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# 基于Pythagorean不确定语言的扩展VIKOR多属性群决策方法

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**摘 要:** 针对属性值为Pythagorean不确定语言变量,属性权重和专家权重完全未知的群决策问题,提出一种扩展VIKOR多属性群决策方法. 首先,给出Pythagorean不确定语言变量的概念,提出考虑语义变化的Pythagorean不确定语言变量运算规则、大小比较方法和Hamming距离测度;其次,提出基于Pythagorean不确定语言模糊熵的属性权重确定方法和基于相似度的专家权重确定方法,进而提出一种新的扩展VIKOR方法;最后,通过国内航空公司服务质量评估实例验证所提出方法的有效性和可行性.

**关键词:** Pythagorean不确定语言变量；VIKOR方法；多属性群决策方法

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# Extended VIKOR method for multi-attribute group decision making based on Pythagorean uncertain linguistic information

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**Abstract:** With respect to the multi-attribute decision making problem with Pythagorean uncertain linguistic variables, in which attribute weights and expert weights are completely unknown, an extended VIKOR method is proposed. Firstly, the concept of Pythagorean uncertain linguistic variables is defined. Combining with the concept of linguistic scale functions, the operational laws, comparison method and Hamming distance of Pythagorean uncertain linguistic variables are also proposed to accommodate different semantic situations. Then, an objective attribute weight determination method is proposed to determinate the attribute weights based on the Pythagorean uncertain linguistic entropy measure, and an objective expert weight determination method is also proposed to determine the weight of experts with respect to each attribute by calculating the similarity degree among evaluation values in which the attribute values are expressed as Pythagorean uncertain linguistic variables. Based on the above research, an extended VIKOR method is proposed to solve Pythagorean uncertain linguistic multi-attribute group decision making problems. Finally, an example of domestic airline service quality evaluation is provided to demonstrate the effectiveness and feasibility of the proposed method.

**Keywords:** Pythagorean uncertain linguistic variables；VIKOR method；multi-attribute group decision making method

## 0 引 言

作为传统模糊集的一种重要拓展,Atanassov提出的直觉模糊集(IFS)[1]可以同时表达隶属度、非隶属度和犹豫度3方面的信息,相比传统模糊集,更适合在实际问题中描述模糊性和不确定性. 最近,Yager在文献[2]中指出,IFS无法描述现实中可能存在的隶属度与非隶属度之和大于1的情况. 例如,专家在评价方案属性时,可能给出的隶属度为$\sqrt{3}/2$,非隶属度为$1/2$,由于$\sqrt{3}/2+1/2 \geqslant 1$,无法直接用传统的IFS来表达. 为此,Yager提出Pythagorean模糊集(PFS)的概念. PFS作为IFS的一种扩展,允许隶属度和非隶属度之和大于等于1,但平方和小于等于1. 由

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于$(\sqrt{3}/2)^2+(1/2)^2 \leqslant 1$,可以直接利用PFS来描述类似信息. 相比直觉模糊集,PFS具有更大范围的约束空间和更强的建模能力. 目前,PFS已经引起国内外学者的广泛关注,并取得了一定的研究成果. Yager[2]提出了一种基于Pythagorean模糊聚合算子的多属性决策方法;Zhang等[3]给出了Pythagorean模糊数(PFN)的定义,提出了Pythagorean模糊TOPSIS多属性决策方法;Peng等[4]进一步定义了PFN的减法和除法操作,并详细讨论了运算性质;Reformat等[5]将PFN应用到协作推荐系统的设计中;Bustince等[6]明确指出了直觉模糊集是PFS的特例.

然而,PFS与直觉模糊集一样,只能粗略表达隶属和非隶属于某个特定模糊概念"好"、"坏"的程度,所属对象不是很清晰和具体[7]. 而在一些现实问题中,人们更习惯对某些准则进行定性评价,比如员工绩效评估考核时,习惯用"优"、"良"、"一般"、"差"等语言术语来评价. 基于文献[7]的研究思路,结合Pythagorean模糊集和不确定语言变量,本文提出Pythagorean不确定语言集的概念. 一方面,它综合了PFS和不确定语言变量的优点;另一方面,直觉语言集[7]、不确定语言变量[8]、Pythagorean语言集[9]都可以看作其特例. 因此,研究Pythagorean不确定语言集的相关理论、方法以及在群决策中的应用更具有普适性,具有重要的研究价值.

VIKOR方法是Opricovic[10]提出的一种基于理想点和折衷规划的排序方法,其最大特点在于同时考虑了群体效用最大化和个体遗憾最小化,并考虑了专家的主观偏好. 之后,Opricovic等[11-12]将VIKOR方法与TOPSIS、ELECTRE和PROMETHEE方法进行对比分析,指出VIKOR方法能够得到更为合理的排序结果. 近年来,许多学者已经将VIKOR方法扩展到了直觉模糊集、二元语义、犹豫模糊语言等决策环境,并应用到供应商选择[13]、能源管理[14]、投资决策[15]等多种领域. 本文则进一步将VIKOR方法扩展到Pythagorean不确定语言多属性决策环境.

综上所述,本文首先研究Pythagorean不确定语言集的基本理论,提出考虑语义变化的Pythagorean不确定语言变量的运算法则、 大小比较方法和Hamming距离测度等;然后提出基于熵权的属性权重确定方法和基于相似度的专家权重确定方法,进而提出扩展的VIKOR方法来解决Pythagorean不确定语言环境下的多属性群决策问题;最后通过航空服务质量评估实例验证所提方法的有效性.

## 1 基本知识

### 1.1 直觉模糊集

**定义1**[1] 设$X$是一个非空有限集合,$X$上的直觉模糊集可以定义为$A=\langle x,\mu_A(x),\nu_A(x)|x\in X\rangle$. 其中,函数$\mu_A(x):X\to[0,1]$和$\nu_A:X\to[0,1]$分别表示元素$x$属于$X$的隶属度和非隶属度,且满足约束条件$0\leqslant\mu_A(x)+\nu_A(x)\leqslant 1$. $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$称为犹豫度.

Xu等[16]对直觉模糊决策理论和方法进行了综述,归纳总结了直觉模糊集在信息聚合、排序方法、属性权重确定方法等方面的研究进展,并指出未来值得关注的研究热点. 然而,直觉模糊集一方面无法表示隶属度和非隶属度之和大于1的情况,另一方面只能粗略表达隶属于或非隶属于某个特定模糊概念的程度,所属对象不是很清晰和具体[7].

### 1.2 不确定语言变量

设$S=\{s_i|i=0,1,\cdots,t\}$是一个有序的奇数个数的离散语言术语集,其中$s_i$表示可能的语言术语值. 通常,$t$可取4、6或者8. $S$需满足以下约束条件.

1) $S$是有序的,即若$i\geqslant j$,则有$s_i\geqslant s_j$.

2) 非负算子. $\mathrm{Neg}(s_i)=s_j$,其中$j=t-1-i$.

3) 最大算子. 若$i\geqslant j$,则有$\max\{s_i,s_j\}=s_i$.

4) 最小算子. 若$i\leqslant j$,则有$\min\{s_i,s_j\}=s_j$.

为了尽量避免和减少计算中可能存在的信息丢失问题,Xu[17]将离散语言术语集$S$扩展成连续语言术语集$\tilde{S}=\{\tilde{s}_i|i\in[0,r]\}$,其中$r\geqslant t$. 此外,为了描述语言评估值可能界于预定义的两个语言术语之间的情况,Xu[8]进一步提出了不确定语言变量的概念.

