

## 非线性扩展结构大系统自适应神经网络跟踪控制

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**摘要:** 针对一类非线性关联大系统在结构扩展时的跟踪控制问题, 提出一种采用自适应神经网络的控制方法. 该方法要求在不改变原结构系统控制律的前提下设计新加入子系统的控制律和自适应律, 使扩展后所有子系统都具有很好的跟踪性能. 这里主要利用神经网络的逼近功能以及 Backstepping 技术来设计自适应律和控制律, 通过 Lyapunov 理论证明在该控制器的作用下闭环系统的所有信号均是有界的, 并可使系统准确跟踪. 仿真结果验证了所提出方法的有效性.

**关键词:** 非线性大系统; 扩展结构; 分散控制; 自适应跟踪控制; Backstepping

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## Adaptive neural network tracking control for a class of nonlinear large-scale systems with expanding construction

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**Abstract:** An adaptive neural network(NN) control approach is presented for the tracking control problem of a class of nonlinear large-scale interconnected systems when its structure is expanded. The method needs to design the control laws and adaptive laws of the newly added subsystems without changing the decentralized control laws and adaptive laws of the original subsystems, so that all subsystems in the expanded system have good tracking performances. The approximation function of the NN and the Backstepping technique are used to design the decentralized control laws and adaptive laws of the newly added subsystems. It is proved by the Lyapunov theory that all signals in the closed-loop expanded system controlled by the proposed controllers are bounded, and the expanded system can track accurately. Simulation results show the effectiveness of the proposed method.

**Keywords:** nonlinear large-scale systems; expanding construction; decentralized control; adaptive tracking control; Backstepping

### 0 引言

关于互联大系统结构重构的控制问题早已是大系统理论研究的方向之一<sup>[1]</sup>. 所谓结构重构, 主要包括大系统中一些子系统脱离原系统或又重新连接上的情况, 以及有新子系统加入的扩展结构情况. 在已有的研究结果中, 对于前一种结构重构的情况研究较多<sup>[2-4]</sup>, 而对于后者的重构情况却仅有有限的几篇文献<sup>[5-7]</sup>. 然而, 大系统结构的扩展如今已越来越普遍, 对其进行研究也变得十分必要. 目前, 在结构扩展方面的研究成果主要是关于线性大系统的关联镇定<sup>[5-7]</sup>, 而对于含有未知非线性函数和非线性关联的非线性

系统尚未见相关研究的报道.

目前, 虽然对于非线性大系统的分散控制的研究已有了很多成果, 其中一类利用 Backstepping 技术来解决非线性大系统的鲁棒镇定及跟踪控制问题<sup>[8-13]</sup>, 也取得了很好的控制效果. 但是, 这类文献均是针对具有固定结构的非线性大系统, 并没有考虑系统的结构重构问题, 更没有考虑有新子系统加入时在不改变原结构系统控制律的前提下如何设计新加入子系统的控制律来保证整个扩展后非线性大系统稳定及准确跟踪的问题. 鉴于此, 本文研究一类严格反馈非线性互联大系统在结构扩展时的跟踪控制问题, 即在不

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改变原有结构的子系统分散控制律和自适应律的基础上, 设计新加入子系统的一种基于 Backstepping 和神经网络的自适应反推跟踪控制方案, 使得新子系统和扩展后的大系统都能够稳定、准确地跟踪给定的参考信号, 并使整个扩展后的大系统的所有信号一致渐近有界。

## 1 问题描述

考虑由  $N$  个子系统构成的大系统, 第  $i$  个子系统方程描述如下:

$$\begin{cases} \dot{x}_{i,j_i} = g_{i,j_i}(\bar{x}_{i,j_i})x_{i,j_i+1} + f_{i,j_i}(\bar{x}_{i,j_i}) + h_{i,j_i}(\bar{y}), \\ \dot{x}_{i,n_i} = g_{i,n_i}(\bar{x}_{i,n_i})u_i + f_{i,n_i}(\bar{x}_{i,n_i}) + h_{i,n_i}(\bar{y}), \\ y_i = x_{i,1}. \end{cases} \quad (1)$$

其中:  $i = 1, 2, \dots, N$ ;  $j_i = 1, 2, \dots, n_i - 1$ ;  $[x_{i,1}, \dots, x_{i,n_i}]^T = x_i \in R^{n_i}$ ;  $y_i \in R$  和  $u_i \in R$  分别表示子系统状态、输出和控制输入;  $\bar{x}_{i,j_i} = [x_{i,1}, x_{i,2}, \dots, x_{i,j_i}]^T \in R^{j_i}$ ,  $\bar{y} = [y_1, y_2, \dots, y_N]$  是子系统间的输出互联信号;  $f_{i,j_i}: R^{j_i} \rightarrow R$  和  $g_{i,j_i}: R^{j_i} \rightarrow R$  是未知的光滑非线性函数;  $h_{i,j_i}: R^N \rightarrow R$  表示子系统间未知的光滑非线性关联函数, 且  $f_{i,j_i}(0) = 0$ ,  $h_{i,j_i}(0) = 0$ 。

考虑在上述非线性大系统的基础上, 加入一个新的非线性子系统, 其数学模型按文献 [14] 的思想建立, 描述如下:

$$\begin{cases} \dot{x}_{N+1,j} = g_{N+1,j}(\bar{x}_{N+1,j})x_{N+1,j+1} + f_{N+1,j}(\bar{x}_{N+1,j}) + \phi_{N+1,j}(\hat{y}), \\ \dot{x}_{N+1,n} = g_{N+1,n}(\bar{x}_{N+1,n})u_{N+1} + f_{N+1,n}(\bar{x}_{N+1,n}) + \phi_{N+1,n}(\hat{y}), \\ y_{N+1} = x_{N+1,1}. \end{cases} \quad (2)$$

其中:  $j = 1, 2, \dots, n - 1$ ;  $[x_{N+1,1}, \dots, x_{N+1,n}]^T = x_{N+1} \in R^n$ ;  $y_{N+1} \in R$  和  $u_{N+1} \in R$  分别表示子系统状态、输出和控制输入;  $\bar{x}_{N+1,j} = [x_{N+1,1}, x_{N+1,2}, \dots, x_{N+1,j}]^T \in R^j$ ;  $\hat{y} = [y_1, \dots, y_N, y_{N+1}]$  是子系统间的输出互联信号;  $f_{N+1,j}: R^j \rightarrow R$  和  $g_{N+1,j}: R^j \rightarrow R$  是未知光滑非线性函数;  $\phi_{N+1,j}: R^{N+1} \rightarrow R$  表示新加入的子系统与原系统中各子系统间未知光滑非线性关联函数;  $f_{N+1,j}(0) = \phi_{N+1,j}(0) = 0$ 。

