} h_{j,k}^{xx} \tilde{x}_{k/k-1} + v_k. \quad (14)$$

式中

$$\tilde{x}_{k-1/k-1} = x_{k-1} - \hat{x}_{k-1/k-1},$$

$$\tilde{x}_{k/k-1} = x_k - f_{k-1}(\hat{x}_{k-1/k-1}),$$

$$A_{k-1} = \frac{\partial f_{k-1}(x_{k-1})}{\partial x_{k-1}} \Big|_{x_{k-1}=\hat{x}_{k-1/k-1}},$$

$$f_{i,k-1}^{xx} = \frac{\partial}{\partial x_{k-1}} \left[\frac{\partial f_{i,k-1}(x_{k-1})}{\partial x_{k-1}} \right]^T \Big|_{x_{k-1}=\hat{x}_{k-1/k-1}},$$

$$C_k = \frac{\partial h_k(x_k)}{\partial x_k} \Big|_{x_k=f_{k-1}(\hat{x}_{k-1/k-1})},$$

$$h_{j,k}^{xx} = \frac{\partial}{\partial x_k} \left[\frac{\partial h_{j,k}(x_k)}{\partial x_k} \right]^T \Big|_{x_k=f_{k-1}(\hat{x}_{k-1/k-1})},$$

$e_i \ R^n$ 和 $e_j \ R^m$ 分别表示第 i 个和第 j 个基向量. 基于式(13) 和(14), 滤波的一步预测及其协方差为

$$\hat{x}_{k/k-1} = f_{k-1}(\hat{x}_{k-1/k-1}) + \mu_{k-1} \bar{w}_{k-1} + a_{k-1}/2, \quad (15)$$

$$P_{k/k-1} = P_{k-1} Q_{k-1}^{-1} + A_{k-1} P_{k-1/k-1} A_{k-1}^T + D_{k-1}/2. \quad (16)$$

式中

$$a_{k-1} = [a_{1,k-1}, a_{2,k-1}, \dots, a_{n,k-1}]^T,$$

$$a_{i,k-1} =$$

$$\begin{aligned} & \text{tr} \left[\frac{\partial}{\partial x_{k-1}} \left[\frac{\partial f_{i,k-1}(x_{k-1})}{\partial x_{k-1}} \right]^T \right]_{x_{k-1}=\hat{x}_{k-1/k-1}} P_{k-1/k-1}, \\ D_{k-1} &= [D_{ij(k-1)}]_{n \times n}, D_{ij(k-1)} = \text{tr}[G], \\ G &= \\ & \frac{\partial}{\partial x_{k-1}} \left[\frac{\partial f_{i,k-1}(x_{k-1})}{\partial x_{k-1}} \right]^T \Big|_{x_{k-1}=\hat{x}_{k-1/k-1}} P_{k-1/k-1} \times \\ & \frac{\partial}{\partial x_{k-1}} \left[\frac{\partial f_{i,k-1}(x_{k-1})}{\partial x_{k-1}} \right]^T \Big|_{x_{k-1}=\hat{x}_{k-1/k-1}} P_{k-1/k-1}, \end{aligned}$$

其中 $\text{tr}(\cdot)$ 为矩阵求迹运算符号. 量测的一步预测及其协方差为

$$\hat{x}_{k/k-1} = h_k(\hat{x}_{k/k-1}) + \bar{v}_k + b_k/2, \quad (17)$$

$$S_k = C_k P_{k/k-1} C_k^T + R_k + B_k/2. \quad (18)$$

式中

$$\begin{aligned} b_k &= [b_{1,k}, b_{2,k}, \dots, b_{m,k}]^T, \\ b_{i,k} &= \text{tr} \left[\frac{\partial}{\partial x_k} \left[\frac{\partial h_{i,k}(x_k)}{\partial x_k} \right]^T \right]_{x_k=\hat{x}_{k/k-1}} P_{k/k-1}, \\ B_k &= [B_{ijk}]_{m \times m}, B_{ijk} = \text{tr}[L], \\ L &= \frac{\partial}{\partial x_k} \left[\frac{\partial h_{i,k}(x_k)}{\partial x_k} \right]^T \Big|_{x_k=\hat{x}_{k/k-1}} P_{k/k-1} \times \\ & \frac{\partial}{\partial x_k} \left[\frac{\partial h_{j,k}(x_k)}{\partial x_k} \right]^T \Big|_{x_k=\hat{x}_{k/k-1}} P_{k/k-1}. \end{aligned}$$

