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交互式凸组合法的混合时滞不确定中立系统的稳定性

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摘 要: 针对带分布时滞和离散时滞的不确定中立型系统进行稳定性研究. 基于交互式凸组合方法和下界引理, 通过构造恰当的李雅普诺夫泛函, 适当分割时滞区间, 处理一组由凸参数逆加权的正函数线性组合 (交互式凸组合), 给出线性矩阵不等式形式的系统鲁棒稳定性判据. 该方法允许离散时滞为变时滞, 增强了系统的鲁棒性能. 数值算例验证了所得结果的有效性和合理性.

关键词: 交互式凸组合；下界引理；时滞分割；线性矩阵不等式

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Stability of uncertain neutral system with mixed time delays based on reciprocally convex combination approach

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Abstract: The stability problem for the uncertain neutral system with distributed and discrete delays is researched. By delay-partitioning and constructing appropriate Lyapunov functional based on reciprocally convex combination and lower bound lemma, which is a way of handling a linear combination of positive functions weighted by the inverse of convex parameters(reciprocally convex combination), a robust stability criterion for the system is obtained in terms of the linear matrix inequalities(LMIs). The proposed approach allows the existence of time-variant delays, which can improve the robust performance of the neutral system. A numerical example is presented to illustrate the effectiveness and rationality of the results.

Keywords: reciprocally convex combination；lower bound lemma；delay-partitioning；linear matrix inequality

0 引 言

时变时滞和不确定现象广泛存在于通讯系统、热交换器、人口生态学和核反应堆等领域, 其存在是导致动力系统不稳定和性能下降的主要原因. 因此, 国内外学者对不确定时变时滞系统的稳定性分析做了大量研究[1-9]. 含分布时滞的中立型系统是一类特殊的时滞系统, 同时含有分布时滞、中立时滞和离散时滞, 表现形式较复杂, 处理起来也比一般的时滞系统更加困难. 在实际工程中, 液体火箭发动机燃烧室燃烧过程即可看作该系统的一个特例[2]. 当前, 时变时滞中立型系统的稳定性研究一般首先在时域上构造合适的李雅普诺夫函数, 并通过时滞分割技术、积分不等式法、自由权矩阵法等方法推导出系统稳定的判据. 惠俊军等[2-3]基于离散化思想, 将离散时滞与分布时滞区间非均匀分割成若干份, 并在对应区间构造适当的李雅普诺夫函数, 推导出带分布时滞的不确定中立型系统的稳定性判据. 李涛等[4-5]基于时滞分割技术和 Jensen 不等式法, 推导出带混合时滞不确定中立型系统稳定的充分条件. 由于以上文献均将时滞区间分成若干份, 虽然能得到保守性较小的结果, 但增加了计算的复杂性. Cheng 等[6]基于时滞分割技术和自由权矩阵法, 将带有扰动的变时滞中立型系统的时滞区间均分成两个子区间, 得到一个与时滞相关的鲁棒稳定判据, 但自由权矩阵法的应用增大了所得结果的保守性. Lu 等[7-8]考虑了变时滞不确定中立型系统的中立时滞和离散时滞的上下界, 在构造李雅普诺夫函数时, 利用时滞上下界与零点的距离中点, 将时滞区间划分成 4 个子区间, 得出一个稳定性判据, 但没

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考虑到分布时滞. Chen 等[9]在研究带分布时滞的不确定中立型系统的稳定性问题时, 没有利用任何边界技术或者模式转换, 得到了保守性较小的结果, 但不能处理含有交互式凸组合项的函数. Park 等[10]介绍了交互式凸组合方法, 并将其运用于变时滞线性系统的稳定性分析, 该方法在 Jensen 不等式引理的基础上再进行一次缩放, 与只运用积分不等式引理相比, 在保证了保守性的同时, 具有更少的决策变量.

综上所述, 目前学者们一般致力于李雅普诺夫函数的创新构造而非创新方法的提出, 这只能解决特定函数的系统稳定性问题, 而利用新的交互式凸组合方法则可以解决几类含有交互式凸组合项的系统稳定性问题. 鉴于此, 本文基于交互式凸组合方法和下界引理, 讨论了带混合时滞不确定中立型系统的鲁棒稳定性, 通过合理构造含三重积分的李雅普诺夫函数, 当函数沿系统对 t 求导时, 适当分割时滞区间, 推导出带混合时滞不确定中立型系统渐近稳定的与时滞相关的充分条件. 最后, 通过数值仿真验证了所提出方法的合理性和有效性.

