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具有干扰输入的大型互联线性系统的分散有限时间镇定

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摘 要: 借助于大型互联线性系统有限时间稳定性的定义, 对具有干扰输入的大型互联线性系统引入了分散有限时间镇定的概念, 并对一类具有干扰输入的大型互联不确定线性系统进行了分散状态反馈和分散动态输出反馈控制器设计, 利用线性矩阵不等式 (LMI) 方法, 提出了一个充分条件. 当反馈控制律作用于该系统时, 闭环系统是有限时间稳定的.

关键词: 有限时间镇定；分散控制；线性矩阵不等式；大型互联线性系统；干扰输入；参数不确定性
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Decentralized finite-time stabilization of large-scale interconnected linear systems with exogenous disturbances

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Abstract: The concept of decentralized finite-time stabilization for large-scale interconnected linear systems with exogenous disturbances is introduced by the definition of finite-time stability. The design of decentralized state feedback controllers and decentralized dynamic output feedback controllers is given for a class of large-scale interconnected linear systems with exogenous disturbances. A sufficient condition is provided by using linear matrix inequality. When the feedback control laws are applied to the systems, the closed-loop systems are finite-time stable.

Key words: finite-time stabilization；decentralized control；linear matrix inequality；large-scale interconnected linear systems；exogenous disturbances；parametric uncertainties

1 引 言

有限时间稳定性[1]刻画的是系统的暂态性能. 由于这种稳定性有别于通常意义下的 Lyapunov 稳定性, 近年来正越来越受到人们的关注. 文献 [2-8] 研究了线性系统的有限时间控制问题, 其中: [2] 对同时带有参数不确定性和外部干扰的线性系统进行了有限时间控制; [3-4]则对不确定线性奇异系统进行了相应的研究; [5] 对具有干扰输入的一类不确定线性系统进行了有限时间观测器设计; [6] 则考虑了一类线性系统基于输出反馈的有限时间镇定问题; [7] 对一类线性离散时间系统进行了有限时间控制设计; [8]则考虑了线性系统的有限时间 H^∞ 控制问题. 上述研究工作大多是基于单一系统开展的.

在许多实际控制问题中, 系统模型大都具有大系统形式, 如电力系统、化工工程、大型空间结构和计算机通讯网络等. 由于其实现的可靠性、实时性和经济性, 分散控制已成为大系统理论中的一个十分活跃的分支[9]. 近年来, 大型互联系统的分散控制设计得到了较深入的研究[10-14], 文献 [15] 将有限时间稳定性等相关概念推广到了大型互联系统, 对一类确定的大型互联线性系统进行了分散有限时间镇定设计. 事实上, 系统往往含有不确定性或受到外部干扰, 所以研究不确定系统的控制设计具有更重要的意义.

本文借助大型互联系统有限时间稳定性的相关定义[15], 对含干扰输入的大型互联不确定线性系统引入了有限时间稳定、镇定等概念, 并将文献 [5] 中研究的含干扰输入的不确定线性系统组合成大系统, 利用线性矩阵不等式的方法, 先构建系统的分散状态反馈

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控制器, 再利用状态反馈控制器构建系统的分散动态输出反馈控制器. 当输出反馈控制律作用于该系统时, 闭环系统是有限时间稳定的.

2 问题描述

考虑如下具有干扰输入的大型互联不确定线性系统:

$$\begin{cases}\dot{x}_i(t)=\\ A_ix_i(t)+B_iu_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ \displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t);\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm{T}}(0)w_i(0)\leqslant d_i;\\ y_i(t)=C_ix_i(t).\end{cases}\tag{1}$$

这里: $i=1,2,\cdots,N$ 表示各个子系统; $x_i(t)\in R^{n_i}$, $u_i(t)\in R^{m_i}$, $y_i(t)\in R^{p_i}$ 分别为第 i 个子系统的状态、控制输入和可量测输出; $w_i(t)\in R^{l_i}$ 为第 i 个子系统的干扰输入; $A_i,B_i,C_i,S_i,D_i,E_{ij}$ 为适维常阵. 对于方阵M, $M<0$表示M是负定的, 记 I 为适维单位阵. 系统(1)中的不确定性矩阵 $\Delta D_i(t),\Delta E_{ij}(t),\Delta S_i(t)$ 满足

$$\Delta D_i(t)=F_i\varDelta_{1i}(t)G_i,\ \Delta E_{ij}(t)=H_{ij}\varDelta_{2ij}(t)L_{ij},$$
$$\Delta S_i(t)=W_i\varDelta_{3i}(t)J_i.\tag{2}$$

其中: F_i, G_i, H_{ij}, L_{ij}, W_i, J_i 为适维常阵; $\varDelta_{1i}$, $\varDelta_{2ij}$, $\varDelta_{3i}$ 的各元素都是Lebesgue可测的, 且满足

$$\varDelta_{1i}(t)\varDelta_{1i}^{\mathrm{T}}(t)\leqslant I,\ \varDelta_{2ij}(t)\varDelta_{2ij}^{\mathrm{T}}(t)\leqslant I,$$
$$\varDelta_{3i}(t)\varDelta_{3i}^{\mathrm{T}}(t)\leqslant I.\tag{3}$$

其中: $i=1,2,\cdots,N$; $j=1,2,\cdots,N$.

注 1 系统(1)是将文献[5]中研究的线性系统组合成了大系统, 且含不确定互联项.

由文献[5,15], 对系统(1)给出如下相应的定义:

定义 1 若

$$\sum_{i=1}^{N}x_{i0}^{\mathrm{T}}R_ix_{i0}\leqslant c_1\Rightarrow\sum_{i=1}^{N}x_i^{\mathrm{T}}(t)R_ix_i(t)\leqslant c_2,\ \forall t\in[0,T],$$

则称大型互联线性系统

$$\begin{cases}\dot{x}_i(t)=\\ A_ix_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ \displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t),\ x_i(0)=x_{i0};\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm{T}}(0)w_i(0)\leqslant d_i\end{cases}$$

关于$(c_1,c_2,T,R_1,\cdots,R_N,d_1,\cdots,d_N)$是有限时间稳定的(FTS). 其中: $i=1,2,\cdots,N$; $c_2>c_1>0$; R_i 为 n_i 阶正定阵.

定义 2 给定一族信号集$\omega_i,i=1,2,\cdots,N$. 若

$$\sum_{i=1}^{N}x_{i0}^{\mathrm{T}}R_ix_{i0}\leqslant c_1\Rightarrow\sum_{i=1}^{N}x_i^{\mathrm{T}}(t)R_ix_i(t)\leqslant c_2,\ \forall t\in[0,T],$$

对一切$\varphi_i(t)\in\omega_i(i=1,2,\cdots,N)$成立, 则称大型互联线性系统

$$\begin{cases}\dot{x}_i(t)=\\ A_ix_i(t)+Y_i\varphi_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ \displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t),\ x_i(0)=x_{i0};\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm{T}}(0)w_i(0)\leqslant d_i\end{cases}$$

关于$(c_1,c_2,\omega_1,\cdots,\omega_N,T,R_1,\cdots,R_N,d_1,\cdots,d_N)$是有限时间有界的(FTB). 其中: $i=1,2,\cdots,N$; $c_2>c_1>0$; R_i 为 n_i 阶正定阵.

