文章编号: 1001-0920(2017)06-1033-08 DOI: 10.13195/j.kzyjc.2016.0431

毕达哥拉斯模糊交叉影响集成算子及其决策应用

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摘 要: 定义毕达哥拉斯模糊数的交叉影响加法、数乘、乘法及幂运算,提出毕达哥拉斯模糊交叉影响加权平均算子(PFIWA)、毕达哥拉斯模糊交叉影响有序加权平均算子(PFIOWA)、毕达哥拉斯模糊交叉影响加权几何算子(PFIWG)及毕达哥拉斯模糊交叉影响有序加权几何算子(PFIOWG),推导出它们的数学表达式,并研究其性质. 提出基于毕达哥拉斯模糊交叉影响集成算子的决策方法,并通过决策实例验证所提出方法的稳定性和有效性.

关键词: 毕达哥拉斯模糊数；交叉影响集成算子；毕达哥拉斯模糊交叉影响有序加权平均算子；毕达哥拉斯模糊交叉影响有序加权几何算子

中图分类号: C934；O223 **文献标志码:** A

Pythagorean fuzzy interaction aggregation operators and applications in decision making

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Abstract: The interaction operations over Pythagorean fuzzy numbers are defined, including addition, scalar multiplication, multiplication and power operations. Then, the Pythagorean fuzzy interaction weighted averaging (PFIWA) operator, Pythagorean fuzzy interaction ordered weighted averaging(PFIOWA) operator, Pythagorean fuzzy interaction weighted geometric(PFIWG) operator, and Pythagorean fuzzy interaction ordered weighted geometric (PFIOWG) operator are proposed, and the mathematical expressions of these operators are obtained by derivation and some properties of these operators are investigated. A method based on Pythagorean fuzzy interaction aggregation operators for decision making is presented, and an illustrative example is given to illustrate the stability and effectiveness of the proposed method.

Keywords: Pythagorean fuzzy number；interaction aggregation operators；PFIOWA operator；PFIOWG operator

0 引 言

作为模糊集[1]的一种推广,Atanassov[2]定义的直觉模糊集从支持、反对、中立3个方面全面描述了客观世界,因此许多学者对其进行了研究[3-16]. 在这些研究中,关于直觉模糊决策的研究是一个热点. 其中Atanassov[8]定义了直觉模糊集的序运算以及加法和乘法运算,De等[9]将直觉模糊集的加法和乘法运算推广到数乘和指数运算,Xu等[10-11]提出了一系列的直觉模糊数加权平均算子和加权几何算子,Zhao等[12]提出了广义直觉模糊集成算子,Tan等[13-14]定义了广义直觉模糊几何算子和直觉模糊Choquet积分集成算子,Yang等[15]研究了拟算术直觉模糊OWA算子等. 但是,何迎东等[16-17]通过分析发现,使用目前的直觉模糊数运算法则[8-9]及相关集成算子[10-11]进行信息集成时,所得到的直觉模糊数的隶属度和非隶属度分别只与原始直觉模糊数的隶属度和非隶属度有关. 而事实上,不同直觉模糊数的隶属度和非隶属度之间也可能存在着某种联系和相互影响,为此他们提出了考虑隶属度和非隶属度交叉影响的直觉模糊数运算及相关集成算子,并通过决策实例说明了这些集成算子的有效性.

此外Yager发现,在直觉模糊决策过程中,可能出现决策者给出的方案满足属性的隶属度和非隶属度之和大于1的情况,为此在研究了模糊集、区间值模

收稿日期: 2016-04-13；**修回日期:** 2016-07-06.

基金项目: 国家自然科学基金项目(11501525)；河南省社科联、河南省经团联调研项目(SKL-2016-3704)；郑州航空工业管理学院青年科研基金项目(2016113001).

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糊集和直觉模糊集的补运算基础上, Yager等[18-19]通过定义毕达哥拉斯模糊补运算,提出了允许隶属度和非隶属度之和超过1,而其平方和不超过1的毕达哥拉斯模糊集,推广了直觉模糊集,使得决策者在决策过程中不必重新修改直觉模糊决策值也可以进行决策;同时, Yager提出了毕达哥拉斯模糊加权平均算子(PFWA)和有序加权几何算子(PFOWG). 在Yager工作基础上,最近一些学者对毕达哥拉斯模糊集进行了研究,其中, Zhang等[20]定义了毕达哥拉斯模糊数的运算及距离,提出了毕达哥拉斯模糊TOPSIS法;彭新东等[21]将毕达哥拉斯模糊集与软集[22]相结合,提出了毕达哥拉斯模糊软集,探讨了其决策应用; Liang等[23]和Peng等[24]研究了区间值毕达哥拉斯模糊集及其决策应用; Gou等[25]研究了毕达哥拉斯模糊数的连续性及微分等内容;刘卫锋等[26]在毕达哥拉斯模糊数运算[20]基础上,提出了一系列的毕达哥拉斯模糊加权平均算子和加权几何算子以及拟加权平均算子和拟加权几何算子.

但是,从目前的毕达哥拉斯模糊数运算[20]和相关集成算子[26]可以看出,无论是毕达哥拉斯模糊加权平均算子还是加权几何算子,其集成结果的隶属度和非隶属度也分别只与原始毕达哥拉斯模糊数的隶属度和非隶属度有关,并没有考虑到不同毕达哥拉斯模糊数的隶属度和非隶属度之间也存在着某种关联和相互影响. 为此,本文在直觉模糊交叉影响运算和集成算子[16-17]启发下, 研究考虑隶属度和非隶属度交叉影响的毕达哥拉斯模糊数运算和集成算子. 首先定义考虑隶属度和非隶属交叉影响的毕达哥拉斯模糊数的加法、数乘、乘法及幂运算,研究它们的运算性质;然后提出毕达哥拉斯模糊交叉影响加权平均算子(PFIWA)、毕达哥拉斯模糊交叉影响有序加权平均算子(PFIOWA)、毕达哥拉斯模糊交叉影响几何算子(PFIWG)和毕达哥拉斯模糊交叉影响有序加权几何算子(PFIOWG),分别给出它们的计算公式,并探讨它们的性质;最后,提出基于毕达哥拉斯模糊交叉影响算子的决策方法,并通过决策实例说明其稳定性和有效性.