1 问题描述

考虑带有混合时滞的不确定中立型系统

$$\dot{x}(t)-C\dot{x}(t-\tau)=$$
$$A_0(t)x(t)+B_0(t)x(t-h(t))+D_0(t)\int_{t-r}^{t}x(s)\mathrm{d}s,\quad(1)$$

满足初始条件

$$x(t)=\phi(t),\ t\in[-\lambda,0],\ \lambda=\max\{\tau,r,h\}.$$

其中: $x(t)\in R^n$ 为系统的状态向量; $\phi(t)$为一个光滑的初始向量函数; 定义域为Banach 空间 $C[-\lambda,0]$ 内的光滑函数集 $\psi:[-\lambda,0]\mapsto R^n$; 变时滞 $h(t)$ 满足

$$0\leqslant h(t)\leqslant h,\ \dot{h}(t)\leqslant\bar{h}<1,\ \bar{h}\geqslant 0;\quad(2)$$

$A_0(t)$、$B_0(t)$、$D_0(t)$ 均为时变不确定性矩阵函数, 即

$$A_0(t)=A+\Delta A(t),\ B_0(t)=B+\Delta B(t),$$
$$D_0(t)=D+\Delta D(t).\quad(3)$$

A、B、C 和 D 为具有适当维数的已知常数矩阵, $\Delta A(t)$、$\Delta B(t)$、$\Delta D(t)$ 为系统内的时变不确定性的矩阵函数, 并假定满足

$$[\Delta A(t),\Delta B(t),\Delta D(t)]=EF(t)[N_A,N_B,N_D],\quad(4)$$

E、N_A、N_B、N_D 为适当维数的常值矩阵, 且满足

$$F^{\mathrm{T}}(t)F(t)\leqslant I.\quad(5)$$

定义算子

$$Dx_t=x(t)-Cx(t-\tau),\quad(6)$$

其中 $Dx_t:C([-\tau,0],R^n)\to R^n$.

定义 1 (交互式凸组合)[10] 令有限个函数 $\psi_1,\psi_2,\cdots,\psi_N:R^m\mapsto R^n$ 在 R^m 的开区间 D 上是正函数, 则 ψ_i 在 D 上的交互式凸组合具有如下形式:

$$\frac{1}{\alpha_1}\psi_1+\frac{1}{\alpha_2}\psi_2+\cdots+\frac{1}{\alpha_N}\psi_N:D\mapsto R^n,\quad(7)$$

其中实数 α_i 满足 $\alpha_i>0$, 且 $\sum\limits_{i=1}^{N}\alpha_i=1$.

引理 1[11] 对于任意常数矩阵 $Z=Z^{\mathrm{T}}>0$ 和常数 $\tau_2>\tau_1>0,\tau_{12}=\tau_2-\tau_1,\tau_s=(\tau_2^2-\tau_1^2)/2$, 有

$$-\tau_{12}\int_{t-\tau_2}^{t-\tau_1}x^{\mathrm{T}}(s)Zx(s)\mathrm{d}s\leqslant$$
$$-\Big(\int_{t-\tau_2}^{t-\tau_1}x(s)\mathrm{d}s\Big)^{\mathrm{T}}Z\Big(\int_{t-\tau_2}^{t-\tau_1}x(s)\mathrm{d}s\Big),\quad(8)$$
$$-\tau_s\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}x^{\mathrm{T}}(s)Zx(s)\mathrm{d}s\mathrm{d}\theta\leqslant$$
$$-\Big(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}x(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}Z\Big(\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}x(s)\mathrm{d}s\mathrm{d}\theta\Big).\quad(9)$$

引理 2 (下界引理)[10] 令函数 $f_1,f_2,\cdots,f_N:R^n\to R$, 且在 R^n 的开子集 D 上恒大于零, 那么 f_i 在 D 上的交互式凸组合有以下性质:

$$\sum_i\frac{1}{\alpha_i}f_i(t)\geqslant\sum_i f_i(t)+\sum_{i\neq j}g_{ij}(t).\quad(10)$$

其中

$$\alpha_i>0,\ \sum_i\alpha_i=1,\ g_{ij}(t):R^n\to R,$$
$$g_{ij}(t)=g_{ji}(t),\ \begin{bmatrix}f_i(t)&g_{ij}(t)\\g_{ji}(t)&f_j(t)\end{bmatrix}\geqslant 0.$$

引理 3[12] 对于任意矩阵 M、B、$F(t)$, 若 $F^{\mathrm{T}}(t)F(t)\leqslant I$, 则对于任意标量 $\varepsilon>0$, 有

$$2M^{\mathrm{T}}F(t)N\leqslant\varepsilon M^{\mathrm{T}}M+\varepsilon^{-1}NN^{\mathrm{T}}.\quad(11)$$

