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T-S 模糊时滞系统的时滞相关镇定

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摘 要: 基于 T-S 模糊模型方法,研究一类非线性区间时滞系统的稳定性分析与镇定问题. 基于一组线性矩阵不等式的可解性,给出了非线性时滞系统时滞相关稳定和镇定的充分条件,并通过构造新的模糊权依赖型 Lyapunov 泛函,降低了稳定性准则的保守性. 最后通过仿真例子表明了所提方法的有效性.

关键词: T-S 模型; 状态时滞; 输入时滞; 模糊控制; 线性矩阵不等式
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Delay-dependent stabilization of T-S fuzzy systems with time-delay

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Abstract: Problem of delay-dependent stability analysis and stabilization for a class of nonlinear delay systems described by fuzzy T-S model is studied. Based on the feasibility solutions of some linear matrix inequalities, the delay-dependent stability criteria and the stabilization design scheme are derived. A new Lyapunov function with fuzzy weighing matrices is developed, and the conservativeness of the stability criterion is decreased. Finally, numerical examples show the effectiveness of the results.

Key words: T-S model; State delay; Input delay; Fuzzy control; Linear matrix inequality

1 引 言

近年来,T-S 模糊模型方法已成为解决非线性问题的重要方法之一. 众多学者投入了大量精力研究模糊控制理论及其应用,并取得了一系列的研究成果. 自文献[1]首先将 T-S 模糊模型方法用于研究非线性时滞系统之后,许多学者在这方面进行了深入研究,并相继发表了许多重要的研究成果[2-9]. 研究时滞系统的稳定性方法通常可分为两类:一类称为时滞独立稳定性;另一类称为时滞相关稳定性. 由于考虑了时滞大小对整个系统的影响,与时滞独立稳定性方法相比较,时滞相关稳定性方法所得到的稳定性准则和控制器设计方案具有较小的保守性,尤其是在时滞较小的情况下. 因此,近年来对于 T-S 模糊模型时滞系统的研究主要集中在时滞相关稳定性分析与镇定问题上. 但是现有的研究结果通常假定时滞的下限为零,很少考虑系统具有区间时滞的情况,这就导致了通常的稳定性条件用于客观存在的区间时滞系统中会产生保守性的现象. 另一方面,现有关于 T-S 模糊时滞系统的控制器设计中很少考虑控制输入中带有时滞的问题.

本文在文献[10,11]所提方法的基础上,通过引入模糊自由权矩阵和模糊权依赖型 Lyapunov 泛函,研究了具有输入时滞和状态时滞的 T-S 模糊系统的镇定问题. 基于线性矩阵不等式的可解性,给出了时滞相关意义下控制器设计的新方案. 仿真例子验证了该方法的有效性.

2 T-S 模糊系统

考虑 T-S 模糊时滞模型,其第 i 条模糊规则为

R_i: If $\theta_1(t)$ is N_{i1} and $\cdots$ and $\theta_g(t)$ is N_{ig};

$$\text{Then}\begin{cases}\dot{x}(t) = A_i x(t) + A_{di} x(t-d(t)) + \\ \qquad B_i u(t-d(t)),\ t>0; \\ x(t) = \phi(t),\ t\in[-\tau_2, 0].\end{cases} \tag{1}$$

其中:$\theta_1(t), \theta_2(t), \cdots, \theta_g(t)$ 是前件变量; $N_{i1}, N_{i2}, \cdots, N_{ig}$ 是模糊集; $x(t)\in R^n$ 是系统状态向量; $u(t)\in R^m$ 是控制输入; A_i, A_{di}, B_i 是具有适当维数的系

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统矩阵; $i \in S := \{1, 2, \cdots, r\}$, r 是模糊规则数; $\phi(t)$ 是定义在 $t \in [-\tau_2, 0]$ 上的初始函数; $d(t)$ 代表时变时滞,且满足

$$\tau_1 \le d(t) \le \tau_2, \ \dot{d}(t) \le \mu, \tag{2}$$

这里 τ_1, τ_2, μ 为正数.

通过中心平均解模糊器、法原理及单点模糊器, T-S 模糊系统(1) 的动态模型可表示为

$$\begin{aligned}\dot{x}(t) = & \\ & \sum_{i=1}^{r} \lambda_i(\theta(t))\{A_i x(t) + A_{di} x(t - d(t)) + \\ & B_i u(t - d(t))\} / \sum_{i=1}^{r} \lambda_i(\theta(t)) = \\ & \sum_{i=1}^{r} h_i(\theta(t))\{A_i x(t) + A_{di} x(t - d(t)) + \\ & B_i u(t - d(t))\}.\end{aligned} \tag{3}$$

其中

$$\theta(t) = [\theta_1(t) \quad \theta_2(t) \quad \cdots \quad \theta_g(t)],$$

$$\lambda_i(\theta(t)) = \prod_{j=1}^{g} N_{ij}(\theta_j(t)),$$

$$h_i(\theta(t)) = \frac{\lambda_i(\theta(t))}{\sum_{i=1}^{r} \lambda_i(\theta(t))}, \ i = 1, 2, \cdots, r.$$

这里 $N_{ij}(\theta_j(t))$ 是 $\theta_j(t)$ 在模糊集 N_{ij} 上的隶属度函数,因此

$$\lambda_i(\theta(t)) \ge 0, \ i \in S, \ \sum_{i=1}^{r} \lambda_i(\theta(t)) > 0. \tag{4}$$

