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非正则分布参数系统的迭代学习控制

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摘 要: 针对一类非正则分布参数系统的迭代学习控制问题进行讨论, 该类分布参数系统由抛物型偏微分方程构成. 基于非正则系统的特点, 使用 D 型学习律构建得到迭代学习控制律, 并基于压缩映射原理, 证明得到输出跟踪误差在 L^2 范数意义下沿迭代轴方向的收敛性结论. 仿真算例表明了所提出结论的有效性.

关键词: 非正则；D 型学习律；迭代学习控制；抛物型分布参数系统

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Iterative learning control for irregular distributed parameter systems

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Abstract: The problem of iterative learning control for a class of irregular distributed parameter systems is considered. The considered distributed parameter systems are composed of parabolic partial differential equations. According to the characteristics of the irregular systems, iterative learning control laws are proposed for such irregular distributed parameter systems by using the D-type learning scheme. Convergence of the output tracking errors in L^2 norm along the iteration axis is proved based on the contraction mapping method. A simulation example shows the feasibility and effectiveness of the obtained conclusion.

Keywords: irregular；D-type learning law；iterative learning control；parabolic distributed parameter systems

0 引 言

自 Arimoto 等[1]于 1984 年首次提出完整的控制算法后, 迭代学习控制设计便成为近年来自动控制领域研究的热点问题, 并引起人们的广泛关注. 针对需要跟踪的理想输出, 迭代学习控制设计的基本思想是: 在重复受控的有限时间段内, 结合系统所满足的性质, 使用合适的学习律, 通过反复迭代, 不断地修正输出跟踪误差, 并最终使得系统的输出完全跟踪给定的理想输出. 在迭代学习的控制设计过程中, 采用较多的是两类学习律: D 型学习律[1-3]和 P 型学习律[4-6].

由偏微分方程, 或偏微分-积分方程, 或偏微分方程与常微分方程耦合的方程描述的控制系统称为分布参数系统. 许多实际问题都可以用分布参数系统模型刻画[7], 如弹性振动系统的控制、温度场的控制、核反应堆的控制、带柔性连杆的机器人等. 近年来, 有关分布参数系统的研究工作已取得了许多成果[8-10]. 由于分布参数系统变量涉及到无穷维函数空间, 至今, 就迭代学习控制而言, 尽管其在由常微分方程描述的集中参数系统中的研究成果已很多, 但在分布参数系统中的研究成果并不多. 文献 [7,11-13] 研究了抛物型分布参数系统的迭代学习控制问题; 文献 [14] 针对一类一阶双曲型分布参数系统, 构建得到了迭代学习控制器; 文献 [15] 利用 P 型学习律, 对一类二阶双曲型分布参数系统进行了迭代学习控制设计. 上述工作针对的均为正则系统 (系统的输入、输出有直输通道), 并采用了适合正则系统的 P 型学习律进行控制设计. 至今, 针对分布参数系统的迭代学习控制设计涉及的多为正则系统. 文献 [16] 利用算子半群理论, 借助于 D 型学习律, 对一类非正则 (系统的输入、输出无直输通道) 抛物型分布参数系统进行了迭代学习控制设计, 然而这类借助于算子半群理论为工具的方法所获得的结论有一个共同的不足之处就是在具体应用中

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难以验证[17], 故对非正则分布参数系统寻求一种实用有效的设计方法一直是分布参数系统迭代学习控制领域的一个重要课题.

本文针对一类非正则抛物型分布参数系统, 研究其迭代学习控制设计问题. 采用适合非正则系统的D型学习律设计得到迭代学习控制器, 借助于泛函分析中的相关知识, 并基于压缩映射原理, 证明得到系统的输出跟踪误差在 L^2 范数意义下沿迭代轴方向的收敛性结论.

1　问题描述

本文给出如下符号约定: 对于二元函数 $Q(x,t)\in R\bigcap L^2[0,1]$, $x\in[0,1]$, $t\in[0,T]$, 记

$$\|Q(x,t)\|_{L^2[0,1]}=\sqrt{\int_0^1 Q^2(x,t)\mathrm{d}x},$$

定义 $\|Q(x,t)\|_{L^2[0,1],s}=\sup\limits_{t\in[0,T]}\|Q(x,t)\|^2_{L^2[0,1]}$, 对于给定的 $\lambda>0$, 定义

$$\|Q(x,t)\|_{L^2[0,1],\lambda}=\sup_{t\in[0,T]}\mathrm{e}^{-\lambda t}\|Q(x,t)\|^2_{L^2[0,1]}.$$

由文献 [6] 可知, 使用 $\|Q(x,t)\|_{L^2[0,1],s}$ 证明收敛性结果与使用 $\|Q(x,t)\|_{L^2[0,1],\lambda}$ 证明收敛性结果是等效的. 对于函数 $Q(x,t)\in R\bigcap L^2(\varOmega)$, $\varOmega=[0,1]\times[0,T]$, 记 $\|Q(x,t)\|_{L^2(\varOmega)}=\sqrt{\int_0^T\int_0^1 Q^2(x,t)\mathrm{d}x\mathrm{d}t}$.

考虑如下形式的非正则抛物型分布参数系统:

$$\begin{cases}\dfrac{\partial Q(x,t)}{\partial t}=a^2\dfrac{\partial^2 Q(x,t)}{\partial x^2}+AQ(x,t)+Bu(x,t),\\ y(x,t)=CQ(x,t).\end{cases}\tag{1}$$

其中: $(x,t)\in(0,1)\times[0,T]$; $Q(x,t),u(x,t),y(x,t)\in R$ 分别为系统的状态、控制输入和输出.

对系统 (1), 给出如下假设条件.

假设 1　对于给定的初值 $Q(x,0)$、边值 $Q(0,t)$ $\left(或\dfrac{\partial Q(x,t)}{\partial x}\Big|_{x=0}\right)$、$Q(1,t)$ $\left(或\dfrac{\partial Q(x,t)}{\partial x}\Big|_{x=1}\right)$ 和控制输入 $u(x,t)$, 系统 (1) 的解 $Q(x,t)$ 在 $(0,1)\times[0,T]$ 内存在唯一.

假设 2　$CB>0$, $A<0$.

假设 3　对于给定的理想轨迹 $y_r(x,t)$, 存在唯一的控制输入 $u_r(x,t)$, 使得

$$\begin{cases}\dfrac{\partial Q_r(x,t)}{\partial t}=a^2\dfrac{\partial^2 Q_r(x,t)}{\partial x^2}+AQ_r(x,t)+Bu_r(x,t),\\ y_r(x,t)=CQ_r(x,t).\end{cases}$$

其中 $(x,t)\in(0,1)\times[0,T]$.