**定义2**[2] 设$\tilde{s}=[s_\alpha,s_\beta]$,$s_\alpha,s_\beta\in\tilde{S}$,且$0\leqslant\alpha\leqslant\beta$,$s_\alpha$和$s_\beta$分别是$\tilde{s}$的下限和上限,则称$\tilde{s}$为不确定语言变量.

设$\tilde{s}_1=[s_{\alpha_1},s_{\beta_1}]$和$\tilde{s}_2=[s_{\alpha_2},s_{\beta_2}]$为任意两个不确定语言变量,$\lambda\geqslant 0$,其运算法则[8]定义如下:

$$\tilde{s}_1\oplus\tilde{s}_2=[s_{\alpha_1},s_{\beta_1}]\oplus[s_{\alpha_2},s_{\beta_2}]=[s_{\alpha_1+\alpha_2},s_{\beta_1+\beta_2}],$$

$$\tilde{s}_1\otimes\tilde{s}_2=[s_{\alpha_1},s_{\beta_1}]\otimes[s_{\alpha_2},s_{\beta_2}]=[s_{\alpha_1\times\alpha_2},s_{\beta_1\times\beta_2}],$$

$$\lambda\tilde{s}_1=\lambda[s_{\alpha_1},s_{\beta_1}]=[s_{\lambda\alpha_1},s_{\lambda\beta_1}],$$

$$(\tilde{s}_1)^\lambda=([s_{\alpha_1},s_{\beta_1}])^\lambda=[(s_{\alpha_1})^\lambda,(s_{\beta_1})^\lambda]=[s_{\alpha_1^\lambda},s_{\beta_1^\lambda}].$$

### 1.3 Pythagorean模糊集

**定义3**[2-3] 设$X$为一个非空有限集合,则$X$上的Pythagorean模糊集(PFS)可以定义为$P=\langle x,P(\mu_P(x),\nu_P(x))\rangle|x\in X$. 其中,函数$\mu_p(x):X\to[0,1]$和$\nu_p(x):X\to[0,1]$分别定义为$x$属于$P$的隶属度和非隶属度,并满足$\mu_P^2(x)+\nu_P^2(x)\leqslant 1$. 对于任意元素$x$和集合$P$,称$\pi_P(x)=\sqrt{1-\mu_P^2(x)-\nu_P^2(x)}$为Pythagorean模糊指标. 为了方便,称$\alpha=P(\mu_\alpha,\nu_\alpha)$

为Pythagorean模糊数(PFN).

设$\alpha = P(\mu_\alpha, \nu_\alpha)$和$\beta = P(\mu_\beta, \nu_\beta)$为任意两个PFNs, Zhang等[3]提出运算法则,定义如下:

$$\alpha^c = P(\nu_\alpha, \mu_\alpha);$$

$$\alpha \oplus \beta = P(\sqrt{\mu_\alpha^2 + \mu_\beta^2 - \mu_\alpha^2\mu_\beta^2}, \nu_\alpha\nu_\beta);$$

$$\alpha \otimes \beta = P(\mu_\alpha\mu_\beta, \sqrt{\nu_\alpha^2 + \nu_\beta^2 - \nu_\alpha^2\nu_\beta^2});$$

$$\lambda\alpha = P(\sqrt{1-(1-\mu_\alpha^2)^\lambda}, (\nu_\alpha)^\lambda), \lambda > 0;$$

$$\alpha^\lambda = P((\mu_\alpha)^\lambda, \sqrt{1-(1-\nu_\alpha^2)^\lambda}), \lambda > 0.$$

## 2 Pythagorean不确定语言集及相关定义

**定义4** 设$X = \{x_1, x_2, \cdots, x_n\}$是有限论域集, $X$上的Pythagorean不确定语言集(PULS)形式如下:

$$\tilde{P} = \{\langle x_i|([s_{\theta(x_i)}, s_{\tau(x_i)}], \tilde{P}(\mu_{\tilde{P}}(x_i), \nu_{\tilde{P}}(x_i)))\rangle|x_i \in X\}.$$

其中: $s_{\theta(x_i)}, s_{\tau(x_i)} \in \tilde{S}$; $\mu_{\tilde{P}} : X \to [0,1]$和$\nu_{\tilde{P}} : X \to [0,1]$分别表示$x_i$隶属于和非隶属于$[s_{\theta(x_i)}, s_{\tau(x_i)}]$的程度,满足$0 \leqslant \mu_{\tilde{P}}^2(x_i)+\nu_{\tilde{P}}^2(x_i) \leqslant 1, \forall x_i \in X$. $\varpi_{\tilde{P}}(x_i) = \sqrt{1-\mu_{\tilde{P}}^2(x_i)-\nu_{\tilde{P}}^2(x_i)}, \forall x_i \in X$,表示$x_i$隶属于$[s_{\theta(x_i)}, s_{\tau(x_i)}]$的犹豫度. 为表示方便,4元组$\tilde{\alpha} = \langle[s_{\theta(x_i)}, s_{\tau(x_i)}], \tilde{P}(\mu_{\tilde{P}}(x_i), \nu_{\tilde{P}}(x_i))\rangle$称为Pythagorean不确定语言数(PULN).

**定义5** [18] 如果任给$\theta_i \in R^+ (R^+ = \{r|r \geqslant 0, r \in R\})$为实数值,则语言刻度函数$f$表示为映射$f : s_i \to \theta_i (i = 0, 1, \cdots, t)$,其中$0 \leqslant \theta_0 < \theta_1 < \cdots < \theta_t$.

显然, $f$是下标$i$的严格单调递增函数. $\theta_i$反映了专家使用$s_i \in S$时的偏好值. Wang等[18]总结了几种可能的语言刻度函数. 例如,简单的基于语言下标的平均函数

$$f(s_i) = \theta_i = \frac{i}{t},\ i = 0, 1, \cdots, t. \tag{1}$$

这种方式虽然简单,易于使用,但是无法反映专家的主观感觉变化. Wang等[18]在文献[19]的基础上,提出一种新的复合语言刻度函数

$$f(s_i) = \theta_i = \begin{cases} \dfrac{\alpha^{\frac{t}{2}} - \alpha^{\frac{t}{2}-i}}{2\alpha^{\frac{t}{2}} - 2},\ 0 \leqslant i \leqslant \dfrac{t}{2}; \\ \dfrac{\alpha^{\frac{t}{2}} + \alpha^{i-\frac{t}{2}} - 2}{2\alpha^{\frac{t}{2}} - 2},\ \dfrac{t}{2} < i \leqslant t. \end{cases} \tag{2}$$

其中$\alpha, \beta \in (0, 1]$. 如果$\alpha = \beta = 1$,则式(2)转换为式(1). 此外,随着术语从中间向两端扩展,式(2)能够表达相邻两个术语绝对偏差的降低. 同样,为了避免信息丢失, $f$可以扩展成$\bar{f} : \tilde{S} \to R^+ (R^+ = \{r|r \geqslant 0, r \in R\})$,满足$\bar{f}(s_i) = \theta_i$,且$\bar{f}$是严格单调递减的连续函数. $\bar{f}$的逆函数表示为$\bar{f}^{-1}$.