显然,下列式子成立:

$$h_i(\theta(t)) \ge 0, \ i \in S, \ \sum_{i=1}^{r} h_i(\theta(t)) = 1. \tag{5}$$

3 T-S 模糊系统的稳定性理论

下面研究当 $u(t - d(t)) = 0$ 时,带有区间时滞的模糊系统的时滞相关稳定性条件. 自治系统模型为

$$\dot{x}(t) = \overline{A}(t) x(t) + \overline{A}_d(t) x(t - d(t)). \tag{6}$$

其中

$$\overline{A}(t) := \sum_{i=1}^{r} h_i(\theta(t)) A_i,$$

$$\overline{A}_d(t) := \sum_{i=1}^{r} h_i(\theta(t)) A_{di}.$$

对于系统(6) 的时滞相关稳定性分析有下述主要结论:

定理 1 设 τ_1, τ_2, μ 是给定的正数,且满足式(2),则区间时滞模糊系统(6) 是渐近稳定的,若存在矩阵 $P = P^{\mathrm{T}} > 0$, $Q_{1j} = Q_{1j}^{\mathrm{T}} \ge 0$, $Q_{2j} = Q_{2j}^{\mathrm{T}} \ge 0$, $Q_{3j} = Q_{3j}^{\mathrm{T}} \ge 0$, $Z_{1j} = Z_{1j}^{\mathrm{T}} \ge 0$, $Z_{2j} = Z_{2j}^{\mathrm{T}} \ge 0$, $M_{1j}, M_{2j}, M_{3j}, S_{1j}, S_{2j}, S_{3j}, N_{1j}, N_{2j}, N_{3j}, T_{1j}, T_{2j}, T_{3j}$, $i, j, l, m, n, k \in S$,满足下列线性矩阵不等式:

$$\Xi_{ijlkmn} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & M_{1j} & -S_{1j} & -S_{1j} & -M_{1j} & -N_{1j} \\ * & -(1-\mu)Q_{3l} + \gamma_{22} & \gamma_{23} & M_{2j} & -S_{2j} & -S_{2j} & -M_{2j} & -N_{2j} \\ * & * & \gamma_{33} & M_{3j} & -S_{3j} & -S_{3j} & -M_{3j} & -Z_{3j} \\ * & * & * & -Q_{1m} & 0 & 0 & 0 & 0 \\ * & * & * & * & -Q_{2n} & 0 & 0 & 0 \\ * & * & * & * & * & -\tau_{12}^{1}(Z_{1k} + Z_{2k}) & 0 & 0 \\ * & * & * & * & * & * & -\tau_{12}^{1} Z_{2k} & 0 \\ * & * & * & * & * & * & * & -\tau_2^{1} Z_{1k} \end{bmatrix} < 0. \tag{7}$$

其中

$$\begin{aligned}\gamma_{11} = & T_{1j} A_i + A_i^{\mathrm{T}} T_{1j}^{\mathrm{T}} + Q_{1j} + \\ & Q_{2j} + Q_{3j} + N_{1j} + N_{1j}^{\mathrm{T}}, \\ \gamma_{12} = & T_{1j} A_{di} + A_i^{\mathrm{T}} T_{2j}^{\mathrm{T}} + N_{2j}^{\mathrm{T}} - \\ & N_{1j} + S_{1j} - M_{1j}, \\ \gamma_{13} = & P + N_{3j}^{\mathrm{T}} - T_{1j} + A_i^{\mathrm{T}} T_{3j}^{\mathrm{T}}, \\ \gamma_{22} = & S_{2j} + S_{2j}^{\mathrm{T}} - N_{2j} - N_{2j}^{\mathrm{T}} - \\ & M_{2j}^{\mathrm{T}} - M_{2j} + T_{2j} A_{di} + A_{di}^{\mathrm{T}} T_{2j}^{\mathrm{T}}, \\ \gamma_{23} = & -N_{3j}^{\mathrm{T}} - M_{3j}^{\mathrm{T}} - T_{2j} + \\ & S_{3j}^{\mathrm{T}} + A_{di}^{\mathrm{T}} T_{3j}^{\mathrm{T}}, \\ \gamma_{33} = & \tau_2 Z_{1j} + (\tau_2 - \tau_1) Z_{2j} - T_{3j} - T_{3j}^{\mathrm{T}}, \\ \tau_{12} = & \tau_2 - \tau_1.\end{aligned}$$

证明 选取 Lyapunov 函数

$$\begin{aligned}V(x_t) = & x^{\mathrm{T}}(t) P x(t) + \\ & \sum_{i=1}^{2} \int_{t-\tau_i}^{t} x^{\mathrm{T}}(s) \overline{Q}_i(s) x(s) \mathrm{d}s + \\ & \int_{t-d(t)}^{t} x^{\mathrm{T}}(s) \overline{Q}_3(s) x(s) \mathrm{d}s + \\ & \int_{-\tau_2}^{0} \int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) \overline{Z}_1(s) \dot{x}(s) \mathrm{d}s \mathrm{d}\theta +\end{aligned}$$

$$\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)\bar{Z}_2(s)\dot{x}(s)\,\mathrm{d}s\mathrm{d}\theta.$$