设动态系统 (1) 在有限区间 $t\in[0,T]$ 内是可重复的, 在迭代学习过程中, 迭代第 k 次时系统 (1) 即为

$$\begin{cases}\dfrac{\partial Q_k(x,t)}{\partial t}=a^2\dfrac{\partial^2 Q_k(x,t)}{\partial x^2}+AQ_k(x,t)+Bu_k(x,t),\\ y_k(x,t)=CQ_k(x,t).\end{cases}\tag{2}$$

其中 $(x,t)\in(0,1)\times[0,T]$.

假设 4　系统的初值定位条件为 $Q_k(x,0)=Q_r(x,0)$, 边值定位条件为

$$Q_k(0,t)=Q_r(0,t)\ 或\ \frac{\partial Q_k(x,t)}{\partial x}\Big|_{x=0}=\frac{\partial Q_r(x,t)}{\partial x}\Big|_{x=0},$$
$$Q_k(1,t)=Q_r(1,t)\ 或\ \frac{\partial Q_k(x,t)}{\partial x}\Big|_{x=1}=\frac{\partial Q_r(x,t)}{\partial x}\Big|_{x=1},$$
$$k=0,1,\cdots.$$

学习控制的目的是寻求适当的学习律, 使得迭代学习序列 $y_k(x,t)$ 在 L^2 空间内一致收敛于理想的输出 $y_r(x,t)$, 即

$$\lim_{k\to\infty}\|e_k(x,t)\|_{L^2[0,1],s}=0,$$

其中 $e_k(x,t)=y_r(x,t)-y_k(x,t)$.

由泛函分析知识[18]可知, 自反 Banach 空间 $\aleph$ 中的任何有界序列 $\{\varphi_n\}$ 必有一个在 $\aleph$ 中弱收敛的子列, 空间 $L^p(\varOmega)(1<p<+\infty)$ 是自反的 Banach 空间. 结合弱收敛的定义[18]和本文中的 $L^2(\varOmega)$ 空间, 注意到 $(L^2(\varOmega))^*=L^2(\varOmega)$($(L^2(\varOmega))^*$ 表示空间 $L^2(\varOmega)$ 的共轭空间), 给出引理 1.

引理 1　设 $\{\varphi_n\}$ 为 $L^2(\varOmega)$ 中的任一有界序列, 即 $\|\varphi_n\|_{L^2(\varOmega)}\leqslant M\ (n=1,2,\cdots)$, 则存在子列 $\{\varphi_{n_j}\}\subset\{\varphi_n\}$, 并存在 $\varphi\in L^2(\varOmega)$, 使得 $\{\varphi_{n_j}\}$ 在 $L^2(\varOmega)$ 中弱收敛于 φ, 即对于任意 $\psi\in(L^2(\varOmega))^*=L^2(\varOmega)$, 都有

$$\lim_{j\to\infty}\psi(\varphi_{n_j})=\lim_{j\to\infty}\int_0^T\int_0^1\varphi_{n_j}\psi\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1\varphi\psi\mathrm{d}x\mathrm{d}t=\psi(\varphi).$$

注意到文献 [19] 中的推论 1, $1<q<+\infty$, 如果 $\{\varphi_n\}$ 在 $L^q(\varOmega)$ 中弱收敛于 φ, $\{\psi_n\}$ 在 $(L^q(\varOmega))^*$ 中弱收敛于 ψ, 则有

$$\lim_{n\to\infty}\int_{\varOmega}\varphi_n\psi_n\mathrm{d}x=\int_{\varOmega}\varphi\psi\mathrm{d}x.$$

结合本文的 $L^2(\varOmega)$ 空间, 给出引理 2.

引理 2　如果 $\{\varphi_n\}$ 在 $L^2(\varOmega)$ 中弱收敛于 φ, $\{\psi_n\}$ 在 $L^2(\varOmega)$ 中弱收敛于 ψ, 则有

$$\lim_{n\to\infty}\int_0^T\int_0^1\varphi_n\psi_n\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1\varphi\psi\mathrm{d}x\mathrm{d}t.$$

特别地, 当 $\psi_n=\varphi_n$ 时, 有

$$\lim_{n\to\infty}\|\varphi_n\|^2_{L^2(\varOmega)}=\lim_{n\to\infty}\int_0^T\int_0^1\varphi_n^2\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1\varphi^2\mathrm{d}x\mathrm{d}t=\|\varphi\|^2_{L^2(\varOmega)}.$$

由收敛序列与其子列间的关系可知, 如果 $\{\varphi_n\}$ 在 $L^2(\varOmega)$ 中弱收敛于 φ, 则 $\{\varphi_n\}$ 的任何子列在 $L^2(\varOmega)$ 中也弱收敛于 φ. 利用有界序列的子列仍为有界序列, 采用子列中再取子列的方法, 反复套用引理 1, 可得性质 1.

性质 1　$L^2(\varOmega)$ 中的 m 个有界序列 $\{\varphi_n^1\},\{\varphi_n^2\},\cdots,\{\varphi_n^m\}$, 可相应找到相同序号的 m 个子列 $\{\varphi_{n_j}^1\}\subset\{\varphi_n^1\},\{\varphi_{n_j}^2\}\subset\{\varphi_n^2\},\cdots,\{\varphi_{n_j}^m\}\subset\{\varphi_n^m\}$, 使得 $\{\varphi_{n_j}^1\}$,

$\{\varphi_{n_j}^2\},\cdots,\{\varphi_{n_j}^m\}$ 在 $L^2(\Omega)$ 中弱收敛.

注 1[18] 空间 $\aleph$ 是自反的, $(\aleph^*)^*=\aleph$.

引理 3[20] 设 $\{a_k\},\{b_k\}$ 是满足 $a_{k+1}\leqslant\rho a_k+b_k$ $(0\leqslant\rho<1)$ 的非负实数数列, 如果有 $\lim\limits_{k\to\infty}b_k=0$, 则有 $\lim\limits_{k\to\infty}a_k=0$.