2 主要结果

定理 1 给定常数 τ、r、h、$\bar{h}$, 满足不等式 (2), 带混合时滞不确定中立型系统 (1) 是渐近稳定的, 如果存在正定矩阵 $P=P^{\mathrm{T}}$、$Q_i(i=1,2,\cdots,6)$、S、R、T、M_1、M_2, 任意矩阵 $\hat{S}$, 使得以下不等式成立:

$$\varXi=\left[\begin{array}{ccccccccc}
\varXi_{11}&\varXi_{12}&0&\varXi_{14}&0&0&\varXi_{17}&\varXi_{18}&\varXi_{19}\\
*&\varXi_{22}&0&\varXi_{24}&0&0&0&0&0\\
&&\varXi_{33}&0&0&0&0&0&0\\
&&*&\varXi_{44}&0&0&\varXi_{47}&0&0\\
&&*&*&\varXi_{55}&0&0&0&0\\
&&*&*&*&\varXi_{66}&0&0&0\\
&&*&*&*&*&\varXi_{77}&0&0\\
&&*&*&*&*&*&\varXi_{88}&0\\
&&*&*&*&*&*&*&\varXi_{99}\\
&&*&*&*&*&*&*&*\\
&&*&*&*&*&*&*&*\\
&&*&*&*&*&*&*&*\\
&&*&*&*&*&*&*&*\\
&&*&*&*&*&*&*&*\\
&&*&*&*&*&*&*&*
\end{array}\right.\to$$

$$
\leftarrow \begin{array}{cccccc}
\Xi_{110} & \Xi_{111} & 0 & N_A^{\mathrm T} & 0 & 0 \\
0 & \Xi_{211} & 0 & 0 & N_B^{\mathrm T} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \Xi_{711} & 0 & 0 & 0 & N_D^{\mathrm T} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\Xi_{910} & 0 & 0 & 0 & 0 & 0 \\
\Xi_{1010} & 0 & 0 & 0 & 0 & 0 \\
* & \Xi_{1111} & \Xi_{1112} & 0 & 0 & 0 \\
* & * & \Xi_{1212} & 0 & 0 & 0 \\
* & * & * & -e_1 I & 0 & 0 \\
* & * & * & * & -e_2 I & 0 \\
* & * & * & * & * & -e_3 I
\end{array} \right] < 0, \tag{12}
$$

$$
\bar S = \begin{bmatrix} S & \hat S \\ * & S \end{bmatrix} \geqslant 0. \tag{13}
$$

其中

$$
\begin{aligned}
&\Xi_{11} = PA + A^{\mathrm T}P + Q_1 + Q_3 + Q_5 + h^2 S + \\
&\quad r^2 T - R - \tau^2 M_1 - h^2 M_2 + \\
&\quad (\varepsilon_1+\varepsilon_2+\varepsilon_3) PEE^{\mathrm T}P, \\
&\Xi_{12} = PB,\ \Xi_{14} = -C^{\mathrm T}PA + R,\ \Xi_{17} = PD, \\
&\Xi_{18} = \tau M_1,\ \Xi_{19} = \Xi_{110} = hM_2,\ \Xi_{111} = Q_2 A, \\
&\Xi_{22} = -(1-\bar h)Q_1,\ \Xi_{24} = -C^{\mathrm T}PB, \\
&\Xi_{211} = Q_2 B,\ \Xi_{33} = -Q_5 + Q_6, \\
&\Xi_{44} = -Q_6 - R + (\varepsilon_1+\varepsilon_2+\varepsilon_3) C^{\mathrm T}PEE^{\mathrm T}PC, \\
&\Xi_{47} = -C^{\mathrm T}PD,\ \Xi_{55} = -Q_3 + Q_4,\ \Xi_{66} = -Q_4, \\
&\Xi_{77} = -T,\ \Xi_{711} = Q_2 D,\ \Xi_{88} = -M_1, \\
&\Xi_{99} = \Xi_{1010} = -S - M_2,\ \Xi_{910} = -\hat S - M_2, \\
&\Xi_{1111} = -Q_2 + Q_2 EE^{\mathrm T}Q_2^{\mathrm T} + \tau^2 R + \frac{\tau^4}{4}M_1 + \\
&\quad \frac{h^4}{4}M_2 + (\varepsilon_4+\varepsilon_5+\varepsilon_6) Q_2 EE^{\mathrm T}Q_2^{\mathrm T}, \\
&\Xi_{1112} = Q_2 C,\ \Xi_{1212} = -Q_2.
\end{aligned}
$$