其中：$\bar{Q}_i(s):=\sum_{j=1}^{r}h_j(\theta(s))Q_{ij}(i=1,2,3)$ 和 $\bar{Z}_k(s):=\sum_{j=1}^{r}h_j(\theta(s))Z_{kj}(k=1,2)$ 是含有隶属度函数的模糊权矩阵，且 $0<P=P^{\mathrm{T}}\in R^{n\times n}$，$0\le Q_{ij}=Q_{ij}^{\mathrm{T}}\in R^{n\times n}$，$0\le Z_{kj}=Z_{kj}^{\mathrm{T}}\in R^{n\times n}$. 则沿模糊系统(6)的时间导数为

$$\begin{aligned}
&\dot{V}(x_t)=\\
&2x^{\mathrm{T}}(t)P\dot{x}(t)+\sum_{i=1}^{3}x^{\mathrm{T}}(t)\bar{Q}_i(t)x(t)-\\
&\sum_{i=1}^{2}x^{\mathrm{T}}(t-\tau_i)\bar{Q}_i(t-\tau_i)x(t-\tau_i)-\\
&(1-\dot{d}(t))x^{\mathrm{T}}(t-d(t))\bar{Q}_3(t-d(t))\times\\
&x(t-d(t))+\tau_2\dot{x}^{\mathrm{T}}(t)\bar{Z}_1(t)\dot{x}(t)-\\
&\int_{t-\tau_2}^{t}\dot{x}^{\mathrm{T}}(s)\bar{Z}_1(s)\dot{x}(s)\mathrm{d}s+\\
&(\tau_2-\tau_1)\dot{x}^{\mathrm{T}}(t)\bar{Z}_2(t)\dot{x}(t)-\\
&\int_{t-\tau_2}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)\bar{Z}_2(s)\dot{x}(s)\mathrm{d}s. \qquad (8)
\end{aligned}$$

由积分性质，下列等式成立：

$$\begin{aligned}
&-\int_{t-\tau_2}^{t}\dot{x}^{\mathrm{T}}(s)\bar{Z}_1(s)\dot{x}(s)\mathrm{d}s=\\
&-\int_{t-d(t)}^{t}\dot{x}^{\mathrm{T}}(s)\bar{Z}_1(s)\dot{x}(s)\mathrm{d}s-\\
&\int_{t-\tau_2}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)\bar{Z}_1(s)\dot{x}(s)\mathrm{d}s, \qquad (9)
\end{aligned}$$

$$\begin{aligned}
&-\int_{t-\tau_2}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)\bar{Z}_2(s)\dot{x}(s)\mathrm{d}s=\\
&-\int_{t-\tau_2}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)\bar{Z}_2(s)\dot{x}(s)\mathrm{d}s-\\
&\int_{t-d(t)}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)\bar{Z}_2(s)\dot{x}(s)\mathrm{d}s. \qquad (10)
\end{aligned}$$

定义自由模糊权矩阵

$$\bar{N}_i(t):=\sum_{j=1}^{r}h_j(\theta(t))N_{ij},$$

$$\tilde{M}_i(t):=\sum_{j=1}^{r}h_j(\theta(t))M_{ij},$$

$$\bar{S}_i(t):=\sum_{j=1}^{r}h_j(\theta(t))S_{ij},$$

$$\bar{T}_i(t):=\sum_{j=1}^{r}h_j(\theta(t))T_{ij}.$$

其中：$N_{ij}\in R^{n\times n}$，$M_{ij}\in R^{n\times n}$，$S_{ij}\in R^{n\times n}$，$T_{ij}\in R^{n\times n}$，$i=1,2,3$.

应用 Leibniz-Newton 定理，可以得到

$$\begin{aligned}
&2[x^{\mathrm{T}}(t)\bar{N}_1(t)+x^{\mathrm{T}}(t-d(t))\bar{N}_2(t)+\\
&\dot{x}^{\mathrm{T}}(t)\bar{N}_3(t)][x(t)-x(t-d(t))-\\
&\int_{t-d(t)}^{t}\dot{x}(s)\mathrm{d}s]=0, \qquad (11)
\end{aligned}$$

$$\begin{aligned}
&2[x^{\mathrm{T}}(t)\bar{S}_1(t)+x^{\mathrm{T}}(t-d(t))\bar{S}_2(t)+\\
&\dot{x}^{\mathrm{T}}(t)\bar{S}_3(t)][x(t-d(t))-x(t-\tau_2)-\\
&\int_{t-\tau_2}^{t-d(t)}\dot{x}(s)\mathrm{d}s]=0, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
&2[x^{\mathrm{T}}(t)\tilde{M}_1(t)+x^{\mathrm{T}}(t-d(t))\tilde{M}_2(t)+\\
&\dot{x}^{\mathrm{T}}(t)\tilde{M}_3(t)][x(t-\tau_1)-x(t-d(t))-\\
&\int_{t-d(t)}^{t-\tau_1}\dot{x}(s)\mathrm{d}s]=0. \qquad (13)
\end{aligned}$$

另外，由系统模型(6)有

$$\begin{aligned}
&2[x^{\mathrm{T}}(t)\bar{T}_1(t)+x^{\mathrm{T}}(t-d(t))\bar{T}_2(t)+\\
&\dot{x}^{\mathrm{T}}(t)\bar{T}_3(t)][\bar{A}(t)x(t)+\\
&\bar{A}_d(t)x(t-d(t))-\dot{x}(t)]=0. \qquad (14)
\end{aligned}$$