2 主要结果

对系统 (1) 构建 D 型学习律

$$u_{k+1}(x,t)=u_k(x,t)+p\frac{\partial(e_k(x,t))}{\partial t},\tag{3}$$

其中 $p>0$ 为学习增益. 记

$$\delta Q_k(x,t)=Q_{k+1}(x,t)-Q_k(x,t),$$
$$\delta u_k(x,t)=u_{k+1}(x,t)-u_k(x,t),$$
$$\delta y_k(x,t)=y_{k+1}(x,t)-y_k(x,t),$$

由式 (2) 有

$$\frac{\partial(\delta y_k(x,t))}{\partial t}=C\frac{\partial(\delta Q_k(x,t))}{\partial t}=C\Big\{a^2\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}+A\delta Q_k(x,t)+B\delta u_k(x,t)\Big\}.$$

再由 $e_{k+1}(x,t)=e_k(x,t)-\delta y_k(x,t)$, 结合式 (3) 可得

$$\frac{\partial(e_{k+1}(x,t))}{\partial t}=\frac{\partial(e_k(x,t))}{\partial t}-\frac{\partial(\delta y_k(x,t))}{\partial t}=\frac{\partial(e_k(x,t))}{\partial t}-C\Big\{a^2\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}+A\delta Q_k(x,t)+B\delta u_k(x,t)\Big\}=(1-CBp)\frac{\partial(e_k(x,t))}{\partial t}-Ca^2\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}-CA\delta Q_k(x,t).\tag{4}$$

选取学习增益 p, 使得

$$0<CBp-1<1\tag{5}$$

成立, 则有以下引理 4.

引理 4 若假设 1 ∼ 假设 4 和式 (5) 成立, 则 D 型学习律 (3) 作用于式 (1) 时, 存在子列 $\Big\{\frac{\partial(e_{k_j}(x,t))}{\partial t}\Big\}$, $\Big\{\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\Big\}$, $\Big\{\frac{\partial^2(e_{k_j}(x,t))}{\partial t\partial x}\Big\}$, $\Big\{\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\Big\}$, $\{\delta Q_{k_j}(x,t)\}$, $\Big\{\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\Big\}$ 在 $L^2(\Omega)$ 中弱收敛, 且 $\{\delta Q_{k_j}(x,t)\}$, $\Big\{\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\Big\}$ 弱收敛于 Ω 上的零函数.

证明 注意到

$$\frac{\partial(e_k(x,t))}{\partial t}-\frac{\partial(e_{k+1}(x,t))}{\partial t}=C\frac{\partial(\delta Q_k(x,t))}{\partial t},$$

式 (4) 两端乘以 $\frac{\partial(e_k(x,t))}{\partial t}-\frac{\partial(e_{k+1}(x,t))}{\partial t}$, 得

$$\Big(\frac{\partial(e_k(x,t))}{\partial t}-\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big)\frac{\partial(e_{k+1}(x,t))}{\partial t}=(1-CBp)\times\Big(\frac{\partial(e_k(x,t))}{\partial t}-\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big)\frac{\partial(e_k(x,t))}{\partial t}-C^2a^2\frac{\partial(\delta Q_k(x,t))}{\partial t}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}-C^2A\frac{\partial(\delta Q_k(x,t))}{\partial t}\delta Q_k(x,t).\tag{6}$$

对式 (6) 右端第 2 项应用分部积分公式, 有

$$\int_0^1\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\mathrm{d}x=\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\Big|_0^1-\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\mathrm{d}x.$$

由假设 4 中的边值定位条件, 可知

$$\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\Big|_0^1=0,$$

所以

$$\int_0^1\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\mathrm{d}x=-\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\mathrm{d}x=-\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial x\partial t}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\mathrm{d}x=-\frac{1}{2}\int_0^1\frac{\partial}{\partial t}\Big(\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\Big)\mathrm{d}x=-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1\Big(\frac{\partial\Big(\delta Q_k(x,t)\Big)}{\partial x}\Big)^2\mathrm{d}x.$$

利用假设 4 中的初值定位条件, 对上式两端从 0 到 t 积分, 有

$$\int_0^t\int_0^1\Big\{\frac{\partial(\delta Q_k(x,\xi))}{\partial\xi}\frac{\partial^2(\delta Q_k(x,\xi))}{\partial x^2}\Big\}\mathrm{d}x\mathrm{d}\xi=-\frac{1}{2}\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x.\tag{7}$$

对式 (6) 右端第 3 项积分, 有

$$\int_0^1\frac{\partial(\delta Q_k(x,t))}{\partial t}\delta Q_k(x,t)\mathrm{d}x=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x.$$

利用假设 4 中的初值定位条件, 对上式两端从 0 到 t 积分, 可得

$$\int_0^t\int_0^1\frac{\partial(\delta Q_k(x,\xi))}{\partial\xi}\delta Q_k(x,\xi)\mathrm{d}x\mathrm{d}\xi=\frac{1}{2}\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x.\tag{8}$$

对式 (6) 两端变量 x 从 0 到 1 积分, 变量 t 从 0 到 t 积分, 结合式 (7) 和 (8), 有

$$\int_0^t\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi-(2-CBp)\int_0^t\int_0^1\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi-(CBp-1)\int_0^t\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi+\frac{C^2a^2}{2}\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x-\frac{C^2A}{2}\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x=0.\tag{9}$$

注意到假设 2 中的 $A<0$, 式 (9) 取 $t=T$, 可得

$$\int_0^T\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi-(2-CBp)\int_0^T\int_0^1\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi-(CBp-1)\int_0^T\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\leqslant 0. \tag{10}$$

由 $L^2(\Omega)$ 范数, 并应用 Cauchy-Schwarz 不等式, 有

$$\int_0^T\int_0^1\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi\leqslant\sqrt{\int_0^T\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi}\times\sqrt{\int_0^T\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi}=\Big\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big\|_{L^2(\Omega)}\Big\|\frac{\partial(e_k(x,t))}{\partial t}\Big\|_{L^2(\Omega)}.$$

由式 (5) 有 $2-CBp>0$, 式 (10) 结合上式, 可得

$$\Big\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big\|^2_{L^2(\Omega)}-(2-CBp)\Big\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big\|_{L^2(\Omega)}\Big\|\frac{\partial(e_k(x,t))}{\partial t}\Big\|_{L^2(\Omega)}-(CBp-1)\Big\|\frac{\partial(e_k(x,t))}{\partial t}\Big\|^2_{L^2(\Omega)}\leqslant\int_0^T\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi-(2-CBp)\int_0^T\int_0^1\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi-(CBp-1)\int_0^T\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\leqslant 0.$$

记 $c_k=\Big\|\frac{\partial(e_k(x,t))}{\partial t}\Big\|_{L^2(\Omega)}$, 上式可化为

$$c_{k+1}^2-(2-CBp)c_{k+1}c_k-(CBp-1)c_k^2\leqslant 0.$$

解之可得

$$-(CBp-1)c_k\leqslant c_{k+1}\leqslant c_k.$$

因为 $c_k\geqslant 0$, 结合式 (5) 的 $CBp-1>0$, 有

$$0\leqslant\cdots\leqslant c_{k+1}\leqslant c_k\leqslant\cdots\leqslant c_0. \tag{11}$$

结论 1　$\Big\{\frac{\partial(e_k(x,t))}{\partial t}\Big\}$、$\Big\{\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big\}$ 为有界序列.

