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基于线性分配和Choquet积分的多值区间中智多属性决策方法

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摘 要: 提出一种新的多值区间中智多属性决策方法. 首先利用多值区间中智集描述不确定信息,定义多值区间中智值之间的余弦值和欧氏距离;然后采用Choquet积分描述属性之间的相关性,采用线性分配方法对方案进行排序,给出具体的方案排序方法;最后通过基于风险的地铁项目建设方案选择表明所提出方法的可行性和有效性.

关键词: 线性分配; 多属性决策; 多值区间中智数; Choquet积分

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New multi-valued interval neutrosophic multiple attribute decision-making method based on linear assignment and Choquet integral

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Abstract: A new multi-valued interval neutrosophic multiple attribute decision making method is developed. Firstly, multi-valued interval neutrosophic numbers are used to model uncertain information, the cossine value and Euclidean distance for multi-valued interval neutrosophic numbers are defined. Then the Choquet integral is used to model correlation of information, and the linear assignment method is used to rank alternatives based on signed distances. The selection of subway construction alternatives based on risk evaluation illustrate the feasibility and effectiveness of the proposed method.

Keywords: linear assignment; multiple attribute decision making; interval neutrosophic number; Choquet integral

0 引 言

随着社会、经济的发展,人们面临的决策问题越来越复杂,所需要的专业知识也越来越多,对于大型复杂决策问题需要来自不同领域的决策者共同参与决策. 由于决策者知识结构和专业领域的限制,决策者不可能掌握决策过程中所需要的全部信息. 为了保证决策结果的科学性和合理性,对于决策者熟悉的属性,决策者可以给出适当的评价值,对于决策者不熟悉的属性,决策者可以拒绝给出评价值. 由于人类思维的模糊性,决策者在评价方案时,很难采用精确值进行评价,更愿意采用区间值. 此外,决策者在评价时会表现出犹豫性和不确定性,采用隶属度、犹豫度和非隶属度可以更加准确地描述决策过程中的模糊性、犹豫性和不确定性. 对于熟悉的属性,决策者给出合理的评价值,对于不熟悉的属性,决策者可以拒绝评价,因此所获得的决策信息可以采用多值区间中智集表示. 例如,在评价某汽车的安全性能时,一位决策者给出的满意度为[0.4,0.5],不满意度为[0.3,0.4],不确定度为[0.2,0.3],另一位决策者给出的满意度为[0.5,0.6],不满意度为[0.3,0.5],不确定度为[0.2,0.3]. 所获得的决策信息为多值区间中智值

$$a=\langle\{[0.4,0.5],[0.5,0.6]\},\{[0.2,0.3]\},\{[0.3,0.4],[0.3,0.5]\}\rangle.$$

中智集由Smarandache[1]首次提出,是对模糊集、直觉模糊集等集合的推广,采用隶属度、犹豫度和非隶属度3个元素描述不确定信息. 目前中智集已经获得了广泛的研究和应用[2-8]. 文献[2]给出了单值中智集. 文献[3]给出了区间值中智集. 文献[4]给出了单值中智环境下的相关系数度量. 文献[5]给出了基于

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多值中智集的TODIM方法. 文献[6]将TOPSIS方法推广到区间值中智环境. 文献[7]给出了区间中智犹豫模糊集,并给出广义混合平均算子. 文献[8]给出了基于广义加权平均算子的区间值中智集结算子. 虽然研究者给出了多种基于中智值的多属性决策方法,但是目前对于多值区间中智集的研究较少. 相对于已有的建模不确定信息的工具,多值区间中智集更加准确、灵活,本文采用多值区间中智集评价方案. 线性分配法为文献[9]提出的决策方法. 该方法针对每个属性对方案排序,建立方案排序的频数矩阵,利用频数矩阵建立线性规划模型,从而获得方案的整体排序. 线性分配法已被推广到模糊环境[10]、区间值直觉模糊环境[11]、二型模糊环境[12],但是多值区间中智模糊环境下的线性分配法的研究较少见到,有必要对其进行相关探讨.

本文采用多值区间中智集建模决策过程中存在的模糊性、犹豫性和不确定信息. 首先定义多值区间中智数之间的余弦值,给出基于余弦值的比较方法,并给出多值区间中智数之间的欧氏距离; 然后给出基于Choquet积分和多值区间中智值的线性分配方法; 最后将所提出的方法应用到地铁项目风险评价上,并将其与基于TOPSIS的方法相比较以表明算法的合理性.

1　基础知识

定义1[1]　设X为对象集,x为对象集X中的一个元素,X的一个多值中智集A可以表示为

$$A=\{\langle x,T_A(x),I_A(x),F_A(x)\rangle \mid x\in X\},$$

其中$T_A(x)$、$I_A(x)$、$F_A(x)$是属于区间$[0,1]$的有限离散值的集合,分别表示隶属度、犹豫度、非隶属度,且满足

$$0\leqslant \gamma,\eta,\xi\leqslant 1,\ \gamma\in T_A(x),\ \eta\in I_A(x),\ \xi\in F_A(x),$$
$$0\leqslant \sup T_A(x)+\sup I_A(x)+\sup F_A(x)\leqslant 3.$$

定义2[3]　设X为对象集,x为对象集X中的一个元素,X的一个多值区间中智集$\tilde{A}$可以表示为

$$\tilde{A}=\{\langle x,\tilde{T}_{\tilde{A}}(x),\tilde{I}_{\tilde{A}}(x),\tilde{F}_{\tilde{A}}(x)\rangle \mid x\in X\},$$