证明　构造李雅普诺夫泛函

$$
V(t) = \sum_{i=1}^{4} V_i(t). \tag{14}
$$

其中

$$
\begin{aligned}
&V_1(t) = (Dx_t)^{\mathrm T}P(Dx_t), \\
&V_2(t) = \int_{t-h(t)}^{t} x^{\mathrm T}(s)Q_1 x(s)\mathrm{d}s + \\
&\quad \int_{t-\tau}^{t} \dot x^{\mathrm T}(s)Q_2 \dot x(s)\mathrm{d}s + \\
&\quad \int_{t-\frac{h}{2}}^{t} \varsigma_1^{\mathrm T}(s) \begin{bmatrix} Q_3 & 0 \\ * & Q_4 \end{bmatrix} \varsigma_1(s)\mathrm{d}s + \\
&\quad \int_{t-\frac{\tau}{2}}^{t} \varsigma_2^{\mathrm T}(s) \begin{bmatrix} Q_5 & 0 \\ * & Q_6 \end{bmatrix} \varsigma_2(s)\mathrm{d}s, \\
&\varsigma_1(t) = \left[x^{\mathrm T}(t), x^{\mathrm T}\left(t-\frac{h}{2}\right)\right]^{\mathrm T}, \\
&\varsigma_2(t) = \left[x^{\mathrm T}(t), x^{\mathrm T}\left(t-\frac{\tau}{2}\right)\right]^{\mathrm T}, \\
&V_3(t) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot x^{\mathrm T}(s)R\dot x(s)\mathrm{d}s\mathrm{d}\theta + \\
&\quad h\int_{-h}^{0}\int_{t+\theta}^{t} x^{\mathrm T}(s)Sx(s)\mathrm{d}s\mathrm{d}\theta + \\
&\quad r\int_{-r}^{0}\int_{t+\theta}^{t} x^{\mathrm T}(s)Tx(s)\mathrm{d}s\mathrm{d}\theta, \\
&V_4(t) = \frac{\tau^2}{2}\int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot x^{\mathrm T}(s)M_1\dot x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta + \\
&\quad \frac{h^2}{2}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot x^{\mathrm T}(s)M_2\dot x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta.
\end{aligned}
$$

将 $V(t)$ 沿着系统 (1) 对时间 t 取导, 有

$$
\begin{aligned}
&\dot V_1(t) = 2(Dx_t)^{\mathrm T}P(\dot Dx_t) = \\
&2(Dx_t)^{\mathrm T}P\Big[A_0(t)x(t) + B_0(t)x(t-h(t)) + \\
&D_0(t)\int_{t-r}^{t} x(s)\mathrm{d}s\Big] = \\
&2(Dx_t)^{\mathrm T}PAx(t) + 2(Dx_t)^{\mathrm T}PBx(t-h(t)) + \\
&2(Dx_t)^{\mathrm T}PD\int_{t-r}^{t} x(s)\mathrm{d}s + 2(Dx_t)^{\mathrm T}P\Delta Ax(t) + \\
&2(Dx_t)^{\mathrm T}P\Delta Bx(t-h(t)) + \\
&2(Dx_t)^{\mathrm T}P\Delta D\int_{t-r}^{t} x(s)\mathrm{d}s.
\end{aligned} \tag{15}
$$

由引理 3 可得, 对于任意标量 ε_1、ε_2、ε_3、ε_4、ε_5、ε_6, 有

$$
\begin{aligned}
&2x^{\mathrm T}(t)P\Delta Ax(t) \leqslant \\
&\varepsilon_1^{-1}x^{\mathrm T}(t)N_A^{\mathrm T}N_A x(t) + \varepsilon_1 x^{\mathrm T}(t)PEE^{\mathrm T}Px(t), \\
&2x^{\mathrm T}(t)P\Delta Bx(t-h(t)) \leqslant \\
&\varepsilon_2^{-1}x^{\mathrm T}(t-h(t))N_B^{\mathrm T}N_B x(t-h(t)) + \\
&\varepsilon_2 x^{\mathrm T}(t)PEE^{\mathrm T}Px(t), \\
&2x^{\mathrm T}(t)P\Delta D\int_{t-r}^{t} x(s)\mathrm{d}s \leqslant \\
&\varepsilon_3 x^{\mathrm T}(t)PEE^{\mathrm T}Px(t) + \\
&\varepsilon_3^{-1}\Big(\int_{t-r}^{t} x(s)\mathrm{d}s\Big)^{\mathrm T} N_D^{\mathrm T}N_D \Big(\int_{t-r}^{t} x(s)\mathrm{d}s\Big), \\
&-2x^{\mathrm T}(t-\tau)C^{\mathrm T}P\Delta Ax(t) \leqslant \\
&\varepsilon_4^{-1}x^{\mathrm T}(t)N_A^{\mathrm T}N_A x(t) + \\
&\varepsilon_4 x^{\mathrm T}(t-\tau)C^{\mathrm T}PEE^{\mathrm T}PCx(t-\tau), \\
&-2x^{\mathrm T}(t-\tau)C^{\mathrm T}P\Delta Bx(t-h(t)) \leqslant \\
&\varepsilon_5^{-1}x^{\mathrm T}(t-h(t))N_B^{\mathrm T}N_B x(t-h(t)) + \\
&\varepsilon_5 x^{\mathrm T}(t-\tau)C^{\mathrm T}PEE^{\mathrm T}PCx(t-\tau), \\
&-2x^{\mathrm T}(t-\tau)C^{\mathrm T}P\Delta D\int_{t-r}^{t} x(s)\mathrm{d}s \leqslant
\end{aligned}
$$

$$\varepsilon_6^{-1}\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}N_D^{\mathrm{T}}N_D\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)+ \varepsilon_6 x^{\mathrm{T}}(t-\tau)C^{\mathrm{T}}PEE^{\mathrm{T}}PCx(t-\tau).$$