加入新子系统后, 原系统的方程描述会变成如下形式:

$$\begin{cases} \dot{x}_{i,j_i} = g_{i,j_i}(\bar{x}_{i,j_i})x_{i,j_i+1} + f_{i,j_i}(\bar{x}_{i,j_i}) + h_{i,j_i}(\bar{y}) + \phi_{i,j_i}(\hat{y}), \\ \dot{x}_{i,n_i} = g_{i,n_i}(\bar{x}_{i,n_i})u_i + f_{i,n_i}(\bar{x}_{i,n_i}) + h_{i,n_i}(\bar{y}) + \phi_{i,n_i}(\hat{y}), \\ y_i = x_{i,1}. \end{cases} \quad (3)$$

其中:  $\phi_{i,j_i}, \phi_{i,n_i}: R^{N+1} \rightarrow R$  表示新增的原子系统与新子系统之间的未知光滑非线性关联函数。

整个扩展结构系统的方程描述如下:

$$\begin{cases} \dot{x}_{i,j_i} = g_{i,j_i}(\bar{x}_{i,j_i})x_{i,j_i+1} + f_{i,j_i}(\bar{x}_{i,j_i}) + h_{i,j_i}(\bar{y}) + \phi_{i,j_i}(\hat{y}), \\ \dot{x}_{i,n_i} = g_{i,n_i}(\bar{x}_{i,n_i})u_i + f_{i,n_i}(\bar{x}_{i,n_i}) + h_{i,n_i}(\bar{y}) + \phi_{i,n_i}(\hat{y}), \\ y_i = x_{i,1}, \\ \dot{x}_{N+1,j} = g_{N+1,j}(\bar{x}_{N+1,j})x_{N+1,j+1} + f_{N+1,j}(\bar{x}_{N+1,j}) + \phi_{N+1,j}(\hat{y}), \\ \dot{x}_{N+1,n} = g_{N+1,n}(\bar{x}_{N+1,n})u_{N+1} + f_{N+1,n}(\bar{x}_{N+1,n}) + \phi_{N+1,n}(\hat{y}), \\ y_{N+1} = x_{N+1,1}. \end{cases} \quad (4)$$

控制目标: 在不改变原结构大系统 (1) 的控制律和自适应律的基础上, 设计新加入子系统 (2) 的自适应神经网络控制律, 使扩展系统 (4) 的所有输出信号准确跟踪其给定的参考信号, 并保证闭环扩展系统的所有信号是一致渐近有界的。

**引理 1** (Young 不等式)<sup>[15]</sup> 对于任意正数  $\varepsilon > 0$ , 下面的不等式成立:

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q, \quad \forall (x, y) \in R^2.$$

其中: 常数  $p > 1, q > 1$ , 满足  $(p-1)(q-1) = 1$ 。

**引理 2**<sup>[15]</sup> 考虑系统 (1), 如果存在一个正定、径向无界、连续可微 Lyapunov 函数  $V: R^n \rightarrow R$ , 常数  $a_0 > 0, b_0 \geq 0$ , 使得下式成立:

$$\dot{V} \leq -a_0V + b_0, \quad t \geq 0.$$

则: 1) 系统存在唯一解; 2) 系统所有信号是一致终极有界的。

**假设 1** 在系统 (4) 中, 对于  $1 \leq i \leq N+1, 1 \leq j \leq n$ , 非线性函数  $g_{i,j}(\bar{x}_{i,j})$  的符号已知, 且满足  $0 < b \leq |g_{i,j}(\bar{x}_{i,j})| \leq \infty$ , 这里  $b$  为常数。

**假设 2**  $y_{di}(t)$  是参考信号, 令  $\bar{y}_{di}^{(j)} = [y_{di}, \dot{y}_{di}, \dots, y_{di}^{(j)}]^T$ , 其中  $y_{di}^{(j)}$  表示  $y_{di}$  的第  $j$  阶导数, 则  $\|\bar{y}_{di}^{(j)}\| \leq d$ ,  $d$  为正数。

**假设 3** 对于系统 (4) 的不确定非线性函数  $h_{i,j}(\bar{y})$  和  $\phi_{i,j}(\hat{y})$ , 存在函数  $\bar{h}_{i,j}(y_i)$  和  $\bar{\phi}_{i,j}(y_i)$  使得如下不等式成立:

$$|h_{i,j}(\bar{y})|^2 \leq y_i^2 \bar{h}_{i,j}^2(y_i), \quad |\phi_{i,j}(\hat{y})|^2 \leq y_{N+1}^2 \bar{\phi}_{i,j}^2(y_i).$$

其中:  $\bar{h}_{i,j}(y_i)$  和  $\bar{\phi}_{i,j}(y_i)$  是满足上述要求的任意选取的未知光滑非线性函数, 且满足  $\bar{h}_{i,j}(0) = 0$  和  $\bar{\phi}_{i,j}(0) = 0$ , 这里  $1 \leq i \leq N+1, 1 \leq j \leq n$ 。

## 2 主要结果

在系统扩展时, 原系统 (1) 已具有自己的自适应神经网络控制器和自适应律, 假设它们可根据文献 [15] 中的设计方法得到, 表述如下:

$$\alpha_{i,j}(Z_{i,j}) = -k_{i,j}z_{i,j} - \frac{1}{2a_{i,j}^2}z_{i,j}\hat{\theta}_i S_{i,j}^T(Z_{i,j})S_{i,j}(Z_{i,j}), \quad (5)$$

$$u_i(Z_{i,n_i}) =$$

$$-k_{i,n_i}z_{i,n_i} - \frac{1}{2a_{i,n_i}^2}z_{i,n_i}^3\hat{\theta}_i S_{i,n_i}^T(Z_{i,n_i})S_{i,n_i}(Z_{i,n_i}), \quad (6)$$

$$\dot{\hat{\theta}}_i = \sum_{j=1}^{n_i} \frac{\lambda_i}{2a_{i,j}^2} z_{i,j}^2 S_{i,j}^T(Z_{i,j})S_{i,j}(Z_{i,j}) - \gamma_i \hat{\theta}_i. \quad (7)$$