将式(15) ~ (18) 带入新息滤波公式, 得到修正的扩展风险性滤波为

$$\begin{aligned} \hat{x}_{k/k} &= f_{k-1}(\hat{x}_{k-1/k-1}) + a_{k-1}/2 + \\ & [P_{k-1}^{-1} + C_k^T (R_k + B_k/2)^{-1} C_k]^{-1} \times \\ & C_k^T (R_k + B_k/2)^{-1} (z_k - \\ & h_k(\hat{x}_{k/k-1}) - b_k/2), \quad (19) \end{aligned}$$

$$\begin{aligned} P_k &= {}_{k-1}Q_{k-1}^{-1} + A_{k-1} (P_{k-1}^{-1} + \\ & C_{k-1}^T (R_{k-1} + B_{k-1}/2)^{-1} \times \\ & C_{k-1} - \mu I)^{-1} A_{k-1}^T + D_{k-1}/2, \quad (20) \end{aligned}$$

$$K_k = [P_k^{-1} + C_k^T (R_k + B_k/2)^{-1} C_k]^{-1} C_k^T (R_k + B_k/2)^{-1}, \quad (21)$$

当且仅当

$$P_i^{-1} + C_i^T (R_i + B_i/2)^{-1} C_i - \mu I > 0, \quad i = 1, 2, \dots, N \quad (22)$$

成立时, 滤波存在.

3.3 基于矩阵分解的 MERSF 及其复杂度估计

MERSF 中有大量的求逆运算, 为保证算法的稳定性, 确保计算时协方差矩阵的正定性, 利用矩阵 Cholesky 分解, 设

$$Q_{k-1} = UU^T, P_{k-1} = W_{k-1}W_{k-1}^T, R_{k-1} = VV^T,$$

$$B_{k-1} = L_{k-1}L_{k-1}^T, D_{k-1} = M_{k-1}M_{k-1}^T,$$

则

$$P_k = N_k^T N_k.$$

其中 N_k 为矩阵 $[{}_{k-1}U \ Y_{k-1} \ M_{k-1}]$ QR 分解中的上三角矩阵.

$$Y_{k-1} =$$

$$A[W_{k-1} \ C_{k-1}V \ C_{k-1}L_{k-1}/\sqrt{2} \ \sqrt{ij}I]^{-T},$$

j 为虚数单位. 滤波增益

$$K_k = H_k^T H_k^{-1} C_k^T J_k^{-1} J_k^{-1},$$

$$H_k = [N_k \ C_kV \ C_kL_k/\sqrt{2}],$$

$$J_k = [C_kV \ C_kL_k/\sqrt{2}].$$

计算时每个时间步需要 6 次 Cholesky 分解、1 次 QR 分解, 所需计算量大约为 $O(n^3/2 + m^3/6)$, 而 EKF 是 $O(n^2)$, 故 MERSF 的计算量比 ERSF 的稍大.

4 仿真结果与分析

选用文献[10]中具有高度非线性的非线性系统, 该系统定义为

$$x_{k+1} = f(x_k) + w_k, z_k = h(x_k) + v_k.$$

其中

$$f(x_k) = 5x_k(1 - x_k^2), w_k \sim N(0, 0.05),$$

$$h(x_k) = 0.01x_k(1 - 0.5x_k), v_k \sim N(0, 0.0001).$$

显然, 系统有 3 个平衡点 0, 1 和 -1. 其中 0 是不平稳的, 其余 2 个是平稳的. 系统初始化条件为: 系统初始真值 $x_0 = -0.2$, 初始滤波值 $\hat{x}_{0/0} = 0.8$, 初始估计协方差 $P_{0/0} = 2$, 风险参数 $\mu = 0.0756$. 考虑 0 ~ 0.8 s 的时间段, 时间步长 $t = 0.01$ s. 一次运行系统的真值与本文算法以及 ERSF 算法的比较如图 1 所示; 本文算法与 ERSF 算法 1000 次蒙特卡罗仿真的均方跟误差 (RMSE) 曲线比较如图 2 所示. μ 是决定风险程度的参数, 由控制增益 计算, $\mu = 1/2^{1/2}$. μ 越小, 模型的不确定性越小, 滤波的稳定