进而可得

$$\begin{aligned}&\dot V_1(t)\leqslant\\&x^{\mathrm{T}}(t)[PA+A^{\mathrm{T}}P+\varepsilon_1^{-1}N_A^{\mathrm{T}}N_A+(\varepsilon_1+\varepsilon_2+\\&\varepsilon_3)PEE^{\mathrm{T}}P]x(t)+2x^{\mathrm{T}}(t)PBx(t-h(t))+\\&2x^{\mathrm{T}}(t)PD\int_{t-r}^{t}x(s)\mathrm{d}s-2x^{\mathrm{T}}(t-\tau)C^{\mathrm{T}}PAx(t)+\\&(\varepsilon_1+\varepsilon_2+\varepsilon_3)x^{\mathrm{T}}(t-\tau)C^{\mathrm{T}}PEE^{\mathrm{T}}PCx(t-\tau)-\\&2x^{\mathrm{T}}(t-\tau)C^{\mathrm{T}}PD\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)-\\&2x^{\mathrm{T}}(t-\tau)C^{\mathrm{T}}PBx(t-h(t))+\\&\varepsilon_3^{-1}\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}N_D^{\mathrm{T}}N_D\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)+\\&\varepsilon_2^{-1}x^{\mathrm{T}}(t-h(t))N_B^{\mathrm{T}}N_Bx(t-h(t)),\end{aligned}\tag{16}$$

$$\begin{aligned}&\dot V_2(t)=\\&x^{\mathrm{T}}(t)Q_1x(t)-(1-\dot h(t))x^{\mathrm{T}}(t-h(t))Q_1x(t-h(t))+\\&\varsigma_1^{\mathrm{T}}(t)\begin{bmatrix}Q_3&0\\ *&Q_4\end{bmatrix}\varsigma_1(t)+\varsigma_2^{\mathrm{T}}(t)\begin{bmatrix}Q_5&0\\ *&Q_6\end{bmatrix}\varsigma_2(t)+\\&\dot x^{\mathrm{T}}(t)Q_2\dot x(t)-\dot x^{\mathrm{T}}(t-\tau)Q_2\dot x(t-\tau)-\\&\varsigma_1^{\mathrm{T}}\Big(t-\frac{h}{2}\Big)\begin{bmatrix}Q_3&0\\ *&Q_4\end{bmatrix}\varsigma_1\Big(t-\frac{h}{2}\Big)-\\&\varsigma_2^{\mathrm{T}}\Big(t-\frac{\tau}{2}\Big)\begin{bmatrix}Q_5&0\\ *&Q_6\end{bmatrix}\varsigma_2\Big(t-\frac{\tau}{2}\Big)\leqslant\\&x^{\mathrm{T}}(t)(Q_1+Q_3+Q_5+\varepsilon_4^{-1}N_A^{\mathrm{T}}N_A)x(t)-\\&x^{\mathrm{T}}(t-h(t))[(1-\bar h)Q_1-\varepsilon_5^{-1}N_B^{\mathrm{T}}N_B]x(t-h(t))+\\&x^{\mathrm{T}}\Big(t-\frac{h}{2}\Big)(Q_4-Q_3)x\Big(t-\frac{h}{2}\Big)+\\&2\dot x^{\mathrm{T}}(t)Q_2C\dot x(t-\tau)+\\&x^{\mathrm{T}}\Big(t-\frac{\tau}{2}\Big)(Q_6-Q_5)x\Big(t-\frac{\tau}{2}\Big)-\\&x^{\mathrm{T}}(t-h)Q_4x(t-h)-\\&\dot x^{\mathrm{T}}(t-\tau)Q_2\dot x(t-\tau)-\dot x^{\mathrm{T}}(t)[Q_2-(\varepsilon_4+\varepsilon_5+\\&\varepsilon_6)Q_2EE^{\mathrm{T}}Q_2^{\mathrm{T}}]\dot x(t)-x^{\mathrm{T}}(t-\tau)Q_6x(t-\tau)+\\&2\dot x^{\mathrm{T}}(t)Q_2Ax(t)+2\dot x^{\mathrm{T}}(t)Q_2Bx(t-h(t))+\\&\varepsilon_6^{-1}\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}N_D^{\mathrm{T}}N_D\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)+\\&2\dot x^{\mathrm{T}}(t)Q_2D\int_{t-r}^{t}x(s)\mathrm{d}s,\end{aligned}\tag{17}$$

$$\begin{aligned}&\dot V_3(t)=\\&x^{\mathrm{T}}(t)[h^2S+r^2T]x(t)+\tau^2\dot x^{\mathrm{T}}(t)R\dot x(t)-\\&\tau\int_{t-\tau}^{t}\dot x^{\mathrm{T}}(s)R\dot x(s)\mathrm{d}s-r\int_{t-r}^{t}x^{\mathrm{T}}(s)Tx(s)\mathrm{d}s-\\&h\int_{t-h}^{t}x^{\mathrm{T}}(s)Sx(s)\mathrm{d}s.\end{aligned}\tag{18}$$