其中$\tilde{T}_{\tilde{A}}(x)$、$\tilde{I}_{\tilde{A}}(x)$、$\tilde{F}_{\tilde{A}}(x)$是由属于区间$[0,1]$的子集构成的集合,分别表示隶属度、犹豫度、非隶属度,且满足

$$[\gamma^L,\gamma^U]\in\tilde{T}_{\tilde{A}}(x),\ [\eta^L,\eta^U]\in\tilde{I}_{\tilde{A}}(x),\ [\xi^L,\xi^U]\in\tilde{F}_{\tilde{A}}(x),$$
$$0\leqslant \sup(\tilde{T}_{\tilde{A}}(x))+\sup(\tilde{I}_{\tilde{A}}(x))+\sup(\tilde{F}_{\tilde{A}}(x))\leqslant 3.$$

采用$a=\langle\tilde{T}_{\tilde{A}},\tilde{I}_{\tilde{A}},\tilde{F}_{\tilde{A}}\rangle$表示多值区间中智集合中的元素,称为多值区间中智数. 如果$\tilde{T}_{\tilde{A}}$、$\tilde{I}_{\tilde{A}}$、$\tilde{F}_{\tilde{A}}$每个集合中只有一个值,则多值区间中智数成为单值区间中智数. 如果$\tilde{I}_{\tilde{A}}=\varnothing,\tilde{F}_{\tilde{A}}=\varnothing$,则多值区间中智数成为区间犹豫模糊数. 如果$\tilde{I}_{\tilde{A}}=\varnothing$,则多值区间中智数成为广义区间犹豫直觉模糊数.

定义3[3]　设a和b分别为多值区间中智数,$a=(\tilde{T}_{\tilde{A}},\tilde{I}_{\tilde{A}},\tilde{F}_{\tilde{A}})$,$b=(\tilde{T}_{\tilde{B}},\tilde{I}_{\tilde{B}},\tilde{F}_{\tilde{B}})$,$[\gamma_a^L,\gamma_a^U]\in\tilde{T}_{\tilde{A}}$,$[\eta_a^L,\eta_a^U]\in\tilde{I}_{\tilde{A}}$,$[\xi_a^L,\xi_a^U]\in\tilde{F}_{\tilde{A}}$,$[\gamma_b^L,\gamma_b^U]\in\tilde{T}_{\tilde{B}}$,$[\eta_b^L,\eta_b^U]\in\tilde{I}_{\tilde{B}}$,$[\xi_b^L,\xi_b^U]\in\tilde{F}_{\tilde{B}}$,$\lambda>0$. 基于区间值直觉模糊集和犹豫模糊集的运算律,定义如下运算律:

1) $a\oplus b=\langle\bigcup\{[\gamma_a^L+\gamma_b^L-\gamma_a^L\gamma_b^L,\gamma_a^U+\gamma_b^U-\gamma_a^U\gamma_b^U]\},\bigcup\{[\eta_a^L\eta_b^L,\eta_a^U\eta_b^U]\},\bigcup\{[\xi_a^L\xi_b^L,\xi_a^U\xi_b^U]\}\rangle$;

2) $a\otimes b=\langle\bigcup\{[\gamma_a^L\gamma_b^L,\gamma_a^U\gamma_b^U]\},\bigcup\{[\eta_a^L+\eta_b^L-\eta_a^L\eta_b^L,\eta_a^U+\eta_b^U-\eta_a^U\eta_b^U]\},\bigcup\{[\xi_a^L+\xi_b^L-\xi_a^L\xi_b^L,\xi_a^U+\xi_b^U-\xi_a^U\xi_b^U]\}\rangle$;

3) $\lambda a=\langle\bigcup\{[1-(1-\gamma_a^L)^\lambda,1-(1-\gamma_a^U)^\lambda]\},\bigcup\{[(\eta_a^L)^\lambda,(\eta_a^U)^\lambda]\},\bigcup\{[(\xi_a^L)^\lambda,(\xi_a^U)^\lambda]\}\rangle$;

4) $a^\lambda=\langle\bigcup\{[(T_a^L)^\lambda,(T_a^U)^\lambda]\},\bigcup\{[1-(1-I_a^L)^\lambda,1-(1-I_a^U)^\lambda]\},\bigcup\{[1-(1-F_a^L)^\lambda,1-(1-F_a^U)^\lambda]\}\rangle$.

定义4[13]　设$m=[m^L,m^U]$和$n=[n^L,n^U]$为区间数,$l_m=m^U-m^L,l_n=n^U-n^L$,则$m>n$的可能度定义为

$$p(a>b)=\max\Big\{1-\max\Big\{\frac{n^U-m^L}{l_m+l_n},0\Big\},0\Big\}.\quad(1)$$

根据区间数的可能度比较两个区间数的大小.

由于不同的多值区间中智数可能含有不同数目的隶属度、犹豫度和非隶属度,为了更加准确地计算两个多值区间中智数a、b的余弦值$\cos(a,b)$和距离$d(a,b)$,根据决策者的风险态度将多值区间中智数进行拓展,使每个多值区间中智数拥有相同数目的隶属度、犹豫度和非隶属度. 如果决策者是风险偏好的,则增加最大的隶属度、最小的犹豫度和非隶属度; 如果决策者是风险厌恶的,则增加最小的隶属度、最大的犹豫度和非隶属度; 如果决策者是风险中性的,则增加隶属度、犹豫度和非隶属度的平均值.