1 相关概念

定义1[1] 设X为论域, $A=\{\langle x,\mu_A(x)\rangle \mid x\in X\}$称为$X$上的一个模糊集,其中$\mu_A: X\to[0,1]$, $\mu_A(x)$表示X上元素x属于A的隶属度.

定义2[2] 设X为论域, $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$称为$X$上的一个直觉模糊集,其中$\mu_A: X\to[0,1]$, $\nu_A: X\to[0,1]$为X上的模糊集, $\mu_A(x)$和$\nu_A(x)$分别表示X上元素x属于A的隶属度和非隶属度,且$\forall x\in X, \mu_A(x),\nu_A(x)\in[0,1]$,有$\mu_A(x)+\nu_A(x)\leqslant 1$.

定义3[18-19] 设X为论域, $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\}$称为$X$上的一个毕达哥拉斯模糊集,其中$\mu_A: X\to[0,1]$, $\nu_A: X\to[0,1]$为X上的模糊集, $\mu_A(x)$和$\nu_A(x)$分别表示X上元素x属于A的隶属度和非隶属度,且$\forall x\in X, \mu_A(x),\nu_A(x)\in[0,1]$,有$\mu_A^2(x)+\nu_A^2(x)\leqslant 1$. 称$\pi_A(x)=\sqrt{1-\mu_A^2(x)-\nu_A^2(x)}$为$x$属于$A$的犹豫度. 称$\alpha=\langle\mu_\alpha,\nu_\alpha\rangle$为毕达哥拉斯模糊数[20]. 全体毕达哥拉斯模糊数集合记作PFN.

由定义2和定义3可以看出,直觉模糊集和毕达哥拉斯模糊集均使用隶属度和非隶属度来描述元素属于和不属于集合的程度,故二者均能从支持、反对、中立3方面描述模糊现象. 但是,二者在隶属度和非隶属度方面要求不同,即直觉模糊集要求$\mu_A(x)+\nu_A(x)\leqslant 1, \forall x\in X$,而毕达哥拉斯模糊集要求$\mu_A^2(x)+\nu_A^2(x)\leqslant 1, \forall x\in X$. 考虑到由$\mu_A(x)+\nu_A(x)\leqslant 1$一定能得到$\mu_A^2(x)+\nu_A^2(x)\leqslant 1$,而反之不成立,故而可知,毕达哥拉斯模糊集不仅推广了直觉模糊集,而且比直觉模糊集具有更强的刻画模糊现象的能力.

定义4[20] 设$\alpha=\langle\mu_\alpha,\nu_\alpha\rangle$, $\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2)$为毕达哥拉斯模糊数,定义:

1) $\alpha_1\oplus\alpha_2=\langle\sqrt{\mu_{\alpha_1}^2+\mu_{\alpha_2}^2-\mu_{\alpha_1}^2\mu_{\alpha_2}^2},\nu_{\alpha_1}\nu_{\alpha_2}\rangle$;

2) $\alpha_1\otimes\alpha_2=\langle\mu_{\alpha_1}\mu_{\alpha_2},\sqrt{\nu_{\alpha_1}^2+\nu_{\alpha_2}^2-\nu_{\alpha_1}^2\nu_{\alpha_2}^2}\rangle$;

3) $\lambda\alpha=\langle\sqrt{1-(1-\mu_\alpha^2)^\lambda},\nu_\alpha^\lambda\rangle,\ \lambda<0$;

4) $\alpha^\lambda=\langle\mu_\alpha^\lambda,\sqrt{1-(1-\nu_\alpha^2)^\lambda}\rangle,\ \lambda<0$.

定义5[20] 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2)$为毕达哥拉斯模糊数,定义$\alpha_1\geqslant\alpha_2\Leftrightarrow\mu_{\alpha_1}\geqslant\mu_{\alpha_2},\nu_{\alpha_1}\leqslant\nu_{\alpha_2}$.

定义6[20] 设$\alpha=\langle\mu_\alpha,\nu_\alpha\rangle$为毕达哥拉斯模糊数,定义$s_\alpha=\mu_\alpha^2-\nu_\alpha^2$为$\alpha$的得分函数.

定义7[20] 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2)$为毕达哥拉斯模糊数,定义: 1) 若$s_{\alpha_1}<s_{\alpha_2}$,则$\alpha_1<\alpha_2$; 2) 若$s_{\alpha_1}=s_{\alpha_2}$,则$\alpha_1\approx\alpha_2$.

2 毕达哥拉斯模糊数交叉影响运算

2.1 加法和数乘运算

设$\alpha_1=\langle\mu_{\alpha_1},0\rangle$, $\alpha_2=\langle\mu_{\alpha_2},\nu_{\alpha_2}\rangle$为毕达哥拉斯模糊数,且$\nu_{\alpha_2}\neq 0$,则由定义4中加法运算可知, $\alpha_1\oplus\alpha_2=\langle\sqrt{\mu_{\alpha_1}^2+\mu_{\alpha_2}^2-\mu_{\alpha_1}^2\mu_{\alpha_2}^2},0\rangle$,即$\nu_{\alpha_2}$在运算中完全不起作用,这与常理不符,为此定义新的加法和数乘运算.

定义8 设$\alpha_i(i=1,2)$为毕达哥拉斯模糊数,若:

1) $\mu_\alpha=\sqrt{\mu_{\alpha_1}^2+\mu_{\alpha_2}^2-\mu_{\alpha_1}^2\mu_{\alpha_2}^2}$;

2) $\nu_\alpha=\sqrt{\nu_{\alpha_1}^2+\nu_{\alpha_2}^2-\nu_{\alpha_1}^2\nu_{\alpha_2}^2-\nu_{\alpha_1}^2\mu_{\alpha_2}^2-\mu_{\alpha_1}^2\nu_{\alpha_2}^2}$;

则称$\alpha=\langle\mu_\alpha,\nu_\alpha\rangle$为$\alpha_1$与$\alpha_2$的和,记作$\alpha=\alpha_1\oplus\alpha_2$.

例1 设$\alpha_1=\langle 0.8,\ 0\rangle$, $\alpha_2=\langle 0.7,\ 0.4\rangle$为两个毕达哥拉斯模糊数,则由定义4中和运算得$\alpha_1\oplus\alpha_2=$

$\langle 0.903\,5, 0\rangle$,而由定义8中的和运算得$\alpha_1 \oplus \alpha_2 = \langle 0.903\,5, 0.24\rangle$. 显然,由定义8计算得到的结果较为合理,其受到了$\mu_{\alpha_1}, \mu_{\alpha_2}, \nu_{\alpha_1}, \nu_{\alpha_2}$之间的交叉作用,结果不完全取决于$\nu_{\alpha_1} = 0$.