运用引理1, 得到

$$\begin{aligned}&-\tau\int_{t-\tau}^{t}\dot x^{\mathrm{T}}(s)R\dot x(s)\mathrm{d}s\leqslant\\&-\Big(\int_{t-\tau}^{t}\dot x(s)\mathrm{d}s\Big)^{\mathrm{T}}R\Big(\int_{t-\tau}^{t}\dot x(s)\mathrm{d}s\Big)\leqslant\\&-[x^{\mathrm{T}}(t)-x^{\mathrm{T}}(t-\tau)]R[x(t)-x(t-\tau)],\end{aligned}\tag{19}$$

$$\begin{aligned}&-r\int_{t-r}^{t}x^{\mathrm{T}}(s)Tx(s)\mathrm{d}s\leqslant\\&-\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}T\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big).\end{aligned}\tag{20}$$

运用引理1和引理2, 得到

$$\begin{aligned}&-h\int_{t-h}^{t}x^{\mathrm{T}}(s)Sx(s)\mathrm{d}s=\\&-h\int_{t-h}^{t-h(t)}x^{\mathrm{T}}(s)Sx(s)\mathrm{d}s-\\&h\int_{t-h(t)}^{t}x^{\mathrm{T}}(s)Sx(s)\mathrm{d}s\leqslant\\&-\frac{h}{h-h(t)}\Big(\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\Big)^{\mathrm{T}}S\Big(\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\Big)-\\&\frac{h}{h(t)}\Big(\int_{t-h(t)}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}S\Big(\int_{t-h(t)}^{t}x(s)\mathrm{d}s\Big)\leqslant\\&-\begin{bmatrix}\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\\ \int_{t-h(t)}^{t}x(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\bar S\begin{bmatrix}\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\\ \int_{t-h(t)}^{t}x(s)\mathrm{d}s\end{bmatrix}.\end{aligned}\tag{21}$$

利用下界引理处理交互式凸组合项可以得到式(21). 因为

$$-\begin{bmatrix}\sqrt{\dfrac{\beta}{\alpha}}\displaystyle\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\\ -\sqrt{\dfrac{\alpha}{\beta}}\displaystyle\int_{t-h(t)}^{t}x(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\times \bar S\begin{bmatrix}\sqrt{\dfrac{\beta}{\alpha}}\displaystyle\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\\ -\sqrt{\dfrac{\alpha}{\beta}}\displaystyle\int_{t-h(t)}^{t}x(s)\mathrm{d}s\end{bmatrix}\leqslant 0,\tag{22}$$

所以只要令 $\alpha=\dfrac{h-h(t)}{h}$, $\beta=\dfrac{h(t)}{h}$ 即可. 有

$$\begin{aligned}&\dot V_3(t)\leqslant\\&x^{\mathrm{T}}(t)(h^2S+r^2T-R)x(t)+\tau^2\dot x^{\mathrm{T}}(t)R\dot x(t)-\\&x^{\mathrm{T}}(t-\tau)Rx(t-\tau)+2x^{\mathrm{T}}(t)Rx(t-\tau)-\\&\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}T\Big(\int_{t-r}^{t}x(s)\mathrm{d}s\Big)-\\&\begin{bmatrix}\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\\ \int_{t-h(t)}^{t}x(s)\mathrm{d}s\end{bmatrix}^{\mathrm{T}}\bar S\begin{bmatrix}\int_{t-h}^{t-h(t)}x(s)\mathrm{d}s\\ \int_{t-h(t)}^{t}x(s)\mathrm{d}s\end{bmatrix},\end{aligned}\tag{23}$$

$$\begin{aligned}\dot V_4(t)=\;&\dot x^{\mathrm{T}}(t)\Big(\frac{\tau^4}{4}M_1+\frac{h^4}{4}M_2\Big)\dot x(t)-\\&\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm{T}}(s)M_1\dot x(s)\mathrm{d}s\mathrm{d}\theta-\\&\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot x^{\mathrm{T}}(s)M_2\dot x(s)\mathrm{d}s\mathrm{d}\theta.\end{aligned}\tag{24}$$

其中: $\bar S=\begin{bmatrix}S&\hat S\\ *&S\end{bmatrix}$, $\hat S$ 为任意适当维数矩阵.