**定义6** 设$\tilde{\alpha} = \langle[s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}], \tilde{P}(\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})\rangle$和$\tilde{\beta} = \langle[s_{\tilde{\beta}_1}, s_{\tilde{\beta}_2}], \tilde{P}(\mu_{\tilde{\beta}}, \nu_{\tilde{\beta}})\rangle$为任意两个PULNs,则考虑语义变化的Pythagorean不确定语言运算法则定义如下:

$$\tilde{\alpha}\tilde{\oplus}\tilde{\beta} = \Big\langle[\bar{f}^{-1}(\bar{f}(s_{\tilde{\alpha}_1}) + f(s_{\tilde{\beta}_1})), \bar{f}^{-1}(\bar{f}(s_{\tilde{\alpha}_2}) + \bar{f}(s_{\tilde{\beta}_2}))], P\Big(\sqrt{\mu_{\tilde{\alpha}}^2 + \mu_{\tilde{\beta}}^2 - \mu_{\tilde{\alpha}}^2\mu_{\tilde{\beta}}^2}, \nu_{\tilde{\alpha}}\nu_{\tilde{\beta}}\Big)\Big\rangle;$$

$$\tilde{\alpha}\tilde{\otimes}\tilde{\beta} = \Big\langle[\bar{f}^{-1}(\bar{f}(s_{\tilde{\alpha}_1}) + \bar{f}(s_{\tilde{\beta}_1})), \bar{f}^{-1}(\bar{f}(s_{\tilde{\alpha}_1}) + \bar{f}(s_{\tilde{\beta}_1}))], P(\mu_{\tilde{\alpha}}\mu_{\tilde{\beta}}, \sqrt{\nu_{\tilde{\alpha}}^2 + \nu_{\tilde{\beta}}^2 - \nu_{\tilde{\alpha}}^2\nu_{\tilde{\beta}}^2})\Big\rangle;$$

$$\lambda\tilde{\alpha} = \Big\langle[\bar{f}^{-1}(\lambda\bar{f}(s_{\tilde{\alpha}_1})), \bar{f}^{-1}(\lambda\bar{f}(s_{\tilde{\alpha}_2}))], P(\sqrt{1-(1-\mu_{\tilde{\alpha}}^2)^\lambda}, (\nu_{\tilde{\alpha}})^\lambda)\Big\rangle;$$

$$\tilde{\alpha}^\lambda = \Big\langle[\bar{f}^{-1}(\lambda\bar{f}(s_{\tilde{\alpha}_1})), \bar{f}^{-1}(\lambda\bar{f}(s_{\tilde{\alpha}_2}))], P((\mu_{\tilde{\alpha}})^\lambda, \sqrt{1-(1-\nu_{\tilde{\alpha}}^2)^\lambda})\Big\rangle;$$

$$\tilde{\alpha}^c = \Big\langle[\bar{f}^{-1}(\bar{f}(s_t) - \bar{f}(s_{\tilde{\alpha}_2})), \bar{f}^{-1}(\bar{f}(s_t) - \bar{f}(s_{\tilde{\alpha}_1}))], P(\nu_\alpha, \mu_\alpha)\Big\rangle$$

**定义7** 设$\tilde{\alpha} = \langle[s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}], \tilde{P}(\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})\rangle$是任意的PULN,则其期望值和精确函数值定义如下:

$$\mathrm{E}(\tilde{\alpha}) = \frac{(\mu^2(\tilde{\alpha}) + 1 - \nu^2(\tilde{\alpha})) \times (\bar{f}(s_{\tilde{\alpha}_1}) + \bar{f}(s_{\tilde{\alpha}_2}))}{4}, \tag{3}$$

$$H(\tilde{\alpha}) = (\mu^2(\tilde{\alpha}) + \nu^2(\tilde{\alpha})) \times \frac{\bar{f}(s_{\tilde{\alpha}_1}) + \bar{f}(s_{\tilde{\alpha}_2})}{2}. \tag{4}$$

**定义8** 设$\tilde{\alpha} = \langle[s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}], \tilde{P}(\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})\rangle$和$\tilde{\beta} = \langle[s_{\tilde{\beta}_1}, s_{\tilde{\beta}_2}], \tilde{P}(\mu_{\tilde{\beta}}, \nu_{\tilde{\beta}})\rangle$为任意两个PULNs,则$\tilde{\alpha}$与$\tilde{\beta}$之间的大小关系可以定义为: 1) 如果$\mathrm{E}(\tilde{\alpha}) > \mathrm{E}(\tilde{\beta})$,则$\tilde{\alpha} \succ \tilde{\beta}$. 2) 如果$\mathrm{E}(\tilde{\alpha}) = \mathrm{E}(\tilde{\beta})$,则若$H(\tilde{\alpha}) > H(\tilde{\beta})$,有$\tilde{\alpha} \succ \tilde{\beta}$;若$H(\tilde{\alpha}) = H(\tilde{\beta})$,有$\tilde{\alpha} = \tilde{\beta}$.

**定义9** 设$\tilde{\alpha} = \langle[s_{\tilde{\alpha}_1}, s_{\tilde{\alpha}_2}], \tilde{P}(\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})\rangle$和$\tilde{\beta} = \langle[s_{\tilde{\beta}_1}, s_{\tilde{\beta}_2}], \tilde{P}(\mu_{\tilde{\beta}}, \nu_{\tilde{\beta}})\rangle$为任意两个PULNs,则$\tilde{\alpha}$与$\tilde{\beta}$之间的Hamming距离定义为

$$d_H(\tilde{\alpha}, \tilde{\beta}) = \frac{1}{4}\big|(1+\mu_\alpha^2-\nu_\alpha^2)(\bar{f}(s_{\tilde{\alpha}_1}) + \bar{f}(s_{\tilde{\alpha}_2})) - (1+\mu_\beta^2-\nu_\beta^2)(\bar{f}(s_{\tilde{\beta}_1}) + \bar{f}(s_{\tilde{\beta}_2}))\big|. \tag{5}$$

**定义10** 设$\alpha_i = \langle[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], P(\mu(\alpha_i), \nu(\alpha_i))\rangle$ $(i = 1, 2, \cdots, n)$为一组PULNs, PULWAA: $\varOmega^n \to \varOmega$,若

$$\mathrm{PULWAA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \sum_{i=1}^n w_i\alpha_i, \tag{6}$$

则称PULWAA为Pythagorean不确定语言加权算术平均算子. 其中: $\varOmega$为Pythagorean不确定语言集合, $w = (w_1, w_2, \cdots, w_n)^{\mathrm{T}}$为$\alpha_i$的权重向量, $w_i \in [0, 1]$且$\sum\limits_{i=1}^n w_i = 1$.