定理1　设$\alpha_i(i = 1, 2)$为毕达哥拉斯模糊数,则$\alpha = \alpha_1 \oplus \alpha_2 = \langle \mu_\alpha, \nu_\alpha\rangle$也是毕达哥拉斯模糊数,且

$$\mu_\alpha = \sqrt{1 - \prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2)},$$

$$\nu_\alpha = \sqrt{\prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2) - \prod_{i=1}^{2}[1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]}.$$

证明　由$\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2\mu_{\alpha_2}^2 = 1 - (1 - \mu_{\alpha_1}^2)(1 - \mu_{\alpha_2}^2)$可得$\mu_\alpha = \sqrt{1 - \prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2)}$.

$$\begin{aligned}\mu_\alpha^2 + \nu_\alpha^2 = {}&\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2\mu_{\alpha_2}^2 + \nu_{\alpha_1}^2 + \nu_{\alpha_2}^2 - \\ &\nu_{\alpha_1}^2\nu_{\alpha_2}^2 - \nu_{\alpha_1}^2\mu_{\alpha_2}^2 - \mu_{\alpha_1}^2\nu_{\alpha_2}^2 = \\ &(\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2) + (\mu_{\alpha_2}^2 + \nu_{\alpha_2}^2) - (\mu_{\alpha_1}^2\mu_{\alpha_2}^2 + \\ &\nu_{\alpha_1}^2\nu_{\alpha_2}^2 + \nu_{\alpha_1}^2\mu_{\alpha_2}^2 + \mu_{\alpha_1}^2\nu_{\alpha_2}^2) = \\ &(\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2) + (\mu_{\alpha_2}^2 + \nu_{\alpha_2}^2) - \\ &(\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2)(\mu_{\alpha_2}^2 + \nu_{\alpha_2}^2) = \\ &1 - [1 - (\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2)][1 - (\mu_{\alpha_2}^2 + \nu_{\alpha_2}^2)],\end{aligned}$$

于是

$$\begin{aligned}&\nu_\alpha^2 = (\mu_\alpha^2 + \nu_\alpha^2) - \mu_\alpha^2 = \\ &1 - [1 - (\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2)][1 - (\mu_{\alpha_2}^2 + \nu_{\alpha_2}^2)] - \\ &[1 - (1 - \mu_{\alpha_1}^2)(1 - \mu_{\alpha_2}^2)] = \\ &\prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2) - \prod_{i=1}^{2}[1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)],\end{aligned}$$

因此

$$\nu_\alpha = \sqrt{\prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2) - \prod_{i=1}^{2}[1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]}.$$

又已知$\mu_{\alpha_1}, \nu_{\alpha_1}, \mu_{\alpha_2}, \nu_{\alpha_2} \in [0, 1]$, $\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2, \mu_{\alpha_2}^2 + \nu_{\alpha_2}^2 \in [0, 1]$,则有

$$\mu_\alpha = \sqrt{1 - \prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2)} \in [0, 1],$$

$$0 \leqslant \nu_\alpha = \sqrt{\prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2) - \prod_{i=1}^{2}[1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]} \leqslant \sqrt{\prod_{i=1}^{2}(1 - \mu_{\alpha_i}^2)} \leqslant 1.$$

于是有

$$0 \leqslant \mu_\alpha^2 + \nu_\alpha^2 = 1 - \prod_{i=1}^{2}[1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)] \leqslant 1.$$

所以$\alpha = \langle \mu_\alpha, \nu_\alpha\rangle$是毕达哥拉斯模糊数. □

定义9　设α为毕达哥拉斯模糊数,$\lambda \geqslant 0$,定义

$$\lambda\alpha = \langle \sqrt{1 - (1 - \mu_\alpha^2)^\lambda},\ \sqrt{(1 - \mu_\alpha^2)^\lambda - [1 - (\mu_\alpha^2 + \nu_\alpha^2)]^\lambda}\rangle.$$

定理2　设α为毕达哥拉斯模糊数,$\lambda \geqslant 0$,则$\lambda\alpha$也是毕达哥拉斯模糊数.

证明　已知$0 \leqslant \mu_\alpha, \nu_\alpha \leqslant 1, 0 \leqslant \mu_\alpha^2 + \nu_\alpha^2 \leqslant 1$, $\lambda \geqslant 0$,则由$\mu_\alpha \in [0, 1]$可知$1 - \mu_\alpha^2 \in [0, 1]$,考虑到$\lambda \geqslant 0$,即得$(1 - \mu_\alpha^2)^\lambda \in [0, 1]$,于是$1 - (1 - \mu_\alpha^2)^\lambda \in [0, 1]$,从而$\sqrt{1 - (1 - \mu_\alpha^2)^\lambda} \in [0, 1]$. 又因为

$$\begin{aligned}&0 \leqslant \sqrt{(1 - \mu_\alpha^2)^\lambda - [1 - (\mu_\alpha^2 + \nu_\alpha^2)]^\lambda} \leqslant \\ &\sqrt{(1 - \mu_\alpha^2)^\lambda} \leqslant 1, \\ &0 \leqslant (\sqrt{1 - (1 - \mu_\alpha^2)^\lambda})^2 + \\ &(\sqrt{(1 - \mu_\alpha^2)^\lambda - [1 - (\mu_\alpha^2 + \nu_\alpha^2)]^\lambda})^2 = \\ &1 - [1 - (\mu_\alpha^2 + \nu_\alpha^2)]^\lambda \leqslant 1,\end{aligned}$$

故$\lambda\alpha$是毕达哥拉斯模糊数. □

定理3　设$\alpha = \langle \mu_\alpha, \nu_\alpha\rangle$, $\alpha_i = \langle \mu_{\alpha_i}, \nu_{\alpha_i}\rangle(i = 1, 2)$为毕达哥拉斯模糊数,$\lambda$, λ_1, $\lambda_2 \geqslant 0$,则有: 1) $\alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1$; 2) $\lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_2 \oplus \lambda\alpha_1$; 3) $(\lambda_1 + \lambda_2)\alpha = \lambda_1\alpha \oplus \lambda_2\alpha$.