其中:  $i=1, 2, \dots, N; j=1, 2, \dots, n_i-1; k_{i,j}, a_{i,j}, \lambda_i$  和  $\gamma_i$  是正的设计参数;  $Z_{i,j}$  是 RBF 神经网络的输入向量,  $Z_{i,1} = [x_{i,1}, y_{di}, \dot{y}_{di}]^T \in \Omega_{Z_{i,1}} \subset R^3$ ,  $Z_{i,j} = [\bar{x}_{i,j}, \hat{\theta}_i, \bar{y}_{di}^{(i)T}]^T \in \Omega_{Z_{i,j}} (j=2, 3, \dots, n_i)$ ,  $\bar{x}_{i,j} = [x_{i,1}, x_{i,2}, \dots, x_{i,j}]^T$ ,  $\hat{\theta}$  是未知常数  $\theta$  的估计值.

$$\theta_i = \max \left\{ \frac{1}{b} \|W_{i,j}^*\|^2; j=1, 2, \dots, n_i \right\},$$

$b$  的定义如假设 1 所示,  $\|W_{i,j}^*\|$  是神经网络理想权向量的范数. 并且定义坐标变换如下:

$$z_{i,j} = x_{i,j} - \alpha_{i,j-1}(\bar{x}_{i,j-1}, \hat{\theta}, \bar{y}_{di}^{(i-1)}), \quad j=1, 2, \dots, n_i. \quad (8)$$

其中:  $\alpha_{i,0} = y_{di}$ ,  $\bar{y}_{di}^{(i)}$  表示  $y_{di}$  及其  $\dot{y}_{di}$  直到  $i$  阶导数的向量,  $\hat{\theta}$  是未知常数  $\theta$  的估计值.

为了简单和清晰起见, 在下面的设计中将非线性函数中的状态向量  $\bar{x}_{i,j}$  省略, 并且令  $S_{i,j}(Z_{i,j}) = S_{i,j}$ . 下面对扩展系统 (4) 中新加入子系统的自适应律和控制律进行设计. 由于加入新子系统后原系统中加入了与新子系统的互联, 而原系统的控制律补偿不了这部分互联的影响, 需要将其考虑在新子系统的控制律设计中, 所以对新子系统的设计仍然要从原系统入手.

第  $(i, 1) (i=1, 2, \dots, N)$  步: 由系统 (4) 可得

$$\dot{z}_{i,1} = g_{i,1}x_{i,2} + f_{i,1} + h_{i,1}(\bar{y}) + \phi_{i,1}(\hat{y}) - \dot{y}_{di}. \quad (9)$$

选择如下 Lyapunov 函数:

$$V_{i,1} = \frac{1}{2}z_{i,1}^2 + \frac{b}{2\lambda_i}\tilde{\theta}_i^2, \quad (10)$$

其中  $\tilde{\theta} = \theta - \hat{\theta}$  是参数误差. 由式 (8) 可得

$$\begin{aligned} \dot{V}_{i,1} &= z_{i,1}(g_{i,1}x_{i,2} + f_{i,1} + h_{i,1}(\bar{y}) + \\ &\phi_{i,1}(\hat{y}) - \dot{y}_{di}) - \frac{b}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \quad (11)$$

由引理 1 和假设 3 可得

$$\begin{aligned} z_{i,1}h_{i,1}(\bar{y}) &\leq \frac{1}{2}z_{i,1}^2y_i^2\bar{h}_{i,1}^2(y_i) + \frac{1}{2}, \\ z_{i,1}\phi_{i,1}(\hat{y}) &\leq \frac{1}{2}z_{i,1}^2y_{N+1}^2\bar{\phi}_{i,1}^2(y_i) + \frac{1}{2}. \end{aligned}$$

式 (11) 可转化为

$$\begin{aligned} \dot{V}_{i,1} &\leq z_{i,1}(g_{i,1}x_{i,2} + \bar{f}_{i,1}(Z_{i,1})) + \frac{1}{2}z_{i,1}^2y_{N+1}^2\bar{\phi}_{i,1}^2(y_i) - \\ &\frac{1}{2}z_{i,1}^2 + 1 - \frac{b}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i, \end{aligned} \quad (12)$$

其中  $\bar{f}_{i,1}(Z_{i,1}) = f_{i,1} + \frac{1}{2}z_{i,1}y_{N+1}^2\bar{h}_{i,1}^2(y_i) - \dot{y}_{di} + \frac{1}{2}z_{i,1}$ .

由于  $f_{i,1}, h_{i,1}$  和  $\phi_{i,1}$  是未知光滑函数, 虚拟控制信号  $\alpha_{i,1}$  不能直接用  $\bar{f}_{i,1}(Z_{i,1})$  构造. 基于 RBF 网络的万能逼近功能, 存在如下形式:

$$\begin{aligned} \bar{f}_{i,1}(Z_{i,1}) &= W_{i,1}^{*T}S_{i,1}(Z_{i,1}) + \delta_{i,1}(Z_{i,1}), \\ |\delta_{i,1}(Z_{i,1})| &\leq \varepsilon_{i,1}. \end{aligned} \quad (13)$$

进而, 由引理 1 和式 (7) 可得

$$\begin{aligned} z_{i,1}\bar{f}_{i,1}(Z_{i,1}) &= \\ z_{i,1}\frac{W_{i,1}^{*T}}{\|W_{i,1}^*\|}\|W_{i,1}^*\|S_{i,1} &+ z_{i,1}\delta_{i,1}(Z_{i,1}) \leq \\ \frac{b}{2a_{i,1}^2}z_{i,1}^2\theta_i S_{i,1}^T S_{i,1} &+ \frac{1}{2}a_{i,1}^2 + \frac{1}{2}z_{i,1}^2 + \frac{1}{2}\varepsilon_{i,1}^2. \end{aligned} \quad (14)$$

将式 (14) 代入 (12), 得

$$\begin{aligned} \dot{V}_{i,1} &\leq z_{i,1}g_{i,1}z_{i,2} + z_{i,1}g_{i,1}\alpha_{i,1} + \\ \frac{1}{2}a_{i,1}^2 + \frac{1}{2}\varepsilon_{i,1}^2 &+ \frac{b}{2a_{i,1}^2}z_{i,1}^2\theta_i S_{i,1}^T S_{i,1} + \\ \frac{1}{2}z_{i,1}^2y_{N+1}^2\bar{\phi}_{i,1}^2(y_i) &+ 1 - \frac{b}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i, \end{aligned} \quad (15)$$