**定理1** 设$\alpha_i = \langle[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], P(\mu(\alpha_i), \nu(\alpha_i))\rangle$ $(i = 1, 2, \cdots, n)$为一组PULNs,则式(6)的计算结果仍为PULN,且有

$$\begin{aligned}&\mathrm{PULWAA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\\&\Big\langle\Big[\bar{f}^{-1}\Big(\sum_{i=1}^{n}w_if(s_{\theta(\alpha_i)}\Big),\bar{f}^{-1}\Big(\sum_{i=1}^{n}w_if(s_{\tau(\alpha_i)})\Big)\Big],\\&\Big(\sqrt{1-\prod_{i=1}^{n}(1-\mu^2(\alpha_i))^{w_i}},\sqrt{\prod_{i=1}^{n}(\nu(\alpha_i))^{2w_i}}\Big)\Big\rangle.\end{aligned}\tag{7}$$

## 3 基于VIKOR的Pythagorean不确定语言多属性群决策方法

### 3.1 决策问题描述

基于PULN的多属性群决策问题描述为:设$X=\{x_1,x_2,\cdots,x_m\}$为有限备选方案集,$C=\{c_1,c_2,\cdots,c_n\}$为属性集合,属性权重向量为$W=\{w_1,w_2,\ldots,w_n\}^{\mathrm{T}},\sum_{i=0}^{n}w_i=1,w_i\geqslant 0,E=\{e_1,e_2,\cdots,e_p\}$为专家集合. 为方便描述,设$M=\{1,2,\cdots,m\}$, $N=\{1,2,\cdots,n\},P=\{1,2,\cdots,p\}$. 由于不同专家拥有的知识、经验及所属行业不同,对不同专家分属性赋予不同权重,并非对所有属性赋予相同权重,可以获得更为合理的决策结果. 设专家权重为$\lambda_k^{(j)}(k\in P,j\in N)$,且$0\leqslant\lambda_k^{(j)}\leqslant 1,\sum_{k=1}^{p}\lambda_k^{(j)}=1$. 设专家$e_k$对方案$x_i$在属性$c_j$下的评估值用PULN表示为$r_{ij}^{(k)}=\langle[s_{\theta^{(k)}(x_{ij})},s_{\tau^{(k)}(x_{ij})}],P(\mu_{ij}^{(k)},\nu_{ij}^{(k)})\rangle$,$e_k$给出的评估矩阵表示为$R^{(k)}=[r_{ij}^{(k)}]_{m\times n},k\in P$.

### 3.2 专家权重确定方法

首先将专家$e_k$提供的决策矩阵$R^{(k)}(k\in P)$转换成关于每个属性的评估矩阵,即$D^{(1)},D^{(2)},\cdots,D^{(n)}$,表示如下:

$$D^{(j)}=(\xi_{ki}^{(j)})_{p\times m}=\begin{array}{c c}& \begin{array}{cccc}x_1 & x_2 & \cdots & x_m\end{array}\\ \begin{array}{c}e_1\\e_2\\\vdots\\e_p\end{array} & \begin{bmatrix}\xi_{11}^{(j)} & \xi_{12}^{(j)} & \cdots & \xi_{1m}^{(j)}\\ \xi_{21}^{(j)} & \xi_{22}^{(j)} & \cdots & \xi_{2m}^{(j)}\\ \vdots & \vdots & \ddots & \vdots\\ \xi_{p1}^{(j)} & \xi_{p2}^{(j)} & \cdots & \xi_{pm}^{(j)}\end{bmatrix}\end{array},\tag{8}$$

其中$\xi_{ki}^{(j)}$等价于$\tilde{r}_{ij}^{(k)}$. 基于文献[20-21],对于矩阵$D^{(j)}$,方案$x_i$关于属性$c_j$的评估均值为

$$\hat{\xi}_i^{(j)}=\langle[s_{\hat{\theta}(x_i^{(j)})},s_{\hat{\tau}(x_i^{(j)})}],P(\hat{\mu}_i^{(j)},\hat{\nu}_i^{(j)})\rangle.\tag{9}$$

其中

$$s_{\hat{\theta}(x_i^{(j)})}=\bar{f}^{-1}\Big(\frac{1}{p}\sum_{k=1}^{p}\bar{f}(s_{\theta^{(j)}(x_{ki})})\Big),$$

$$s_{\hat{\tau}(x_i^{(j)})}=\bar{f}^{-1}\Big(\frac{1}{p}\sum_{k=1}^{p}\bar{f}(s_{\tau^{(j)}(x_{ki})})\Big),$$

$$\hat{\mu}_i^{(j)}=\frac{1}{p}\sum_{k=1}^{p}\mu_{ki}^{(j)},$$

$$\hat{\nu}_i^{(j)}=\frac{1}{p}\sum_{k=1}^{p}\nu_{ki}^{(j)}.$$

对于方案$x_i\in X$,$e_k$提供的$\xi_{ki}^{(j)}$与均值$\hat{\xi}_i^{(j)}$之间的相似度$s_{ki}^{(j)}$可以定义为

$$s_{ki}^{(j)}=1-\frac{d(\xi_{ki}^{(j)},\hat{\xi}_i^{(j)})}{\sum_{k=1}^{p}d(\xi_{ki}^{(j)},\hat{\xi}_i^{(j)})},\tag{10}$$

$$k\in P,i\in M,j\in N,$$

其中$d(\xi_{ki}^{(j)},\hat{\xi}_i^{(j)})$为式(5)定义的$\xi_{ki}^{(j)}$与均值$\hat{\xi}_i^{(j)}$之间的距离. 对于属性$c_j\in C$,利用相似度$s_{ki}^{(j)}$可以构造相似度矩阵为

$$S^{(j)}=(s_{ki}^{(j)})_{p\times m}=\begin{array}{c c}& \begin{array}{cccc}x_1 & x_2 & \cdots & x_m\end{array}\\ \begin{array}{c}e_1\\e_2\\\vdots\\e_p\end{array} & \begin{bmatrix}s_{11}^{(j)} & s_{12}^{(j)} & \cdots & s_{1m}^{(j)}\\ s_{21}^{(j)} & s_{22}^{(j)} & \cdots & s_{2m}^{(j)}\\ \vdots & \vdots & \ddots & \vdots\\ s_{p1}^{(j)} & s_{p2}^{(j)} & \cdots & s_{pm}^{(j)}\end{bmatrix}\end{array},\tag{11}$$

其中$j\in N$. 基于相似度矩阵$S^{(j)}$,专家$e_k(k\in P)$关于属性$c_j\in C$的总体相似度计算如下:

$$\gamma_k^{(j)}=\sum_{i=1}^{m}s_{ki}^{(j)},\ k\in P,\ i\in M,\ j\in N.\tag{12}$$

最后,专家$e_i$关于属性$c_j$的权重可以计算如下:

$$\lambda_k^{(j)}=\frac{\gamma_k^{(j)}}{\sum_{k=1}^{p}\gamma_k^{(j)}},\ k\in P,\ j\in N.\tag{13}$$

显然,$0\leqslant\lambda_k^{(j)}\leqslant 1$,且$\sum_{k=1}^{p}\lambda_k^{(j)}=1$.

### 3.3 属性权重确定方法

设$A=\{\langle x_i|([s_{\theta(x_i)},s_{\tau(x_i)}],P(\mu(x_i),\nu(x_i)))\rangle x_i\in X\}$和$B=\{\langle x_i|([s_{\tilde{\theta}(x_i)},s_{\tilde{\tau}(x_i)}],P(\tilde{\mu}(x_i),\tilde{\nu}(x_i)))\rangle x_i\in X\}$为任意两个PULNs,则当且仅当$s_{\theta(x_i)}\leqslant s_{\tilde{\theta}(x_i)}$, $s_{\tau(x_i)}\geqslant s_{\tilde{\tau}(x_i)},\mu(x_i)\leqslant\tilde{\mu}(x_i),\tilde{\nu}(x_i)\geqslant\nu(x_i),\forall x_i\in X$时,有$A\subseteq B$. $X$上的Pythagorean不确定语言数集合用$\mathrm{PULN}(X)$表示.