定义 3 若存在分散状态反馈控制律$u_i(t)=K_ix_i(t)$, 使得闭环系统

$$\begin{cases}\dot{x}_i(t)=\\ (A_i+B_iK_i)x_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ \displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t),\ x_i(0)=x_{i0};\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm{T}}(0)w_i(0)\leqslant d_i\end{cases}\tag{4}$$

关于$(c_1,c_2,T,R_1,\cdots,R_N,d_1,\cdots,d_N)$是有限时间稳定的(FTS), 则称大型互联线性系统

$$\begin{cases}\dot{x}_i(t)=\\ A_ix_i(t)+B_iu_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ \displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t),\ x_i(0)=x_{i0};\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm{T}}(0)w_i(0)\leqslant d_i\end{cases}$$

是基于状态反馈分散有限时间镇定的. 其中$i=1,2,\cdots,N$.

3 基于状态反馈的分散有限时间镇定

同文献[5], 首先考虑确定性系统的分散有限时间镇定, 即在系统(1)中令$\Delta D_i(t),\Delta E_{ij}(t),\Delta S_i(t)$均为零矩阵, 则系统(4)为

$$\begin{cases}\dot{x}_i(t)=\\ (A_i+B_iK_i)x_i(t)+D_iw_i(t)+\\ \displaystyle\sum_{j=1,j\neq i}^{N}E_{ij}x_j(t),\ x_i(0)=x_{i0};\\ \dot{w}_i(t)=S_iw_i(t),\ w_i^{\mathrm{T}}(0)w_i(0)\leqslant d_i.\end{cases}\tag{5}$$

定理 1 记

$$\bar{Q}_{11i}=R_i^{-1/2}Q_{11i}R_i^{-1/2},$$
$$U_{11i}=A_i\bar{Q}_{11i}+\bar{Q}_{11i}A_i^{\mathrm{T}}+B_iM_i+M_i^{\mathrm{T}}B_i^{\mathrm{T}}-\alpha\bar{Q}_{11i},$$
$$U_{12i}=S_iQ_{12i}+Q_{12i}S_i^{\mathrm{T}}-\alpha Q_{12i}.$$

若存在非负常数α, 正定矩阵$Q_{11i}\in R^{n_i\times n_i}$, $Q_{12i}\in R^{l_i\times l_i}$和矩阵$M_i\in R^{m_i\times n_i}$, 使得如下不等式成立:

$X=$

$$\begin{bmatrix} U_{111} & D_1Q_{121} & E_{12}\bar{Q}_{112}+\bar{Q}_{111}E_{21}^{\mathrm{T}} \\ Q_{121}D_1^{\mathrm{T}} & U_{121} & 0 \\ \bar{Q}_{112}E_{12}^{\mathrm{T}}+E_{21}\bar{Q}_{111} & 0 & U_{112} \\ 0 & 0 & Q_{122}D_2^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \bar{Q}_{11N}E_{1N}^{\mathrm{T}}+E_{N1}\bar{Q}_{111} & 0 & \bar{Q}_{11N}E_{2N}^{\mathrm{T}}+E_{N2}\bar{Q}_{112} \\ 0 & 0 & 0 \end{matrix} \rightarrow$$

$$\leftarrow \begin{matrix} 0 & \cdots & E_{1N}\bar{Q}_{11N}+\bar{Q}_{111}E_{N1}^{\mathrm{T}} & 0 \\ 0 & \ddots & 0 & 0 \\ D_2Q_{122} & \cdots & E_{2N}\bar{Q}_{11N}+\bar{Q}_{112}E_{N2}^{\mathrm{T}} & 0 \\ U_{122} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & U_{11N} & D_NQ_{12N} \\ 0 & \cdots & Q_{12N}D_N^{\mathrm{T}} & U_{12N} \end{bmatrix} < 0, \tag{6}$$

$$\frac{\hat{\lambda}_{11}}{\lambda_{11}}+\frac{\hat{\lambda}_{11}}{c_1\lambda_{12}}\sum_{i=1}^{N}d_i<\frac{c_2}{c_1}\mathrm{e}^{-\alpha T}. \tag{7}$$

其中: $\lambda_{11}=\min\limits_{1\leqslant i\leqslant N}\{\lambda_{11i}\}$, $\hat{\lambda}_{11}=\max\limits_{1\leqslant i\leqslant N}\{\hat{\lambda}_{11i}\}$, $\lambda_{11i},\hat{\lambda}_{11i}$ 分别为 Q_{11i} 的最小和最大特征值; $\lambda_{12}=\min\limits_{1\leqslant i\leqslant N}\{\lambda_{12i}\}$, 而 λ_{12i} 为 Q_{12i} 的最小特征值, $i=1,2,\cdots,N$. 则分散状态反馈控制 $u_i(t)=K_ix_i(t)$ 使得系统 (5) 关于 $(c_1, c_2, T, R_1,\cdots,R_N,d_1,\cdots,d_N)$ 是有限时间稳定的 (FTS), 而 $K_i=M_i\bar{Q}_{11i}^{-1}$, $i=1,2,\cdots,N$.

证明　记

$$z(t)=[x_1^{\mathrm{T}}(t),w_1^{\mathrm{T}}(t),\cdots,x_N^{\mathrm{T}}(t),w_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$P=\begin{bmatrix} \bar{Q}_{111}^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & Q_{121}^{-1} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \bar{Q}_{112}^{-1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & Q_{122}^{-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \bar{Q}_{11N}^{-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & Q_{12N}^{-1} \end{bmatrix},$$

则 $\sum\limits_{i=1}^{N}x_i^{\mathrm{T}}(t)R_i^{1/2}Q_{11i}^{-1}R_i^{1/2}x_i(t)+\sum\limits_{i=1}^{N}w_i^{\mathrm{T}}(t)Q_{12i}^{-1}w_i(t)=\sum\limits_{i=1}^{N}x_i^{\mathrm{T}}(t)\bar{Q}_{11i}^{-1}x_i(t)+\sum\limits_{i=1}^{N}w_i^{\mathrm{T}}(t)Q_{12i}^{-1}w_i(t)=z^{\mathrm{T}}(t)Pz(t)=:V(z(t))$. 由式 (6), 进行合同变换, 可知 $PXP<0$, 即

$$\begin{bmatrix} V_{111} & \bar{Q}_{111}^{-1}D_1 & \bar{Q}_{111}^{-1}E_{12}+E_{21}^{\mathrm{T}}\bar{Q}_{112}^{-1} \\ D_1^{\mathrm{T}}\bar{Q}_{111}^{-1} & V_{121} & 0 \\ E_{12}^{\mathrm{T}}\bar{Q}_{111}^{-1}+\bar{Q}_{112}^{-1}E_{21} & 0 & V_{112} \\ 0 & 0 & D_2^{\mathrm{T}}\bar{Q}_{112}^{-1} \\ \vdots & \vdots & \vdots \\ E_{1N}^{\mathrm{T}}\bar{Q}_{111}^{-1}+\bar{Q}_{11N}^{-1}E_{N1} & 0 & E_{2N}^{\mathrm{T}}\bar{Q}_{112}^{-1}+\bar{Q}_{11N}^{-1}E_{N2} \\ 0 & 0 & 0 \end{matrix} \rightarrow$$

$$\leftarrow \begin{matrix} 0 & \cdots & \bar{Q}_{111}^{-1}E_{1N}+E_{N1}^{\mathrm{T}}\bar{Q}_{11N}^{-1} & 0 \\ 0 & \cdots & 0 & 0 \\ \bar{Q}_{112}^{-1}D_2 & \cdots & \bar{Q}_{112}^{-1}E_{2N}+E_{N2}^{\mathrm{T}}\bar{Q}_{11N}^{-1} & 0 \\ V_{122} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & V_{11N} & \bar{Q}_{11N}^{-1}D_N \\ 0 & \cdots & D_N^{\mathrm{T}}\bar{Q}_{11N}^{-1} & V_{12N} \end{bmatrix} < 0, \tag{8}$$