注 1　由式 (21) 到 (22) 减少了相关决策变量的数量, 时滞分割成子区间的数量越多, 决策变量数量减少得越多.

同样运用引理 1, 有

$$-\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)M_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leqslant\\ -\Big(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}M_1\Big(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\Big)\leqslant\\ -\Big(\tau x(t)-\int_{t-\tau}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}M_1\Big(\tau x(t)-\int_{t-\tau}^{t}x(s)\mathrm{d}s\Big),\tag{25}$$

$$-\frac{h^2}{2}\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)M_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\leqslant\\ -\Big(\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}M_2\Big(\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\Big)\leqslant\\ -\Big(hx(t)-\int_{t-h}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}M_2\Big(hx(t)-\int_{t-h}^{t}x(s)\mathrm{d}s\Big),\tag{26}$$

进而得到

$$\dot{V}_4(t)\leqslant x^{\mathrm{T}}(t)(-\tau^2M_1-h^2M_2)x(t)+\\ 2hx^{\mathrm{T}}(t)M_2\Big(\int_{t-h}^{t}x(s)\mathrm{d}s\Big)+\\ 2\tau x^{\mathrm{T}}(t)M_1\int_{t-\tau}^{t}x(s)\mathrm{d}s+\\ \dot{x}^{\mathrm{T}}(t)\Big(\frac{\tau^4}{4}M_1+\frac{h^4}{4}M_2\Big)\dot{x}(t)-\\ \Big(\int_{t-h}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}M_2\Big(\int_{t-h}^{t}x(s)\mathrm{d}s\Big)-\\ \Big(\int_{t-\tau}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}M_1\Big(\int_{t-\tau}^{t}x(s)\mathrm{d}s\Big).\tag{27}$$

因此, 结合不等式 (16)、(17)、(23) 和 (27), 得到

$$\dot{V}(t)=\sum_{i=1}\dot{V}_i(t)\leqslant\chi^{\mathrm{T}}(t)\varGamma\chi(t).\tag{28}$$

其中

$$\chi(t)=\mathrm{col}\Big\{x(t),x(t-h),x\Big(t-\frac{\tau}{2}\Big),x(t-\tau),\\ x\Big(t-\frac{\tau}{2}\Big),x(t-h),\int_{t-r}^{t}x(s)\mathrm{d}s,\int_{t-\tau}^{t}x(s)\mathrm{d}s,\\ \int_{t-h}^{t-h(t)}x(s)\mathrm{d}s,\int_{t-h(t)}^{t}x(s)\mathrm{d}s,\dot{x}(t),\dot{x}(t-\tau)\Big\},$$

$$\varGamma=\begin{bmatrix}\varGamma_{11} & PB & 0 & R-C^{\mathrm{T}}PA & 0 & 0 & PD & \tau M_1 & hM_2 & hM_2 & Q_2A & 0\\ * & \varGamma_{22} & 0 & -C^{\mathrm{T}}PB & 0 & 0 & 0 & 0 & 0 & 0 & Q_2B & 0\\ * & * & \varXi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & \varXi_{44} & 0 & 0 & -C^{\mathrm{T}}PD & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \varXi_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & \varXi_{66} & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & \varGamma_{77} & 0 & 0 & 0 & Q_2D & 0\\ * & * & * & * & * & * & * & \varXi_{88} & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & \varXi_{99} & -\hat{S}-M_2 & 0 & 0\\ * & * & * & * & * & * & * & * & * & \varXi_{1010} & 0 & 0\\ * & * & * & * & * & * & * & * & * & * & \varXi_{1111} & Q_2C\\ * & * & * & * & * & * & * & * & * & * & * & \varXi_{1212}\end{bmatrix},\tag{29}$$

$$\varGamma_{11}=PA+A^{\mathrm{T}}P+Q_1+Q_3+Q_5+h^2S+r^2T-\\ R-\tau^2M_1-h^2M_2+(\varepsilon_1+\varepsilon_2+\varepsilon_3)PEE^{\mathrm{T}}P+\\ (\varepsilon_1^{-1}+\varepsilon_4^{-1})N_A^{\mathrm{T}}N_A,$$

$$\varGamma_{22}=-(1-\bar{h})Q_1+(\varepsilon_2^{-1}+\varepsilon_5^{-1})N_B^{\mathrm{T}}N_B,$$

$$\varGamma_{77}=-T+(\varepsilon_3^{-1}+\varepsilon_6^{-1})N_D^{\mathrm{T}}N_D,$$

$\varXi_{ii}(i=1,2,\cdots,12)$ 如前面定理所规定.