**定义11** 设$A=\{\langle x_i|([s_{\theta(x_i)},s_{\tau(x_i)}],P(\mu(x_i),\nu(x_i)))\rangle,x_i\in X\}$,令

$$\begin{aligned}\mathrm{E}(A)=\frac{1}{n}\sum_{i=1}^{n}&[2-|\bar{f}(s_{\theta(x_i)})-\bar{f}(s_{\tau(x_i)})|-|\mu^2(x_i)-\\&\nu^2(x_i)|+\pi_P(x_i)]/[2+|\bar{f}(s_{\theta(x_i)})-\\&\bar{f}(s_{\tau(x_i)})|+|\mu^2(x_i)-\nu^2(x_i)|+\pi_P(x_i)].\end{aligned}$$

显然,当方案在某一属性下的评价信息熵值越小时,赋予该属性的权重应越大;反之,权重应越小. 依据这一基本原则及文献[22],给出属性客观权重确定

方法如下:

首先,利用式(14)计算专家评估信息的熵值,建立Pythagorean不确定语言熵矩阵为

$$D_E=\begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_p \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_{1n} \end{array} \\ \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1m} \\ E_{21} & E_{22} & \cdots & E_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ E_{m1} & E_{m2} & \cdots & E_{pm} \end{bmatrix} \end{array};$$

然后,聚合和标准化Pythagorean不确定语言模糊熵值

$$S_j=\frac{\sum\limits_{i=1}^{m}E_{ij}}{\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n}E_{ij}},j\in N, \tag{14}$$

其中$E_{ij}$为备选方案$x_i$在属性$c_j$下的Pythagorean不确定语言模糊熵;

最后,计算属性权重

$$w_j=\frac{1-S_j}{\sum\limits_{j=1}^{n}(1-S_j)},j\in N. \tag{15}$$

### 3.4 决策步骤

Step 1: 构建Pythagorean不确定语言决策矩阵$R^{(k)}=(r_{ij}^{(k)})_{m\times n}(k\in P)$,并进行标准化处理,获得标准化决策矩阵$\tilde{R}^{(k)}=(\tilde{r}_{ij}^{(k)})_{m\times n}(k\in P)$.

$$\tilde{r}_{ij}^{(k)}=\begin{cases} r_{ij}^{(k)}, \text{效益型属性}I_b; \\ \left(r_{ij}^{(k)}\right)^c, \text{成本型属性}I_c. \end{cases} \tag{16}$$

其中:$i\in M,j\in N$; $(r_{ij}^{(k)})^c$是$r_{ij}^{(k)}$的补集,且$(r_{ij}^{(k)})^c=\langle[\bar{f}^{-1}(\bar{f}(s_t)-\bar{f}(s_{\tau(x_{ij})}^{(k)})),\bar{f}^{-1}(\bar{f}(s_t)-\bar{f}(s_{\theta(x_{ij})}^{(k)}))],P(\nu_{ij}^{(k)},\mu_{ij}^{(k)})\rangle$.

Step 2: 利用3.2节中专家权重确定方法,计算专家对不同属性的权重$\lambda_k^{(j)}(k\in P,j\in N)$.

Step 3: 利用Pythagorean不确定语言加权平均算子PULWAA(式(7))对方案$x_i$在属性$c_j$下的评估信息进行集结,将个体决策矩阵$\tilde{R}^{(k)}$聚合为群体决策矩阵$G=\{\eta_{ij}\}_{m\times n}=\langle[s_{\tilde{\theta}(\eta_{ij})},s_{\tilde{\tau}(\eta_{ij})}],P(\tilde{\mu}_{\eta_{ij}},\tilde{\nu}_{\eta_{ij}})\rangle$.

Step 4: 利用3.3节中的属性权重确定方法,基于群体决策矩阵$G$和Pythagorean不确定语言模糊熵公式,求解属性权重$w_j(j\in N)$.

Step 5: 对群体决策矩阵$G$,确定方案正理想解$g^+=\{g_1^+,g_2^+,\cdots,g_n^+\}$和负理想解$g^-=\{g_1^-,g_2^-,\cdots,g_n^-\}$,其中

$$g_j^+=\langle[\max_i s_{\theta(\eta_{ij})},\max_i s_{\tau(\eta_{ij})}],\ P(\max_i\tilde{\mu}_{\eta_{ij}},\min_i\tilde{\nu}_{\eta_{ij}})\rangle,$$

$$g_j^-=\langle[\min_i s_{\theta(\eta_{ij})},\min_i s_{\tau(\eta_{ij})}],\ P(\min_i\tilde{\mu}_{\eta_{ij}},\max_i\tilde{\nu}_{\eta_{ij}})\rangle.$$

Step 6: 将VIKOR方法扩展到Pythagorean不确定语言环境,计算各方案的群体效益值$S(x_i)$、个体遗憾值$R(x_i)$和折衷值$Q(x_i)$,有

$$S(x_i)=\sum_{j=1}^{n}w_j\frac{d(g_j^+,\eta_{ij})}{d(g_j^+,g_j^-)}, \tag{17}$$

$$R(x_i)=\max_j\Big\{w_j\Big(\frac{d(g_j^+,\eta_{ij})}{d(g_j^+,g_j^-)}\Big)\Big\}, \tag{18}$$

$$Q(x_i)=\phi\frac{S(x_i)-S^+}{S^--S^+}+(1-\phi)\frac{R(x_i)-R^+}{R^--R^+}. \tag{19}$$

其中:$i\in M,j\in N$; $w_j$表示属性权重,$d$表示任意两个PULNs之间的距离; $S^+=\min\limits_i\{S(x_i)\},S^-=\max\limits_i\{S(x_i)\},R^+=\min\limits_i\{R(x_i)\},R^-=\max\limits_i\{R(x_i)\}$; $\phi\in[0,1]$表示折衷系数,$1-\phi$表示个体遗憾值权重. 当$\phi>0.5$时,专家倾向于按最大化群体效用方式进行决策;当$\phi<0.5$时,专家倾向于按个体遗憾最小方式进行决策;当$\phi=0.5$时,专家按均衡方式,即群体效用和个体遗憾同等重要的方式进行决策.

Step 7: 分别根据$S(x_i)$、$R(x_i)$和$Q(x_i)$对方案进行升序排列,得到3个排序,数值越小表明方案越优.

Step 8: 确定折衷方案. 设按照$Q(x_i)$值升序的排列结果为$x^{(1)},x^{(2)},\cdots,x^{(m)}$. 如果$x^{(1)}$同时满足以下两个条件,则$x^{(1)}$为折衷方案:

1) $Q(x^{(2)})-Q(x^{(1)})\geqslant\dfrac{1}{m-1}$;

2) 在依据$S(x_i)$和$R(x_i)$进行排列时,$x^{(1)}$至少有一个依然排列为最小值.

如果上述条件不能同时满足,则可以依据以下情况分别得到妥协解方案: 1)如果不满足条件2,则方案$x^{(1)}$和$x^{(2)}$均是折衷方案; 2)如果不满足条件1,则折衷方案为$x^{(1)},x^{(2)},\cdots,x^{(d)}$,其中$x^{(d)}$满足$Q(x^{(d)})-Q(x^{(1)})d<\dfrac{1}{m-1}$.

## 4 实例分析

由于高速铁路的快速发展,国内航空市场面临着激烈的竞争,越来越多的航空公司意识到服务质量是在激烈的航空市场中生存和竞争的关键因素. 现由4位专家$(e_1\sim e_4)$组成的评估委员会需要对4家航空公司$(x_1\sim x_4)$进行客服质量评估. 选择文献[23]中提出的有型性、可靠性、保证性和服务人员作为评价指标$(c_1\sim c_4)$. 专家以PULN形式给出评估值,用到的语言术语集$S=\{s_0=\text{非常差},s_1=\text{差},s_2=\text{稍微差},s_3=\text{一般},s_4=\text{稍微好},s_5=\text{好},s_6=\text{非常好}\}$. 专家给出的决策矩阵为$R^{(k)}=\langle[s_{\theta^{(k)}(x_{ij})},$

$s_{\tau^{(k)}(x_{ij})}], P(\mu_{ij}^{(k)}, \nu_{ij}^{(k)})\rangle, i,j,k = 1,2,\cdots,4$, 如表1～表4所示. 现要求选出服务质量最优的公司.