利用等式(9)～(14)可得

$$\begin{aligned}
&\dot{V}(x_t)\le\\
&2x^{\mathrm{T}}(t)P\dot{x}(t)+x^{\mathrm{T}}(t)\sum_{i=1}^{3}\bar{Q}_i(t)x(t)-\\
&\sum_{i=1}^{2}x^{\mathrm{T}}(t-\tau_i)Q_i(t-\tau_i)x(t-\tau_i)-(1-\mu)\times\\
&x^{\mathrm{T}}(t-d(t))\bar{Q}_3(t-d(t))x(t-d(t))+\\
&\dot{x}^{\mathrm{T}}(t)[\tau_2\bar{Z}_1(t)+(\tau_2-\tau_1)\bar{Z}_2(t)]\dot{x}(t)-\\
&\int_{t-d(t)}^{t}\dot{x}^{\mathrm{T}}(s)\bar{Z}_1(s)\dot{x}(s)\mathrm{d}s-\\
&\int_{t-\tau_2}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)(\bar{Z}_1(s)+\bar{Z}_2(s))\dot{x}(s)\mathrm{d}s-\\
&\int_{t-d(t)}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)\bar{Z}_2(s)\dot{x}(s)\mathrm{d}s+\\
&2[x^{\mathrm{T}}(t)\bar{N}_1(t)+x^{\mathrm{T}}(t-d(t))\bar{N}_2(t)+\\
&\dot{x}^{\mathrm{T}}(t)\bar{N}_3(t)][x(t)-x(t-d(t))-\\
&\int_{t-d(t)}^{t}\dot{x}(s)\mathrm{d}s]+2[x^{\mathrm{T}}(t)\bar{S}_1(t)+\\
&x^{\mathrm{T}}(t-d(t))\bar{S}_2(t)+\dot{x}^{\mathrm{T}}(t)\bar{S}_3(t)]\times\\
&[x(t-d(t))-x(t-\tau_2)-\int_{t-\tau_2}^{t-d(t)}\dot{x}(s)\mathrm{d}s]+\\
&2[x^{\mathrm{T}}(t)\tilde{M}_1(t)+x^{\mathrm{T}}(t-d(t))\tilde{M}_2(t)+\\
&\dot{x}^{\mathrm{T}}(t)\tilde{M}_3(t)][x(t-\tau_1)-x(t-d(t))-\\
&\int_{t-d(t)}^{t-\tau_1}\dot{x}(s)\mathrm{d}s]+2[x^{\mathrm{T}}(t)\bar{T}_1(t)+\\
&x^{\mathrm{T}}(t-d(t))\bar{T}_2(t)+\dot{x}^{\mathrm{T}}(t)\bar{T}_3(t)]\times\\
&[\bar{A}(t)x(t)+\bar{A}_d(t)x(t-d(t))-\dot{x}(t)]\le\\
&\frac{1}{d(t)(\tau_2-d(t))(d(t)-\tau_1)}\times\\
&\int_{t-d(t)}^{t}\int_{t-d(t)}^{t-\tau_1}\int_{t-\tau_2}^{t-d(t)}\zeta^{\mathrm{T}}(t,\alpha,\beta,\gamma)\times\\
&\Delta\zeta(t,\alpha,\beta,\gamma)\,\mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}\gamma.
\end{aligned}$$

其中

$$\zeta^{\mathrm{T}}(t,\alpha,\beta,\gamma) = \{x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-d(t)), \dot{x}^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau_1), x^{\mathrm{T}}(t-\tau_2), \dot{x}^{\mathrm{T}}(\alpha), \dot{x}^{\mathrm{T}}(\beta), \dot{x}^{\mathrm{T}}(\gamma)\},$$

$$\Delta = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \widetilde{M}_1(t) & -\overline{S}_1(t) & -\overline{S}_1(t) & -\widetilde{M}_1(t) & -\overline{N}_1(t) \\ * & \Phi_{22} & \Phi_{23} & \widetilde{M}_2(t) & -\overline{S}_2(t) & -\overline{S}_2(t)) & -\widetilde{M}_2(t) & -\overline{N}_2(t) \\ * & * & \Phi_{33} & \widetilde{M}_3(t) & -\overline{S}_3(t) & -\overline{S}_3(t) & -\widetilde{M}_3(t) & -\overline{N}_3(t) \\ * & * & * & -\overline{Q}_1(t-\tau_1) & 0 & 0 & 0 & 0 \\ * & * & * & * & -\overline{Q}_2(t-\tau_2) & 0 & 0 & 0 \\ * & * & * & * & * & -\tau_{12}^{-1}(\overline{Z}_1(s)+\overline{Z}_2(s)) & 0 & 0 \\ * & * & * & * & * & * & -\tau_{12}^{-1}\overline{Z}_2(s) & 0 \\ * & * & * & * & * & * & * & -\tau_2^{-1}\overline{Z}_1(s) \end{bmatrix}. \quad (15)$$