其中  $z_{i,2} = x_{i,2} - \alpha_{i,1}$ . 由所给出的虚拟控制器 (5) 和假设 1, 可得

$$\begin{aligned} z_{i,1}g_{i,1}\alpha_{i,1} &\leq \\ -k_{i,1}bz_{i,1}^2 - \frac{b}{2a_{i,1}^2}z_{i,1}^2\hat{\theta}_i S_{i,1}^T &S_{i,1}(Z_{i,1})S_{i,1}(Z_{i,1}). \end{aligned} \quad (16)$$

根据引理 1, 有

$$z_{i,1}g_{i,1}z_{i,2} \leq \frac{1}{2}g_{i,1}z_{i,1}^2 + \frac{1}{2}g_{i,1}z_{i,2}^2. \quad (17)$$

考虑式 (16) 和 (17), 式 (15) 可转化为

$$\begin{aligned} \dot{V}_{i,1} &\leq \\ -c_{i,1}z_{i,1}^2 + \frac{1}{2}g_{i,1}z_{i,2}^2 &+ \frac{1}{2}a_{i,1}^2 + \frac{1}{2}\varepsilon_{i,1}^2 + 1 + \\ \frac{b}{\lambda_i}\tilde{\theta}_i\left(\frac{\lambda_i}{2a_{i,1}^2}z_{i,1}^2 S_{i,1}^T S_{i,1} - \hat{\theta}_i\right) &+ \frac{1}{2}z_{i,1}^2y_{N+1}^2\bar{\phi}_{i,1}^2(y_i), \end{aligned} \quad (18)$$

其中  $c_{i,1} = \left(k_{i,1} - \frac{1}{2}\right)b > 0$ .

根据自适应律 (7), 式 (18) 可转化为

$$\begin{aligned} \dot{V}_{i,1} &\leq -c_{i,1}z_{i,1}^2 + \frac{1}{2}g_{i,1}z_{i,2}^2 + \frac{1}{2}a_{i,1}^2 + \frac{1}{2}\varepsilon_{i,1}^2 + 1 + \\ \frac{\gamma_i b}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i &+ \frac{1}{2}z_{i,1}^2y_{N+1}^2\bar{\phi}_{i,1}^2(y_i). \end{aligned} \quad (19)$$

第  $(i, j)$  步 ( $2 \leq j \leq n_i - 1$ ): 与第  $(i, 1)$  步相似, 考虑系统 (4) 和坐标变换 (8), 有

$$\dot{z}_{i,j} = g_{i,j}x_{i,j+1} + f_{i,j} + \phi_{i,j}(\hat{y}) + h_{i,j}(\bar{y}) - \dot{\alpha}_{i,j-1}, \quad (20)$$

其中

$$\begin{aligned} \dot{\alpha}_{i,j-1} &= \\ \sum_{q=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,q}}(g_{i,q}x_{i,q+1} &+ f_{i,q} + h_{i,q}(\bar{y}) + \\ \phi_{i,q}(\hat{y})) &+ \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i + \sum_{q=0}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial y_{di}^{(q)}}y_{di}^{(q+1)}. \end{aligned}$$

选取 Lyapunov 函数为  $V_{i,j} = V_{i,j-1} + \frac{1}{2}z_{i,j}^2$ , 应用式 (20), 可得

$$\dot{V}_{i,j} = \dot{V}_{i,j-1} + z_{i,j}(g_{i,j}x_{i,j+1} + f_{i,j} + \phi_{i,j}(\hat{y}) + h_{i,j}(\bar{y}) - \dot{\alpha}_{i,j-1}). \quad (21)$$

类似于第  $(i, 1)$  步的式 (11)~(19) 的过程, 可得到下面的结果:

$$\begin{aligned} \dot{V}_{i,j} \leq & - \sum_{q=1}^j c_{i,q} z_{i,q}^2 + \frac{1}{2} \sum_{q=1}^j a_{i,q}^2 + \frac{1}{2} \sum_{q=1}^j z_{i,q}^2 y_{N+1}^2 \bar{\phi}_{i,q}^2(y_i) + \\ & \frac{1}{2} \sum_{q=1}^j \varepsilon_{i,q}^2 + \frac{b\gamma_i}{\lambda_i} \tilde{\theta}_i \hat{\theta}_i + \frac{1}{2} g_{i,j} z_{i,j+1}^2 + 2j - 1 + \\ & \frac{1}{2} \sum_{q=2}^j z_{i,q}^2 \left( \sum_{q=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,q}} \right)^2 y_{N+1}^2 \bar{\phi}_{i,q}^2(y_i). \end{aligned} \quad (22)$$

其中:  $c_{i,q} = \left(k_{i,j} - \frac{1}{2}\right)b > 0, q = 1, 2, \dots, j$ .

第  $(i, n_i)$  步: 考虑在系统 (4) 和坐标变换 (8), 有

$$\dot{z}_{i,n_i} = g_{i,n_i} u_i + f_{i,n_i} + \phi_{i,n_i}(\hat{y}) + h_{i,n_i}(\bar{y}) - \dot{\alpha}_{i,n_i-1},$$

其中

$$\begin{aligned} \dot{\alpha}_{i,n_i-1} = & \sum_{q=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,q}} (g_{i,q} x_{i,q+1} + f_{i,q} + h_{i,q}(\bar{y}) + \\ & \phi_{i,q}(\hat{y})) + \frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i + \sum_{q=0}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial y_{di}^{(q)}} y_{di}^{(q+1)}. \end{aligned}$$

选取 Lyapunov 函数为  $V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2} z_{i,n_i}^2$ . 与第  $(i, j)$  步  $(2 \leq j \leq n_i - 1)$  的推导过程相似, 可得

$$\begin{aligned} \dot{V}_{i,n_i} \leq & - \sum_{q=1}^{n_i} c_{i,q} z_{i,q}^2 + \frac{1}{2} \left( \sum_{q=1}^{n_i} a_{i,q}^2 + \sum_{q=1}^{n_i} z_{i,q}^2 y_{N+1}^2 \bar{\phi}_{i,q}^2(y_i) \right) + \\ & \frac{b\gamma_i}{\lambda_i} \tilde{\theta}_i \hat{\theta}_i + 2n_i - 1 + \frac{1}{2} \sum_{q=1}^{n_i} \varepsilon_{i,q}^2 + \\ & \frac{1}{2} \sum_{j=2}^{n_i} z_{i,q}^2 \left( \sum_{q=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,q}} \right)^2 y_{N+1}^2 \bar{h}_{i,q}^2(y_i). \end{aligned} \quad (23)$$

其中:  $c_{i,q} = \left(k_{i,j} - \frac{1}{2}\right)b > 0, q = 1, 2, \dots, n_i - 1, c_{i,n_i} = k_{i,n_i} b$ .