其中 $V_{11i}=\bar{Q}_{11i}^{-1}A_i+A_i^{\mathrm{T}}\bar{Q}_{11i}^{-1}+\bar{Q}_{11i}^{-1}B_iM_i\bar{Q}_{11i}^{-1}+\bar{Q}_{11i}^{-1}M_i^{\mathrm{T}}B_i^{\mathrm{T}}\bar{Q}_{11i}^{-1}-\alpha\bar{Q}_{11i}^{-1}$. 由 $M_i=K_i\bar{Q}_{11i}$, 有

$$V_{11i}=\bar{Q}_{11i}^{-1}(A_i+B_iK_i)+(A_i+B_iK_i)^{\mathrm{T}}\bar{Q}_{11i}^{-1}-\alpha\bar{Q}_{11i}^{-1},$$

而 $V_{12i}=\bar{Q}_{12i}^{-1}S_i+S_i^{\mathrm{T}}\bar{Q}_{12i}^{-1}-\alpha\bar{Q}_{12i}^{-1}$, 所以由系统 (5), 利用式 (8), 可得

$$\dot{V}(z(t))<\alpha V(z(t)). \tag{9}$$

对式 (9) 从 0 到 t 积分, 得

$$V(z(t))<V(z(0))\mathrm{e}^{\alpha t}. \tag{10}$$

由 $V(z(t))$ 的定义, 可知

$$V(z(t))\geqslant \sum_{i=1}^{N}\frac{1}{\hat{\lambda}_{11i}}x_i^{\mathrm{T}}(t)R_ix_i(t)\geqslant\frac{1}{\hat{\lambda}_{11}}\sum_{i=1}^{N}x_i^{\mathrm{T}}(t)R_ix_i(t), \tag{11}$$

$$\begin{aligned} V(z(0))\mathrm{e}^{\alpha t}\leqslant&\Big(\sum_{i=1}^{N}\frac{1}{\lambda_{11i}}x_i^{\mathrm{T}}(0)R_ix_i(0)\Big)\mathrm{e}^{\alpha t}+\\ &\Big(\sum_{i=1}^{N}\frac{1}{\lambda_{12i}}w_i^{\mathrm{T}}(0)w_i(0)\Big)\mathrm{e}^{\alpha t}\leqslant\\ &\frac{1}{\lambda_{11}}\Big(\sum_{i=1}^{N}x_i^{\mathrm{T}}(0)R_ix_i(0)\Big)\mathrm{e}^{\alpha t}+\\ &\frac{1}{\lambda_{12}}\Big(\sum_{i=1}^{N}w_i^{\mathrm{T}}(0)w_i(0)\Big)\mathrm{e}^{\alpha t}\leqslant\\ &\Big(\frac{c_1}{\lambda_{11}}+\frac{1}{\lambda_{12}}\sum_{i=1}^{N}d_i\Big)\mathrm{e}^{\alpha t}. \end{aligned} \tag{12}$$

由式 (7) 和 (10) ~ (12), 得 $\sum\limits_{i=1}^{N}x_i^{\mathrm{T}}(t)R_ix_i(t)\leqslant c_2$ 对一切 $t\in[0,T]$ 成立. □

由定理 1 可知, 当系统含有不确定项 $\Delta D_i(t)$, $\Delta E_{ij}(t)$, $\Delta S_i(t)$ 时, 式 (6) 相应为

$$\bar{X}=$$

$$\begin{bmatrix} U_{111} & \tilde{D}_1Q_{121} & \tilde{E}_{12}\bar{Q}_{112}+\bar{Q}_{111}\tilde{E}_{21}^{\mathrm{T}} \\ Q_{121}\tilde{D}_1^{\mathrm{T}} & \bar{U}_{121} & 0 \\ \bar{Q}_{112}\tilde{E}_{12}^{\mathrm{T}}+\tilde{E}_{21}\bar{Q}_{111} & 0 & U_{112} \\ 0 & 0 & Q_{122}\tilde{D}_2^{\mathrm{T}} \\ \vdots & \vdots & \vdots \\ \bar{Q}_{11N}\tilde{E}_{1N}^{\mathrm{T}}+\tilde{E}_{N1}\bar{Q}_{111} & 0 & \bar{Q}_{11N}\tilde{E}_{2N}^{\mathrm{T}}+\tilde{E}_{N2}\bar{Q}_{112} \\ 0 & 0 & 0 \end{matrix} \rightarrow$$

$$\begin{bmatrix} 0 & \cdots & \tilde{E}_{1N}\bar{Q}_{11N}+\bar{Q}_{111}\tilde{E}_{N1}^{\mathrm{T}} & 0 \\ 0 & \cdots & 0 & 0 \\ \tilde{D}_2Q_{122} & \cdots & \tilde{E}_{2N}\bar{Q}_{11N}+\bar{Q}_{112}\tilde{E}_{N2}^{\mathrm{T}} & 0 \\ \leftarrow \bar{U}_{122} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & U_{11N} & \tilde{D}_NQ_{12N} \\ 0 & \cdots & Q_{12N}\tilde{D}_N^{\mathrm{T}} & \bar{U}_{12N} \end{bmatrix} < 0. \tag{13}$$

其中: $\tilde{D}_i = D_i + \Delta D_i(t)$, $\tilde{E}_{ij} = E_{ij} + \Delta E_{ij}(t)$, $\bar{U}_{12i} = \tilde{S}_iQ_{12i} + Q_{12i}\tilde{S}_i^{\mathrm{T}} - \alpha Q_{12i}$, 而 $\tilde{S}_i = S_i + \Delta S_i(t)$. 由式(2)及矩阵的分块运算知

$$\begin{aligned}
&\bar{X} = \\
&X + \begin{bmatrix} F_1 \\ 0 \end{bmatrix} \varDelta_{11}[0 \quad G_1Q_{121} \quad 0] + \\
&\begin{bmatrix} 0 \\ Q_{121}G_1^{\mathrm{T}} \\ 0 \end{bmatrix} \varDelta_{11}^{\mathrm{T}}[F_1^{\mathrm{T}} \quad 0] + \\
&\begin{bmatrix} 0 \\ F_2 \\ 0 \end{bmatrix} \varDelta_{12}[0 \quad G_2Q_{122} \quad 0] + \\
&\begin{bmatrix} 0 \\ Q_{122}G_2^{\mathrm{T}} \\ 0 \end{bmatrix} \varDelta_{12}^{\mathrm{T}}[0 \quad F_2^{\mathrm{T}} \quad 0] + \cdots + \\
&\begin{bmatrix} 0 \\ F_{N-1} \\ 0 \end{bmatrix} \varDelta_{1(N-1)}[0 \quad G_{N-1}Q_{12(N-1)} \quad 0] + \\
&\begin{bmatrix} 0 \\ Q_{12(N-1)}G_{N-1}^{\mathrm{T}} \\ 0 \end{bmatrix} \varDelta_{1(N-1)}^{\mathrm{T}}[0 \quad F_{N-1}^{\mathrm{T}} \quad 0] + \\
&\begin{bmatrix} 0 \\ F_N \\ 0 \end{bmatrix} \varDelta_{1N}[0 \quad G_NQ_{12N}] + \\
&\begin{bmatrix} 0 \\ Q_{12N}G_N^{\mathrm{T}} \end{bmatrix} \varDelta_{1N}^{\mathrm{T}}[0 \quad F_N^{\mathrm{T}} \quad 0] + \\
&\begin{bmatrix} H_{12} \\ 0 \end{bmatrix} \varDelta_{212}[0 \quad L_{12}\bar{Q}_{112} \quad 0] + \\
&\begin{bmatrix} 0 \\ \bar{Q}_{112}L_{12}^{\mathrm{T}} \\ 0 \end{bmatrix} \varDelta_{212}^{\mathrm{T}}[H_{12}^{\mathrm{T}} \quad 0] + \\
&\begin{bmatrix} \bar{Q}_{111}L_{21}^{\mathrm{T}} \\ 0 \end{bmatrix} \varDelta_{221}^{\mathrm{T}}[0 \quad H_{21}^{\mathrm{T}} \quad 0] + \\
&\begin{bmatrix} 0 \\ H_{21} \\ 0 \end{bmatrix} \varDelta_{221}[L_{21}\bar{Q}_{111} \quad 0] + \\
&\begin{bmatrix} 0 \\ W_1 \\ 0 \end{bmatrix} \varDelta_{31}[0 \quad J_1Q_{121} \quad 0] + \\
&\begin{bmatrix} 0 \\ Q_{121}J_1^{\mathrm{T}} \\ 0 \end{bmatrix} \varDelta_{31}^{\mathrm{T}}[0 \quad W_1^{\mathrm{T}} \quad 0] + \cdots + \\
&\begin{bmatrix} 0 \\ W_N \end{bmatrix} \varDelta_{3N}[0 \quad J_NQ_{12N}] + \\
&\begin{bmatrix} 0 \\ Q_{12N}J_N^{\mathrm{T}} \end{bmatrix} \varDelta_{3N}^{\mathrm{T}}[0 \quad W_N^{\mathrm{T}}].
\end{aligned}$$