2.2　乘法和幂运算

设$\alpha_1 = \langle 0, \nu_{\alpha_1}\rangle$, $\alpha_2 = \langle \mu_{\alpha_2}, \nu_{\alpha_2}\rangle$为毕达哥拉斯模糊数,且$\mu_{\alpha_2} \neq 0$,则由定义4中乘法运算可知,$\alpha_1 \otimes \alpha_2 = \langle 0, \sqrt{\nu_{\alpha_1}^2 + \nu_{\alpha_2}^2 - \nu_{\alpha_1}^2\nu_{\alpha_2}^2}\rangle$,即$\mu_{\alpha_2}$在运算中完全不起作用,这也不符常理,为此定义新的乘法和幂运算.

定义10　设$\alpha_i(i = 1, 2)$为毕达哥拉斯模糊数,若: 1) $\mu_\alpha = \sqrt{\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2\mu_{\alpha_2}^2 - \mu_{\alpha_1}^2\nu_{\alpha_2}^2 - \nu_{\alpha_1}^2\mu_{\alpha_2}^2}$; 2) $\nu_\alpha = \sqrt{\nu_{\alpha_1}^2 + \nu_{\alpha_2}^2 - \nu_{\alpha_1}^2\nu_{\alpha_2}^2}$. 则称$\alpha = \langle \mu_\alpha, \nu_\alpha\rangle$为$\alpha_1$与$\alpha_2$的乘积,记作$\alpha = \alpha_1 \otimes \alpha_2$.

定理4　设$\alpha_i(i = 1, 2)$为毕达哥拉斯模糊数,则$\alpha = \alpha_1 \otimes \alpha_2 = \langle \mu_\alpha, \nu_\alpha\rangle$也是毕达哥拉斯模糊数,且

$$\mu_\alpha = \sqrt{\prod_{i=1}^{2}(1 - \nu_{\alpha_i}^2) - \prod_{i=1}^{2}[1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]},$$

$$\nu_\alpha = \sqrt{1 - \prod_{i=1}^{2}(1 - \nu_{\alpha_i}^2)}.$$

定义11　设α为毕达哥拉斯模糊数,$\lambda \geqslant 0$,定义

$$\begin{aligned}\alpha^\lambda = {}&\langle \sqrt{(1 - \nu_\alpha^2)^\lambda - [1 - (\mu_\alpha^2 + \nu_\alpha^2)]^\lambda}, \\ &\sqrt{1 - (1 - \nu_\alpha^2)^\lambda}\rangle.\end{aligned}$$

定理5　设α为毕达哥拉斯模糊数,$\lambda \geqslant 0$,则α^λ

也是毕达哥拉斯模糊数.

定理6 设$\alpha = \langle \mu_\alpha, \nu_\alpha \rangle$, $\alpha_i = \langle \mu_{\alpha_i}, \nu_{\alpha_i} \rangle (i = 1, 2)$为毕达哥拉斯模糊数, λ, λ_1, $\lambda_2 \geqslant 0$,则: 1) $\alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1$; 2) $(\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda$; 3) $\alpha^{\lambda_1 + \lambda_2} = \alpha^{\lambda_1} \otimes \alpha^{\lambda_2}$.

3 毕达哥拉斯模糊交叉影响集成算子

3.1 毕达哥拉斯模糊交叉影响平均算子

定义12 设$\alpha_i = \langle \mu_{\alpha_i}, \nu_{\alpha_i} \rangle (i = 1, 2, \cdots, n)$为一组毕达哥拉斯模糊数,若函数PFIWA : $\mathrm{PFN}^n \to \mathrm{PFN}$, $\mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \bigoplus_{i=1}^{n} w_i \alpha_i$, 则称PFIWA为毕达哥拉斯模糊交叉影响加权平均算子,简称PFIWA算子,其中权重向量$(w_1, w_2, \cdots, w_n)$满足$w_i \geqslant 0$且$\sum_{i=1}^{n} w_i = 1$.

定理7 设$\alpha_i = \langle \mu_{\alpha_i}, \nu_{\alpha_i} \rangle (i = 1, 2, \cdots, n)$为一组毕达哥拉斯模糊数,则

$$\mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \Big\langle \sqrt{1 - \prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i}}, \sqrt{\prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{n} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i}} \Big\rangle. \quad (1)$$

证明 当$n = 2$时,有

$$\mathrm{PFIWA}(\alpha_1, \alpha_2) = \bigoplus_{i=1}^{2} w_i \alpha_i = \langle \sqrt{1 - (1 - \mu_{\alpha_1}^2)^{w_1}}, \sqrt{(1 - \mu_{\alpha_1}^2)^{w_1} - [1 - (\mu_{\alpha_1}^2 + \nu_{\alpha_1}^2)]^{w_1}} \rangle \oplus \langle \sqrt{1 - (1 - \mu_{\alpha_2}^2)^{w_2}}, \sqrt{(1 - \mu_{\alpha_2}^2)^{w_2} - [1 - (\mu_{\alpha_2}^2 + \nu_{\alpha_2}^2)]^{w_2}} \rangle = \Big\langle \sqrt{1 - \prod_{i=1}^{2} (1 - \mu_{\alpha_i}^2)^{w_i}}, \sqrt{\prod_{i=1}^{2} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{n} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i}} \Big\rangle.$$

假设当$n = k$时,式(1)成立,则当$n = k + 1$时,有

$$\mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_{k+1}) = \bigoplus_{i=1}^{k+1} w_i \alpha_i = \bigoplus_{i=1}^{k} w_i \alpha_i \oplus w_{k+1} \alpha_{k+1} = \Big\langle \sqrt{1 - \prod_{i=1}^{k} (1 - \mu_{\alpha_i}^2)^{w_i}}, \sqrt{\prod_{i=1}^{k} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{k} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i}} \Big\rangle \oplus \langle \sqrt{1 - (1 - \mu_{\alpha_{k+1}}^2)^{w_{k+1}}}, \sqrt{(1 - \mu_{\alpha_{k+1}}^2)^{w_{k+1}} - [1 - (\mu_{\alpha_{k+1}}^2 + \nu_{\alpha_{k+1}}^2)]^{w_{k+1}}} \rangle = \Big\langle \sqrt{1 - \prod_{i=1}^{k+1} (1 - \mu_{\alpha_i}^2)^{w_i}}, \sqrt{\prod_{i=1}^{k+1} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{k+1} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i}} \Big\rangle.$$

由数学归纳法可知,式(1)成立.