由上面推导可见, 原系统 (1) 的自适应律及控制律 (5)~(7) 不能抵消掉新互联项的影响, 要将其放在新加入子系统的控制律设计中加以考虑. 对于新子系统, 设计过程如下.

第  $(N + 1, 1)$  步: 由系统 (4), 可得

$$\begin{aligned} \dot{z}_{N+1,1} = & g_{N+1,1} x_{N+1,2} + f_{N+1,1} + \phi_{N+1,1}(\hat{y}) - \dot{y}_{dN+1}. \end{aligned}$$

选择如下 Lyapunov 函数:

$$V_{N+1,1} = \frac{1}{2} z_{N+1,1}^2 + \frac{b}{2\lambda_{N+1}} \tilde{\theta}_{N+1}^2.$$

采用与第  $(i, 1)$  ( $i = 1, 2, \dots, N$ ) 步相同的过程, 选取合适的 RBF 神经网络  $W_{N+1,1}^* S_{N+1,1}(Z_{N+1,1})$  来逼近

未知函数  $\bar{f}_{N+1,1}(Z_{N+1,1})$ , 可得

$$\begin{aligned} \dot{V}_{N+1,1} \leq & z_{N+1,1} g_{N+1,1} z_{N+1,2} + z_{N+1,1} g_{N+1,1} \alpha_{N+1,1} + \\ & \frac{1}{2} a_{N+1,1}^2 + \frac{b}{2a_{N+1,1}^2} z_{N+1,1}^2 \theta_{N+1} S_{N+1,1}^T S_{N+1,1} + \\ & \frac{1}{2} \varepsilon_{N+1,1}^2 - \frac{b}{\lambda_{N+1}} \tilde{\theta}_{N+1} \dot{\hat{\theta}}_{N+1} + 1 + \\ & \frac{1}{2} z_{N+1,1}^2 y_{N+1}^2 \bar{\phi}_{N+1,1}^2(y_{N+1}). \end{aligned} \quad (24)$$

根据式 (24), 设计第  $N + 1$  个子系统的虚拟控制器为

$$\begin{aligned} \alpha_{N+1,1} = & -k_{N+1,1} z_{N+1,1} - \\ & \frac{1}{2a_{N+1,1}^2} z_{N+1,1} \hat{\theta}_{N+1} S_{N+1,1}^T S_{N+1,1} - \\ & \frac{1}{2} z_{N+1,1} \hat{\theta}_{N+1} S_{N+1,1}^T S_{N+1,1}, \end{aligned} \quad (25)$$

其中  $k_{N+1,1}$  和  $a_{N+1,1}$  是正的设计参数.

类似于式 (16)~(18) 的推导过程, 式 (24) 可以转化为

$$\begin{aligned} \dot{V}_{N+1,1} \leq & -c_{N+1,1} z_{N+1,1}^2 + \frac{1}{2} g_{N+1,1} z_{N+1,2}^2 + \\ & \frac{1}{2} a_{N+1,1}^2 + \frac{1}{2} \varepsilon_{N+1,1}^2 + \\ & \frac{b}{\lambda_{N+1}} \tilde{\theta}_{N+1} \left( \frac{\lambda_{N+1}}{2a_{N+1,1}^2} z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1} - \right. \\ & \left. \dot{\hat{\theta}}_{N+1} + \frac{\lambda_{N+1}}{2} z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1} \right) + \\ & 1 + \frac{1}{2} z_{N+1,1}^2 y_{N+1}^2 \bar{\phi}_{N+1,1}^2(y_{N+1}), \end{aligned} \quad (26)$$

其中  $c_{N+1,1} = \left(k_{N+1,1} - \frac{1}{2}\right)b > 0$ .

第  $(N + 1, j)$  步  $(2 \leq j \leq n - 1)$ : 设计过程与第  $(N + 1, 1)$  步的设计过程类似. 由系统 (4) 可得

$$\begin{aligned} \dot{z}_{N+1,j} = & g_{N+1,j} x_{N+1,j+1} + f_{N+1,j} + \phi_{N+1,j}(\hat{y}) - \dot{\alpha}_{N+1,j-1}. \end{aligned}$$

选择 Lyapunov 函数  $V_{N+1,j} = V_{N+1,j-1} + \frac{1}{2} z_{N+1,j}^2$ . 设计过程与第  $(i, j)$  步的设计过程类似, 选取合适的 RBF 神经网络  $W_{N+1,j}^* S_{N+1,j}(Z_{N+1,j})$  来逼近未知函数  $\bar{f}_{N+1,j}(Z_{N+1,j})$ , 可得

$$\begin{aligned} \dot{V}_{N+1,j} \leq & \dot{V}_{N+1,j-1} + z_{N+1,j} g_{N+1,j} z_{N+1,j+1} + \\ & z_{N+1,j} g_{N+1,j} \alpha_{N+1,j} + \frac{1}{2} a_{N+1,j}^2 + \\ & \frac{b}{2a_{N+1,j}^2} z_{N+1,j}^2 \theta_{N+1} S_{N+1,j}^T S_{N+1,j} + \frac{1}{2} + \frac{1}{2} \varepsilon_{N+1,j}^2 - \\ & \frac{b}{\lambda_{N+1}} \tilde{\theta}_{N+1} \dot{\hat{\theta}}_{N+1} + \sum_{j=1}^{n-1} \frac{1}{2} z_{N+1,j}^2 y_{N+1}^2 \bar{\phi}_{N+1,j}^2(y_{N+1}). \end{aligned}$$

设计虚拟控制器为

$$\alpha_{N+1,j} = -k_{N+1,j}z_{N+1,j} - \frac{1}{2a_{N+1,j}^2}z_{N+1,j}\hat{\theta}_{N+1}S_{N+1,j}^T S_{N+1,j}. \quad (27)$$

类似于式(16)~(18)的推导过程, 可得

$$\begin{aligned} \dot{V}_{N+1,j} \leq & -\sum_{q=1}^j c_{N+1,q}z_{N+1,q}^2 + \frac{1}{2}\sum_{q=1}^j a_{N+1,q}^2 + \\ & \frac{1}{2}\sum_{q=1}^j \varepsilon_{N+1,q}^2 + \frac{1}{2}g_{N+1,j}z_{N+1,j+1}^2 + 2j - 1 + \\ & \frac{b}{\lambda_{N+1}}\tilde{\theta}\left(\sum_{q=1}^j \frac{\lambda_{N+1}}{2a_{N+1,q}^2}z_{N+1,q}^2 S_{N+1,q}^T S_{N+1,q} - \right. \\ & \left. \dot{\theta}_{N+1} + \frac{\lambda_{N+1}}{2}z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1}\right) + \\ & \sum_{j=1}^{n-1} \frac{1}{2}z_{N+1,j}^2 y_{N+1}^2 \bar{\phi}_{N+1,1}^2(y_{N+1}). \quad (28) \end{aligned}$$

其中:  $c_{N+1,q} = \left(k_{N+1,q} - \frac{1}{2}\right)b > 0, q = 1, 2, \dots, j$ .

