文章编号: 1001-0920(2018)01-0119-07 DOI: 10.13195/j.kzyjc.2016.1431

区间直觉语言Frank集结算子及其在决策中的应用

杜玉琴[1,2†], 侯福均[2]

(1. 中国社会科学院大学 经济学院，北京 100089；2. 北京理工大学 管理与经济学院，北京 100081)

摘 要: 基于区间直觉语言变量和Frank算子的概念,首先提出区间直觉语言环境下Frank算子的运算规则;然后介绍几种区间直觉语言Frank信息集成算子,如:区间直觉语言Frank加权算术平均算子、区间直觉语言Frank加权几何平均算子、区间直觉语言广义Frank加权算术平均算子等,并介绍各算子具有的性质,同时,基于上述算子提出两种属性权重为实数且属性值为区间直觉语言变量的多属性决策方法;最后,结合示例表明所提出方法的有效性和实用性.

关键词: 区间直觉语言变量；Frank算子；多属性群决策；集结算子；模糊信息

中图分类号: C934 **文献标志码:** A

Interval intuitionistic linguistic Frank aggregation operators and their application in decision making

DU Yu-qin[1,2†], *HOU Fu-jun*[2]

(1. School of Economics, University of Chinese Academy of Social Sciences, Beijing 100089，China；2. School of Management and Economics, Beijing Institute of Technology, Beijing 100081，China)

Abstract: Based on the interval intuitionistic linguistic variables and the Frank operator, the operational laws of the Frank operator under the interval intuitionistic linguistic circumstance are firstly defined. Then, several interval intuitionistic linguistic Frank aggregation operators are introduced, such as the interval intuitionistic linguistic Frank weighted average operator, the interval intuitionistic linguistic Frank weighted geometric operator, and the interval intuitionistic linguistic Frank generalized weighted average operator, etc. and some of their properties are discussed. Moreover, two approaches are introduced on the basis of the interval intuitionistic linguistic Frank aggregation operators, and an example is given to illustrate the feasibility and effectiveness of the proposed approaches.

Keywords: interval intuitionistic linguistic variables；Frank operator；multiple attribute group decision making；aggregation operator；fuzzy information

0 引 言

1986年,Atanassov[1]在模糊集[2]的基础上定义了直觉模糊集,由于其能较好地描述客观世界的模糊性和不确定性,已被广泛应用于金融、物流管理、保险、模糊决策等诸多领域.

在自然环境中,很多现象的属性信息难以用数字进行描述,而语言评价[3]能较好地表达这种情况. 针对此类问题,徐泽水[4-5]定义了三角模糊语言变量和不确定语言变量;Herrera等[6]定义了二元语义集;Ju等[7]定义了梯形二元语义集;王坚强等[8]定义了直觉模糊语言集和直觉二元语义的距离公式;Liu[9]定义了广义直觉模糊语言集结算子、二维语言广义集结算子[10]、直觉不确定语言集[11]、区间直觉模糊不确定语言集[12];王佩[13]在区间直觉模糊不确定语言变量的基础上,提出了区间直觉语言数的概念,并定义了区间直觉语言数的加权算术平均算子(IVILWA)、区间直觉语言数的加权几何平均算子(IVILWG).

阿基米德T模和阿基米德S模包含代数T模和代数S模[14-16],Enistein T模和Enistein S模[17],Hamacher T模和Hamacher S模[18],Frank T模和Frank S模[19]等,其中Frank T模和Frank S模具有一定的兼容性,可依据参数的变化解决问题,能处理更多的多属性群决策问题. 例如:Casasnovas等[20]针对Frank T模标量基数介绍了一种公理化方法,并证明这个公理

收稿日期: 2016-11-13；**修回日期:** 2017-03-12.

基金项目: 国家自然科学基金项目(71571019)；中国青年政治学院学术创新支持计划项目(189100113).

作者简介: 杜玉琴(1976−), 女, 讲师, 博士, 从事决策理论与方法的研究；侯福均(1967−), 男, 副教授, 博士生导师, 从事决策理论与方法、运筹与优化等研究.

†通讯作者. E-mail: duyq1234567@163.com

化特性适合其他的标准T模; Yager[21] 依据两种不同的方法介绍Frank T模和Frank S模公式; Sarkoci[22] 通过研究指出Hamacher T模和Frank T模属于相同类型的集结算子; Wang等[23] 证明了Frank T模和Frank S模运算规则满足概率测度公理,具有连续单调特性; Deschrijver[24] 介绍了区间模糊集扩展运算规则; Qin等[25] 介绍了三角区间二型模糊Frank加权集结算子,以及算子所具有的性质; Qin等[26] 定义了模糊犹豫环境下Frank算子运算规则,并介绍了几类犹豫模糊Frank信息集成算子,如加权平均算子(HFFWA)、有序加权平均算子(HFFOWA)、混合几何算子(HFFHG)等概念; Zhang等[27] 定义了几种区间直觉模糊Frank信息集结算子,并研究了算子所具有的相关性质; Zhang等[28] 定义了几类直觉模糊Frank幂信息集结算子, 并研究了算子的性质.