由μ_{α_i}, $\nu_{\alpha_i} \in [0, 1]$且$\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2 \in [0, 1]$可知, $1 - \mu_{\alpha_i}^2 \in [0, 1]$,即有$1 - \prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i} \in [0, 1]$,从而$\sqrt{1 - \prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i}} \in [0, 1]$. 同时,有$0 \leqslant \prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{n} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i} \leqslant \prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i} \leqslant 1$,即得到

$$\sqrt{\prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{n} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i}} \in [0, 1].$$

又由

$$\Big(\sqrt{1 - \prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i}} \Big)^2 + \Big(\sqrt{\prod_{i=1}^{n} (1 - \mu_{\alpha_i}^2)^{w_i} - \prod_{i=1}^{n} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i}} \Big)^2 = 1 - \prod_{i=1}^{n} [1 - (\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2)]^{w_i} \in [0, 1]$$

可知, $\mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_n)$是毕达哥拉斯模糊数. □

定理8 设$\alpha_i = \langle \mu_{\alpha_i}, \nu_{\alpha_i} \rangle (i = 1, 2, \cdots, n)$为一组毕达哥拉斯模糊数,则:

1) 幂等性. 若$\alpha_i = \alpha = \langle \mu_\alpha, \nu_\alpha \rangle (i = 1, 2, \cdots, n)$,则$\mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \alpha$.

2) 有界性. $\alpha^- \leqslant \mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) \leqslant \alpha^+$,其中

$$\alpha^- = \langle \min_i \{\mu_{\alpha_i}\}, \sqrt{\max_i \{\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2\} - \min_i \{\mu_{\alpha_i}^2\}} \rangle,$$

$$\alpha^+ = \langle \max_i \{\mu_{\alpha_i}\}, \sqrt{\max\{0, \min_i \{\mu_{\alpha_i}^2 + \nu_{\alpha_i}^2\} - \max_i \{\mu_{\alpha_i}^2\}\}} \rangle.$$

3) 单调性. 设$\beta_i = \langle \mu_{\beta_i}, \nu_{\beta_i} \rangle (i = 1, 2, \cdots, n)$为另一组毕达哥拉斯模糊数,若$\mu_{\alpha_i} \leqslant \mu_{\beta_i}$, $\pi_{\alpha_i} \leqslant \pi_{\beta_i}$, $i = 1, 2, \cdots, n$,则$\mathrm{PFIWA}(\alpha_1, \alpha_2, \cdots, \alpha_n) \leqslant \mathrm{PFIWA}(\beta_1, \beta_2, \cdots, \beta_n)$.

证明 1) 当$\alpha_i = \alpha = \langle \mu_\alpha, \nu_\alpha \rangle (i = 1, 2, \cdots, n)$

时,有

$$\mathrm{PFIWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=$$

$$\Big\langle\sqrt{1-\prod_{i=1}^{n}(1-\mu_\alpha^2)^{w_i}},$$

$$\sqrt{\prod_{i=1}^{n}(1-\mu_\alpha^2)^{w_i}-\prod_{i=1}^{n}[1-(\mu_\alpha^2+\nu_\alpha^2)]^{w_i}}\Big\rangle=$$

$$\Big\langle\sqrt{1-(1-\mu_\alpha^2)^{\sum\limits_{i=1}^{n}w_i}},$$

$$\sqrt{(1-\mu_\alpha^2)^{\sum\limits_{i=1}^{n}w_i}-[1-(\mu_\alpha^2+\nu_\alpha^2)]^{\sum\limits_{i=1}^{n}w_i}}\Big\rangle=$$

$$\langle\sqrt{\mu_\alpha^2},\sqrt{\nu_\alpha^2}\rangle=\langle\mu_\alpha,\nu_\alpha\rangle=\alpha.$$

2) 设$\alpha=\mathrm{PFIWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)$. 由$\min\limits_i\{\mu_{\alpha_i}\}\leqslant\mu_{\alpha_i}$可得$1-(\min\limits_i\{\mu_{\alpha_i}\})^2\geqslant1-\mu_{\alpha_i}^2$,即有

$$(1-(\min_i\{\mu_{\alpha_i}\})^2)^{w_i}\geqslant(1-\mu_{\alpha_i}^2)^{w_i},$$

从而有$1-(\min\limits_i\{\mu_{\alpha_i}\})^2=\prod\limits_{i=1}^{n}(1-(\min\limits_i\{\mu_{\alpha_i}\})^2)^{w_i}\geqslant\prod\limits_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}$,于是$(\min\limits_i\{\mu_{\alpha_i}\})^2\leqslant1-\prod\limits_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}$,即得

$$\min_i\{\mu_{\alpha_i}\}\leqslant\sqrt{1-\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}}.$$

类似地,由$\mu_{\alpha_i}\leqslant\max\limits_i\{\mu_{\alpha_i}\}$,可得

$$\max_i\{\mu_{\alpha_i}\}\geqslant\sqrt{1-\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}}.$$

由于

$$\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}-\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i}\geqslant$$

$$\prod_{i=1}^{n}(1-(\max_i\{\mu_{\alpha_i}\})^2)^{w_i}-$$

$$\prod_{i=1}^{n}[1-\min_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}]^{w_i}=$$

$$\min_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}-(\max_i\{\mu_{\alpha_i}\})^2,$$

考虑到$(\max\limits_i\{\mu_{\alpha_i}\})^2=\max\limits_i\{\mu_{\alpha_i}^2\}$及$\min\limits_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}-(\max\limits_i\{\mu_{\alpha_i}\})^2$可能为负数,则有

$$\sqrt{\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}-\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i}}\geqslant$$

$$\sqrt{\max\{0,\min_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}-\max_i\{\mu_{\alpha_i}^2\}\}}.$$

类似可得

$$\sqrt{\max_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}-\min_i\{\mu_{\alpha_i}^2\}}\geqslant$$

$$\sqrt{\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}-\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i}}.$$

于是有

$$\min_i\{\mu_{\alpha_i}^2\}-[\max_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}-\min_i\{\mu_{\alpha_i}^2\}]\leqslant$$

$$\Big[1-\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}\Big]-\Big[\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}-$$

$$\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i}\Big]\leqslant$$

$$\max_i\{\mu_{\alpha_i}^2\}-\max\{0,\min_i\{\mu_{\alpha_i}^2+\nu_{\alpha_i}^2\}-\max_i\{\mu_{\alpha_i}^2\}\}.$$

即有$s_{\alpha^-}\leqslant s_\alpha\leqslant s_{\alpha^+}$,即得$\alpha^-\leqslant\mathrm{PFIWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\alpha^+$.