第  $(N+1, n)$  步: 选取 Lyapunov 函数  $V_{N+1,n} = V_{N+1,n-1} + \frac{1}{2}z_{N+1,n}^2$ , 推导过程与第  $(i, n_i)$  步类似, 有  $u_{N+1} = -k_{N+1,n}z_{N+1,n} - \frac{1}{2a_{N+1,n}^2}z_{N+1,n}\hat{\theta}_{N+1}S_{N+1,n}^T S_{N+1,n}$ , (29)

$$\begin{aligned} \dot{V}_{N+1,n} \leq & -\sum_{q=1}^n c_{N+1,q}z_{N+1,q}^2 + \frac{1}{2}\sum_{q=1}^n a_{N+1,q}^2 + \frac{1}{2}\sum_{q=1}^n \varepsilon_{N+1,q}^2 + \\ & \frac{b}{\lambda_{N+1}}\tilde{\theta}\left(\sum_{q=1}^n \frac{\lambda_{N+1}}{2a_{N+1,q}^2}z_{N+1,q}^2 S_{N+1,q}^T S_{N+1,q} + \right. \\ & \left. \frac{\lambda_{N+1}}{2}z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1} - \dot{\theta}_{N+1}\right) + \\ & 2n - 1 + \sum_{j=1}^n \frac{1}{2}z_{N+1,j}^2 y_{N+1}^2 \bar{\phi}_{N+1,1}^2(y_{N+1}). \quad (30) \end{aligned}$$

其中:  $c_{N+1,q} = \left(k_{N+1,q} - \frac{1}{2}\right)b > 0, q = 1, 2, \dots, n-1, c_{N+1,n} = k_{N+1,n}b$ .

选取闭环系统的 Lyapunov 函数, 令  $n_{N+1} = n$ .

$$V = \sum_{i=1}^{N+1} V_{i,n_i} = \sum_{i=1}^N \left(\frac{1}{2}\sum_{j=1}^{n_i} z_{i,j}^2 + \frac{b}{2\lambda_i}\tilde{\theta}_i^2\right) + \frac{1}{2}\sum_{j=1}^n z_{N+1,j}^2 + \frac{b}{2\lambda_{N+1}}\tilde{\theta}_{N+1}^2.$$

结合不等式(23)和(30), 可得

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \left(-\sum_{j=1}^{n_i} c_{i,j}z_{i,j}^2 - \frac{\gamma_i b}{2\lambda_i}\tilde{\theta}_i^2\right) + \sum_{i=1}^N \frac{\gamma_i b}{\lambda_i}\theta_i^2 + \\ & \sum_{i=1}^N \left(\frac{1}{2}\sum_{j=1}^{n_i} a_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{n_i} \varepsilon_{i,j}^2 + 2n_i - 1 + \right. \\ & \left. \frac{\gamma_i b}{\lambda_i}\theta_i^2\right)\bar{f}_{N+1}(Z_{N+1}) - \sum_{j=1}^n c_{N+1,j}z_{N+1,j}^2 + \end{aligned}$$

$$\begin{aligned} & \frac{1}{2}\sum_{j=1}^n a_{N+1,j}^2 + \frac{1}{2}\sum_{j=1}^n \varepsilon_{N+1,j}^2 + 2n - 1 + \\ & \frac{b}{\lambda_{N+1}}\tilde{\theta}_{N+1}\left(\sum_{j=1}^n \frac{\lambda_{N+1}}{2a_{N+1,j}^2}z_{N+1,j}^2 S_{N+1,j}^T S_{N+1,j} + \right. \\ & \left. \frac{\lambda_{N+1}}{2}z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1} - \dot{\theta}_{N+1}\right). \quad (31) \end{aligned}$$

令

$$\begin{aligned} \bar{f}_{N+1}(Z_{N+1}) = & \frac{1}{2}\sum_{i=1}^N \left(\sum_{j=1}^{n_i} z_{i,j}^2 y_i^2 \bar{\phi}_{i,j}^2(y_i) + \right. \\ & \left. \sum_{j=2}^{n_i} z_{i,j}^2 \left(\sum_{q=1}^{n_i} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,q}}\right)^2 y_i^2 \bar{h}_{i,q}^2(y_i)\right) + \\ & \sum_{j=1}^n \frac{1}{2}z_{N+1,j}^2 y_{N+1}^2 \bar{\phi}_{N+1,1}^2(y_{N+1}), \end{aligned}$$

根据式(13)以及杨氏不等式, 式(31)可转化为

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \left(-\sum_{j=1}^{n_i} c_{i,j}z_{i,j}^2 - \frac{\gamma_i b}{2\lambda_i}\tilde{\theta}_i^2\right) + \sum_{i=1}^N \frac{\gamma_i b}{\lambda_i}\theta_i^2 + \\ & \sum_{i=1}^N \left(\frac{1}{2}\sum_{j=1}^{n_i} a_{i,j}^2 + \frac{1}{2}\sum_{j=1}^{n_i} \varepsilon_{i,j}^2 + n_i + \frac{\gamma_i b}{\lambda_i}\theta_i^2\right) + \\ & \frac{1}{2}b\theta_{N+1}S_{N+1}^T S_{N+1} + \frac{1}{2}\varepsilon_{N+1}^2 + 1 - \sum_{j=1}^n c_{N+1,j}z_{N+1,j}^2 + \\ & \frac{1}{2}\sum_{j=1}^n a_{N+1,j}^2 + \frac{1}{2}\sum_{j=1}^n \varepsilon_{N+1,j}^2 + 2n - 1 + \\ & \frac{b}{\lambda_{N+1}}\tilde{\theta}_{N+1}\left(\sum_{j=1}^n \frac{\lambda_{N+1}}{2a_{N+1,j}^2}z_{N+1,j}^2 S_{N+1,j}^T S_{N+1,j} + \right. \\ & \left. \frac{\lambda_{N+1}}{2}z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1} - \dot{\theta}_{N+1}\right). \quad (32) \end{aligned}$$

根据式(32), 可以得到系统(4)中新加入子系统的自适应律为

$$\begin{aligned} \dot{\theta}_{N+1} = & \sum_{j=1}^n \frac{\lambda_{N+1}}{2a_{N+1,j}^2}z_{N+1,j}^2 S_{N+1,j}^T S_{N+1,j} - \\ & \gamma_{N+1}\hat{\theta}_{N+1} + \frac{\lambda_{N+1}}{2}z_{N+1,1}^2 S_{N+1,1}^T S_{N+1,1}, \quad (33) \end{aligned}$$

其中  $a_{N+1}$ 、 $\lambda_{N+1}$  和  $\gamma_{N+1}$  是正的设计参数.