定义5　设a和b分别为多值区间中智数,a与b之间的余弦值$\cos(a,b)$定义为

$$\cos(a,b)=\frac{\gamma+\eta+\xi}{\parallel a\parallel\cdot\parallel b\parallel}.\quad(2)$$

其中

$$\gamma=\frac{1}{l_k}\sum_{t=1}^{l_k}(\gamma_{a\sigma(t)}^L\gamma_{b\sigma(t)}^L+\gamma_{a\sigma(t)}^U\gamma_{b\sigma(t)}^U),$$

$$\eta=\frac{1}{l_i}\sum_{t=1}^{l_i}(\eta_{a\sigma(t)}^L\eta_{b\sigma(t)}^L+\eta_{a\sigma(t)}^U\eta_{b\sigma(t)}^U),$$

$$\xi=\frac{1}{l_j}\sum_{t=1}^{l_j}(\xi_{a\sigma(t)}^L\xi_{b\sigma(t)}^L+\xi_{a\sigma(t)}^U\xi_{b\sigma(t)}^U),$$

$$\|a\|=$$
$$\Big(\frac{1}{l_k}\sum_{t=1}^{l_k}\big((\gamma^L_{a\sigma(t)})^2+(\gamma^U_{a\sigma(t)})^2\big)+\frac{1}{l_i}\sum_{t=1}^{l_i}((\eta^L_{a\sigma(t)})^2+$$
$$(\eta^U_{a\sigma(t)})^2)+\frac{1}{l_j}\sum_{t=1}^{l_j}((\xi^L_{a\sigma(t)})^2+(\xi^U_{a\sigma(t)})^2)\Big)^{1/2},$$
$$\|b\|=\Big(\frac{1}{l_k}\sum_{t=1}^{l_k}\big((\gamma^L_{b\sigma(t)})^2+(\gamma^U_{b\sigma(t)})^2\big)+$$
$$\frac{1}{l_i}\sum_{t=1}^{l_i}((\eta^L_{b\sigma(t)})^2+(\eta^U_{b\sigma(t)})^2)+$$
$$\frac{1}{l_j}\sum_{t=1}^{l_j}((\xi^L_{b\sigma(t)})^2+(\xi^U_{b\sigma(t)})^2)\Big)^{1/2}.$$

当b为理想解$S=\langle\{[1,1],\cdots,[1,1]\},\{[0,0],\cdots,[0,0]\},\{[0,0],\cdots,[0,0]\}\rangle$时,$a$与$S$之间的$\cos(a,S)$的值越大,$a$与$S$的方向越一致,有

$$\cos(a,S)=\frac{\dfrac{1}{l_k}\sum\limits_{t=1}^{l_k}(\gamma^L_{a\sigma(t)}+\gamma^U_{a\sigma(t)})}{\sqrt{2}\,\|a\|}. \tag{3}$$

定义6 设a和b为多值区间中智数,如果$\cos(a,I)\leqslant\cos(b,I)$,则$a\leqslant b$.

定义7 设a和b为多值区间中智数,a与b之间的欧氏距离定义为

$$d(a,b)=$$
$$\Big(\frac{1}{3}\Big(\frac{1}{2l_k}\sum_{t=1}^{l_k}((\gamma^L_{a\sigma(t)}-\gamma^L_{b\sigma(t)})^2+(\gamma^U_{a\sigma(t)}-$$
$$\gamma^U_{b\sigma(t)})^2)+\frac{1}{2l_i}\sum_{t=1}^{l_i}((\eta^L_{a\sigma(t)}-\eta^L_{b\sigma(t)})^2+$$
$$(\eta^U_{a\sigma(t)}-\eta^U_{b\sigma(t)})^2)+\frac{1}{2l_j}\sum_{t=1}^{l_j}((\xi^L_{a\sigma(t)}-$$
$$\xi^L_{b\sigma(t)})^2+(\xi^U_{a\sigma(t)}-\xi^U_{b\sigma(t)})^2)\Big)\Big)^{1/2}. \tag{4}$$

定理1 设a和b为多值区间中智数,则由定义7定义的距离满足如下性质:

1) $d(a,b)=0$当且仅当$a=b$;

2) $d(a,b)=d(b,a)$;

3) $d(a,b)\leqslant d(a,c)+d(c,b)$.

定义8[14] 设$P(X)$为集合X的幂集,即X的所有子集构成的集合,集合X上的测度μ是一个集值函数$\mu:P(X)\to[0,1]$,满足如下公理:

1) $\mu(\phi)=0,\ \mu(X)=1$;

2) 对于任意的$B,C\subseteq X$,$B\subseteq C$,有$\mu(B)\leqslant\mu(C)$;

3) 对于所有的$B,C\subseteq X$和$B\bigcap C=\varnothing$,$\mu(B\bigcup C)=\mu(B)+\mu(C)+\rho\mu(B)\mu(C)$,其中$\rho\in(-1,+\infty)$.

设$X=\{x_1,x_2,\cdots,x_n\}$为一个有限集. 给出如下等式确定集合X上的模糊测度以避免复杂的计算[15]:

$$\mu(X)=\begin{cases}\dfrac{1}{\rho}\Big(\prod\limits_{i=1}^{n}(1+\rho\mu(x_i))-1\Big),\ \rho\neq0;\\ \sum\limits_{i=1}^{n}\mu(x_i),\ \rho=0.\end{cases} \tag{5}$$

ρ可以利用$\mu(X)=1$和下式确定:

$$\rho=\prod_{i=1}^{n}(1+\rho\mu(x_i))-1. \tag{6}$$

2 基于Choquet积分的多值区间中智线性分配法

随着社会经济的发展,决策问题变得越来越复杂,需要来自不同专业的多位决策者参与决策. 设有一多属性决策问题,$A=\{A_1,A_2,\cdots,A_m\}$为方案集,$C=\{C_1,C_2,\cdots,C_n\}$为属性集. 多位决策者根据自己的知识和经验给出方案相对于属性的区间型隶属度、犹豫度和非隶属度构成多值区间中智评价值,所得决策矩阵为$D=(a_{ij})_{m\times n}$.