3) 设 $\alpha=\mathrm{PFIWA}(\alpha_1,\alpha_2,\cdots,\alpha_n),\beta=\mathrm{PFIWA}(\beta_1,\beta_2,\cdots,\beta_n)$,则有

$$s_\alpha=1-2\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}+\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i},$$

$$s_\beta=1-2\prod_{i=1}^{n}(1-\mu_{\beta_i}^2)^{w_i}+\prod_{i=1}^{n}[1-(\mu_{\beta_i}^2+\nu_{\beta_i}^2)]^{w_i}.$$

由$\mu_{\alpha_i}\leqslant\mu_{\beta_i}$可得$1-\mu_{\alpha_i}^2\geqslant1-\mu_{\beta_i}^2$,继而有

$$\prod_{i}^{n}(1-\mu_{\alpha_i}^2)^{w_i}\geqslant\prod_{i}^{n}(1-\mu_{\beta_i}^2)^{w_i},$$

于是得到$1-2\prod\limits_{i}^{n}(1-\mu_{\alpha_i}^2)^{w_i}\leqslant1-2\prod\limits_{i}^{n}(1-\mu_{\beta_i}^2)^{w_i}$.

由$\pi_{\alpha_i}\leqslant\pi_{\beta_i}$可知

$$\sqrt{1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)}\leqslant\sqrt{1-(\mu_{\beta_i}^2+\nu_{\beta_i}^2)},$$

即得到$1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)\leqslant1-(\mu_{\beta_i}^2+\nu_{\beta_i}^2)$,从而有

$$\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i}\leqslant\prod_{i=1}^{n}[1-(\mu_{\beta_i}^2+\nu_{\beta_i}^2)]^{w_i}.$$

于是得到

$$1-2\prod_{i=1}^{n}(1-\mu_{\alpha_i}^2)^{w_i}+\prod_{i=1}^{n}[1-(\mu_{\alpha_i}^2+\nu_{\alpha_i}^2)]^{w_i}\leqslant$$

$$1-2\prod_{i=1}^{n}(1-\mu_{\beta_i}^2)^{w_i}+\prod_{i=1}^{n}[1-(\mu_{\beta_i}^2+\nu_{\beta_i}^2)]^{w_i},$$

即有$s_\alpha\leqslant s_\beta$,所以$\mathrm{PFIWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\mathrm{PFIWA}(\beta_1,\beta_2,\cdots,\beta_n)$. □

定义13 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,若函数$\mathrm{PFIOWA}:\mathrm{PFN}^n\to\mathrm{PFN}$, $\mathrm{PFIOWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\bigoplus\limits_{j=1}^{n}w_j\alpha_{\sigma(j)}$,则称PFIOWA为毕达哥拉斯模糊交叉影响有序加权平均算子,简称PFIOWA算子,其中$(\sigma(1),\sigma(2),\cdots,\sigma(n))$为$(1,2,\cdots,n)$的一个置换,且$(\sigma(j-1)\geqslant\sigma(j)(j=1,2,\cdots,n)$,权重向量$(w_1,w_2,\cdots,w_n)$满足$w_j\geqslant0$且$\sum\limits_{j=1}^{n}w_j=1$.

定理9 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,则

$$\mathrm{PFIOWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=$$
$$\Big\langle\sqrt{1-\prod_{j=1}^{n}(1-\mu^2_{\alpha_{\sigma(j)}})^{w_j}},$$
$$\sqrt{\prod_{j=1}^{n}(1-\mu^2_{\alpha_{\sigma(j)}})^{w_j}-\prod_{j=1}^{n}[1-(\mu^2_{\alpha_{\sigma(j)}}+\nu^2_{\alpha_{\sigma(j)}})]^{w_j}}\Big\rangle.$$

定理10 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,则:

1) 幂等性.若$\alpha_i=\alpha=\langle\mu_\alpha,\nu_\alpha\rangle(i=1,2,\cdots,n)$,则$\mathrm{PFIOWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\alpha$.

2) 有界性.$\alpha^-\leqslant\mathrm{PFIOWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\alpha^+$,其中

$$\alpha^-=\langle\min_i\{\mu_{\alpha_i}\},\sqrt{\max_i\{\mu^2_{\alpha_i}+\nu^2_{\alpha_i}\}-\min_i\{\mu^2_{\alpha_i}\}}\rangle,$$
$$\alpha^+=$$
$$\langle\max_i\{\mu_{\alpha_i}\},\sqrt{\max\{0,\min_i\{\mu^2_{\alpha_i}+\nu^2_{\alpha_i}\}-\max_i\{\mu^2_{\alpha_i}\}\}}\rangle.$$

3) 单调性.设$\beta_i=\langle\mu_{\beta_i},\nu_{\beta_i}\rangle(i=1,2,\cdots,n)$为另一组毕达哥拉斯模糊数,若$\mu_{\alpha_{\sigma(j)}}\leqslant\mu_{\beta_{\sigma(j)}},\pi_{\alpha_{\sigma(j)}}\leqslant\pi_{\beta_{\sigma(j)}}$, $j=1,2,\cdots,n$,则$\mathrm{PFIWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\mathrm{PFIWA}(\beta_1,\beta_2,\cdots,\beta_n)$.

4) 置换不变性.设$(\beta_1,\beta_2,\cdots,\beta_n)$为$(\alpha_1,\alpha_2,\cdots,\alpha_n)$的一个置换,则$\mathrm{PFIOWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\mathrm{PFIOWA}(\beta_1,\beta_2,\cdots,\beta_n)$.