由式(3)及文献[5]中引理1可知式(13)成立, 当且仅当存在正数 $\varepsilon_i, \varepsilon_{ij}, \delta_i, \delta_{ij}$, $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, N$, $j \neq i$, 满足不等式

$$\begin{bmatrix} \tilde{U}_{111} & D_1Q_{121} & E_{12}\bar{Q}_{112}+\bar{Q}_{111}E_{21}^{\mathrm{T}} \\ Q_{121}D_1^{\mathrm{T}} & \tilde{U}_{121} & 0 \\ \bar{Q}_{112}E_{12}^{\mathrm{T}}+E_{21}\bar{Q}_{111} & 0 & \tilde{U}_{112} \\ 0 & 0 & Q_{122}D_2^{\mathrm{T}} & \rightarrow \\ \vdots & \vdots & \vdots \\ \bar{Q}_{11N}E_{1N}^{\mathrm{T}}+E_{N1}\bar{Q}_{111} & 0 & \bar{Q}_{11N}E_{2N}^{\mathrm{T}}+E_{N2}\bar{Q}_{112} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & \cdots & E_{1N}\bar{Q}_{11N}+\bar{Q}_{111}E_{N1}^{\mathrm{T}} & 0 \\ 0 & \cdots & 0 & 0 \\ D_2Q_{122} & \cdots & E_{2N}\bar{Q}_{11N}+\bar{Q}_{112}E_{N2}^{\mathrm{T}} & 0 \\ \leftarrow \tilde{U}_{122} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \tilde{U}_{11N} & D_NQ_{12N} \\ 0 & \cdots & Q_{12N}D_N^{\mathrm{T}} & \tilde{U}_{12N} \end{bmatrix} < 0.$$

其中

$$\begin{aligned}
\tilde{U}_{11i} = {} & U_{11i} + \varepsilon_iF_iF_i^{\mathrm{T}} + \sum_{j=i+1}^{N}\varepsilon_{ij}H_{ij}H_{ij}^{\mathrm{T}} + \\
& \sum_{j=1}^{i-1}\varepsilon_{ji}^{-1}\bar{Q}_{11i}L_{ji}^{\mathrm{T}}L_{ji}\bar{Q}_{11i} + \\
& \sum_{j=i+1}^{N}\delta_{ij}^{-1}\bar{Q}_{11i}L_{ji}^{\mathrm{T}}L_{ji}\bar{Q}_{11i} + \sum_{j=1}^{i-1}\delta_{ji}H_{ij}H_{ij}^{\mathrm{T}}, \\
\tilde{U}_{12i} = {} & U_{12i} + \varepsilon_i^{-1}Q_{12i}G_i^{\mathrm{T}}G_iQ_{12i} + \\
& \delta_i^{-1}Q_{12i}J_i^{\mathrm{T}}J_iQ_{12i} + \delta_iW_iW_i^{\mathrm{T}}.
\end{aligned}$$

由Schur补引理可得:

定理 2 若存在非负常数 α, 正数 $\varepsilon_i, \varepsilon_{ij}, \delta_i, \delta_{ij}$, 正定矩阵 $Q_{11i} \in R^{n_i \times n_i}$, $Q_{12i} \in R^{l_i \times l_i}$ 和矩阵 $M_i \in R^{m_i \times n_i}$, 使得下面的LMI成立:

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix} < 0.$$

其中

$X_{11} =$

$$\left[\begin{array}{ccc} \hat{U}_{111} & D_1Q_{121} & E_{12}\bar{Q}_{112}+\bar{Q}_{111}E_{21}^{\rm T} \\ Q_{121}D_1^{\rm T} & \hat{U}_{121} & 0 \\ \bar{Q}_{112}E_{12}^{\rm T}+E_{21}\bar{Q}_{111} & 0 & \hat{U}_{112} \\ 0 & 0 & Q_{122}D_2^{\rm T} \\ \vdots & \vdots & \vdots \\ \bar{Q}_{11N}E_{1N}^{\rm T}+E_{N1}\bar{Q}_{111} & 0 & \bar{Q}_{11N}E_{2N}^{\rm T}+E_{N2}\bar{Q}_{112} \\ 0 & 0 & 0 \end{array}\right. \rightarrow$$

$$\leftarrow \left.\begin{array}{cccc} 0 & \cdots & E_{1N}\bar{Q}_{11N}+\bar{Q}_{111}E_{N1}^{\rm T} & 0 \\ 0 & \cdots & 0 & 0 \\ D_2Q_{122} & \cdots & E_{2N}\bar{Q}_{11N}+\bar{Q}_{112}E_{N2}^{\rm T} & 0 \\ \hat{U}_{122} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \hat{U}_{11N} & D_NQ_{12N} \\ 0 & \cdots & Q_{12N}D_N^{\rm T} & \hat{U}_{12N} \end{array}\right],$$

$X_{12} =$

$$\left[\begin{array}{cccccc} \bar{Q}_{111}L_{21}^{\rm T} & \cdots & \bar{Q}_{111}L_{N1}^{\rm T} & 0 & 0 & 0 \\ 0 & \cdots & 0 & Q_{121}G_1^{\rm T} & Q_{121}J_1^{\rm T} & 0 \\ 0 & \cdots & 0 & 0 & 0 & \bar{Q}_{112}L_{12}^{\rm T} \\ 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 \end{array}\right. \rightarrow$$

$$\leftarrow \begin{array}{cccccc} 0 & \cdots & 0 & 0 & 0 & \cdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots \\ \bar{Q}_{112}L_{32}^{\rm T} & \cdots & \bar{Q}_{112}L_{N2}^{\rm T} & 0 & 0 & \cdots \\ 0 & \cdots & 0 & Q_{122}G_2^{\rm T} & Q_{122}J_2^{\rm T} & \cdots \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots \\ 0 & \cdots & 0 & 0 & 0 & \cdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots \end{array} \rightarrow$$

$$\leftarrow \left.\begin{array}{ccccc} 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \bar{Q}_{11N}L_{1N}^{\rm T} & \cdots & \bar{Q}_{11N}L_{(N-1)N}^{\rm T} & 0 & 0 \\ 0 & \cdots & 0 & Q_{12N}G_N^{\rm T} & Q_{12N}J_N^{\rm T} \end{array}\right],$$