Step 1: 决策者给出方案关于属性的区间型评价值构成多值区间中智决策矩阵$D=(a_{ij})_{m\times n}$,其中$a_{ij}=\langle\tilde{T}_{a_{ij}},\tilde{I}_{a_{ij}},\tilde{F}_{a_{ij}}\rangle$,$\tilde{T}_{a_{ij}}$、$\tilde{I}_{a_{ij}}$、$\tilde{F}_{a_{ij}}$分别为决策者给出的方案$A_i$满足属性$C_j$的区间型隶属度、犹豫度和非隶属度. 根据决策者的风险态度拓展决策矩阵,使得每个多值区间中智数都含有相同数目的隶属度、犹豫度和非隶属度. 拓展矩阵记为$D'=(a'_{ij})_{m\times n}$.

Step 2: 利用$\rho=\prod\limits_{i=1}^{n}(1+\rho\mu(x_i))-1$计算$\rho$值,并根据式(5)计算属性集上的模糊测度.

Step 3: 计算多值区间中智加权决策矩阵$D''=(a''_{ij})_{m\times n}$. 其中: $a''_{i\sigma(j)}=(\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)}))a'_{i\sigma(j)}$, $A_{\sigma(j)}=\{C_{\sigma(1)},C_{\sigma(2)},\cdots,C_{\sigma(j)}\}$,$C_{\sigma(k)}$为评价值$a'_{i1}$, $a'_{i2},\cdots,a'_{in}$中第k大的评价值对应的属性.

Step 4: 确定多值区间中智正理想解为

$$H^+=\{a_1^+,a_2^+,\cdots,a_n^+\}=$$
$$\{\max_i a''_{i1},\max_i a''_{i2},\cdots,\max_i a''_{in}\}, \tag{7}$$

多值区间中智负理想解为

$$H^-=\{a_1^-,a_2^-,\cdots,a_n^-\}=$$
$$\{\min_i a''_{i1},\min_i a''_{i2},\cdots,\min_i a''_{in}\}. \tag{8}$$

Step 5: 计算符号测度S_{ij},有

$$S_{ij}=\frac{d(a''_{ij},a_j^-)}{d(a''_{ij},a_j^-)+d(a''_{ij},a_j^+)},$$
$$i=1,2,\cdots,m,j=1,2,\cdots,n. \tag{9}$$

Step 6: 针对每个属性C_j,将S_{ij} $(i=1,2,\cdots,m,$

$j=1,2,\cdots,n$)排序,并根据排序结果给出相应的方案排序.

Step 7: 确定排序频数矩阵$\varPi=(\varPi_{ij})_{m\times m}$,有

$$\varPi=\begin{bmatrix}\varPi_{11} & \varPi_{12} & \cdots & \varPi_{1m}\\ \varPi_{21} & \varPi_{22} & \cdots & \varPi_{2m}\\ \vdots & \vdots & \ddots & \vdots\\ \varPi_{m1} & \varPi_{m2} & \cdots & \varPi_{mm}\end{bmatrix},$$

其中$\varPi_{ij}$表示方案A_i排在第j位的次数.

Step 8: 定义排列矩阵$P_{m\times m}$,并建立如下线性分配模型:

$$\text{(M-1)}\ \max\ \sum_{i=1}^{m}\sum_{j=1}^{m}\varPi_{ij}P_{ij}.$$
$$\text{s.t.}\ \sum_{j=1}^{m}P_{ij}=1,\ i=1,2,\cdots,m;$$
$$\sum_{i=1}^{m}P_{ij}=1,\ j=1,2,\cdots,m;$$
$$P_{ij}=0\text{ 或者 }1,\ i,j=1,2,\cdots,m.$$

Step 9: 利用单纯形法求解线性规划模型(M-1)获得最优矩阵P,并利用排列矩阵计算出方案的最优排序.

3　实例分析

随着社会经济和城市化进程的发展,城市的规模越来越大. 人们生活水平提高,私家车的拥有量越来越大,随之而来的交通拥堵现象越来越严重. 地铁有着准时、快捷、舒适、占地小等特点,已经成为缓解拥堵的有效方法,但是地铁的建设过程中存在着各种风险. 西安地处黄土高原,并且地下有着丰富的历史遗迹,因此在地铁的建设中存在着与其他地区不同的风险. 设有一条地铁项目需要建设,决策者在5种建设方案$A_i(i=1,2,\cdots,5)$中选取一种方案,主要考虑如下风险因素: C_1技术风险, C_2环境风险, C_3公众干预风险, C_4监管风险. 采用所提出的方法对方案排序.

Step 1: 决策者给出方案关于属性的多值区间中智评价值构成决策矩阵D如表1所示. 假设决策者是风险厌恶的,对每个多值区间中智数增加最小的隶属度、最大的犹豫度和最大的非隶属度,拓展矩阵$D'=(a'_{ij})_{5\times 4}$如表2所示.