对式 (9) 两端变量 t 从 0 到 T 积分, 得到

$$\int_0^T\int_0^t\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\mathrm{d}t-(2-CBp)\int_0^T\int_0^t\int_0^1\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi\mathrm{d}t-(CBp-1)\int_0^T\int_0^t\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\mathrm{d}t+\frac{C^2a^2}{2}\int_0^T\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x\mathrm{d}t-\frac{C^2A}{2}\int_0^T\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x\mathrm{d}t=0. \tag{12}$$

应用 Cauchy-Schwarz 不等式, 有

$$\int_0^T\int_0^t\int_0^1\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi\mathrm{d}t\leqslant\sqrt{\int_0^T\int_0^t\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\mathrm{d}t}\times\sqrt{\int_0^T\int_0^t\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\mathrm{d}t}.$$

记

$$d_k=\sqrt{\int_0^T\int_0^t\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\mathrm{d}t},$$

注意到 $2-CBp>0$, 式 (12) 结合上式, 有

$$d_{k+1}^2-(2-CBp)d_{k+1}d_k-(CBp-1)d_k^2+\frac{C^2a^2}{2}\int_0^T\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x\mathrm{d}t-\frac{C^2A}{2}\int_0^T\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x\mathrm{d}t\leqslant 0. \tag{13}$$

注意到 $A<0$, 由式 (13) 可得

$$d_{k+1}^2-(2-CBp)d_{k+1}d_k-(CBp-1)d_k^2\leqslant 0.$$

同前面过程, 有 $0\leqslant\cdots\leqslant d_{k+1}\leqslant d_k\leqslant\cdots\leqslant d_0$, 由此可知序列 $\{d_k\}$ 的极限存在, 记 $\lim\limits_{k\to\infty}d_k=\theta\geqslant 0$. 由式 (13) 和 $A<0$, 可得

$$0\leqslant\frac{C^2a^2}{2}\int_0^T\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x\mathrm{d}t-\frac{C^2A}{2}\int_0^T\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x\mathrm{d}t\leqslant-d_{k+1}^2+(2-CBp)d_{k+1}d_k+(CBp-1)d_k^2. \tag{14}$$

而

$$\lim_{k\to\infty}\{-d_{k+1}^2+(2-CBp)d_{k+1}d_k+(CBp-1)d_k^2\}=-\theta^2+(2-CBp)\theta\theta+(CBp-1)\theta^2=0,$$

利用夹逼准则, 由式 (14) 可得

$$\lim_{k\to\infty}\Big\{\frac{C^2a^2}{2}\int_0^T\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x\mathrm{d}t-\frac{C^2A}{2}\int_0^T\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x\mathrm{d}t\Big\}=0.$$

又因为 $A<0$, 所以有

$$\lim_{k\to\infty}\|\delta Q_k(x,t)\|^2_{L^2(\Omega)}=\lim_{k\to\infty}\int_0^T\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x\mathrm{d}t=0, \tag{15}$$

$$\lim_{k\to\infty}\Big\|\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\|^2_{L^2(\Omega)}=\lim_{k\to\infty}\int_0^T\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x\mathrm{d}t=0. \tag{16}$$

结论 2　序列 $\{\delta Q_k(x,t)\}$、$\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}$ 有界.

结论 3　序列 $\{\delta Q_k(x,t)\}$、$\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}$ 的任一弱收敛子列必弱收敛于 Ω 上的零函数.

设 $\{\delta Q_{k_i}(x,t)\}$ 为 $\{\delta Q_k(x,t)\}$ 的一个弱收敛子列, 且弱收敛于 $f(x,t)\in L^2(\Omega)$, 则由引理 2 有

$$\lim_{i\to\infty}\|\delta Q_{k_i}(x,t)\|^2_{L^2(\Omega)}=\lim_{i\to\infty}\int_0^T\int_0^1(\delta Q_{k_i}(x,t))^2\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1(f(x,t))^2\mathrm{d}x\mathrm{d}t=\|f(x,t)\|^2_{L^2(\Omega)}.$$

由式 (15) 可知

$$\lim_{i\to\infty}\|\delta Q_{k_i}(x,t)\|^2_{L^2(\Omega)}=0,$$

所以 $\|f(x,t)\|^2_{L^2(\Omega)}=0$, 由此有 $f(x,t)=0(\forall(x,t)\in\Omega)$, 即 $\{\delta Q_k(x,t)\}$ 的任一弱收敛子列弱收敛于 Ω 上的零函数. 同样, 由式 (16) 可得, $\left\{\dfrac{\partial(\delta Q_k(x,t))}{\partial x}\right\}$ 的任一弱收敛子列弱收敛于 Ω 上的零函数.

式 (4) 两端对变量 x 求偏导, 有

$$\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}=(1-CBp)\frac{\partial^2(e_k(x,t))}{\partial t\partial x}-Ca^2\frac{\partial^3(\delta Q_k(x,t))}{\partial x^3}-CA\frac{\partial(\delta Q_k(x,t))}{\partial x}.\qquad(17)$$

注意到

$$\frac{\partial^2(e_k(x,t))}{\partial t\partial x}-\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}=C\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x},$$

式 (17) 两端乘以 $\dfrac{\partial^2(e_k(x,t))}{\partial t\partial x}-\dfrac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}$, 得到

$$\Big(\frac{\partial^2(e_k(x,t))}{\partial t\partial x}-\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\Big)\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}=(1-CBp)\times\Big(\frac{\partial^2(e_k(x,t))}{\partial t\partial x}-\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\Big)\frac{\partial^2(e_k(x,t))}{\partial t\partial x}-C^2a^2\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^3(\delta Q_k(x,t))}{\partial x^3}-C^2A\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_k(x,t))}{\partial x}.\qquad(18)$$

对式 (18) 右端第 2 项应用分部积分公式, 有

$$\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^3(\delta Q_k(x,t))}{\partial x^3}\Big\}\mathrm{d}x=\lim_{\substack{\mu\to1^-\\ \omega\to0^+}}\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\Big|_{x=\omega}^{x=\mu}-\int_0^1\Big\{\frac{\partial^3(\delta Q_k(x,t))}{\partial t\partial x^2}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\mathrm{d}x.\qquad(19)$$