下面给出本文的主要结果.

**定理 1** 考虑满足假设 1~假设 3 的非线性扩展结构关联大系统(4), 在保证原系统(1)的自适应神经网络控制律(5)~(7)不变的前提下, 设计新加入子系统的神经网络控制律和自适应律如式(27)、(29)和(33), 它们可使得扩展后的闭环系统的所有信号一致终极有界.

**证明** 选取闭环系统的 Lyapunov 函数

$$V = \sum_{i=1}^{N+1} V_{i,n_i} = \sum_{i=1}^{N+1} \left(\frac{1}{2}\sum_{j=1}^{n_i} z_{i,j}^2 + \frac{b}{2\lambda_i}\tilde{\theta}_i^2\right).$$

应用式(32)以及自适应律(33), 可得

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^N \left(-\sum_{j=1}^{n_i} c_{i,j}z_{i,j}^2 - \frac{\gamma_i b}{2\lambda_i}\tilde{\theta}_i^2\right) + \sum_{i=1}^N \frac{\gamma_i b}{\lambda_i}\theta_i^2 + \end{aligned}$$

$$\begin{aligned} & \sum_{i=1}^N \left( \frac{1}{2} \sum_{j=1}^{n_i} a_{i,j}^2 + \frac{1}{2} \sum_{j=1}^{n_i} \varepsilon_{i,j}^2 + 2n_i - 1 + \frac{\gamma_i b}{\lambda_i} \theta_i^2 \right) - \\ & \sum_{j=1}^n c_{N+1,j} z_{N+1,j}^2 + \frac{1}{2} \sum_{j=1}^n a_{N+1,j}^2 + \frac{1}{2} \sum_{j=1}^n \varepsilon_{N+1,j}^2 + \\ & 2n - 1 + \frac{1}{2} \varepsilon_{N+1}^2 + 1 + \frac{\gamma_{N+1} b}{\lambda_{N+1}} \tilde{\theta}_{N+1} \hat{\theta}_{N+1}. \end{aligned} \quad (34)$$

对于  $\frac{\gamma_i b}{\lambda_i} \tilde{\theta}_i \hat{\theta}_i$ , 下式成立:

$$\frac{\gamma_i b}{\lambda_i} \tilde{\theta}_i \hat{\theta}_i = -\frac{\gamma_i b}{\lambda_i} \tilde{\theta}_i^2 + \frac{\gamma_i b}{\lambda_i} \tilde{\theta}_i \theta_i \leq -\frac{\gamma_i b}{2\lambda_i} \tilde{\theta}_i^2 + \frac{\gamma_i b}{\lambda_i} \theta_i^2. \quad (35)$$

将式 (33) 代入 (34), 可得

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^{N+1} \left( -\sum_{j=1}^{n_i} c_{i,j} z_{i,j}^2 - \frac{\gamma_i b}{2\lambda_i} \tilde{\theta}_i^2 \right) + \sum_{i=1}^{N+1} \frac{\gamma_i b}{\lambda_i} \theta_i^2 + \\ & \sum_{i=1}^{N+1} \left( \frac{1}{2} \sum_{j=1}^{n_i} a_{i,j}^2 + \frac{1}{2} \sum_{j=1}^{n_i} \varepsilon_{i,j}^2 + \frac{\gamma_i b}{\lambda_i} \theta_i^2 + 2n_i - 1 \right) + 1. \end{aligned} \quad (36)$$

令  $a_0 = \min\{2c_{i,j}, \gamma_i, j = 1, 2, \dots, n\} = \min\{2(k_{i,j} - 1/2), \gamma_i\}$ ,  $b_0 = \sum_{i=1}^{N+1} \frac{\gamma_i b}{\lambda_i} \theta_i^2 + \sum_{i=1}^{N+1} \left( \frac{1}{2} \sum_{j=1}^{n_i} a_{i,j}^2 + \frac{1}{2} \sum_{j=1}^{n_i} \varepsilon_{i,j}^2 + \frac{\gamma_i b}{\lambda_i} \theta_i^2 + 2n_i - 1 \right) + 1$ , 式 (36) 可化为  $\dot{V} \leq -a_0 V + b_0, t \geq 0$ . 因此, 由引理 2 可知, 扩展后的闭环系统中所有信号都是一致终极有界的. □

### 3 仿真算例

考虑一个具有两个子系统的非线性互联大系统作为原结构系统, 其方程描述如下:

$$\begin{cases} \dot{x}_{1,1} = (1 + \sin x_{1,1})^2 x_{1,2} + x_{1,1} \sin x_{1,1} + 0.5 y_1 y_2, \\ \dot{x}_{1,2} = (1 + \sin(x_{1,1} x_{1,2})) u_1 + x_{1,1} x_{1,2} + y_1 y_2, \\ y_1 = x_{1,1}; \\ \dot{x}_{2,1} = (3 + \sin x_{2,1}) x_{2,2} + 0.5 x_{2,1} + y_1 y_2^2, \\ \dot{x}_{2,2} = (2 + \sin^2(x_{2,1} x_{2,2})) u_2 + x_{2,1} \sin x_{2,2} + y_1 y_2, \\ y_2 = x_{2,1}. \end{cases}$$

其中:  $x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}$  是系统的状态;  $y_1$  和  $y_2$  是系统的输出. 参考信号设定为

$$\begin{aligned} y_{d1} &= 0.5 \sin t + 0.5 \sin(0.5t), \\ y_{d2} &= 0.5 \sin t + \sin(0.5t). \end{aligned}$$

根据文献 [15] 给出的设计方案, 计算出虚拟控制信号  $\alpha_{i,j}$ , 实际控制律  $u_i$  和自适应律  $\hat{\theta}_i$ . 其中:  $i = 1, 2; j = 1, 2$ . 在仿真中, 选择初始条件  $[x_{1,1}(0), x_{1,2}(0), x_{2,1}(0), x_{2,2}(0)]^T = [0, 0, 0, 0]^T$ ,  $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0, 0]^T$ . 选择设计参数  $k_{1,1} = 40, k_{1,2} = 50, k_{2,1} = 40, k_{2,2} = 40, \gamma_1 = \gamma_2 = 10, a_{1,1} = a_{1,2} = a_{2,1} = a_{2,2} = 20$  和  $\lambda_1 = \lambda_2 = 20$ . 系统跟踪的仿真结果如图 1 ~ 图 3 所示.