Step 2: 假设$\mu(C_1)=0.25,\mu(C_2)=0.3,\mu(C_3)=0.35,\mu(C_4)=0.2$. 利用式(6)计算$\rho=-0.2286$,再利用式(5)计算属性集的测度,有

$$\mu(C_1)=0.25,\ \mu(C_2)=0.3,\ \mu(C_3)=0.35,$$
$$\mu(C_4)=0.2,\ \mu(\{C_1,C_2\})=0.5329,$$
$$\mu(\{C_1,C_3\})=0.5800,\ \mu(\{C_1,C_4\})=0.4386,$$
$$\mu(\{C_2,C_3\})=0.6260,\ \mu(\{C_2,C_4\})=0.6260,$$
$$\mu(\{C_3,C_4\})=0.5340,\ \mu(\{C_1,C_2,C_3\})=0.8402,$$
$$\mu(\{C_1,C_2,C_4\})=0.7085,$$
$$\mu(\{C_1,C_3,C_4\})=0.7535,$$
$$\mu(\{C_2,C_3,C_4\})=0.7974,$$
$$\mu(\{C_1,C_2,C_3,C_4\})=1.0.$$

Step 3: 计算多值区间中智加权决策矩阵$D''=(a'')_{5\times 4}$如表3所示,其中

$$a''_{i\sigma(j)}=(\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)}))a'_{i\sigma(j)}.$$

为了计算a''_{11},需要对a'_{11}、a'_{12}、a'_{13}、a'_{14}排序. 由于

$$\cos(a'_{11},S)=0.6523,\ \cos(a'_{12},S)=0.4180,$$
$$\cos(a'_{13},S)=0.6275,\ \cos(a'_{14},S)=0.7798,$$

其中

$$S=\langle\{[1,1],[1,1]\},\{[0,0],[0,0]\},\{[0,0],[0,0]\}\rangle,$$

表1　决策矩阵D

	C_1	C_2
A_1	$\langle\{[0.5,0.6],[0.4,0.5]\},\{[0.2,0.3]\},\{[0.3,0.5],[0.3,0.4]\}\rangle$	$\langle\{[0.3,0.4],[0.2,0.3]\},\{[0.1,0.2]\},\{[0.5,0.6],[0.4,0.5]\}\rangle$
A_2	$\langle\{[0.6,0.7]\},\{[0.1,0.2]\},\{[0.2,0.3]\}\rangle$	$\langle\{[0.6,0.7]\},\{[0.1,0.2]\},\{[0.1,0.2]\}\rangle$
A_3	$\langle\{[0.5,0.6]\},\{[0.2,0.3]\},\{[0.3,0.4]\}\rangle$	$\langle\{[0.5,0.6]\},\{[0.2,0.4]\},\{[0.2,0.3]\}\rangle$
A_4	$\langle\{[0.7,0.8],[0.6,0.7]\},\{[0.0,0.1]\},\{[0.2,0.4]\}\rangle$	$\langle\{[0.3,0.5]\},\{[0.3,0.4]\},\{[0.3,0.4],[0.1,0.2]\}\rangle$
A_5	$\langle\{[0.7,0.8]\},\{[0.3,0.4]\},\{[0.1,0.2]\}\rangle$	$\langle\{[0.4,0.5]\},\{[0.4,0.5]\},\{[0.5,0.6]\}\rangle$

	C_3	C_4
A_1	$\langle\{[0.5,0.6],[0.4,0.5]\},\{[0.1,0.2]\},\{[0.3,0.4]\}\rangle$	$\langle\{[0.8,0.9],[0.6,0.7]\},\{[0.1,0.2]\},\{[0.2,0.3]\}\rangle$
A_2	$\langle\{[0.3,0.4]\},\{[0.1,0.2]\},\{[0.2,0.3],[0.1,0.2]\}\rangle$	$\langle\{[0.3,0.4]\},\{[0.4,0.5],[0.3,0.4]\},\{[0.5,0.6],[0.4,0.5]\}\rangle$
A_3	$\langle\{[0.5,0.6],[0.3,0.4]\},\{[0.2,0.4]\},\{[0.2,0.3]\}\rangle$	$\langle\{[0.6,0.8]\},\{[0.5,0.6]\},\{[0.3,0.4]\}\rangle$
A_4	$\langle\{[0.7,0.8]\},\{[0.0,0.1]\},\{[0.1,0.2]\}\rangle$	$\langle\{[0.3,0.4],[0.2,0.3]\},\{[0.2,0.3]\},\{[0.5,0.6]\}\rangle$
A_5	$\langle\{[0.2,0.3]\},\{[0.4,0.5]\},\{[0.3,0.5]\}\rangle$	$\langle\{[0.4,0.5]\},\{[0.2,0.3],[0.1,0.3]\},\{[0.2,0.3]\}\rangle$

表 2　决策矩阵 D'