由式 (2) 和 (3) 可得

$$\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}=\frac{1}{a^2}\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}-A\delta Q_k(x,t)-Bp\frac{\partial e_k(x,t)}{\partial t}\Big\}.$$

进而有

$$\lim_{\substack{\mu\to1^-\\ \omega\to0^+}}\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\Big|_{x=\omega}^{x=\mu}=\frac{1}{a^2}\lim_{\substack{\mu\to1^-\\ \omega\to0^+}}\Big\{\Big\{\frac{\partial}{\partial t}\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)\Big\}\times\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}-A\delta Q_k(x,t)-Bp\frac{\partial e_k(x,t)}{\partial t}\Big\}\Big\}\Big|_{x=\omega}^{x=\mu}=$$

$$\frac{1}{a^2}\Big\{\Big\{\frac{\partial}{\partial t}\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)\Big\}\times\Big\{\frac{\partial(\delta Q_k(x,t))}{\partial t}-A\delta Q_k(x,t)-Bp\frac{\partial e_k(x,t)}{\partial t}\Big\}\Big\}\Big|_0^1.$$

利用假设 4 中的边值定位条件, 有

$$\lim_{\substack{\mu\to1^-\\ \omega\to0^+}}\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\Big|_{x=\omega}^{x=\mu}=0.$$

将上式代入式 (19), 可得

$$\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^3(\delta Q_k(x,t))}{\partial x^3}\Big\}\mathrm{d}x=-\int_0^1\Big\{\Big(\frac{\partial}{\partial t}\Big(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big)\Big)\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\mathrm{d}x=-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1\Big(\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big)^2\mathrm{d}x.$$

利用假设 4 中的初值定位条件, 对上式两端从 0 到 T 积分, 有

$$\int_0^T\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial^3(\delta Q_k(x,t))}{\partial x^3}\Big\}\mathrm{d}x\mathrm{d}t=-\frac{1}{2}\int_0^1\Big(\frac{\partial^2(\delta Q_k(x,T))}{\partial x^2}\Big)^2\mathrm{d}x.\qquad(20)$$

对式 (18) 右端第 3 项积分, 有

$$\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\mathrm{d}x=\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x.$$

利用假设 4 中的初值定位条件, 对上式两端从 0 到 T 积分, 可得

$$\int_0^T\int_0^1\Big\{\frac{\partial^2(\delta Q_k(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\mathrm{d}x\mathrm{d}t=\frac{1}{2}\int_0^1\Big(\frac{\partial(\delta Q_k(x,T))}{\partial x}\Big)^2\mathrm{d}x.\qquad(21)$$

对式 (18) 两端变量 x 从 0 到 1 积分, 变量 t 从 0 到 T 积分, 结合式 (20) 和 (21), 有

$$\int_0^T\int_0^1\Big(\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\Big)^2\mathrm{d}x\mathrm{d}t-(2-CBp)\int_0^T\int_0^1\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\frac{\partial^2(e_k(x,t))}{\partial t\partial x}\mathrm{d}x\mathrm{d}t-(CBp-1)\int_0^T\int_0^1\Big(\frac{\partial^2(e_k(x,t))}{\partial t\partial x}\Big)^2\mathrm{d}x\mathrm{d}t+\frac{C^2a^2}{2}\int_0^1\Big(\frac{\partial^2(\delta Q_k(x,T))}{\partial x^2}\Big)^2\mathrm{d}x-\frac{C^2A}{2}\int_0^1\Big(\frac{\partial(\delta Q_k(x,T))}{\partial x}\Big)^2\mathrm{d}x=0.$$

注意到 $A<0$, 由上式可得

$$\int_0^T\int_0^1\Big(\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\Big)^2\mathrm{d}x\mathrm{d}t-(2-CBp)\int_0^T\int_0^1\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\frac{\partial^2(e_k(x,t))}{\partial t\partial x}\mathrm{d}x\mathrm{d}t-(CBp-1)\int_0^T\int_0^1\Big(\frac{\partial^2(e_k(x,t))}{\partial t\partial x}\Big)^2\mathrm{d}x\mathrm{d}t\leqslant0.$$

记 $g_k=\Big\|\dfrac{\partial^2(e_k(x,t))}{\partial t\partial x}\Big\|_{L^2(\Omega)}$, 结合上式和前面的过程, 可得 $0\leqslant\cdots\leqslant g_{k+1}\leqslant g_k\leqslant\cdots\leqslant g_0$, 进而有结论 4.

结论 4 序列 $\left\{\dfrac{\partial^2(e_k(x,t))}{\partial t\partial x}\right\}$、$\left\{\dfrac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\right\}$ 有界.

综合结论 1 ~ 结论 4, 并应用性质 1, 针对 6 个序列 $\left\{\frac{\partial(e_k(x,t))}{\partial t}\right\}$、$\left\{\frac{\partial(e_{k+1}(x,t))}{\partial t}\right\}$、$\left\{\frac{\partial^2(e_k(x,t))}{\partial t\partial x}\right\}$、$\left\{\frac{\partial^2(e_{k+1}(x,t))}{\partial t\partial x}\right\}$、$\left\{\delta Q_k(x,t)\right\}$、$\left\{\frac{\partial(\delta Q_k(x,t))}{\partial x}\right\}$, 可相应找到相同序号的 6 个子列 $\left\{\frac{\partial(e_{k_j}(x,t))}{\partial t}\right\}$、$\left\{\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\right\}$、$\left\{\frac{\partial^2(e_{k_j}(x,t))}{\partial t\partial x}\right\}$、$\left\{\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\right\}$、$\left\{\delta Q_{k_j}(x,t)\right\}$、$\left\{\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\right\}$, 使 6 个子列在 $L^2(\Omega)$ 中弱收敛, 且 $\{\delta Q_{k_j}(x,t)\}$、$\left\{\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\right\}$ 弱收敛于 Ω 上的零函数. □

引理 5　若假设 1 ~ 假设 4 和式 (5) 成立, 则当 D 型学习律 (3) 作用于系统 (1) 时, 有

$$\lim_{k\to\infty}\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}=\lim_{k\to\infty}\int_0^T\int_0^1\left(\frac{\partial(e_k(x,t))}{\partial t}\right)^2\mathrm{d}x\mathrm{d}t=0.$$

证明　由式 (11) 可知序列 $\{c_k\}$ 的极限存在, 记

$$\lim_{k\to\infty}c_k^2=\lim_{k\to\infty}\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}=K\geqslant 0. \tag{22}$$

设引理 4 中的子列 $\left\{\frac{\partial(e_{k_j}(x,t))}{\partial t}\right\}$、$\left\{\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\right\}$、$\left\{\frac{\partial^2(e_{k_j}(x,t))}{\partial t\partial x}\right\}$、$\left\{\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\right\}$ 分别弱收敛于 $L^2(\Omega)$ 中的函数 $\varphi(x,t),\psi(x,t),\sigma(x,t),\tau(x,t)$.