**表 1 专家 $e_1$ 给出的决策矩阵 $R^{(1)}$**

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | $\langle [s_2,s_3],(0.7,0.4)\rangle$ | $\langle [s_2,s_2],(0.6,0.3)\rangle$ | $\langle [s_2,s_4],(0.8,0.3)\rangle$ | $\langle [s_4,s_6],(0.8,0.4]\rangle$ |
| $x_2$ | $\langle [s_2,s_4],(0.6,0.3)\rangle$ | $\langle [s_1,s_3],(0.7,0.4)\rangle$ | $\langle [s_2,s_4],(0.6,0.2)\rangle$ | $\langle [s_3,s_4],(0.9,0.2)\rangle$ |
| $x_3$ | $\langle [s_1,s_2],(0.5,0.6)\rangle$ | $\langle [s_1,s_2],(0.6,0.3)\rangle$ | $\langle [s_3,s_5],(0.7,0.6)\rangle$ | $\langle (s_2,s_4],(0.6,0.4)\rangle$ |
| $x_4$ | $\langle [s_3,s_4],(0.8,0.3)\rangle$ | $\langle [s_3,s_3],(0.5,0.4)\rangle$ | $\langle [s_1,s_4],(0.9,0.3)\rangle$ | $\langle [s_1,s_3],(0.5,0.3)\rangle$ |

**表 2 专家 $e_2$ 给出的决策矩阵 $R^{(2)}$**

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | $\langle [s_2,s_4],(0.6,0.3)\rangle$ | $\langle [s_1,s_2],(0.8,0.3)\rangle$ | $\langle [s_3,s_4],(0.7,0.4)\rangle$ | $\langle [s_5,s_6],(0.5,0.3]\rangle$ |
| $x_2$ | $\langle [s_3,s_4],(0.8,0.4)\rangle$ | $\langle [s_2,s_4],(0.6,0.2)\rangle$ | $\langle [s_1,s_3],(0.6,0.5)\rangle$ | $\langle [s_3,s_4],(0.7,0.3)\rangle$ |
| $x_3$ | $\langle [s_1,s_3],(0.7,0.5)\rangle$ | $\langle [s_2,s_3],(0.7,0.4)\rangle$ | $\langle [s_2,s_5],(0.8,0.4)\rangle$ | $\langle [s_4,s_5],(0.6,0.5)\rangle$ |
| $x_4$ | $\langle [s_2,s_3],(0.7,0.2)\rangle$ | $\langle [s_1,s_3],(0.5,0.3)\rangle$ | $\langle [s_1,s_3],(0.5,0.4)\rangle$ | $\langle [s_2,s_4],(0.8,0.3)\rangle$ |

**表 3 专家 $e_3$ 给出的决策矩阵 $R^{(3)}$**

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | $\langle [s_1,s_2],(0.8,0.1)\rangle$ | $\langle [s_2,s_4],(0.7,0.5)\rangle$ | $\langle [s_2,s_4],(0.5,0.4)\rangle$ | $\langle [s_4,s_5],(0.7,0.5]\rangle$ |
| $x_2$ | $\langle [s_2,s_2],(0.9,0.2)\rangle$ | $\langle [s_1,s_2],(0.5,0.3)\rangle$ | $\langle [s_3,s_4],(0.7,0.2)\rangle$ | $\langle [s_4,s_5],(0.8,0.4)\rangle$ |
| $x_3$ | $\langle [s_2,s_4],(0.6,0.3)\rangle$ | $\langle [s_1,s_4],(0.8,0.3)\rangle$ | $\langle [s_3,s_3],(0.9,0.1)\rangle$ | $\langle [s_3,s_4],(0.7,0.3)\rangle$ |
| $x_4$ | $\langle [s_2,s_4],(0.5,0.3)\rangle$ | $\langle [s_3,s_4],(0.6,0.5)\rangle$ | $\langle [s_2,s_2],(0.8,0.2)\rangle$ | $\langle [s_5,s_6],(0.9,0.3)\rangle$ |

**表 4 专家 $e_4$ 给出的决策矩阵 $R^{(4)}$**

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | $\langle [s_4,s_4],(0.9,0.2)\rangle$ | $\langle [s_2,s_4],(0.5,0.4)\rangle$ | $\langle [s_2,s_3],(0.8,0.3)\rangle$ | $\langle [s_3,s_4],(0.8,0.5]\rangle$ |
| $x_2$ | $\langle [s_1,s_4],(0.7,0.2)\rangle$ | $\langle [s_1,s_3],(0.7,0.4)\rangle$ | $\langle [s_3,s_3],(0.7,0.1)\rangle$ | $\langle [s_4,s_6],(0.8,0.4)\rangle$ |
| $x_3$ | $\langle [s_3,s_5],(0.8,0.2)\rangle$ | $\langle [s_3,s_4],(0.9,0.2)\rangle$ | $\langle [s_3,s_4],(0.7,0.2)\rangle$ | $\langle [s_3,s_4],(0.7,0.2)\rangle$ |
| $x_4$ | $\langle [s_1,s_2],(0.8,0.4)\rangle$ | $\langle [s_2,s_4],(0.7,0.4)\rangle$ | $\langle [s_3,s_4],(0.9,0.1)\rangle$ | $\langle [s_3,s_5],(0.7,0.4)\rangle$ |

### 4.1 评估方案排序

下面给出基于上述方法进行评估的具体步骤,其中语言刻度函数选择式(2), 参数$\alpha$取值为1.4[18].

Step 1: 由于4个属性均是效益型,不需要进行规范化处理,即

$$\tilde{R}^{(k)} = R^{(k)} = \langle [s_{\tilde{\theta}^{(k)}(x_{ij})}, s_{\tilde{\tau}^{(k)}(x_{ij})}], P(\tilde{\mu}_{ij}^{(k)}, \tilde{\nu}_{ij}^{(k)})\rangle, \quad i,j,k = 1,2,\cdots,4.$$

Step 2: 基于3.2节中提出的基于相似度的专家权重确定方法,计算专家权重,结果如下:

$$\lambda_1^{(1)} = 0.2715, \lambda_2^{(1)} = 0.2188,$$
$$\lambda_3^{(1)} = 0.2567, \lambda_4^{(1)} = 0.253;$$
$$\lambda_1^{(2)} = 0.2205, \lambda_2^{(2)} = 0.2612,$$
$$\lambda_3^{(2)} = 0.2726, \lambda_4^{(2)} = 0.2457;$$
$$\lambda_1^{(3)} = 0.219, \lambda_2^{(3)} = 0.2787,$$
$$\lambda_3^{(3)} = 0.2455, \lambda_4^{(3)} = 0.2567;$$
$$\lambda_1^{(4)} = 0.2715, \lambda_2^{(4)} = 0.2188,$$
$$\lambda_3^{(4)} = 0.2567, \lambda_4^{(4)} = 0.253.$$

Step 3: 将个体决策矩阵$\tilde{R}^{(k)} (k = 1,2,\cdots,4)$利用PULWAA算子(式(7))聚合成群体决策矩阵$G = \{\eta_{ij}\}_{m\times n} = \langle [s_{\tilde{\theta}(\eta_{ij})}, s_{\tilde{\tau}(\eta_{ij})}], P(\tilde{\mu}_{\eta_{ij}}, \tilde{\nu}_{\eta_{ij}})\rangle$,结果如表5所示.