	C_1
A_1	$\langle\{[0.5,0.6],[0.4,0.5]\},\{[0.2,0.3],[0.5,0.6]\},\{[0.3,0.5],[0.3,0.4]\}\rangle$
A_2	$\langle\{[0.6,0.7],[0.2,0.3]\},\{[0.1,0.2],[0.5,0.6]\},\{[0.2,0.3],[0.5,0.6]\}\rangle$
A_3	$\langle\{[0.5,0.6],[0.2,0.3]\},\{[0.2,0.3],[0.5,0.6]\},\{[0.3,0.4],[0.5,0.6]\}\rangle$
A_4	$\langle\{[0.7,0.8],[0.6,0.7]\},\{[0.0,0.1],[0.5,0.6]\},\{[0.2,0.4],[0.5,0.6]\}\rangle$
A_5	$\langle\{[0.7,0.8],[0.2,0.3]\},\{[0.3,0.4],[0.5,0.6]\},\{[0.1,0.2],[0.5,0.6]\}\rangle$
	C_2
A_1	$\langle\{[0.3,0.4],[0.2,0.3]\},\{[0.1,0.2],[0.5,0.6]\},\{[0.5,0.6],[0.4,0.5]\}\rangle$
A_2	$\langle\{[0.6,0.7],[0.2,0.3]\},\{[0.1,0.2],[0.5,0.6]\},\{[0.1,0.2],[0.5,0.6]\}\rangle$
A_3	$\langle\{[0.5,0.6],[0.2,0.3]\},\{[0.2,0.4],[0.5,0.6]\},\{[0.2,0.3],[0.5,0.6]\}\rangle$
A_4	$\langle\{[0.3,0.5],[0.2,0.3]\},\{[0.3,0.4],[0.5,0.6]\},\{[0.3,0.4],[0.1,0.2]\}\rangle$
A_5	$\langle\{[0.4,0.5],[0.2,0.3]\},\{[0.4,0.5],[0.5,0.6]\},\{[0.5,0.6],[0.5,0.6]\}\rangle$
	C_3
A_1	$\langle\{[0.5,0.6],[0.4,0.5]\},\{[0.1,0.2],[0.5,0.6]\},\{[0.3,0.4],[0.5,0.6]\}\rangle$
A_2	$\langle\{[0.3,0.4],[0.2,0.3]\},\{[0.1,0.2],[0.5,0.6]\},\{[0.2,0.3],[0.1,0.2]\}\rangle$
A_3	$\langle\{[0.5,0.6],[0.3,0.4]\},\{[0.2,0.4],[0.5,0.6]\},\{[0.2,0.3],[0.5,0.6]\}\rangle$
A_4	$\langle\{[0.7,0.8],[0.2,0.3]\},\{[0.0,0.1],[0.5,0.6]\},\{[0.1,0.2],[0.5,0.6]\}\rangle$
A_5	$\langle\{[0.2,0.3],[0.2,0.3]\},\{[0.4,0.5],[0.5,0.6]\},\{[0.3,0.5],[0.5,0.6]\}\rangle$
	C_4
A_1	$\langle\{[0.8,0.9],[0.6,0.7]\},\{[0.1,0.2],[0.5,0.6]\},\{[0.2,0.3],[0.5,0.6]\}\rangle$
A_2	$\langle\{[0.3,0.4],[0.2,0.3]\},\{[0.4,0.5],[0.3,0.4]\},\{[0.5,0.6],[0.4,0.5]\}\rangle$
A_3	$\langle\{[0.6,0.8],[0.2,0.3]\},\{[0.5,0.6],[0.5,0.6]\},\{[0.3,0.4],[0.5,0.6]\}\rangle$
A_4	$\langle\{[0.3,0.4],[0.2,0.3]\},\{[0.2,0.3],[0.5,0.6]\},\{[0.5,0.6],[0.5,0.6]\}\rangle$
A_5	$\langle\{[0.4,0.5],[0.2,0.3]\},\{[0.2,0.3],[0.1,0.3]\},\{[0.2,0.3],[0.5,0.6]\}\rangle$

表 3　决策矩阵 D''

	C_1
A_1	$\langle\{[0.1524,0.1964],[0.1147,0.1524]\},\{[0.0518,0.0816],[0.1524,0.1964]\},\{[0.0816,0.1524],[0.0816,0.1147]\}\rangle$
A_2	$\langle\{[0.1922,0.2445],[0.0506,0.0797]\},\{[0.0242,0.0506],[0.1491,0.1922]\},\{[0.0506,0.0797],[0.1491,0.1922]\}\rangle$
A_3	$\langle\{[0.1310,0.1694],[0.0442,0.0697]\},\{[0.0442,0.0697],[0.1310,0.1694]\},\{[0.0697,0.0983],[0.1310,0.1694]\}\rangle$
A_4	$\langle\{[0.2599,0.3313],[0.2047,0.2599]\},\{[0.0000,0.0260],[0.1591,0.2047]\},\{[0.0543,0.1199],[0.1591,0.2047]\}\rangle$
A_5	$\langle\{[0.2599,0.3313],[0.0543,0.0853]\},\{[0.0853,0.1199],[0.1591,0.2047]\},\{[0.0260,0.0543],[0.1591,0.2047]\}\rangle$
	C_2
A_1	$\langle\{[0.0842,0.1183],[0.0535,0.0842]\},\{[0.0256,0.0535],[0.0256,0.0535]\},\{[0.1571,0.2022],[0.1183,0.1571]\}\rangle$
A_2	$\langle\{[0.2403,0.3032],[0.0648,0.1015]\},\{[0.0311,0.0648],[0.1877,0.2403]\},\{[0.0311,0.0648],[0.1877,0.2403]\}\rangle$
A_3	$\langle\{[0.1577,0.2403],[0.0648,0.1015]\},\{[0.0648,0.1421],[0.0648,0.1015]\},\{[0.0648,0.1015],[0.1877,0.2403]\}\rangle$
A_4	$\langle\{[0.0886,0.1650],[0.0564,0.0886]\},\{[0.0886,0.1245],[0.1650,0.2121]\},\{[0.0886,0.1245],[0.0270,0.0564]\}\rangle$
A_5	$\langle\{[0.1288,0.1706],[0.0584,0.0918]\},\{[0.0586,0.1288],[0.1706,0.2191]\},\{[0.1706,0.2191],[0.1706,0.2191]\}\rangle$
	C_3
A_1	$\langle\{[0.1961,0.2506],[0.1486,0.1961]\},\{[0.0326,0.0679],[0.1961,0.2506]\},\{[0.1062,0.1486],[0.1961,0.2506]\}\rangle$
A_2	$\langle\{[0.1038,0.1453],[0.0663,0.1038]\},\{[0.0319,0.0663],[0.1918,0.2454]\},\{[0.0663,0.1038],[0.0319,0.0319]\}\rangle$
A_3	$\langle\{[0.2023,0.2582],[0.1098,0.1534]\},\{[0.0702,0.1534],[0.2023,0.2582]\},\{[0.0702,0.1098],[0.2023,0.2582]\}\rangle$
A_4	$\langle\{[0.3279,0.4121],[0.0710,0.1110]\},\{[0.0000,0.0342],[0.2045,0.2609]\},\{[0.0342,0.0710],[0.2045,0.2609]\}\rangle$
A_5	$\langle\{[0.0630,0.0987],[0.0630,0.0987]\},\{[0.1383,0.1829],[0.1829,0.2344]\},\{[0.0987,0.1829],[0.1829,0.2344]\}\rangle$
	C_4
A_1	$\langle\{[0.2752,0.3690],[0.1674,0.2140]\},\{[0.0209,0.0436],[0.1294,0.1674]\},\{[0.0436,0.0689],[0.1294,0.1674]\}\rangle$
A_2	$\langle\{[0.0554,0.0784],[0.0350,0.0554]\},\{[0.0784,0.1049],[0.0554,0.0784]\},\{[0.1049,0.1362],[0.0784,0.1049]\}\rangle$
A_3	$\langle\{[0.1453,0.2411],[0.0375,0.0593]\},\{[0.1120,0.1453],[0.1120,0.1453]\},\{[0.0593,0.0838],[0.1120,0.1453]\}\rangle$
A_4	$\langle\{[0.0554,0.0784],[0.0350,0.0554]\},\{[0.0350,0.0554],[0.1049,0.1362]\},\{[0.1049,0.1362],[0.1049,0.1362]\}\rangle$
A_5	$\langle\{[0.0918,0.1225],[0.0412,0.0651]\},\{[0.0412,0.0651],[0.0197,0.0651]\},\{[0.0412,0.0651],[0.1225,0.1587]\}\rangle$