由引理 2 有

$$\lim_{j\to\infty}\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\frac{\partial(e_{k_j}(x,t))}{\partial t}\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1\psi(x,t)\varphi(x,t)\mathrm{d}x\mathrm{d}t. \tag{23}$$

利用引理 2 和式 (22), 可得

$$K=\lim_{j\to\infty}\left\|\frac{\partial(e_{k_j}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}=\lim_{j\to\infty}\int_0^T\int_0^1\left(\frac{\partial(e_{k_j}(x,t))}{\partial t}\right)^2\mathrm{d}x\mathrm{d}t=\|\varphi(x,t)\|^2_{L^2(\Omega)}, \tag{24}$$

$$K=\lim_{j\to\infty}\left\|\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}=\lim_{j\to\infty}\int_0^T\int_0^1\left(\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\right)^2\mathrm{d}x\mathrm{d}t=\|\psi(x,t)\|^2_{L^2(\Omega)}. \tag{25}$$

注意到, 引理 4 中 $\{\delta Q_{k_j}(x,t)\}$、$\left\{\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\right\}$ 弱收敛于零函数的结论, 应用引理 2, 有

$$\lim_{j\to\infty}\int_0^T\int_0^1\delta Q_{k_j}(x,t)\frac{\partial(e_{k_j}(x,t))}{\partial t}\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1 0\varphi(x,t)\mathrm{d}x\mathrm{d}t=0,$$

$$\lim_{j\to\infty}\int_0^T\int_0^1\delta Q_{k_j}(x,t)\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1 0\psi(x,t)\mathrm{d}x\mathrm{d}t=0,$$

$$\lim_{j\to\infty}\int_0^T\int_0^1\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\frac{\partial^2(e_{k_j}(x,t))}{\partial t\partial x}\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1 0\sigma(x,t)\mathrm{d}x\mathrm{d}t=0,$$

$$\lim_{j\to\infty}\int_0^T\int_0^1\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\mathrm{d}x\mathrm{d}t=\int_0^T\int_0^1 0\tau(x,t)\mathrm{d}x\mathrm{d}t=0. \tag{26}$$

利用序号 k_j 替换式 (4) 中的 k, 即得到

$$\frac{\partial(e_{k_j+1}(x,t))}{\partial t}=(1-CBp)\frac{\partial(e_{k_j}(x,t))}{\partial t}-Ca^2\frac{\partial^2(\delta Q_{k_j}(x,t))}{\partial x^2}-CA\delta Q_{k_j}(x,t). \tag{27}$$

对式 (27) 两端乘以 $\frac{\partial(e_{k_j+1}(x,t))}{\partial t}$, 并在 Ω 上积分, 可得

$$\int_0^T\int_0^1\left(\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\right)^2\mathrm{d}x\mathrm{d}t=(1-CBp)\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\frac{\partial(e_{k_j}(x,t))}{\partial t}\mathrm{d}x\mathrm{d}t-Ca^2\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\frac{\partial^2(\delta Q_{k_j}(x,t))}{\partial x^2}\mathrm{d}x\mathrm{d}t-CA\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\delta Q_{k_j}(x,t)\mathrm{d}x\mathrm{d}t. \tag{28}$$

应用分部积分公式, 并利用假设 4 中的边值定位条件, 有

$$\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\frac{\partial^2(\delta Q_{k_j}(x,t))}{\partial x^2}\mathrm{d}x\mathrm{d}t=\int_0^T\left\{\left\{\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\right\}\Big|_0^1-\int_0^1\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\mathrm{d}x\right\}\mathrm{d}t=-\int_0^T\int_0^1\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\mathrm{d}x\mathrm{d}t.$$

将上式代入式 (28), 得到

$$\int_0^T\int_0^1\left(\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\right)^2\mathrm{d}x\mathrm{d}t=(1-CBp)\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\frac{\partial(e_{k_j}(x,t))}{\partial t}\mathrm{d}x\mathrm{d}t+Ca^2\int_0^T\int_0^1\frac{\partial^2(e_{k_j+1}(x,t))}{\partial t\partial x}\frac{\partial(\delta Q_{k_j}(x,t))}{\partial x}\mathrm{d}x\mathrm{d}t-CA\int_0^T\int_0^1\frac{\partial(e_{k_j+1}(x,t))}{\partial t}\delta Q_{k_j}(x,t)\mathrm{d}x\mathrm{d}t.$$

令 $j\to\infty$, 由式 (23)、(25) 和 (26), 可得

$$K=(1-CBp)\int_0^T\int_0^1\psi(x,t)\varphi(x,t)\mathrm{d}x\mathrm{d}t. \tag{29}$$

对式 (27) 两端乘以 $\partial(e_{k_j}(x,t))/\partial t$, 并在 Ω 上积分, 借助于式 (23)、(24) 和 (26), 同样可得

$$\int_0^T\int_0^1\varphi(x,t)\psi(x,t)\mathrm{d}x\mathrm{d}t=(1-CBp)K. \tag{30}$$

结合式 (29) 和 (30), 有

$$K=(1-CBp)^2K.$$

由式 (5) 可知, $0<(1-CBp)^2<1$, 由此得到 $K=0$. □

定理1 若假设1～假设4和式(5)成立, 则系统(1)在D型学习律(3)作用下是收敛的, 即

$$\lim_{k\to\infty}\|e_k(x,t)\|_{L^2[0,1],s}=0.$$

证明 注意到$e_k(x,t)-e_{k+1}(x,t)=C\delta Q_k(x,t)$, 式(4)两端乘以$e_k(x,t)-e_{k+1}(x,t)$, 得到

$$\begin{aligned}&(e_k(x,t)-e_{k+1}(x,t))\frac{\partial(e_{k+1}(x,t))}{\partial t}=\\&(1-CBp)(e_k(x,t)-e_{k+1}(x,t))\frac{\partial(e_k(x,t))}{\partial t}-\\&C^2a^2\delta Q_k(x,t)\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}-C^2A(\delta Q_k(x,t))^2.\end{aligned}\tag{31}$$