目前,关于Frank T模和Frank S模的研究主要集中在基础理论部分,在多属性群决策应用方面还很少见. 由于区间直觉模糊语言能更客观、更准确地表达客观世界的不确定性和模糊性,研究在区间直觉模糊语言环境下的Frank T模和Frank S模的应用问题具有一定的理论价值. 本文首先定义Frank算子在区间直觉语言环境下的计算公式;然后定义几类区间直觉语言Frank信息集成算子,并研究算子所具有的几种性质;最后,将这些算子应用于解决区间直觉语言模糊环境下的多属性群决策问题. 示例计算结果表明了本文方法的有效性.

1 预备知识

定义1 [13] 令 $\bar{S}=\{s_i|s_0\leqslant s_i\leqslant s_g, i\in[0,g]\}$ 为 $S=\{s_0,s_1,\cdots,s_g\}$ 的连续表达式, $s_{\theta(x)}\in\bar{S}$, X 是一个给定的论域,则

$$A=\{x\langle s_{\tau(x)},([u_A^L(x),u_A^U(x)],[v_A^L(x),v_A^U(x)])\rangle|x\in X\}$$

为区间直觉语言集. 其中: $u_A(x):X\to[0,1]$ 和 $v_A(x):X\to[0,1]$ 分别表示元素 x 对语言评价值 $s_{\theta(x)}$ 的隶属度和非隶属度,且有 $0\leqslant u_A(x)+v_A(x)\leqslant 1,\forall x\in X$.

定义2 [13] 令 $\alpha_i=\{s_{\tau(a_i)},([u^L(a_i),u^U(a_i)],[v^L(a_i),v^U(a_i)])\}$ 为一个区间直觉语言变量, α_i 的期望函数定义为

$$\mathrm{E}(\alpha_i)=\frac{1}{2}\Big(\frac{u^L(a_i)+u^U(a_i)}{2}+1-\frac{v^L(a_i)+v^U(a_i)}{2}\Big)s_{\tau(a_i)}=s_{\tau(a_i)\times(u^L(a_i)+u^U(a_i)+2-v^L(a_i)-v^U(a_i))/4}. \tag{1}$$

α_i 的精确值函数 $H(\alpha_i)$ 定义为

$$H(\alpha_i)=\Big(\frac{u^L(a_i)+u^U(a_i)}{2}+\frac{v^L(a_i)+v^U(a_i)}{2}\Big)\times s_{\tau(a_i)}=s_{\tau(a_i)\times(u^L(a_i)+u^U(a_i)+v^L(a_i)+v^U(a_i))/2}. \tag{2}$$

定义3 [13] 令 α_1 和 α_2 为任意两个区间直觉语言变量,则有:

1) 如果 $E(\alpha_1)>E(\alpha_2)$,则 $\alpha_1>\alpha_2$;

2) 当 $E(\alpha_1)=E(\alpha_2)$ 时,如果 $H(\alpha_1)>H(\alpha_2)$,则 $\alpha_1>\alpha_2$;如果 $H(\alpha_1)=H(\alpha_2)$,则 $\alpha_1=\alpha_2$.

定义4 [24] Frank T-模和Frank S-模定义如下:

$$T(x,y)=x\otimes y=\log_\theta\Big(1+\frac{(\theta^x-1)(\theta^y-1)}{\theta-1}\Big),\ \forall(x,y)\in[0,1]\times[0,1]; \tag{3}$$

$$S(x,y)=x\oplus y=1-\log_\theta\Big(1+\frac{(\theta^{1-x}-1)(\theta^{1-y}-1)}{\theta-1}\Big),\ \forall(x,y)\in[0,1]\times[0,1]. \tag{4}$$

2 区间直觉语言环境下Frank 算子的运算规则

根据区间直觉语言变量、Frank T模和Frank S模的定义,本文定义区间直觉语言环境下Frank算子的运算规则.