有

$$a'_{14} > a'_{11} > a'_{13} > a'_{12},$$

$$w_{11} = (m(\{C_1, C_4\}) - m(\{C_4\})) = 0.4386 - 0.2000 = 0.2386,$$

$$a''_{11} = w_{11}a'_{11} = \langle\{[0.1524, 0.1964], [0.1147, 0.1524]\}, \{[0.0518, 0.0816], [0.1524, 0.1964]\}, \{[0.0816, 0.1524], [0.0816, 0.1147]\}\rangle,$$

其他值可以类似计算.

Step 4: 确定多值区间中智正理想解H^+和多值区间中智负理想解H^-分别为

$$\begin{aligned}H^+ = \{&\langle\{[0.2599, 0.3313], [0.2047, 0.2599]\},\\ &\{[0.0000, 0.0260], [0.1591, 0.2047]\},\\ &\{[0.0543, 0.1199], [0.1591, 0.2047]\}\rangle,\\ &\langle\{[0.2403, 0.3032], [0.0648, 0.1015]\},\\ &\{[0.0311, 0.0648], [0.1877, 0.2403]\},\\ &\{[0.0311, 0.0648], [0.1877, 0.2403]\}\rangle,\\ &\langle\{[0.3279, 0.4121], [0.0710, 0.1110]\},\\ &\{[0.0000, 0.0342], [0.2045, 0.2609]\},\\ &\{[0.0342, 0.0710], [0.2045, 0.2609]\}\rangle,\\ &\langle\{[0.2752, 0.3690], [0.1674, 0.2140]\},\\ &\{[0.0209, 0.0436], [0.1294, 0.1674]\},\\ &\{[0.0436, 0.0689], [0.1294, 0.1674]\}\rangle\},\end{aligned}$$

$$\begin{aligned}H^- = \{&\langle\{[0.1310, 0.1694], [0.0442, 0.0697]\},\\ &\{[0.0442, 0.0697], [0.1310, 0.1694]\},\\ &\{[0.0697, 0.0983], [0.1310, 0.1694]\}\rangle,\\ &\langle\{[0.0842, 0.1183], [0.0535, 0.0842]\},\\ &\{[0.0256, 0.0535], [0.0256, 0.0535]\},\\ &\{[0.1571, 0.2022], [0.1183, 0.1571]\}\rangle,\\ &\langle\{[0.0630, 0.0987], [0.0630, 0.0987]\},\\ &\{[0.1383, 0.1829], [0.1829, 0.2344]\},\\ &\{[0.0987, 0.1829], [0.1829, 0.2344]\}\rangle,\\ &\langle\{[0.0554, 0.0784], [0.0350, 0.0554]\},\\ &\{[0.0784, 0.1049], [0.0554, 0.0784]\},\\ &\{[0.1049, 0.1362], [0.0784, 0.1049]\}\rangle\}.\end{aligned}$$

Step 5: 计算每个加权属性值与正负理想解的距离,并利用式(9)计算符号测度$S_{ij}(i = 1, 2, \cdots, 5,\ j = 1, 2, \cdots, 4)$为

$$S = \begin{bmatrix} 0.3618 & 0.0000 & 0.5222 & 1.0000\\ 0.2961 & 1.0000 & 0.4095 & 0.1913\\ 0.0000 & 0.5635 & 0.4978 & 0.4294\\ 1.0000 & 0.3706 & 1.0000 & 0.0000\\ 0.4631 & 0.2484 & 0.0000 & 0.2890 \end{bmatrix}.$$

Step 6: 对于每个属性$C_j(j = 1, 2, \cdots, 4)$,根据S_{ij}值对方案排序,得

(针对C_1) $A_4 > A_5 > A_1 > A_2 > A_3$,

(针对C_2) $A_2 > A_3 > A_4 > A_5 > A_1$,

(针对C_3) $A_4 > A_1 > A_3 > A_2 > A_5$,

(针对C_4) $A_1 > A_3 > A_5 > A_2 > A_4$.