$X_{22} =$

$${\rm diag}(-\delta_{12}I,\cdots,-\delta_{1N}I,-\varepsilon_1 I,-\delta_1 I,-\varepsilon_{12}I,$$
$$-\delta_{23}I,\cdots,-\delta_{2N}I,-\varepsilon_2 I,-\delta_2 I,\cdots,$$
$$-\varepsilon_{1N}I,\cdots,-\varepsilon_{(N-1)N}I,-\varepsilon_N I,-\delta_N I),$$

$$\hat{U}_{111} = U_{111} + \varepsilon_1 F_1 F_1^{\rm T} + \sum_{j=2}^{N} \varepsilon_{1j} H_{1j} H_{1j}^{\rm T},$$

$$\hat{U}_{112} = U_{112} + \varepsilon_2 F_2 F_2^{\rm T} + \sum_{j=3}^{N} \varepsilon_{2j} H_{2j} H_{2j}^{\rm T} + \delta_{12} H_{21} H_{21}^{\rm T},$$

$$\hat{U}_{11N} = U_{11N} + \varepsilon_N F_N F_N^{\rm T} + \sum_{j=1}^{N-1} \delta_{jN} H_{Nj} H_{Nj}^{\rm T},$$

$$\hat{U}_{121} = U_{121} + \delta_1 W_1 W_1^{\rm T},$$

$$\hat{U}_{122} = U_{122} + \delta_2 W_2 W_2^{\rm T},$$

$$\hat{U}_{12N} = U_{12N} + \delta_N W_N W_N^{\rm T}.$$

而

$$\frac{\hat{\lambda}_{11}}{\lambda_{11}} + \frac{\hat{\lambda}_{11}}{c_1\lambda_{12}} \sum_{i=1}^{N} d_i < \frac{c_2}{c_1}\,{\rm e}^{-\alpha T},$$

$\lambda_{11}, \lambda_{12}, \hat{\lambda}_{11}, \bar{Q}_{11i}, U_{11i}, U_{12i}$ 同定理 1. 则分散状态反馈控制 $u_i(t)=K_ix_i(t)$ 使得系统 (4) 关于 $(c_1, c_2, T, R_1, \cdots, R_N, d_1, \cdots, d_N)$ 是有限时间稳定的 (FTS), 而 $K_i = M_i\bar{Q}_{11i}^{-1}, i=1,2,\cdots,N$.

4　基于输出反馈的分散有限时间镇定

构建系统 (1) 的分散动态输出反馈控制器如下:

$$\begin{aligned} &\dot{\xi}_i(t) = \\ &A_i\xi_i(t) + B_iu_i(t) + L_i(C_i\xi_i(t) - y_i(t)) = \\ &A_i\xi_i(t) + B_iu_i(t) + L_iC_i(\xi_i(t) - x_i(t)),\ \xi_i(0)=0; \\ &u_i(t) = K_i\xi_i(t). \end{aligned}$$

记 $e_i(t)=x_i(t)-\xi_i(t)$, 则

$$\begin{aligned} &\dot{x}_i(t) = \\ &(A_i + B_iK_i)x_i(t) - B_iK_ie_i(t) + \\ &(D_i + \Delta D_i(t))w_i(t) + \\ &\sum_{j=1,j\neq i}^{N} (E_{ij} + \Delta E_{ij}(t))x_j(t),\ x_i(0) = x_{i0}. \end{aligned} \tag{14}$$

而

$$\begin{aligned} &\dot{e}_i(t) = \\ &(A_i + L_iC_i)e_i(t) + (D_i + \Delta D_i(t))w_i(t) + \\ &\sum_{j=1,j\neq i}^{N} (E_{ij} + \Delta E_{ij}(t))(e_j(t) + \xi_j(t)),\ e_i(0) = x_{i0}; \end{aligned} \tag{15}$$

$$\dot{w}_i(t) = (S_i + \Delta S_i(t))w_i(t),\ w_i^{\rm T}(0)w_i(0) \leqslant d_i; \tag{16}$$

$$\dot{\xi}_i(t) = (A_i + B_iK_i)\xi_i(t) - L_iC_ie_i(t),\ \xi_i(0) = 0. \tag{17}$$

显然, 对于给定的 $x_{i0}, i=1,2,\cdots,N$, 组合系统 (15) ~ (17) 关于 $e_i(t), w_i(t), \xi_i(t)\ (i=1,2,\cdots,N)$ 是适定的, 即解是存在唯一的. 由此, 提出系统 (1) 的基于输出反馈的分散有限时间镇定问题: 给定 K_i, 使得系统

$$\begin{cases}\dot{x}_i(t)=(A_i+B_iK_i)x_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ \qquad\displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t);\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm T}(0)w_i(0)\leqslant d_i\end{cases}$$

关于 $(c_1,c_2,T,R_1,\cdots,R_N,d_1,\cdots,d_N)$ 是FTS的. 寻求观测器增益 L_i, 使得系统

$$\begin{cases}\dot{x}_i(t)=\\ (A_i+B_iK_i)x_i(t)-B_iK_ie_i(t)+\\ (D_i+\Delta D_i(t))w_i(t)+\displaystyle\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))x_j(t);\\ \dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),\ w_i^{\mathrm T}(0)w_i(0)\leqslant d_i\end{cases}$$

关于 $(c_1,c_2,\omega_{1L},\cdots,\omega_{NL},T,R_1,\cdots,R_N,d_1,\cdots,d_N)$ 是FTB的, 而

$$\begin{aligned}&\omega_{iL}=\\ &\Big\{e_i(t)|\dot{e}_i(t)=(A_i+L_iC_i)e_i(t)+(D_i+\Delta D_i(t))w_i(t)+\\ &\sum_{j=1,j\neq i}^{N}(E_{ij}+\Delta E_{ij}(t))(e_j(t)+\xi_j(t)),e_i(0)=x_{i0},\\ &\dot{w}_i(t)=(S_i+\Delta S_i(t))w_i(t),w_i^{\mathrm T}(0)w_i(0)\leqslant d_i,\\ &\dot{\xi}_i(t)=(A_i+B_iK_i)\xi_i(t)-L_iC_ie_i(t),\xi_i(0)=0,\\ &\sum_{i=1}^{N}x_{i0}^{\mathrm T}R_ix_{i0}\leqslant c_1\Big\},\ i=1,2,\cdots,N.\end{aligned}$$

同样, 先考虑确定性系统的分散有限时间镇定, 即在系统(1)中令 $\Delta D_i(t),\Delta E_{ij}(t),\Delta S_i(t)$ 均为零矩阵, 则有如下定理:

定理 3 记

$$\begin{aligned}&\bar{Q}_{21i}=R_i^{1/2}Q_{21i}R_i^{1/2},\ \bar{Q}_{22i}=R_i^{1/2}Q_{22i}R_i^{1/2},\\ &U_{21i}=(A_i+B_iK_i)^{\mathrm T}\bar{Q}_{21i}+\bar{Q}_{21i}(A_i+B_iK_i)-\alpha\bar{Q}_{21i},\\ &U_{22i}=A_i^{\mathrm T}\bar{Q}_{22i}+\bar{Q}_{22i}A_i+C_i^{\mathrm T}\bar{M}_i^{\mathrm T}+\bar{M}_iC_i-\alpha\bar{Q}_{22i},\\ &U_{23i}=Q_{23i}S_i+S_i^{\mathrm T}Q_{23i}-\alpha Q_{23i}.\end{aligned}$$