3.2 毕达哥拉斯模糊交叉影响几何算子

定义14 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,若函数$\mathrm{PFIWG}:\mathrm{PFN}^n\to\mathrm{PFN}$, $\mathrm{PFIWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\bigotimes_{i=1}^{n}\alpha_i^{w_i}$,则称PFIWG为毕达哥拉斯模糊交叉影响加权几何算子,简称PFIWG算子,其中权重向量$(w_1,w_2,\cdots,w_n)$满足$w_i\geqslant0$且$\sum_{i=1}^{n}w_i=1$.

定理11 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,则

$$\mathrm{PFIWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=$$
$$\Big\langle\sqrt{\prod_{i=1}^{n}(1-\nu^2_{\alpha_i})^{w_i}-\prod_{i=1}^{n}[1-(\mu^2_{\alpha_i}+\nu^2_{\alpha_i})]^{w_i}},$$
$$\sqrt{1-\prod_{i=1}^{n}(1-\nu^2_{\alpha_i})^{w_i}}\Big\rangle. \tag{2}$$

定理12 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,则:

1) 幂等性. 若

$$\alpha_i=\alpha=\langle\mu_\alpha,\nu_\alpha\rangle,\ i=1,2,\cdots,n),$$

则$\mathrm{PFIWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\alpha$.

2) 有界性.$\alpha^-\leqslant\mathrm{PFIWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\alpha^+$,其中

$$\alpha^-=$$
$$\langle\sqrt{\max\{0,\min_i\{\mu^2_{\alpha_i}+\nu^2_{\alpha_i}\}-\max_i\{\nu^2_{\alpha_i}\}\}},\max_i\{\nu_{\alpha_i}\}\rangle,$$
$$\alpha^+=\langle\sqrt{\max_i\{\mu^2_{\alpha_i}+\nu^2_{\alpha_i}\}-\min_i\{\nu^2_{\alpha_i}\}},\min_i\{\nu_{\alpha_i}\}\rangle.$$

3) 单调性.设$\beta_i=\langle\mu_{\beta_i},\nu_{\beta_i}\rangle(i=1,2,\cdots,n)$为另一组毕达哥拉斯模糊数,若$\nu_{\alpha_i}\leqslant\nu_{\beta_i},\pi_{\alpha_i}\leqslant\pi_{\beta_i}$, $i=1,2,\cdots,n$,则$\mathrm{PFIWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)\geqslant\mathrm{PFIWG}(\beta_1,\beta_2,\cdots,\beta_n)$.

定义15 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,若函数$\mathrm{PFIOWG}:\mathrm{PFN}^n\to\mathrm{PFN}$, $\mathrm{PFIOWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\bigoplus_{j=1}^{n}\alpha_{\sigma(j)}^{w_j}$,则称PFIOWG为毕达哥拉斯模糊交叉影响有序加权几何算子,简称PFIOWG算子,其中$(\sigma(1),\sigma(2),\cdots,\sigma(n))$为$(1,2,\cdots,n)$的一个置换,且$\sigma(j-1)\geqslant\sigma(j)(j=1,2,\cdots,n)$,权重向量$(w_1,w_2,\cdots,w_n)$满足$w_j\geqslant0$且$\sum_{j=1}^{n}w_j=1$.

定理13 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,则:

$$\mathrm{PFIOWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=$$
$$\Big\langle\sqrt{\prod_{j=1}^{n}(1-\nu^2_{\alpha_{\sigma(j)}})^{w_j}-\prod_{j=1}^{n}[1-(\mu^2_{\alpha_{\sigma(j)}}+\nu^2_{\alpha_{\sigma(j)}})]^{w_j}},$$
$$\sqrt{1-\prod_{j=1}^{n}(1-\nu^2_{\alpha_{\sigma(j)}})^{w_j}}\Big\rangle. \tag{3}$$

定理14 设$\alpha_i=\langle\mu_{\alpha_i},\nu_{\alpha_i}\rangle(i=1,2,\cdots,n)$为一组毕达哥拉斯模糊数,则:

1) 幂等性.若$\alpha_i=\alpha=\langle\mu_\alpha,\nu_\alpha\rangle(i=1,2,\cdots,n)$,则$\mathrm{PFIOWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\alpha$.

2) 有界性.$\alpha^-\leqslant\mathrm{PFIOWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\alpha^+$,其中

$$\alpha^-=$$
$$\langle\sqrt{\max\{0,\min_i\{\mu^2_{\alpha_i}+\nu^2_{\alpha_i}\}-\max_i\{\nu^2_{\alpha_i}\}\}},\max_i\{\nu_{\alpha_i}\}\rangle,$$
$$\alpha^+=\langle\sqrt{\max_i\{\mu^2_{\alpha_i}+\nu^2_{\alpha_i}\}-\min_i\{\nu^2_{\alpha_i}\}},\min_i\{\nu_{\alpha_i}\}\rangle.$$

3) 单调性.设$\beta_i=\langle\mu_{\beta_i},\nu_{\beta_i}\rangle(i=1,2,\cdots,n)$为另一组毕达哥拉斯模糊数,若$\nu_{\alpha_i}\leqslant\nu_{\beta_i},\pi_{\alpha_i}\leqslant\pi_{\beta_i}$, $i=1,2,\cdots,n$,则$\mathrm{PFIOWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)\geqslant\mathrm{PFIOWG}(\beta_1,\beta_2,\cdots,\beta_n)$.

4) 置换不变性.设$(\beta_1,\beta_2,\cdots,\beta_n)$为$(\alpha_1,\alpha_2,\cdots,\alpha_n)$的一个置换,则$\mathrm{PFIOWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\mathrm{PFIOWG}(\beta_1,\beta_2,\cdots,\beta_n)$.

4 决策应用

设方案集为$X = \{x_1, x_2, \cdots, x_m\}$,属性集为$C = \{c_1, c_2, \cdots, c_n\}$,$(w_1, w_2, \cdots, w_n)$为属性权重向量,其中$w_j \geqslant 0$且$\sum_{j=1}^{n} w_j = 1$. 决策者给出的方案$x_i$在属性$c_j$下的属性值$\alpha_{ij}$为毕达哥拉斯模糊数,于是得到毕达哥拉斯模糊数决策矩阵$M = (\alpha_{ij})_{mn}$. 下面给出基于毕达哥拉斯模糊交叉影响集成算子的决策方法,具体步骤如下.

Step 1: 由决策者根据实际情况建立毕达哥拉斯模糊决策矩阵;

Step 2: 利用毕达哥拉斯模糊交叉影响集成算子求出各方案的综合属性值;

Step 3: 由综合属性值求出各方案的得分函数;

Step 4: 根据各方案得分函数实现方案排序择优.