**表 5 专家群体决策矩阵 $G$**

| | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|
| $x_1$ | $\langle [s_{1.96},s_{3.19}],(0.77,0.22)\rangle$ | $\langle [s_{1.75},s_{3.04}],(0.67,0.37)\rangle$ | $\langle [s_{2.19},s_{3.77}],(0.73,0.34)\rangle$ | $\langle [s_{4.11},s_{5.35}],(0.72,0.42)\rangle$ |
| $x_2$ | $\langle [s_{2.02},s_{3.45}],(0.80,0.27)\rangle$ | $\langle [s_{1.19},s_{2.94}],(0.64,0.32)\rangle$ | $\langle [s_{2.17},s_{3.57}],(0.66,0.21)\rangle$ | $\langle [s_{3.55},s_{4.88}],(0.81,0.32)\rangle$ |
| $x_3$ | $\langle [s_{1.57},s_{3.54}],(0.66,0.38)\rangle$ | $\langle [s_{1.60},s_{3.21}],(0.78,0.29)\rangle$ | $\langle [s_{2.75},s_{4.36}],(0.79,0.27)\rangle$ | $\langle [s_{3.06},s_{4.31}],(0.66,0.33)\rangle$ |
| $x_4$ | $\langle [s_{1.98},s_{3.39}],(0.71,0.28)\rangle$ | $\langle [s_{2.20},s_{3.65}],(0.59,0.40)\rangle$ | $\langle [s_{1.65},s_{3.33}],(0.84,0.22)\rangle$ | $\langle [s_{2.80},s_{4.75}],(0.78,0.32)\rangle$ |

Step 4: 利用熵公式(14), 建立专家群体决策Pythagorean不确定语言熵矩阵$D_E$, 进而利用式(16)计算属性权重为

$$w_1 = 0.2510, w_2 = 0.2424,$$
$$w_3 = 0.2547, w_4 = 0.2519.$$

Step 5: 基于矩阵$G$,确定方案正负理想解为

$$\begin{aligned} g^+ = \{ & \langle [s_{2.0163}, s_{3.5421}], (0.7968, 0.2155)\rangle, \\ & \langle [s_{2.1975}, s_{3.56}], (0.7835, 0.2893)\rangle, \\ & \langle [s_{2.7504}, s_{4.3604}], (0.8365, 0.2046)\rangle, \\ & \langle [[s_{4.1061}, s_{5.347}], (0.8099, 0.3179)\rangle\}, \end{aligned}$$

$$\begin{aligned} g^- = \{ & \langle [s_{1.5727}, s_{3.194}], (0.6626, 0.379)\rangle, \\ & \langle [s_{1.1934}, s_{2.9388}], (0.5888, 0.399)\rangle, \\ & \langle [s_{1.6479}, s_{3.3262}], (0.655, 0.3429)\rangle, \\ & \langle [s_{2.7926}, s_{4.3065}], (0.6557, 0.4145)\rangle\}. \end{aligned}$$

Step 6: 根据式(18)和(19),结合距离公式(5),计算各方案的群体效益值$S(x_i)$和个体遗憾值$R(x_i)$为

$$S(x_1)=0.4695, S(x_2)=0.4905,$$
$$S(x_3)=0.5515, S(x_4)=0.5426,$$
$$R(x_1)=0.1626, R(x_2)=0.2064,$$
$$R(x_3)=0.2209, R(x_4)=0.1547;$$

根据式(20), $\lambda=0.5$,计算折衷值为

$$Q(x_1)=0.0592, Q(x_2)=0.5179,$$
$$Q(x_3)=1.0, Q(x_4)=0.4455.$$

Step 7: 根据$S(x_i)$、$R(x_i)$和$Q(x_i)$的值进行升序排列,分别得到备选方案的3个排序,结果见表6.

表 6　按$S_i, R_i, Q_i$的值升序排列结果

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 按$S_i$ | $x_1$ | $x_2$ | $x_4$ | $x_3$ |
| 按$R_i$ | $x_4$ | $x_1$ | $x_2$ | $x_3$ |
| 按$Q_i$ | $x_1$ | $x_4$ | $x_2$ | $x_3$ |

Step 8: 由表6可以看出, $x_1$满足$Q(x_1)-Q(x_4)=0.3863 \geqslant \dfrac{1}{(4-1)}=0.3333$,且$x_1$在依据$S(x_i)$排序时仍然是最小值,即$x_1$同时满足条件1和条件2. 因此, $\lambda=0.5$时, $x_1$为折衷方案.

### 4.2　敏感性分析

在实际决策中,专家可能有不同的决策态度,进而采取不同的折衷系数,即$\phi$可取[0, 1]之间的任何值. 下面分析折衷参数$\phi$的变化对最终排序结果的影响,计算结果如表7所示.

表 7　折衷系数$\phi$对方案排序的影响

| $\phi$ | $Q(x_1)$ | $Q(x_2)$ | $Q(x_3)$ | $Q(x_4)$ | 方案排序 | 折衷方案 |
|---|---|---|---|---|---|---|
| 0.0 | 0.1183 | 0.7803 | 1.0 | 0.0 | $x_4 \succ x_1 \succ x_2 \succ x_3$ | $x_1, x_4$ |
| 0.1 | 0.1065 | 0.7278 | 1.0 | 0.0891 | $x_4 \succ x_1 \succ x_2 \succ x_3$ | $x_1, x_4$ |
| 0.3 | 0.0828 | 0.6229 | 1.0 | 0.2673 | $x_1 \succ x_4 \succ x_2 \succ x_3$ | $x_1, x_4$ |
| 0.5 | 0.0592 | 0.5179 | 1.0 | 0.4455 | $x_1 \succ x_4 \succ x_2 \succ x_3$ | $x_1$ |
| 0.7 | 0.0355 | 0.4130 | 1.0 | 0.6237 | $x_1 \succ x_2 \succ x_4 \succ x_3$ | $x_1$ |
| 0.9 | 0.0118 | 0.3080 | 1.0 | 0.8019 | $x_1 \succ x_2 \succ x_4 \succ x_3$ | $x_1, x_2$ |
| 1.0 | 0.0 | 0.2555 | 1.0 | 0.8910 | $x_1 \succ x_2 \succ x_4 \succ x_3$ | $x_1, x_2$ |

显然,从表7可以看出$\phi$会对结果产生影响. 当$\phi$取值为$\{0, 0.1, 0.3\}$,即专家较多考虑个体遗憾时,折衷方案为$\{x_1, x_4\}$;当$\phi=0.5$,即专家认为群体效用和个体遗憾同等重要时,折衷方案为$x_1$;当$\phi$取值为0.9或1,即较多考虑群体效用时,折衷方案为$\{x_1, x_2\}$. 此外,当$\phi$分别取两个极端值,即当$\phi=0$,只考虑个体遗憾时,排序结果等价于依据个体遗憾最小$R(x_i)$进行排序;当$\phi=1$,即只考虑群体效用时,排序结果等价于依据群体效用最大$S(x_i)$进行排序. 由此可见, $\phi$描述了最大群体效用和最小个体遗憾之间的妥协, $\phi$值的变化可以表达专家不同的主观偏好,提高决策的灵活性和可用性.