对式(31)右端第2项应用分部积分公式, 并利用假设4中的边值定位条件, 有

$$\begin{aligned}&\int_0^1\Big\{\delta Q_k(x,t)\frac{\partial^2(\delta Q_k(x,t))}{\partial x^2}\Big\}\mathrm{d}x=\\&\Big\{\delta Q_k(x,t)\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big\}\Big|_0^1-\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x=\\&-\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x.\end{aligned}$$

对式(31)两端变量x从0到1积分, 结合上式, 有

$$\begin{aligned}&\int_0^1 e_k(x,t)\frac{\partial(e_{k+1}(x,t))}{\partial t}\mathrm{d}x-\\&\int_0^1 e_{k+1}(x,t)\frac{\partial(e_{k+1}(x,t))}{\partial t}\mathrm{d}x=\\&(1-CBp)\Big\{\int_0^1 e_k(x,t)\frac{\partial(e_k(x,t))}{\partial t}\mathrm{d}x-\\&-\int_0^1 e_{k+1}(x,t)\frac{\partial(e_k(x,t))}{\partial t}\mathrm{d}x\Big\}+\\&C^2a^2\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x-\\&C^2A\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x.\end{aligned}$$

由此得

$$\begin{aligned}&\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(e_{k+1}(x,t))^2\mathrm{d}x+\\&C^2a^2\int_0^1\Big(\frac{\partial(\delta Q_k(x,t))}{\partial x}\Big)^2\mathrm{d}x-\\&C^2A\int_0^1(\delta Q_k(x,t))^2\mathrm{d}x=\\&\frac{CBp-1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(e_k(x,t))^2\mathrm{d}x+\\&(1-CBp)\int_0^1 e_{k+1}(x,t)\frac{\partial(e_k(x,t))}{\partial t}\mathrm{d}x+\\&\int_0^1 e_k(x,t)\frac{\partial(e_{k+1}(x,t))}{\partial t}\mathrm{d}x.\end{aligned}$$

注意到$A<0$, 由上式可推得

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(e_{k+1}(x,t))^2\mathrm{d}x\leqslant\\&(CBp-1)\frac{\mathrm{d}}{\mathrm{d}t}\int_0^1(e_k(x,t))^2\mathrm{d}x+\\&2(1-CBp)\int_0^1 e_{k+1}(x,t)\frac{\partial(e_k(x,t))}{\partial t}\mathrm{d}x+\\&2\int_0^1 e_k(x,t)\frac{\partial(e_{k+1}(x,t))}{\partial t}\mathrm{d}x.\end{aligned}$$

对上式两端变量t从0到t积分, 并利用假设4中的初值定位条件, 得

$$\begin{aligned}&\int_0^1(e_{k+1}(x,t))^2\mathrm{d}x\leqslant\\&(CBp-1)\int_0^1(e_k(x,t))^2\mathrm{d}x+\\&2(1-CBp)\int_0^t\int_0^1 e_{k+1}(x,\xi)\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi+\\&2\int_0^t\int_0^1 e_k(x,\xi)\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi.\end{aligned}\tag{32}$$

由基本不等式, 有

$$\begin{aligned}&2\Big|\int_0^t\int_0^1 e_{k+1}(x,\xi)\frac{\partial(e_k(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi\Big|\leqslant\\&\int_0^t\int_0^1(e_{k+1}(x,\xi))^2\mathrm{d}x\mathrm{d}\xi+\\&\int_0^t\int_0^1\Big(\frac{\partial(e_k(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\leqslant\\&\int_0^t\int_0^1(e_{k+1}(x,\xi))^2\mathrm{d}x\mathrm{d}\xi+\Big\|\frac{\partial(e_k(x,t))}{\partial t}\Big\|^2_{L^2(\Omega)},\\&2\Big|\int_0^t\int_0^1 e_k(x,\xi)\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\mathrm{d}x\mathrm{d}\xi\Big|\leqslant\\&\int_0^t\int_0^1(e_k(x,\xi))^2\mathrm{d}x\mathrm{d}\xi+\\&\int_0^t\int_0^1\Big(\frac{\partial(e_{k+1}(x,\xi))}{\partial\xi}\Big)^2\mathrm{d}x\mathrm{d}\xi\leqslant\\&\int_0^t\int_0^1(e_k(x,\xi))^2\mathrm{d}x\mathrm{d}\xi+\Big\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big\|^2_{L^2(\Omega)}.\end{aligned}$$

将上两式代入式(32), 并注意到$CBp-1>0$, 有

$$\begin{aligned}&\int_0^1(e_{k+1}(x,t))^2\mathrm{d}x\leqslant\\&(CBp-1)\int_0^1(e_k(x,t))^2\mathrm{d}x+\\&(CBp-1)\int_0^t\int_0^1(e_{k+1}(x,\xi))^2\mathrm{d}x\mathrm{d}\xi+\\&(CBp-1)\Big\|\frac{\partial(e_k(x,t))}{\partial t}\Big\|^2_{L^2(\Omega)}+\\&\int_0^t\int_0^1(e_k(x,\xi))^2\mathrm{d}x\mathrm{d}\xi+\Big\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\Big\|^2_{L^2(\Omega)}.\end{aligned}\tag{33}$$

而

$$\begin{aligned}&\int_0^t\int_0^1(e_{k+1}(x,\xi))^2\mathrm{d}x\mathrm{d}\xi=\\&\int_0^t\{\mathrm{e}^{\lambda\xi}\mathrm{e}^{-\lambda\xi}\int_0^1(e_{k+1}(x,\xi))^2\mathrm{d}x\}\mathrm{d}\xi\leqslant\\&\int_0^t\mathrm{e}^{\lambda\xi}\mathrm{d}\xi\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}=\\&\frac{\mathrm{e}^{\lambda t}-1}{\lambda}\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda},\end{aligned}\tag{34}$$

同样有

$$\begin{aligned}&\int_0^t\int_0^1(e_k(x,\xi))^2\mathrm{d}x\mathrm{d}\xi\leqslant\\&\frac{\mathrm{e}^{\lambda t}-1}{\lambda}\|e_k(x,t)\|_{L^2[0,1],\lambda}.\end{aligned}\tag{35}$$

将式(34)和(35)代入(33), 得到

$$\begin{aligned}&\int_0^1(e_{k+1}(x,t))^2\mathrm{d}x\leqslant\\&(CBp-1)\int_0^1(e_k(x,t))^2\mathrm{d}x+\\&(CBp-1)\frac{\mathrm{e}^{\lambda t}-1}{\lambda}\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}+\end{aligned}$$

$$(CBp-1)\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}+$$
$$\frac{e^{\lambda t}-1}{\lambda}\|e_k(x,t)\|_{L^2[0,1],\lambda}+\left\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}.$$