这里

$$\Phi_{11} = \overline{T_1}(t)\overline{A}(t) + \overline{A}^{\mathrm{T}}(t)\overline{T_1}^{\mathrm{T}}(t) + \sum_{i=1}^{3}\overline{Q}_i(t) + \overline{N}_1(t) + \overline{N}_1^{\mathrm{T}}(t),$$

$$\Phi_{12} = \overline{T_1}(t)\overline{A}_d(t) + \overline{A}^{\mathrm{T}}(t)\overline{T_2}^{\mathrm{T}}(t) + \overline{N}_2^{\mathrm{T}}(t) - \overline{N}_1(t) + \overline{S}_1(t) - \widetilde{M}_1(t),$$

$$\Phi_{13} = P + \overline{N}_3^{\mathrm{T}}(t) - \overline{T_1}(t) + \overline{A}^{\mathrm{T}}(t)\overline{T_3}^{\mathrm{T}}(t),$$

$$\Phi_{22} = -(1-\mu)\overline{Q}_3(t-d(t)) + \overline{S}_2(t) + \overline{S}_2^{\mathrm{T}}(t) - \overline{N}_2(t) - \overline{N}_2^{\mathrm{T}}(t) - \widetilde{M}_2^{\mathrm{T}}(t) - \widetilde{M}_2(t) + \overline{T_2}(t)\overline{A}_d(t) + \overline{A}_d^{\mathrm{T}}(t)\overline{T_2}^{\mathrm{T}}(t),$$

$$\Phi_{23} = -\overline{N}_3^{\mathrm{T}}(t) - \widetilde{M}_3^{\mathrm{T}}(t) - \overline{T_2}(t) + \overline{S}_3^{\mathrm{T}}(t) + \overline{A}_d^{\mathrm{T}}(t)\overline{T_3}^{\mathrm{T}}(t),$$

$$\Phi_{33} = \tau_2\overline{Z}_1(t) + (\tau_2-\tau_1)\overline{Z}_2(t) - \overline{T_3}(t) - \overline{T_3}^{\mathrm{T}}(t),$$

$$\tau_{12} = \tau_2 - \tau_1.$$

故当 $\Delta < 0$ 时,$\dot{V}(x_t) < 0$. 由 Lyapunov 稳定性判定定理可知,模糊系统(6)是渐近稳定的. 又因式(15)还可以表示为

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\sum_{l=1}^{r}h_i(\theta(t))h_j(\theta(t))h_k(\theta(s)) \times h_l(\theta(t-d(t)))h_m(\theta(t-\tau_1))h_n(\theta(t-\tau_2))\Xi_{ijlkmn}, \quad (16)$$

故可得线性矩阵不等式(7). □

4 模糊控制器设计

为讨论状态反馈镇定问题,引入下述控制律规则,其中第 i 个规则为

R_i: If $\theta_1(t)$ is N_{i1} and $\cdots$ and $\theta_g(t)$ is N_{ig},

Then $u(t) = K_i x(t)$, $i = 1,2,\cdots,r$.

于是全局模糊控制律为

$$u(t) = \sum_{i=1}^{r}h_i(\theta(t))K_i x(t),$$

其中 K_i 为局部控制增益矩阵,且 $K_i \in R^{m\times n}$.

因为控制向量中带有时变时滞,故有

$$u(t-d(t)) = \sum_{i=1}^{r}h_i(\theta(t-d(t)))K_i x(t-d(t)). \quad (17)$$

进一步假设下面关系式成立:

$$\sum_{i=1}^{r}h_i(\theta(t-d(t))) = 1,$$

$$h_i(\theta(t-d(t))) \geq 0,\ t \in [-\tau_2, 0].$$

结合式(17),全局模糊闭环控制系统(3)可表示为

$$\dot{x}(t) = \overline{A}(t)x(t) + \overline{A}_f(t)x(t-d(t)). \quad (18)$$

其中

$$\overline{A}_f(t) = \overline{A}_d(t) + \overline{B}(t)\overline{K}(t-d(t)),$$

$$\overline{B}(t) = \sum_{i=1}^{r}h_i(\theta(t))B_i,$$

$$\overline{K}(t-d(t)) = \sum_{i=1}^{r}h_i(\theta(t-d(t)))K_i.$$

由定理 1 的证明过程可知,闭环模糊系统(18)是渐近稳定的,当且仅当式(15)中的 $\overline{A}_d(t)$ 用 $\overline{A}_f(t)$ 替代.

下面求控制增益 K_i. 假设 $\overline{T_1}$ 是非奇异的,故 $\overline{T_1}^{-1}$ 存在,并定义矩阵

$$\Gamma = \mathrm{diag}\{\overline{T_1}^{-1}(t), \overline{T_1}^{-1}(t), \overline{T_1}^{-1}(t-d(t)), \overline{T_1}^{-1}(t-\tau_1), \overline{T_1}^{-1}(t-\tau_2), \overline{T_1}^{-1}(s), \overline{T_1}^{-1}(s), \overline{T_1}^{-1}(s)\},$$

$$\widetilde{Q}_i(t) = \overline{T_1}^{-1}(t)\overline{Q}_i(t)\overline{T_1}^{-\mathrm{T}}(t),$$

$$\widetilde{N}_i(t) = \overline{T_1}^{-1}(t)\overline{N}_i(t)\overline{T_1}^{-\mathrm{T}}(t),$$

$$\widetilde{M}_i(t) = \overline{T_1}^{-1}(t)\widetilde{M}_i(t)\overline{T_1}^{-\mathrm{T}}(t),$$

$$\widetilde{S}_i(t) = \overline{T_1}^{-1}(t)\overline{S}_i(t)\overline{T_1}^{-\mathrm{T}}(t),$$

$$\widetilde{Z}_i(t) = \overline{T_1}^{-1}(t)\overline{Z}_i(t)\overline{T_1}^{-\mathrm{T}}(t),$$

$$\overline{T_2}(t) = \lambda_2\overline{T_1}(t),$$

$$\overline{T_3}(t) = \lambda_3\overline{T_1}(t),$$

$$\overline{T_1}^{\mathrm{T}}(t) = \overline{T_1}(t) = \lambda_1 X,$$
$$X = P^{-1}.$$