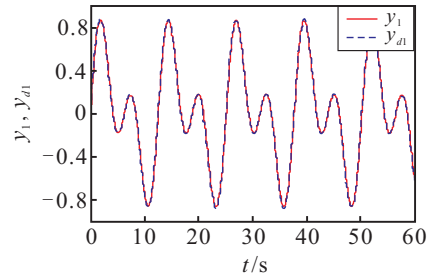


图 1 子系统 1 的输出  $y_1$  和参考信号  $y_{d1}$  曲线

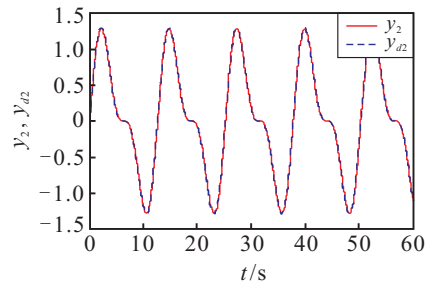


图 2 子系统 2 的输出  $y_2$  和参考信号  $y_{d2}$  曲线

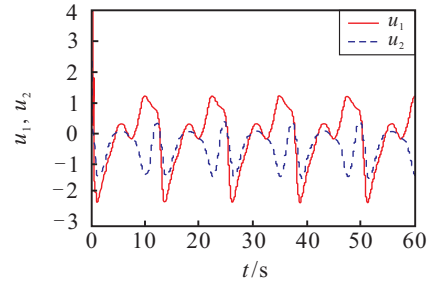


图 3 原系统的实际控制信号  $u_1$  和  $u_2$  曲线

新加入的子系统方程描述如下:

$$\begin{cases} \dot{x}_{3,1} = (1 + \sin x_{3,1}) x_{3,2} + 3x_{3,1} x_{3,2} + y_3 (y_1 + y_2), \\ \dot{x}_{3,2} = (2 + \cos(x_{3,1} x_{3,2})) u_3 + x_{3,1}^2 x_{3,2} + y_2^2 y_3, \\ y_3 = x_{3,1}. \end{cases}$$

加入新的子系统后, 原系统变成如下形式:

$$\begin{cases} \dot{x}_{1,1} = g_{1,1}(\bar{x}_{1,1}) x_{1,2} + \sin x_{1,1} + 0.5 y_1 y_2 + y_1 \cos y_3, \\ \dot{x}_{1,2} = g_{1,2}(\bar{x}_{1,2}) u_1 + x_{1,1} x_{1,2} + y_1 y_2 + y_1 \sin y_2 \cos y_3, \\ y_1 = x_{1,1}; \\ \dot{x}_{2,1} = g_{2,1}(\bar{x}_{2,1}) x_{2,2} + 0.5 x_{2,1} + y_1 y_2^2 + y_2 y_3, \\ \dot{x}_{2,2} = g_{2,2}(\bar{x}_{2,2}) u_2 + x_{2,1} \sin x_{2,2} + y_1 y_2 + y_1 y_2 \cos y_3, \\ y_2 = x_{2,1}. \end{cases}$$

其中

$$\begin{aligned} g_{1,1}(\bar{x}_{1,1}) &= (1 + \sin x_{1,1})^2, \\ g_{1,2}(\bar{x}_{1,2}) &= 1 + \sin(x_{1,1} x_{1,2}), \\ g_{2,1}(\bar{x}_{2,1}) &= 3 + \sin x_{2,1}, \\ g_{2,2}(\bar{x}_{2,2}) &= 2 + \sin^2(x_{2,1} x_{2,2}). \end{aligned}$$

新子系统的参考信号设定为  $y_{d3} = 0.5 \cos t - \cos(0.5t)$ . 在仿真中, 选择初始条件  $[x_{3,1}(0), x_{3,2}(0)]^T = [0, 0]^T$ ,  $\hat{\theta}_3(0) = 0$ . 选择设计参数  $k_{3,1} = 50, k_{3,2} = 50, a_{3,1} = a_{3,2} = 10, \gamma_3 = 10, \lambda_3 = 20$ . 新子系统的仿真结果如图 4

和图5所示. 由于原系统的两个子系统的跟踪效果没有发生变化, 为了节省空间, 这里略去了.

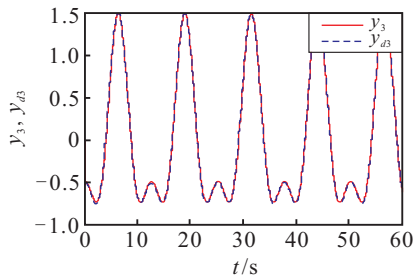


图4 新加入子系统3的输出 $y_3$ 参考信号 $y_{d3}$ 曲线

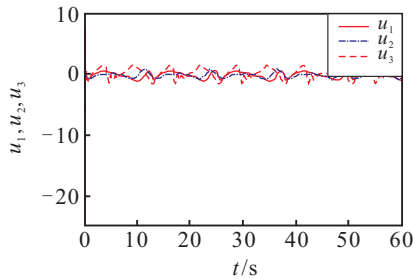


图5 扩展结构系统(4)的实际控制信号 $u_1$ 、 $u_2$ 和 $u_3$ 曲线

从图4和图5可以看出, 加入新子系统后, 在所设计的自适应神经网络控制律作用下, 系统中各子系统均具有良好的跟踪性能, 验证了所提出的控制方案的有效性.

## 4 结 论

本文研究了一类非线性大系统在结构扩展时的自适应神经网络反推跟踪控制问题. 在不改变原结构非线性大系统的自适应律及控制律的前提下, 利用神经网络、Backstepping方法和Lyapunov稳定性理论设计了新加入非线性子系统的神经网络控制器和自适应律. 该控制方案使扩展后的非线性大系统中的各个子系统都能很好地跟踪给定信号. 最后, 针对严格反馈非线性扩展大系统进行了仿真研究, 结果表明所提出的方法是有效的.

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