进而有

$$\begin{aligned}&\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}=\\&\max_{t\in[0,T]}\left\{e^{-\lambda t}\int_0^1(e_{k+1}(x,t))^2dx\right\}\leqslant\\&(CBp-1)\max_{t\in[0,T]}\left\{e^{-\lambda t}\int_0^1(e_k(x,t))^2dx\right\}+\\&(CBp-1)\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}\max_{t\in[0,T]}\left\{\frac{1-e^{-\lambda t}}{\lambda}\right\}+\\&\|e_k(x,t)\|_{L^2[0,1],\lambda}\max_{t\in[0,T]}\left\{\frac{1-e^{-\lambda t}}{\lambda}\right\}+\\&\left((CBp-1)\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}\right)\max_{t\in[0,T]}\{e^{-\lambda t}\}+\\&\left(\left\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}\right)\max_{t\in[0,T]}\{e^{-\lambda t}\}=\\&(CBp-1)\|e_k(x,t)\|_{L^2[0,1],\lambda}+\\&(CBp-1)\frac{1-e^{-\lambda T}}{\lambda}\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}+\\&\frac{1-e^{-\lambda T}}{\lambda}\|e_k(x,t)\|_{L^2[0,1],\lambda}+\\&(CBp-1)\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}+\left\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}.\end{aligned}$$

取 $\lambda>1$, 由 $(CBp-1)\frac{1-e^{-\lambda T}}{\lambda}<1$, 上式可化为

$$\begin{aligned}&\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}\leqslant\\&\frac{CBp-1+\frac{1-e^{-\lambda T}}{\lambda}}{1-(CBp-1)\frac{1-e^{-\lambda T}}{\lambda}}\|e_k(x,t)\|_{L^2[0,1],\lambda}+\\&\frac{1}{1-(CBp-1)\frac{1-e^{-\lambda T}}{\lambda}}\times\\&\left\{(CBp-1)\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}+\right.\\&\left.\left\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}\right\}.\end{aligned}\tag{36}$$

记

$$\frac{CBp-1+\frac{1-e^{-\lambda T}}{\lambda}}{1-(CBp-1)\frac{1-e^{-\lambda T}}{\lambda}}=\rho,$$
$$\begin{aligned}&\frac{1}{1-(CBp-1)\frac{1-e^{-\lambda T}}{\lambda}}\times\\&\left\{(CBp-1)\left\|\frac{\partial(e_k(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}+\right.\\&\left.\left\|\frac{\partial(e_{k+1}(x,t))}{\partial t}\right\|^2_{L^2(\Omega)}\right\}=b_k,\end{aligned}$$

式 (36) 可记为

$$\|e_{k+1}(x,t)\|_{L^2[0,1],\lambda}\leqslant\rho\|e_k(x,t)\|_{L^2[0,1],\lambda}+b_k,\tag{37}$$

由式 (5) $0<CBp-1<1$, 取足够大的 λ, 能够使得 $0<\rho<1$. 由引理 5 可知, $\lim_{k\to\infty}b_k=0$, 对式 (37) 应用引理 3, 可得 $\lim_{k\to\infty}\|e_k(x,t)\|_{L^2[0,1],\lambda}=0$, 由此得到 $\lim_{k\to\infty}\|e_k(x,t)\|_{L^2[0,1],s}=0$. □

注 2　由证明过程可知, 若将假设 2 中的 $CB>0$ 改为 $CB<0$, 或将 $A<0$ 改为 $A=0$, 则结论同样成立, 若系统 (1) 中的 $A>0$, 则无法推得结论.

注 3　由式 (5) 可知, 本文所得结论是易验证的.

3　仿真算例

对系统 (1) 取 $a^2=1$, $A=-1$, $B=1$, $C=1$, 有

$$\begin{cases}\frac{\partial Q(x,t)}{\partial t}=\frac{\partial^2Q(x,t)}{\partial x^2}-Q(x,t)+u(x,t),\\y(x,t)=Q(x,t).\end{cases}$$

考虑区域 $(x,t)\in(0,1)\times[0,1]$, 构建理想输出 $y_r(x,t)=\sin(x+t)$, 则有 $Q_r(x,t)=\sin(x+t)$, $u_r(x,t)=2\sin(x+t)+\cos(x+t)$. 迭代第 k 次时, 有

$$\begin{cases}\frac{\partial Q_k(x,t)}{\partial t}=\frac{\partial^2Q_k(x,t)}{\partial x^2}-Q_k(x,t)+u_k(x,t),\\y_k(x,t)=Q_k(x,t).\end{cases}$$

结合理想状态 $Q_r(x,t)=\sin(x+t)$, 取初、边值 (第 1 边值) $Q_k(x,0)=\sin(x)$, $Q_k(0,t)=\sin(t)$, $Q_k(1,t)=\sin(1+t)$. 取初始控制 $u_0(x,t)=1$, 构建迭代学习控制 $u_{k+1}(x,t)=u_k(x,t)+p\frac{\partial(e_k(x,t))}{\partial t}$, $k=0,1,\cdots$.

由式 (5) 可知, 选取学习增益 $1<p<2$, 则能保证迭代收敛, 由此取 $p=1.5$. 利用 Mathematica 软件进行仿真分析可得, 当 $k\to\infty$ 时, 有 $\|e_k(x,t)\|_{L^2,s}\to 0$, 结果见图 1.

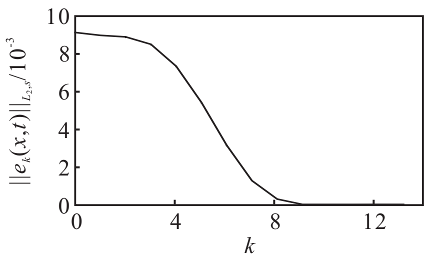

图 1　跟踪误差随迭代次数的变化曲线

4　结　论

本文针对一类非正则分布参数系统, 进行了迭代学习控制设计. 该系统由时不变线性抛物型偏微分方程构建而成, 并具有适定的初值、边值 (第 1 边值或第 2 边值) 定解条件. 本文采用适用于非正则系统的 D 型学习律进行迭代学习控制设计, 并借助于泛函分析知识, 证明系统的输出跟踪误差于 L^2 空间内沿迭代轴方向收敛, 得到了较好的结果. 如何将本文的结论作到系统中 $A>0$ 的情形, 并进一步推广到时变的、非线性的等非正则分布参数系统上, 尚有待作进一步的探究.

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