根据 Lyapunov 稳定性理论, 如果 $\varGamma<0$, 则系统 (1) 是渐近稳定的. 令 $e_1^{-1}=\varepsilon_1^{-1}+\varepsilon_4^{-1}$, $e_2^{-1}=\varepsilon_2^{-1}+\varepsilon_5^{-1}$, $e_3^{-1}=\varepsilon_3^{-1}+\varepsilon_6^{-1}$, 由 Schur 补引理可知, $\varGamma<0$ 等价于不等式 (13). □

3　数值仿真

例 1　考虑带有如下参数的时滞不确定中立型系统 (1), 有

$$A=\begin{bmatrix}-2 & 1\\ 1 & -2\end{bmatrix},\ B=\begin{bmatrix}-1 & 0.1\\ 0.1 & 1\end{bmatrix},$$

$$C=\begin{bmatrix}-0.2 & 0.1\\ 0.1 & -0.2\end{bmatrix},\ D=\begin{bmatrix}-1 & 0.1\\ 0.1 & -1\end{bmatrix},$$

$$E=N_A=N_B=N_D=\mathrm{diag}(\sqrt{0.2},\sqrt{0.2}),$$

$$\varepsilon_j=0.1,\ j=1,2,\cdots,6,$$

$$h=\tau=r=\bar{h}=0.9.$$

通过 Matlab LMI 工具箱求解不等式 (12) 和 (13), 得到

$$P=\begin{bmatrix}0.0998 & -0.0278\\ -0.0278 & 0.1014\end{bmatrix},\ M_1=\begin{bmatrix}1.4579 & -0.0125\\ -0.0125 & 1.4590\end{bmatrix},$$

$$M_2=\begin{bmatrix}0.6130 & -0.0036\\ -0.0036 & 0.6143\end{bmatrix},\ T=\begin{bmatrix}0.1969 & 0.0008\\ 0.0008 & 0.1964\end{bmatrix},$$

$$Q_1=\begin{bmatrix}1.5107 & -0.1926\\ -0.1926 & 1.5219\end{bmatrix},\ Q_2=\begin{bmatrix}0.0228 & 0.0012\\ 0.0012 & 0.0236\end{bmatrix},$$

$$Q_3=\begin{bmatrix}0.5077 & 0.0006\\ 0.0006 & 0.5073\end{bmatrix},\ Q_4=\begin{bmatrix}0.3164 & 0.0003\\ 0.0003 & 0.3161\end{bmatrix},$$

$$Q_5=\begin{bmatrix}0.4342 & 0.0032\\ 0.0032 & 0.4334\end{bmatrix},\ Q_6=\begin{bmatrix}0.2298 & 0.0035\\ 0.0035 & 0.2292\end{bmatrix},$$

$$R=\begin{bmatrix}2.1180 & 0.0183\\ 0.0183 & 2.1302\end{bmatrix},\ \hat{S}=\begin{bmatrix}-0.1168 & 0.0011\\ 0.0011 & -0.1187\end{bmatrix},$$

$$S=\begin{bmatrix}0.1969 & 0.0008\\ 0.0008 & 0.1964\end{bmatrix}.$$

矩阵 $P=P^{\mathrm{T}}$、$Q_i(i=1,2,\cdots,6)$、M_1、M_2、S、R、T 均为正定矩阵.

系统仿真如图1所示. 由图1可见, 从初始条件 $(1,-1)$ 出发, 系统的状态运动轨迹在 $t=5\,\mathrm{s}$ 时趋于稳定. 图2的状态运动轨迹在 $t=18\,\mathrm{s}$ 时才趋于稳定. 因此, 利用本文方法能使系统状态更快趋于稳定.

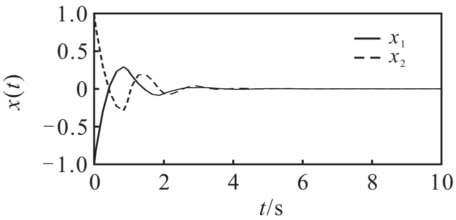

图 1 状态运动轨迹 (例 1)

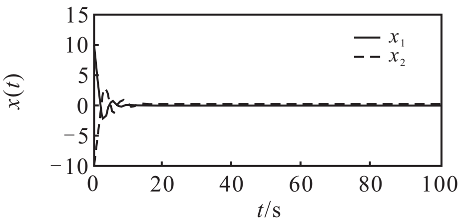

图 2 状态运动轨迹 (文献 [3])

4 结 论

本文讨论了带混合变时滞不确定中立型系统的鲁棒稳定性问题. 基于交互式凸组合方法和下界引理, 通过适当分割时滞区间, 合理构造包含三重积分的李雅普诺夫函数, 有效处理了交互式凸组合项, 而不是直接忽略该项, 推导出带混合时滞不确定中立型系统的稳定性判据. 该判据在保证保守性的同时, 减少了相关的决策变量数量, 避免了数值计算的复杂性. 数值算例验证了所得结果的有效性和合理性.

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