例2[20] 现对国内4家航空公司$X = \{x_1, x_2, x_3, x_4\}$的服务质量进行评价,经专家确定评价指标有4个,它们分别为c_1: 定售票服务; c_2: 登记程序; c_3: 客舱服务; c_4: 响应性. 指标权重向量为(0.15, 0.25, 0.35, 0.25). 经过专家评估,建立了毕达哥拉斯模糊数决策矩阵$M = (\alpha_{ij})_{44}$,其中评价值$\alpha_{ij} = \langle \mu_{\alpha_{ij}}, \nu_{\alpha_{ij}} \rangle (i, j = 1, 2, 3, 4)$为毕达哥拉斯模糊数. 根据专家提供的决策矩阵,评价这4家国内航空公司的服务质量.

Step 1: 建立毕达哥拉斯模糊决策矩阵,见表1.

表1 毕达哥拉斯模糊决策矩阵

	c_1	c_2	c_3	c_4
x_1	$\langle 0.9, 0.3 \rangle$	$\langle 0.7, 0.6 \rangle$	$\langle 0.5, 0.8 \rangle$	$\langle 0.6, 0.3 \rangle$
x_2	$\langle 0.4, 0.7 \rangle$	$\langle 0.9, 0.2 \rangle$	$\langle 0.8, 0.1 \rangle$	$\langle 0.5, 0.3 \rangle$
x_3	$\langle 0.8, 0.4 \rangle$	$\langle 0.7, 0.5 \rangle$	$\langle 0.6, 0.2 \rangle$	$\langle 0.7, 0.4 \rangle$
x_4	$\langle 0.7, 0.2 \rangle$	$\langle 0.8, 0.2 \rangle$	$\langle 0.8, 0.4 \rangle$	$\langle 0.6, 0.6 \rangle$

Step 2: 由PFIWA算子进行集成得到各方案综合属性值为

$$x_1 = \langle 0.6836, 0.5979 \rangle,$$

$$x_2 = \langle 0.7625, 0.2945 \rangle,$$

$$x_3 = \langle 0.6899, 0.4041 \rangle,$$

$$x_4 = \langle 0.7497, 0.3998 \rangle.$$

Step 3: 各方案的得分函数为

$$s_{x_1} = 0.1098,\ s_{x_2} = 0.4947,$$

$$s_{x_3} = 0.3127,\ s_{x_4} = 0.4022.$$

Step 4: 根据得分函数,方案优劣排序为$x_2 \succ x_4 \succ x_3 \succ x_1$,即$x_2$最优.

为了考察不同集成算子和不同决策方法对集成算子的影响,分别选择文献[18]中的PFWA算子,文献[26]中的PFWA算子以及文[20]中的TOPSIS方法,计算出方案综合属性值,并将综合属性值和排序结果列入表2.

表2 方案综合属性值和方案排序

集成算子	x_1	x_2	x_3	x_4	方案排序
本文中PFIWA	$\langle 0.6836, 0.5979 \rangle$	$\langle 0.7625, 0.2945 \rangle$	$\langle 0.6899, 0.4041 \rangle$	$\langle 0.7497, 0.3998 \rangle$	$x_2 \succ x_4 \succ x_3 \succ x_1$
文献[18]中PFWA	$\langle 0.6350, 0.5500 \rangle$	$\langle 0.6900, 0.2650 \rangle$	$\langle 0.6800, 0.3550 \rangle$	$\langle 0.7350, 0.3700 \rangle$	$x_2 \succ x_4 \succ x_3 \succ x_1$
文献[26]中PFWA	$\langle 0.6836, 0.5029 \rangle$	$\langle 0.7625, 0.2096 \rangle$	$\langle 0.6899, 0.3318 \rangle$	$\langle 0.7497, 0.3355 \rangle$	$x_2 \succ x_4 \succ x_3 \succ x_1$
文献[20]中TOPSIS	—	—	—	—	$x_2 \succ x_3 \succ x_4 \succ x_1$

由表2可知,虽然使用考虑交叉影响的PFIWA算子与使用文献[18]中PFWA算子和文献[26]中PFWA算子所得方案综合属性值不同,但是最终排序完全相同,均为$x_2 \succ x_4 \succ x_3 \succ x_1$. 同时,利用文献[20]中TOPSIS方法得到方案排序结果与3种集成算子结果不尽相同,但是最优方案都是x_2,这表明了利用交叉影响PFIWA算子进行决策的有效性.

现在讨论交叉影响PFIWA算子的稳定性. 不妨将方案x_4的第1个属性值$\langle 0.7, 0.2 \rangle$变为$\langle 0.7, 0 \rangle$,其余属性值不发生变化时,使用文献[26]中的PFWA算子,可得方案x_4的综合属性值为$\langle 0.7497, 0 \rangle$,得分函数为$s_{x_4} = 0.5621$,从而方案排序为$x_4 \succ x_2 \succ x_3 \succ x_1$. 而考虑本文提出的PFIWA算子,可得方案$x_4$的综合属性值为$\langle 0.7497, 0.3955 \rangle$,得分函数为$s_{x_4} = 0.4056$,方案排序仍为$x_2 \succ x_4 \succ x_3 \succ x_1$. 这说明交叉影响PFIWA算子的集成结果具有很好的稳定性,特别是当某个集成元素的非隶属度为0时,其优越性尤为突出. 因此,根据此例计算结果,提出如下建议: 在进行信息集成时,若非隶属度中没有0,则使用交叉影响集成算子和不考虑交叉影响的集成算子均可;但是,当非隶属度中存在0时,优先考虑本文提出的交叉影响集成算子.

5 结 论

本文定义了体现出隶属度和非隶属度之间可能存在的交叉影响的毕达哥拉斯模糊数的加法、乘法、数乘和幂运算;然后,提出了PFIWA算子、PFIOWA算子、PFIWG算子和PFIOWG算子,推导出其计算公式,

并研究了它们的性质;最后通过决策实例和比较分析,表明了基于交叉影响毕达哥拉斯模糊集成算子的有效性和稳定性. 本文的研究使得毕达哥拉斯模糊数运算更加合理,同时也丰富了毕达哥拉斯模糊集成算子理论.

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（责任编辑：孙艺红）