若存在非负常数 α, 正定矩阵 $Q_{21i},Q_{22i}\in R^{n_i\times n_i}$, $Q_{23i}\in R^{l_i\times l_i}$ 和矩阵 $\bar{M}_i\in R^{n_i\times p_i}$ 以及正常数 λ_{21}, $\hat{\lambda}_{21}$, $\hat{\lambda}_{22}$, $\hat{\lambda}_{23}$, 使得如下不等式成立:

$$\begin{bmatrix}U_{211} & -\bar{Q}_{211}B_1K_1 & \bar{Q}_{211}D_1 & \bar{Q}_{211}E_{12}+E_{21}^{\mathrm T}\bar{Q}_{212} & E_{21}^{\mathrm T}\bar{Q}_{222} & 0 & \cdots & \bar{Q}_{211}E_{1N}+E_{N1}^{\mathrm T}\bar{Q}_{21N} & E_{N1}^{\mathrm T}\bar{Q}_{22N} & 0\\ -K_1^{\mathrm T}B_1^{\mathrm T}\bar{Q}_{211} & U_{221} & \bar{Q}_{221}D_1 & \bar{Q}_{221}E_{12} & 0 & 0 & \cdots & \bar{Q}_{221}E_{1N} & 0 & 0\\ D_1^{\mathrm T}\bar{Q}_{211} & D_1^{\mathrm T}\bar{Q}_{221} & U_{231} & 0 & 0 & 0 & \cdots & 0 & 0 & 0\\ E_{12}^{\mathrm T}\bar{Q}_{211}+\bar{Q}_{212}E_{21} & E_{12}^{\mathrm T}\bar{Q}_{221} & 0 & U_{212} & -\bar{Q}_{212}B_2K_2 & \bar{Q}_{212}D_2 & \cdots & \bar{Q}_{212}E_{2N}+E_{N2}^{\mathrm T}\bar{Q}_{21N} & E_{N2}^{\mathrm T}\bar{Q}_{22N} & 0\\ \bar{Q}_{222}E_{21} & 0 & 0 & -K_2^{\mathrm T}B_2^{\mathrm T}\bar{Q}_{212} & U_{222} & \bar{Q}_{222}D_2 & \cdots & \bar{Q}_{222}E_{2N} & 0 & 0\\ 0 & 0 & 0 & D_2^{\mathrm T}\bar{Q}_{212} & D_2^{\mathrm T}\bar{Q}_{222} & U_{232} & \cdots & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ E_{1N}^{\mathrm T}\bar{Q}_{211}+\bar{Q}_{21N}E_{N1} & E_{1N}^{\mathrm T}\bar{Q}_{221} & 0 & E_{2N}^{\mathrm T}\bar{Q}_{212}+\bar{Q}_{21N}E_{N2} & E_{2N}^{\mathrm T}\bar{Q}_{222} & 0 & \cdots & U_{21N} & -\bar{Q}_{21N}B_NK_N & \bar{Q}_{21N}D_N\\ \bar{Q}_{22N}E_{N1} & 0 & 0 & \bar{Q}_{22N}E_{N2} & 0 & 0 & \cdots & -K_N^{\mathrm T}B_N^{\mathrm T}\bar{Q}_{21N} & U_{22N} & \bar{Q}_{22N}D_N\\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots & D_N^{\mathrm T}\bar{Q}_{21N} & D_N^{\mathrm T}\bar{Q}_{22N} & U_{23N}\end{bmatrix}<0; \tag{18}$$

$$\lambda_{21}I\leqslant Q_{21i}\leqslant\hat{\lambda}_{21}I,\ i=1,2,\cdots,N; \tag{19}$$

$$0<Q_{22i}\leqslant\hat{\lambda}_{22}I,\ i=1,2,\cdots,N; \tag{20}$$

$$0<Q_{23i}\leqslant\hat{\lambda}_{23}I,\ i=1,2,\cdots,N; \tag{21}$$

$$c_1(\hat{\lambda}_{21}+\hat{\lambda}_{22})+\hat{\lambda}_{23}\sum_{i=1}^{N}d_i\leqslant c_2\,\mathrm{e}^{-\alpha T}\lambda_{21}. \tag{22}$$

则系统(1) ($\Delta D_i(t),\Delta E_{ij}(t),\Delta S_i(t)$ 均为零矩阵时) 的基于输出反馈的分散有限时间镇定问题是可解的, 而 $L_i=\bar{Q}_{22i}^{-1}\bar{M}_i,i=1,2,\cdots,N$.

证明 记

$$\begin{aligned}&\bar{z}(t)=[x_1^{\mathrm T}(t),e_1^{\mathrm T}(t),w_1^{\mathrm T}(t),x_2^{\mathrm T}(t),e_2^{\mathrm T}(t),w_2^{\mathrm T}(t),\\ &\qquad\cdots,x_N^{\mathrm T}(t),e_N^{\mathrm T}(t),w_N^{\mathrm T}(t)]^{\mathrm T},\\ &\bar{P}=\mathrm{diag}(\bar{Q}_{211},\bar{Q}_{221},Q_{231},\bar{Q}_{212},\bar{Q}_{222},\\ &\qquad Q_{232},\cdots,\bar{Q}_{21N},\bar{Q}_{22N},Q_{23N}),\end{aligned}$$

则

$$\begin{aligned}&\sum_{i=1}^{N}(x_i^{\mathrm T}(t)R_i^{1/2}Q_{21i}R_i^{1/2}x_i(t)+\\ &e_i^{\mathrm T}(t)R_i^{1/2}Q_{22i}R_i^{1/2}e_i(t)+w_i^{\mathrm T}(t)Q_{23i}w_i(t))=\\ &\sum_{i=1}^{N}(x_i^{\mathrm T}(t)\bar{Q}_{21i}x_i(t)+\\ &e_i^{\mathrm T}(t)\bar{Q}_{22i}e_i(t)+w_i^{\mathrm T}(t)Q_{23i}w_i(t))=\\ &\bar{z}^{\mathrm T}(t)\bar{P}\bar{z}(t)=:V(\bar{z}(t)).\end{aligned}$$

将 $\bar{M}_i=\bar{Q}_{22i}L_i$ 代入式 (18), 得

$$\left[\begin{array}{ccc} U_{211} & -\bar{Q}_{211}B_1K_1 & \bar{Q}_{211}D_1 \\ -K_1^{\rm T}B_1^{\rm T}\bar{Q}_{211} & V_{221} & \bar{Q}_{221}D_1 \\ D_1^{\rm T}\bar{Q}_{211} & D_1^{\rm T}\bar{Q}_{221} & U_{231} \\ E_{12}^{\rm T}\bar{Q}_{211}+\bar{Q}_{212}E_{21} & E_{12}^{\rm T}\bar{Q}_{221} & 0 \\ \bar{Q}_{222}E_{21} & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ E_{1N}^{\rm T}\bar{Q}_{211}+\bar{Q}_{21N}E_{N1} & E_{1N}^{\rm T}\bar{Q}_{221} & 0 \\ \bar{Q}_{22N}E_{N1} & 0 & 0 \\ 0 & 0 & 0 \end{array}\right. \rightarrow$$