其中

$$\tilde{Q}_i(t) := \sum_{j=1}^{r} h_i(\theta(t))\tilde{Q}_{ij},$$
$$\overline{N_i}(t) := \sum_{j=1}^{r} h_i(\theta(t))\overline{N_{ij}},$$
$$\tilde{M}_i(t) := \sum_{j=1}^{r} h_i(\theta(t))\tilde{M}_{ij},$$
$$\tilde{S}_i(t) := \sum_{j=1}^{r} h_i(\theta(t))\tilde{S}_{ij},$$
$$\tilde{Z}_i(t) := \sum_{j=1}^{r} h_i(\theta(t))\tilde{Z}_{ij}.$$

然后用矩阵 Γ 和 Γ^{T} 分别左右乘以不等式(15)(其中的 $\overline{A_d}(t)$ 用 $\overline{A_d}(t) + \overline{B}(t)\overline{K}(t-d(t))$ 来代替),可得不等式

$$\begin{bmatrix}
\tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & \tilde{M}_1(t) & -\tilde{S}_1(t) & -\tilde{S}_1(t) & -\tilde{M}_1(t) & -\overline{N_1}(t) \\
* & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} & \tilde{M}_2(t) & -\tilde{S}_2(t) & -\tilde{S}_2(t) & -\tilde{M}_2(t) & -\overline{N_2}(t) \\
* & * & \tilde{\Phi}_{33} & \tilde{M}_3(t) & -\tilde{S}_3(t) & -\tilde{S}_3(t) & -\tilde{M}_3(t) & -\overline{N_3}(t) \\
* & * & * & -\tilde{Q}_1(t-\tau_1) & 0 & 0 & 0 & 0 \\
* & * & * & * & -\tilde{Q}_2(t-\tau_2) & 0 & 0 & 0 \\
* & * & * & * & * & -\tau_{12}^{-1}(\tilde{Z}_1(s)+\tilde{Z}_2(s)) & 0 & 0 \\
* & * & * & * & * & * & -\tau_{12}^{-1}\tilde{Z}_2(s) & 0 \\
* & * & * & * & * & * & * & -\tau_2^{-1}\tilde{Z}_1(s)
\end{bmatrix} < 0. \quad (19)$$

其中

$$\tilde{\Phi}_{11} = \sum_{i=1}^{3}\tilde{Q}_i(t) + \overline{N_1}(t) + \overline{N_1}^{\mathrm{T}}(t) + \lambda_1\overline{A}(t)X + \lambda_1 X\overline{A}^{\mathrm{T}}(t),$$

$$\tilde{\Phi}_{12} = -\overline{N_1}(t) + \overline{N_2}^{\mathrm{T}}(t) + \tilde{S}_1(t) - \tilde{M}_1(t) + \lambda_1\overline{A_d}(t)X + \lambda_1\lambda_2 X\overline{A}^{\mathrm{T}}(t) + \lambda_1\overline{B}(t)\overline{K}(t-d(t))X,$$

$$\tilde{\Phi}_{13} = \lambda_1^2 X + \overline{N_3}^{\mathrm{T}}(t) - \lambda_1 X + \lambda_1\lambda_3 X\overline{A}^{\mathrm{T}}(t),$$

$$\tilde{\Phi}_{22} = -(1-\mu)\tilde{Q}_3(t-d(t)) + \tilde{S}_2(t) + \tilde{S}_2^{\mathrm{T}}(t) - \overline{N_2}(t) - \overline{N_2}^{\mathrm{T}}(t) - \tilde{M}_2^{\mathrm{T}}(t) - \tilde{M}_2(t) + \lambda_1\lambda_2\overline{A_d}(t)X + \lambda_1\lambda_2 X\overline{A}_d^{\mathrm{T}}(t) + \lambda_1\lambda_2\overline{B}(t)\overline{K}(t-d(t))X + \lambda_1\lambda_2 X\overline{K}^{\mathrm{T}}(t-d(t))\overline{B}^{\mathrm{T}}(t),$$

$$\tilde{\Phi}_{23} = -\overline{N_3}^{\mathrm{T}}(t) - \tilde{M}_3^{\mathrm{T}}(t) - \lambda_1\lambda_2 X + \tilde{S}_3^{\mathrm{T}}(t) + \lambda_1\lambda_3 X\overline{A}_d^{\mathrm{T}}(t) + \lambda_1\lambda_3 X\overline{K}^{\mathrm{T}}(t-d(t))\overline{B}^{\mathrm{T}}(t),$$

$$\tilde{\Phi}_{33} = \tau_2\tilde{Z}_1(t) + (\tau_2-\tau_1)\tilde{Z}_2(t) - 2\lambda_1\lambda_3 X,$$

$$\tau_{12} = \tau_2 - \tau_1.$$

同样,式(19)可以表示为式(16)小于零的形式,故可得到如下定理:

定理 2　设 τ_1,τ_2,μ 是给定的正数,且满足式(2),则带有区间时滞的 T-S 模糊系统(3)是渐近稳定的,若存在常数 $\lambda_1 \neq 0,\lambda_2,\lambda_3$ 和矩阵 $X = X^{\mathrm{T}} > 0$, $\tilde{Q}_{1j} = \tilde{Q}_{1j}^{\mathrm{T}} \geq 0$, $\tilde{Q}_{2j} = \tilde{Q}_{2j}^{\mathrm{T}} \geq 0$, $\tilde{Q}_{3j} = \tilde{Q}_{3j}^{\mathrm{T}} \geq 0$, $\tilde{M}_{1j}$, $\tilde{M}_{2j}$, $\tilde{M}_{3j}$, $\tilde{S}_{1j}$, $\tilde{S}_{2j}$, $\tilde{S}_{3j}$, $\overline{N_{1j}}$, $\overline{N_{2j}}$, $\overline{N_{3j}}$, Y_j, $i,j,k,l,m,n \in S$ 满足如下线性矩阵不等式:

$$\begin{bmatrix}
\tilde{\gamma}_{11} & \tilde{\gamma}_{12} & \tilde{\gamma}_{13} & \tilde{M}_{1j} & -\tilde{S}_{1j} & -\tilde{S}_{1j} & \rightarrow \\
* & -(1-\mu)\tilde{Q}_{3l} + \tilde{\gamma}_{22} & \tilde{\gamma}_{23} & \tilde{M}_{2j} & -\tilde{S}_{2j} & -\tilde{S}_{2j} & \\
* & * & \tilde{\gamma}_{23} & \tilde{M}_{3j} & -\tilde{S}_{3j} & -\tilde{S}_{3j} & \\
* & * & * & -\tilde{Q}_{1m} & 0 & 0 & \\
* & * & * & * & -\tilde{Q}_{2n} & 0 & \\
* & * & * & * & * & -\tau_{12}^{-1}(\tilde{Z}_{1k} + \tilde{Z}_{2k}) & \\
* & * & * & * & * & * & \\
* & * & * & * & * & * &
\end{bmatrix}$$

$$
\leftarrow
\begin{bmatrix}
\cdots & -\widetilde{M}_{1j} & -\overline{N_{1j}} \\
\cdots & -\widetilde{M}_{2j} & -\overline{N_{2j}} \\
\cdots & -\widetilde{M}_{3j} & -\overline{N_{3j}} \\
\cdots & 0 & 0 \\
\cdots & 0 & 0 \\
\cdots & 0 & 0 \\
\cdots & -\tau_{12}^{-1}\widetilde{Z}_{2k} & 0 \\
\cdots & * & -\tau_2^{-1}\widetilde{Z}_{1k}
\end{bmatrix} < 0. \qquad (20)
$$

其中

$$
\begin{aligned}
\widetilde{\gamma}_{11} &= \lambda_1 A_i X + \lambda_1 X A_i^{\mathrm T} + \widetilde{Q}_{1j} + \widetilde{Q}_{2j} + \\
&\quad \widetilde{Q}_{3j} + \overline{N_{1j}} + \overline{N_{1j}^{\mathrm T}}, \\
\widetilde{\gamma}_{12} &= \lambda_1 A_{di} X + \lambda_1 \lambda_2 X A_i^{\mathrm T} + \overline{N_{2j}^{\mathrm T}} - \\
&\quad \overline{N_{1j}} + \widetilde{S}_{1j} - \widetilde{M}_{1j} + \lambda_1 B_i Y_j, \\
\widetilde{\gamma}_{13} &= \lambda_1^2 X + \overline{N_{3j}^{\mathrm T}} - \lambda_1 X + \lambda_1 \lambda_3 X A_i^{\mathrm T}, \\
\widetilde{\gamma}_{22} &= \widetilde{S}_{2j} + \widetilde{S}_{2j}^{\mathrm T} - \overline{N_{2j}} - \overline{N_{2j}^{\mathrm T}} - \widetilde{M}_{2j} - \\
&\quad \widetilde{M}_{2j}^{\mathrm T} + \lambda_1 \lambda_2 A_{di} X + \lambda_1 \lambda_2 X A_{di}^{\mathrm T} + \\
&\quad \lambda_1 \lambda_2 B_i Y_j + \lambda_1 \lambda_2 Y_j^{\mathrm T} B_i^{\mathrm T}, \\
\widetilde{\gamma}_{23} &= -\overline{N_{3j}^{\mathrm T}} - \widetilde{M}_{3j}^{\mathrm T} - \lambda_1 \lambda_2 X + \widetilde{S}_{3j}^{\mathrm T} + \\
&\quad \lambda_1 \lambda_3 X A_{di}^{\mathrm T} + \lambda_1 \lambda_3 Y_j^{\mathrm T} B_i^{\mathrm T}, \\
\widetilde{\gamma}_{33} &= \tau_2 \widetilde{Z}_{1j} + (\tau_2 - \tau_1) \widetilde{Z}_{2j} - 2\lambda_1 \lambda_3 X, \\
\tau_{12} &= \tau_2 - \tau_1.
\end{aligned}
$$

且控制增益矩阵设计为 $K_j = Y_j X^{-1}, j = 1, 2, \cdots, r$.