定义5 令 $\alpha_i=\langle s_{\tau(a_i)},([u^L(a_i),u^U(a_i)],[v^L(a_i),v^U(a_i)])\rangle(i=1,2)$ 是两个区间直觉语言变量,则有:

1) $\alpha_1\oplus\alpha_2=$

$$\Big\langle s_{\tau(a_1)+\tau(a_2)},\Big(\Big[1-\log_\theta\Big(1+\frac{(\theta^{1-u^L(a_1)}-1)(\theta^{1-u^L(a_2)}-1)}{(\theta-1)}\Big),1-\log_\theta\Big(1+\frac{(\theta^{1-u^U(a_1)}-1)(\theta^{1-u^U(a_2)}-1)}{(\theta-1)}\Big)\Big],\Big[\log_\theta\Big(1+\frac{(\theta^{v^L(a_1)}-1)(\theta^{v^L(a_2)}-1)}{(\theta-1)}\Big),\log_\theta\Big(1+\frac{(\theta^{v^U(a_1)}-1)(\theta^{v^U(a_2)}-1)}{(\theta-1)}\Big)\Big]\Big)\Big\rangle;$$

2) $\alpha_1\otimes\alpha_2=$

$$\Big\langle s_{\tau(a_1)\tau(a_2)},\Big(\Big[\log_\theta\Big(1+\frac{(\theta^{u^L(a_1)}-1)(\theta^{u^L(a_2)}-1)}{(\theta-1)}\Big),\log_\theta\Big(1+\frac{(\theta^{u^U(a_1)}-1)(\theta^{u^U(a_2)}-1)}{(\theta-1)}\Big)\Big],\Big[1-\log_\theta\Big(1+\frac{(\theta^{1-v^L(a_1)}-1)(\theta^{1-v^L(a_2)}-1)}{(\theta-1)}\Big),1-\log_\theta\Big(1+$$

$$\frac{(\theta^{1-v^U(a_1)}-1)(\theta^{1-v^U(a_2)}-1)}{(\theta-1)}\Big)\Big]\Big)\Big\rangle;$$

3) $\lambda\alpha=$

$$\Big\langle s_{\lambda\tau(a)},\Big(\Big[1-\log_\theta\Big(1+\frac{(\theta^{1-u^L(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big),$$
$$1-\log_\theta\Big(1+\frac{(\theta^{1-u^U(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big)\Big],$$
$$\Big[\log_\theta\Big(1+\frac{(\theta^{v^L(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big),\log_\theta\Big(1+$$
$$\frac{(\theta^{v^U(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big)\Big]\Big)\Big\rangle,\ \lambda>0;$$

4) $\alpha^\lambda=$

$$\Big\langle s_{(\tau(a))^\lambda},\Big(\Big[\log_\theta\Big(1+\frac{(\theta^{u^L(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big),$$
$$\log_\theta\Big(1+\frac{(\theta^{u^U(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big)\Big],$$
$$\Big[1-\log_\theta\Big(1+\frac{(\theta^{1-v^L(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big),$$
$$1-\log_\theta\Big(1+\frac{(\theta^{1-v^U(a)}-1)^\lambda}{(\theta-1)^{\lambda-1}}\Big)\Big]\Big)\Big\rangle,\ \lambda>0.$$

易证上述计算结果仍为区间直觉语言变量.

定义6　令$\alpha_i=\langle s_{\tau(a_i)},([u^L(a_i),u^U(a_i)],[v^L(a_i),v^U(a_i)])\rangle(i=1,2,3)$是任意三个区间直觉语言变量,且$\lambda,\lambda_1,\lambda_2\geqslant 0$,则:

1) $\alpha_1\oplus\alpha_2=\alpha_2\oplus\alpha_1$;

2) $\alpha_1\otimes\alpha_2=\alpha_2\otimes\alpha_1$;

3) $(\alpha_1\oplus\alpha_2)\oplus\alpha_3=\alpha_1\oplus(\alpha_2\oplus\alpha_3)$;

4) $(\alpha_1\otimes\alpha_2)\otimes\alpha_3=\alpha_1\otimes(\alpha_2\otimes\alpha_3)$;

5) $\lambda(\alpha_1\oplus\alpha_2)=\lambda\alpha_2\oplus\lambda\alpha_1$;

6) $\lambda_1\alpha_1\oplus\lambda_2\alpha_1=(\lambda_1+\lambda_2)\otimes\alpha_1$;

7) $\alpha_1^{\lambda_1}\otimes\alpha_1^{\lambda_2}=\alpha_1^{\lambda_1+\lambda_2}$;

8) $(\alpha_1\otimes\alpha_2)^\lambda=\alpha_1^\lambda\otimes\alpha_2^\lambda$.

3　区间直觉语言Frank信息集成算子

基于区间直觉语言变量Frank算子的运算规则,下面定义几类信息集成算子.