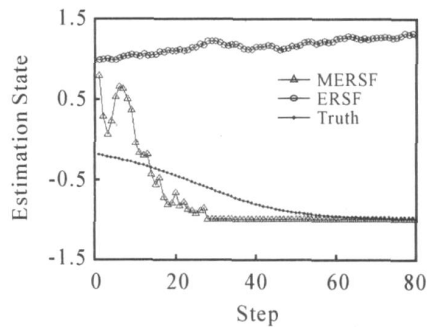


图 1 MERSF 与 ERSF 状态估计的比较

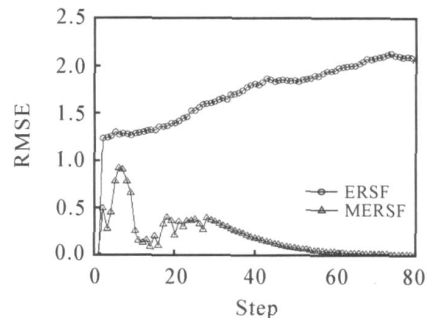
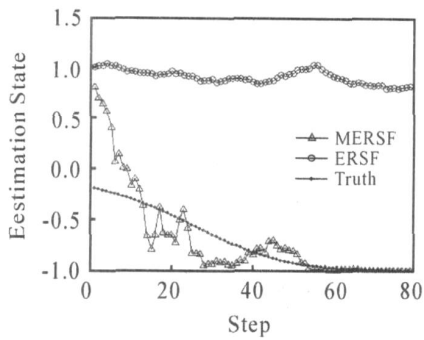
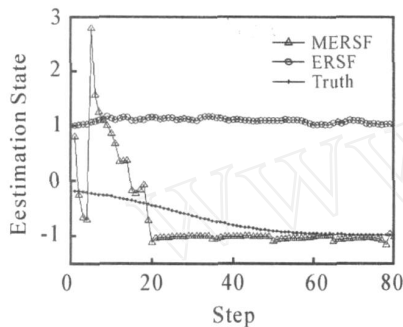
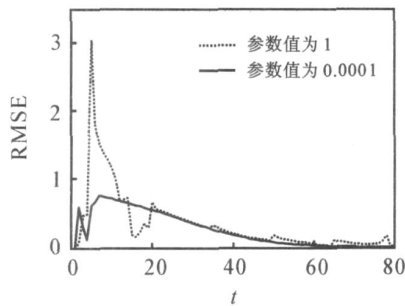


图 2 MERSF 与 ERSF 误差估计的比较

图3 $\mu = 0.0001$ 时的状态估计图4 $\mu = 1$ 时的状态估计图5 不同 μ 值对滤波精度的影响

性越好; μ 越大,模型的不确定性越大,滤波越不稳定^[13]。图3和图4分别给出了不同风险参数取值的情形下,本文算法与ERSF算法的估计曲线;图5给出了不同 μ 值对滤波精度的影响。

由仿真结果可以看出,本文算法的跟踪性能优于ERSF算法,其均方跟踪误差小于ERSF算法。这主要是由于本文算法利用了二阶Taylor展开的高阶项,对一阶EKF的状态估计及其协方差阵进行了适当修正,在一定程度上减小了截断误差,使得算法收敛的稳定性得到增强。风险性参数 μ 对估计精度有较大影响, μ 越小,估计精度越高。

5 结论

本文针对扩展风险性滤波算法精度较低的缺点,提出一种利用二阶Taylor展开的高阶项对其状态估计及其协方差阵进行适当修正的算法。该算法在一定程度上减小了截断误差,增加了算法的收敛性和稳定性。仿真结果表明,本文算法与ERSF算法

相比较,本文算法的计算量增加较少,而估计精度明显提高。

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