$$\leftarrow \begin{array}{cccc} \bar{Q}_{211}E_{12}+E_{21}^{\rm T}\bar{Q}_{212} & E_{21}^{\rm T}\bar{Q}_{222} & 0 & \cdots \\ \bar{Q}_{221}E_{12} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ U_{212} & -\bar{Q}_{212}B_2K_2 & \bar{Q}_{212}D_2 & \cdots \\ -K_2^{\rm T}B_2^{\rm T}\bar{Q}_{212} & V_{222} & \bar{Q}_{222}D_2 & \cdots \\ D_2^{\rm T}\bar{Q}_{212} & D_2^{\rm T}\bar{Q}_{222} & U_{232} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ E_{2N}^{\rm T}\bar{Q}_{212}+\bar{Q}_{21N}E_{N2} & E_{2N}^{\rm T}\bar{Q}_{222} & 0 & \cdots \\ \bar{Q}_{22N}E_{N2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \end{array} \rightarrow$$

$$\leftarrow \left.\begin{array}{ccc} \bar{Q}_{211}E_{1N}+E_{N1}^{\rm T}\bar{Q}_{21N} & E_{N1}^{\rm T}\bar{Q}_{22N} & 0 \\ \bar{Q}_{221}E_{1N} & 0 & 0 \\ 0 & 0 & 0 \\ \bar{Q}_{212}E_{2N}+E_{N2}^{\rm T}\bar{Q}_{21N} & E_{N2}^{\rm T}\bar{Q}_{22N} & 0 \\ \bar{Q}_{222}E_{2N} & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ U_{21N} & -\bar{Q}_{21N}B_NK_N & \bar{Q}_{21N}D_N \\ -K_N^{\rm T}B_N^{\rm T}\bar{Q}_{21N} & V_{22N} & \bar{Q}_{22N}D_N \\ D_N^{\rm T}\bar{Q}_{21N} & D_N^{\rm T}\bar{Q}_{22N} & U_{23N} \end{array}\right] < 0. \tag{23}$$

其中 $V_{22i}=(A_i+L_iC_i)^{\rm T}\bar{Q}_{22i}+\bar{Q}_{22i}(A_i+L_iC_i)-\alpha\bar{Q}_{22i}$, $i=1,2,\cdots,N$. 由 $e_j(t)+\xi_j(t)=x_j(t)$, 联合式 (14) ∼ (16), $\Delta D_i(t), \Delta E_{ij}(t), \Delta S_i(t)$ 均为零矩阵时, 有

$$\begin{cases} \dot{x}_i(t)=(A_i+B_iK_i)x_i(t)-B_iK_ie_i(t)+D_iw_i(t)+ \\ \qquad \sum\limits_{j=1,j\neq i}^{N}E_{ij}x_j(t),\ x_i(0)=x_{i0}; \\ \dot{e}_i(t)=(A_i+L_iC_i)e_i(t)+D_iw_i(t)+ \\ \qquad \sum\limits_{j=1,j\neq i}^{N}E_{ij}x_j(t),\ e_i(0)=x_{i0}; \\ \dot{w}_i(t)=S_iw_i(t), w_i^{\rm T}(0)w_i(0)\leqslant d_i. \end{cases} \tag{24}$$

因此, 由式 (24) 对 $V(\bar{z}(t))$ 求导, 并利用式 (23) 可得到, 对于一切 $t\in[0,T]$, 所有 $e_i(t)\in\omega_{iL}(i=1,2,\cdots,N)$, $\Delta D_i(t), \Delta E_{ij}(t), \Delta S_i(t)$ 均为零, 有

$$\dot{V}(\bar{z}(t))<\alpha V(\bar{z}(t)). \tag{25}$$

对式 (25) 从 0 到 t 积分, 得

$$V(\bar{z}(t))<V(\bar{z}(0))\,{\rm e}^{\alpha t}. \tag{26}$$

由 $V(\bar{z}(t))$ 的定义及式 (19)∼(21), 可知

$$\begin{aligned} V(\bar{z}(t)) \geqslant & \sum_{i=1}^{N}(\lambda_{\min}(Q_{21i})x_i^{\rm T}(t)R_ix_i(t)+ \\ & \lambda_{\min}(Q_{22i})e_i^{\rm T}(t)R_ie_i(t)+ \\ & \lambda_{\min}(Q_{23i})w_i^{\rm T}(t)w_i(t)) \geqslant \\ & \lambda_{21}\sum_{i=1}^{N}x_i^{\rm T}(t)R_ix_i(t), \end{aligned} \tag{27}$$

$$\begin{aligned} V(\bar{z}(0))\,{\rm e}^{\alpha t} \leqslant & \Big(\sum_{i=1}^{N}(\lambda_{\max}(Q_{21i})x_i^{\rm T}(0)R_ix_i(0)+ \\ & \lambda_{\max}(Q_{22i})e_i^{\rm T}(0)R_ie_i(0)+ \\ & \lambda_{\max}(Q_{23i})w_i^{\rm T}(0)w_i(0))\Big)\,{\rm e}^{\alpha t} \leqslant \\ & \Big[(\hat{\lambda}_{21}+\hat{\lambda}_{22})c_1+\hat{\lambda}_{23}\sum_{i=1}^{N}d_i\Big]\,{\rm e}^{\alpha T}. \end{aligned} \tag{28}$$

由式 (22) 和 (26) ∼ (28), 得

$$\sum_{i=1}^{N}x_i^{\rm T}(t)R_ix_i(t)\leqslant c_2$$

对一切 $t\in[0,T]$ 均成立. □

同上节, 由 Schur 补引理, 可得:

定理 4　若存在非负常数 α, 正数 $\bar{\varepsilon}_i, \bar{\varepsilon}_{ij}, \bar{\delta}_i, \bar{\delta}_{ij}, \eta_i, \eta_{ij}$, 正定矩阵 $Q_{21i}, Q_{22i}\in R^{n_i\times n_i}$, $Q_{23i}\in R^{l_i\times l_i}$ 和矩阵 $\bar{M}_i\in R^{n_i\times p_i}$, 使得如下的 LMI 成立:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^{\rm T} & Y_{22} \end{bmatrix}<0.$$

其中

$$Y_{11}=$$

$$\left[\begin{array}{ccc} \hat{U}_{211} & -\bar{Q}_{211}B_1K_1 & \bar{Q}_{211}D_1 \\ -K_1^{\rm T}B_1^{\rm T}\bar{Q}_{211} & \hat{U}_{221} & \bar{Q}_{221}D_1 \\ D_1^{\rm T}\bar{Q}_{211} & D_1^{\rm T}\bar{Q}_{221} & \hat{U}_{231} \\ E_{12}^{\rm T}\bar{Q}_{211}+\bar{Q}_{212}E_{21} & E_{12}^{\rm T}\bar{Q}_{221} & 0 \\ \bar{Q}_{222}E_{21} & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ E_{1N}^{\rm T}\bar{Q}_{211}+\bar{Q}_{21N}E_{N1} & E_{1N}^{\rm T}\bar{Q}_{221} & 0 \\ \bar{Q}_{22N}E_{N1} & 0 & 0 \\ 0 & 0 & 0 \end{array}\right. \rightarrow$$