### 4.3　方法对比分析

将本文提出的方法(PUL-VIKOR)同Pythagorean不确定语言TOPSIS方法(PUL-TOPSIS,扩展自文献[3])和Pythagorean不确定语言算术加权平均方法(PULWAA)进行比较,计算结果如表8所示.

表 8　方法对比

| 方案 | PUL-VIKOR($\phi=0$) | | PUL-VIKOR($\phi=0.5$) | | PUL-VIKOR($\phi=0$) | | PUL-TOPSIS | | PULWAA | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $Q(x_i)$ | 排序 | $Q(x_i)$ | 排序 | $Q(x_i)$ | 排序 | 贴近度 | 排序 | 得分 | 排序 |
| $x_1$ | 0.1183 | 2 | 0.0592 | 1 | 0 | 1 | 0.5071 | 1 | 0.3757 | 1 |
| $x_2$ | 0.7803 | 3 | 0.5179 | 3 | 0.2555 | 2 | 0.4763 | 2 | 0.3660 | 2 |
| $x_3$ | 1.0 | 4 | 1.0 | 4 | 1.0 | 4 | 0.4541 | 3 | 0.3662 | 4 |
| $x_4$ | 0.0 | 1 | 0.4455 | 2 | 0.8910 | 3 | 0.4333 | 4 | 0.3654 | 3 |

从表7和表8可以看出:当$\phi=0$时, PUL-VIKOR方法得到的排序为$x_4 \succ x_1 \succ x_2 \succ x_3$,折衷方案为$x_1$和$x_4$,而当$\phi=0.5$或1时,折衷方案均为$x_1$; PUL-TOPSIS方法和PULWAA方法得到的最优选择均为$x_1$. 虽然3种方法产生的排序结果存在一定差异,但得到的最优选择均为$x_1$,这也验证了本文所提方法的有效性. 产生差异的原因可能在于PUL-TOPSIS方法和PULWAA方法都没有考虑专家的主观偏好. 此外, PUL-VIKOR方法考虑了群体最大和个体遗憾最小来获得妥协解,同时结合语言刻度函数,可以适应不同语义环境.

## 5　结　论

Pythagorean不确定语言变量集成了Pythagorean模糊数和不确定语言变量的优点,是直觉语言数的扩展和一般化,更适合表达语言信息,具有更广泛的使用场合和较强的使用价值. 本文提出了一种扩展的VIKOR方法来处理Pythagorean不确定语言环境下的多属性群决策问题,通过航空公司客服质量评估

示例分析,说明了该方法的有效性,并进行了参数的敏感性分析以及对比分析. 下一步将针对专家和属性之间可能存在关联关系、权重部分未知的多属性决策问题,提出相应的Pythagorean不确定语言多属性群决策方法.

## 参考文献(References)


[1] Atanassov K T. Intuitionistic fuzzy sets[J]. Fuzzy Sets and Systems, 1986, 20(1): 87-96.

[2] Yager R R. Pythagorean membership grades in multi-criteria decision making[J]. IEEE Trans on Fuzzy Systems, 2014, 22(4): 958-965.

[3] Zhang X L, Xu Z S. Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets[J]. Int J of Intelligent Systems, 2014, 29(12): 1061-1078.

[4] Peng X, Yang Y. Some results for pythagorean fuzzy sets[J]. Int J of Intelligent Systems, 2015, 30(11): 1133-1160.

[5] Reformat M Z, Yager R R. Suggesting recommendations using Pythagorean fuzzy sets illustrated using netflix movie data[C]. Information Processing and Management of Uncertainty in Knowledge-Based Systems. Berlin: Springer, 2014: 546-556.

[6] Bustince H, Barrenechea E, Miguel Pagola, et al. A historical account of types of fuzzy sets and their relationships[J]. IEEE Trans on Fuzzy Systems, 2016, 24(1): 179-194.

[7] 王坚强, 李寒波. 基于直觉语言集结算子的多准则决策方法[J]. 控制与决策, 2010, 25(10): 1571-1574.
(Wang J Q, Li H B. Multi-criteria decision-making method based on aggregation operators for intuitionistic linguistic fuzzy numbers[J]. Control and Decision, 2010, 25(10): 1571-1574.)

[8] Xu Z S. Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment[J]. Information Sciences, 2004, 168(1): 171-184.

[9] 彭新东, 杨勇. 基于Pythagorean模糊语言集多属性群决策方法[J]. 计算机工程与应用, 2016, 52(23): 50-54.
(Peng X D, Yang Y. Multiple attribute group decision making methods based on Pythagorean fuzzy linguistic set computer engineering and applications[J]. Computer Engineering and Applications, 2016, 52(23): 50-54.)

[10] Opricovic S. Multicriteria optimization of civil engineering systems[J]. Faculty of Civil Engineering, Belgrade, 1998, 2(1): 5-21.

[11] Opricovic S, Tzeng G H. Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS[J]. European J of Operational Research, 2004, 156(2): 445-455.

[12] Opricovic S, Tzeng G H. Extended VIKOR method in comparison with outranking methods[J]. European J of Operational Research, 2007, 178(2): 514-529.

[13] Shemshadi A, Shirazi H, Toreihi M, et al. A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting[J]. Expert Systems with Applications, 2011, 38(10): 12160-12167.

[14] Dai C Y, Zhang X L, Wang E C, et al. Multi-criteria renewable energy planning decision-making model based on VIKOR[J]. Advanced Materials Research, 2012, 512: 1174-1180.

[15] Qin J, Liu X, Pedrycz W. An extended VIKOR method based on prospect theory for multiple attribute decision making under interval type-2 fuzzy environment[J]. Knowledge-Based Systems, 2015, 86: 116-130.

[16] Xu Z S, Zhao N. Information fusion for intuitionistic fuzzy decision making: An overview[J]. Information Fusion, 2016, 28: 10-23.

[17] Xu Z S. A method based on linguistic aggregation operators for group decision making with linguistic preference relations[J]. Information Sciences, 2004, 166(1): 19-30.

[18] Wang J Q, Wu J T, Wang J, et al. Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems[J]. Information Sciences, 2014, 288: 55-72.

[19] 鲍广宇, 连向磊, 何明, 等. 基于新型语言评估标度的二元语义改进模型[J]. 控制与决策, 2010, 25(5): 780-784.
(Bao G Y, Lian X L, He M, et al. Improved two-tuple linguistic representation model based on new linguistic evaluation scale[J]. Control and Decision, 2010, 25(5): 780-784.)

[20] Yue Z L. Approach to group decision making based on determining the weights of experts by using projection method[J]. Applied Mathematical Modelling, 2012, 36(7): 2900-2910.

[21] Chen Z P, Yang W. A new multiple attribute group decision making method in intuitionistic fuzzy setting[J]. Applied Mathematical Modelling, 2011, 35(9): 4424-4437.

[22] Chen T Y, Li C H. Objective weights with intuitionistic fuzzy entropy measures and computational experiment analysis[J]. Applied Soft Computing, 2011, 11(8): 5411-5423.

[23] 章玲, 张露平, 周鹏. 航空公司服务质量评价——基于非可加直觉模糊VIKOR方法[J]. 技术经济与管理研究, 2014(4): 8-14.
(Zhang L, Zhang L P, Zhou P. Evaluating airline service quality—A non-additive VIKOR methods in intuitionistic fuzzy settings[J]. Technoeconomics & Management Research, 2014(4): 8-14.)


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