Step 7: 确定排序频数矩阵$\Pi = (\Pi_{ij})_{5\times5}$如下:

$$\Pi = \begin{bmatrix} 1&1&1&0&1\\ 1&0&0&3&0\\ 0&2&1&0&1\\ 2&0&1&0&1\\ 0&1&1&1&1 \end{bmatrix}.$$

Step 8: 求解0-1排列矩阵$P = (p_{ij})_{5\times5}$如下:

$$\text{(M-2)}\ \max\ p_{11}+p_{12}+p_{13}+p_{15}+p_{21}+3p_{24}+ 2p_{32}+p_{33}+p_{35}+2p_{41}+p_{43}+p_{45}+ p_{52}+p_{53}+p_{54}+p_{55};$$

$$\begin{aligned}\text{s.t.}\ & p_{11}+p_{12}+p_{13}+p_{14}+p_{15}=1,\\ & p_{21}+p_{22}+p_{23}+p_{24}+p_{25}=1,\\ & p_{31}+p_{32}+p_{33}+p_{34}+p_{35}=1,\\ & p_{41}+p_{42}+p_{43}+p_{44}+p_{45}=1,\\ & p_{51}+p_{52}+p_{53}+p_{54}+p_{55}=1,\\ & p_{11}+p_{21}+p_{31}+p_{41}+p_{51}=1,\\ & p_{12}+p_{22}+p_{32}+p_{42}+p_{52}=1,\\ & p_{13}+p_{23}+p_{33}+p_{43}+p_{53}=1,\\ & p_{14}+p_{24}+p_{34}+p_{44}+p_{54}=1,\\ & p_{15}+p_{25}+p_{35}+p_{45}+p_{55}=1,\\ & p_{ij}=0\ \text{或者}\ 1,\ i,j=1,2,\cdots,5.\end{aligned}$$

Step 9: 求解规划模型得到矩阵$P = (p_{ij})_{5\times5}$为

$$P = \begin{bmatrix} 0&0&1&0&0\\ 0&0&0&1&0\\ 0&1&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&1 \end{bmatrix}.$$

令$A = [A_1, A_2, A_3, A_4, A_5]$,由$AP = [A_4, A_3, A_1, A_2, A_5]$得到方案的排序为$A_4 > A_3 > A_1 > A_2 > A_5$,最优方案为$A_4$.

为了表明算法的有效性和合理性,将基于Choquet积分的TOPSIS方法[16]推广到多值区间中智环境下,采用基于Choquet积分的TOPSIS方法对方案排序. 算法前4步相同,Step 5利用等式$d_i^+ = d_{i1}^+ + d_{i2}^+ + d_{i3}^+ + d_{i4}^+$和$d_i^- = d_{i1}^- + d_{i2}^- + d_{i3}^- + d_{i4}^-$ $(i = 1, 2, \cdots, 5)$计算每个方案与正负理想解的距离,得到$d_1^+ = 0.2485$, $d_2^+ = 0.3416$, $d_3^+ = 0.3245$, $d_4^+ = 0.2237$, $d_5^+ = 0.4124$, $d_1^- = 0.2523$, $d_2^- = 0.2501$, $d_3^- = 0.2236$, $d_4^- = 0.2935$, $d_5^- = 0.1514$. Step 6利用等式$C_i = d_i^-/(d_i^- + d_i^+)$计算每个方案的相对贴近度,

得到$C_1 = 0.503\,8, C_2 = 0.422\,7, C_3 = 0.407\,9, C_4 = 0.567\,5, C_5 = 0.272\,0$. Step 7 根据相对贴近度对方案进行排序,可得$A_4 > A_1 > A_2 > A_3 > A_5$,最优方案为$A_4$. 两种方法得到的最优方案都是$A_4$. 从决策矩阵可以看出,方案$A_4$在属性$C_1$和属性$C_3$都有该属性上最大的评价值,而$C_2$上的评价值排在中间位置,总体上$A_4$的评价值较高,因此其成为最优方案合理. 虽然两种方法中的最优方案一致,但是排序结果并不完全相同. 从频数矩阵可以看出,A_3排在A_2的次数较多,因此在整体排序中A_3排在A_2之前更加合理. 但是在基于TOPSIS的方法中,A_3排在A_2之后. 不同的排序结果是由于两种方法基于不同的排序思想,TOPSIS方法中最优方案靠近正理想解同时远离负理想解,对评价值相对敏感. 线性分配方法基于频数矩阵,这样可以减少主观因素的影响. 因此本文所提出的基于线性分配的方法的结果更加合理,更加鲁棒.

4 结 论

本文定义了多值区间中智集上的余弦值和欧氏距离,给出了多值区间中智数的比较方法,并给出了基于Choquet积分和线性分配方法的多值区间中智多属性决策方法. 多值区间中智集能够更加灵活地描述不确定信息,决策者可以根据自己的情况决定给出或者拒绝给出评价值,这样可以避免由于不熟悉属性而给出过高或者过低的评价值. 采用Choquet积分能够描述属性之间的相关性,采用线性分配法对方案排序可以避免决策者的主观性对于决策结果的影响,得到更加合理的决策结果. 最后通过实例表明了算法的可行性和有效性,与其他方法的比较结果说明了所提出方法的合理性.

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