5　仿真实例

例 1　考虑模糊系统(6),其模糊规则如下:

Plant Rule1:

If $x_1(t)$ is M_1,

Then $\dot{x}(t) = A_1 x(t) + A_{d1} x(t - d(t))$;

Plant Rule 2:

If $x_2(t)$ is M_2,

Then $\dot{x}(t) = A_2 x(t) + A_{d2} x(t - d(t))$.

其中:隶属度函数为

$$
h_1(x_1(t)) = \frac{1}{1 + \exp(-2x_1(t))},
$$

$$
h_2(x_2(t)) = 1 - h_1(x_1(t));
$$

系统矩阵为

$$
A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},
$$

$$
A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},\ A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.
$$

当 $\mu = 0$ 时,取时滞下限 $\tau_1 = 0$,与文献[9]进行比较,结果如表 1 所示.

表 1　$\mu = 0$ 时最大时滞上限比较

方　　法	得到的最大时滞上限
文献[9]推论 1	1.597
本文定理 1	1.826

对该系统选取时滞 $\tau = 1.826$,初始条件 $\phi(t) = [1.5\quad -1.5]^{\mathrm T}, t \in [-1.826\quad 0]$. 仿真结果见图 1.

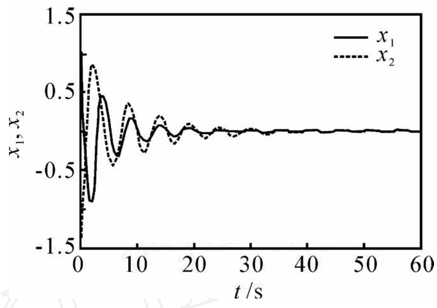

图 1　例 1 的状态响应曲线

当 $\mu = 1.5$ 时,与文献[9]的比较结果如表 2 所示.

表 2　$\mu = 1.5$ 时最大时滞上限比较

求得的时滞上限	时滞下限				
	0	0.4	0.8	1.0	1.2
文献[9]	0.721	0.883	1.093	1.211	1.336
本文定理 1	1.009	1.080	1.211	1.308	1.419

例 2　考虑下述 T-S 模糊系统(其隶属度函数同例 1):

$$
\dot{x}(t) = \sum_{i=1}^{2} h_i(x_i(t))\{A_i x(t) + A_{di} x(t - d(t)) + B_i u(t - d(t))\},
$$

其系统矩阵为

$$
A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},
$$

$$
A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix},
$$

$$
B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
$$

当 $\mu = 0, \tau_1 = 0$ 时,取 $\lambda_1 = 3, \lambda_2 = 1.5, \lambda_3 = 3$. 与文献[2]中定理 3 所提方法进行比较,结果见表 3.

表 3　$\mu = 0, \tau_1 = 0$ 时最大时滞上限和反馈增益矩阵比较

方　　法	所得到的最大时滞上限	反馈增益矩阵
文献[2]中定理 3	0.1150	$K_1 = [1.6930\quad -6.5193]$ $K_2 = [-1.3967\quad -6.9037]$
本文定理 2	0.3120	$K_1 = [1.0598\quad -5.6598]$ $K_2 = [-1.3068\quad -4.1167]$

对该系统选取时滞 $\tau = 0.3120$,初始条件 $\phi(t) = [1.5\quad -1.5]^{\mathrm T}, t \in [-0.3120\quad 0]$. 图 2 和图 3 给出了仿真结果.

从例 1 和例 2 可以看出,本文方法相对其他文献具有较小的保守性,而且得到了比较小的反馈增

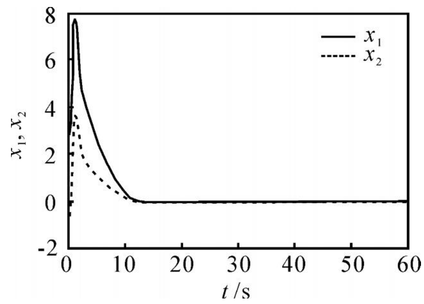

图 2　例 2 的状态响应曲线

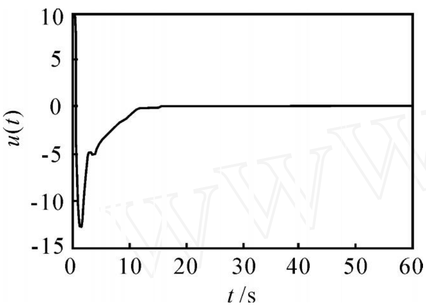

图 3　例 2 的控制输入曲线

益矩阵.

6　结　论

本文主要研究了区间时滞 T-S 模糊系统的时滞相关镇定问题. 基于线性矩阵不等式可行解的存在性, 给出了由 T-S 模糊模型描述的非线性系统时滞相关稳定性准则, 以及时滞相关控制器的设计方案, 并通过仿真验证了该方法的有效性. 另外, 只需对 Lyapunov 函数稍作改动, 本文所提出的结果便可推广到含有不确定性的时滞系统中.

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