$$
\leftarrow\begin{array}{cccc}
\bar{Q}_{211}E_{12}+E_{21}^{\mathrm{T}}\bar{Q}_{212} & E_{21}^{\mathrm{T}}\bar{Q}_{222} & 0 & \cdots \\
\bar{Q}_{221}E_{12} & 0 & 0 & \cdots \\
0 & 0 & 0 & \cdots \\
\hat{U}_{212} & -\bar{Q}_{212}B_2K_2 & \bar{Q}_{212}D_2 & \cdots \\
-K_2^{\mathrm{T}}B_2^{\mathrm{T}}\bar{Q}_{212} & \hat{U}_{222} & \bar{Q}_{222}D_2 & \cdots \\
D_2^{\mathrm{T}}\bar{Q}_{212} & D_2^{\mathrm{T}}\bar{Q}_{222} & \hat{U}_{232} & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
E_{2N}^{\mathrm{T}}\bar{Q}_{212}+\bar{Q}_{21N}E_{N2} & E_{2N}^{\mathrm{T}}\bar{Q}_{222} & 0 & \cdots \\
\bar{Q}_{22N}E_{N2} & 0 & 0 & \cdots \\
0 & 0 & 0 & \cdots
\end{array}\rightarrow
$$

$$
\leftarrow\begin{array}{ccc}
\bar{Q}_{211}E_{1N}+E_{N1}^{\mathrm{T}}\bar{Q}_{21N} & E_{N1}^{\mathrm{T}}\bar{Q}_{22N} & 0 \\
\bar{Q}_{221}E_{1N} & 0 & 0 \\
0 & 0 & 0 \\
\bar{Q}_{212}E_{2N}+E_{N2}^{\mathrm{T}}\bar{Q}_{21N} & E_{N2}^{\mathrm{T}}\bar{Q}_{22N} & 0 \\
\bar{Q}_{222}E_{2N} & 0 & 0 \\
0 & 0 & 0 \\
\vdots & \vdots & \vdots \\
\hat{U}_{21N} & -\bar{Q}_{21N}B_NK_N & \bar{Q}_{21N}D_N \\
-K_N^{\mathrm{T}}B_N^{\mathrm{T}}\bar{Q}_{21N} & \hat{U}_{22N} & \bar{Q}_{22N}D_N \\
D_N^{\mathrm{T}}\bar{Q}_{21N} & D_N^{\mathrm{T}}\bar{Q}_{22N} & \hat{U}_{23N}
\end{array}\right];
$$

$Y_{12}=$

$$
\left[\begin{array}{ccccc}
\bar{Q}_{211}F_1 & \bar{Q}_{211}H_{12} & \cdots & \bar{Q}_{211}H_{1N} & 0 \\
0 & 0 & \cdots & 0 & \bar{Q}_{221}F_1 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0
\end{array}\right.\rightarrow
$$

$$
\leftarrow\begin{array}{cccccc}
0 & \cdots & 0 & 0 & 0 & 0 \\
\bar{Q}_{221}H_{12} & \cdots & \bar{Q}_{221}H_{1N} & 0 & 0 & 0 \\
0 & \cdots & 0 & Q_{231}W_1 & 0 & 0 \\
0 & \cdots & 0 & 0 & \bar{Q}_{212}H_{21} & \bar{Q}_{212}F_2 \\
0 & \cdots & 0 & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 & 0
\end{array}\rightarrow
$$

$$
\leftarrow\begin{array}{cccccc}
\cdots & 0 & 0 & 0 & \cdots & 0 \\
\cdots & 0 & 0 & 0 & \cdots & 0 \\
\cdots & 0 & 0 & 0 & \cdots & 0 \\
\cdots & \bar{Q}_{212}H_{2N} & 0 & 0 & \cdots & 0 \\
\cdots & 0 & \bar{Q}_{222}H_{21} & \bar{Q}_{222}F_2 & \cdots & \bar{Q}_{222}H_{2N} \\
\cdots & 0 & 0 & 0 & \cdots & 0 \\
\ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\cdots & 0 & 0 & 0 & \cdots & 0 \\
\cdots & 0 & 0 & 0 & \cdots & 0 \\
\cdots & 0 & 0 & 0 & \cdots & 0
\end{array}\rightarrow
$$

$$
\leftarrow\begin{array}{cccccc}
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
Q_{232}W_2 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \bar{Q}_{21N}H_{N1} & \bar{Q}_{21N}H_{N2} & \cdots & \bar{Q}_{21N}F_N \\
0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0
\end{array}\rightarrow
$$

$$
\leftarrow\left.\begin{array}{ccccc}
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 \\
\bar{Q}_{22N}H_{N1} & \bar{Q}_{22N}H_{N2} & \cdots & \bar{Q}_{22N}F_N & 0 \\
0 & 0 & \cdots & 0 & Q_{23N}W_N
\end{array}\right];
$$

$Y_{22}=$

$\mathrm{diag}(-\bar{\varepsilon}_1 I,-\bar{\varepsilon}_{12}I,\cdots,-\bar{\varepsilon}_{1N}I,-\eta_1 I,-\eta_{12}I,\cdots,$
$-\eta_{1N}I,-\bar{\delta}_1 I,-\bar{\delta}_{21}I,-\bar{\varepsilon}_2 I,\cdots,-\bar{\varepsilon}_{2N}I,-\eta_{21}I,$
$-\eta_2 I,\cdots,-\eta_{2N}I,-\bar{\delta}_2 I,\cdots,-\bar{\delta}_{N1}I,-\bar{\delta}_{N2}I,$
$\cdots,-\bar{\varepsilon}_N I,-\eta_{N1}I,-\eta_{N2}I,\cdots,-\eta_N I,-\bar{\delta}_N I)$;

$$\hat{U}_{211}=U_{211}+\sum_{i=2}^{N}(\bar{\delta}_{i1}+\eta_{i1})L_{i1}^{\mathrm{T}}L_{i1};$$

$$\hat{U}_{221}=U_{221};$$

$$\hat{U}_{231}=U_{231}+\bar{\delta}_1 J_1^{\mathrm{T}}J_1+(\bar{\varepsilon}_1+\eta_1)G_1^{\mathrm{T}}G_1;$$

$$\hat{U}_{212}=U_{212}+(\bar{\varepsilon}_{12}+\eta_{12})L_{12}^{\mathrm{T}}L_{12}+\sum_{i=3}^{N}(\bar{\delta}_{i2}+\eta_{i2})L_{i2}^{\mathrm{T}}L_{i2};$$

$\hat{U}_{222}=U_{222}$;

$\hat{U}_{232}=U_{232}+\bar{\delta}_2 J_2^{\mathrm{T}} J_2+(\bar{\varepsilon}_2+\eta_2) G_2^{\mathrm{T}} G_2$;

$\hat{U}_{21N}=U_{21N}+\sum\limits_{i=1}^{N-1}(\bar{\varepsilon}_{iN}+\eta_{iN}) L_{iN}^{\mathrm{T}} L_{iN}$;

$\hat{U}_{22N}=U_{22N}$;

$\hat{U}_{23N}=U_{23N}+\bar{\delta}_N J_N^{\mathrm{T}} J_N+(\bar{\varepsilon}_N+\eta_N) G_N^{\mathrm{T}} G_N$.

$\bar{Q}_{21i},\bar{Q}_{22i},U_{21i},U_{22i},U_{23i}$ 同定理 3, 且式(19)∼(22) 成立, 则系统 (1) 的基于输出反馈的分散有限时间镇定问题是可解的, 而 $L_i=\bar{Q}_{22i}^{-1}\bar{M}_i,\ i=1,2,\cdots,N$.

5 结　论

本文考虑了含干扰输入的大型互联线性系统的分散有限时间镇定问题. 与文献 [5,15] 相对比, 本文的主要贡献是: 将 [15] 中分散有限时间镇定的概念和控制设计方法推广应用到含干扰输入的不确定大系统; 也是将 [5] 中的系统组合成大系统, 并基于 LMI 设计得到系统的状态反馈和输出反馈控制律. 当反馈控制律作用于该系统时, 闭环系统是有限时间稳定的.

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