3.1　区间直觉语言Frank加权算术平均算子

定义7　设$\alpha_i(i=1,2,\cdots,n)$是一组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i$的权重向量,且有$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$,其中$\sum_{i=1}^n w_i=1$,$\Omega$为所有区间直觉语言变量集合,则区间直觉语言Frank加权算术平均算子(IVILFWA)可定义如下(IVILFWA: $\Omega^n\to\Omega$):

$$\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\sum_{i=1}^n w_i\alpha_i=$$
$$\Big\langle s_{\sum_{i=1}^n w_i\cdot\tau(a_i)},\Big(\Big[1-\log_\theta\Big(1+\prod_{i=1}^n(\theta^{1-u^L(a_i)}-$$
$$1)^{w_i}\Big),1-\log_\theta\Big(1+\prod_{i=1}^n(\theta^{1-u^U(a_i)}-1)^{w_i}\Big)\Big],$$
$$\Big[\log_\theta\Big(1+\prod_{i=1}^n(\theta^{v^L(a_i)}-1)^{w_i}\Big),$$
$$\log_\theta\Big(1+\prod_{i=1}^n(\theta^{v^U(a_i)}-1)^{w_i}\Big)\Big]\Big)\Big\rangle.\tag{5}$$

定理1 (有界性)　设$\alpha_i(i=1,2,\cdots,n)$为一组区间直觉语言变量,若

$$s_{\tau^-}=\min_{1\leqslant i\leqslant n}\{s_{\tau(a_i)}\},\ s_{\tau^+}=\max_{1\leqslant i\leqslant n}\{s_{\tau(a_i)}\},$$
$$u_-^L=\min_{1\leqslant i\leqslant n}\{u^L(a_i)\},\ u_-^U=\min_{1\leqslant i\leqslant n}\{u^U(a_i)\},$$
$$u_+^L=\max_{1\leqslant i\leqslant n}\{u^L(a_i)\},\ u_+^U=\max_{1\leqslant i\leqslant n}\{u^U(a_i)\},$$
$$v_-^L=\min_{1\leqslant i\leqslant n}\{v^L(a_i)\},\ v_+^L=\max_{1\leqslant i\leqslant n}\{v^L(a_i)\},$$
$$v_-^U=\min_{1\leqslant i\leqslant n}\{v^U(a_i)\},\ v_+^U=\max_{1\leqslant i\leqslant n}\{v^U(a_i)\},$$

则$\langle s_{\tau^-},([u_-^L,u_-^U],[v_+^L,v_+^U])\rangle\leqslant\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\leqslant\langle s_{\tau^+},([u_+^L,u_+^U],[v_-^L,v_-^U])\rangle$.

证明过程与文献[27]类似, 此处省略.

定理2 (幂等性)　设$\alpha_i(i=1,2,\cdots,n)$为一组区间直觉语言变量,若$\alpha_i=\alpha_0(i=1,2,\cdots,n)$,则$\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\alpha_0$.

定理3 (单调性)　设$\alpha_i(i=1,2,\cdots,n)$与$\alpha_i'(i=1,2,\cdots,n)$为两组区间直觉语言变量,若$\alpha_i'\leqslant\alpha_i,i=1,2,\cdots,n$,则$\mathrm{IVILFWA}(\alpha_1',\alpha_2',\cdots,\alpha_n')=\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)$.

定理4　设$\alpha_i(i=1,2,\cdots,n)$为一组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i(i=1,2,\cdots,n)$的权重向量,其中$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$且$\sum_{i=1}^n w_i=1$,如果$r>0$,$\alpha$是一个区间直觉语言变量,则:

1) $\mathrm{IVILFWA}(r\alpha_1,r\alpha_2,\cdots,r\alpha_n)=r\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)$;

2) $\mathrm{IVILFWA}(\alpha_1\oplus\alpha,\alpha_2\oplus\alpha,\cdots,\alpha_n\oplus\alpha)=\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\oplus\alpha$;

3) $\mathrm{IVILFWA}(r\alpha_1\oplus\alpha,r\alpha_2\oplus\alpha,\cdots,r\alpha_n\oplus\alpha)=r\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\oplus\alpha$.

证明过程与文献[27]类似, 此处省略.

定理5　设$\alpha_i(i=1,2,\cdots,n)$和$\alpha_i'(i=1,2,\cdots,n)$是两组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i(i=1,2,\cdots,n)$的权重向量,其中$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$且$\sum_{i=1}^n w_i=1$,则

$$\text{IVILFWA}(\alpha_1\oplus\alpha_1',\alpha_2\oplus\alpha_2',\cdots,\alpha_n\oplus\alpha_n')=\text{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)\oplus\text{IVILFWA}(\alpha_1',\alpha_2',\cdots,\alpha_n').$$

证明过程可参考文献[27],此处省略.

下面讨论IVILFWA算子的特殊情况.

1) 当$\theta\to 1$时,IVILFWA算子简化为IVILWA算子,即

$$\lim_{\theta\to 1}\text{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\text{IVILWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\Big\langle s_{\sum\limits_{i=1}^n w_i\cdot\tau(a_i)},\Big(\Big[1-\prod_{i=1}^n(1-u^L(a_i))^{w_i},1-\prod_{i=1}^n(1-u^U(a_i))^{w_i}\Big],\Big[(v^L(a_i))^{w_i},(v^U(a_i))^{w_i}\Big]\Big)\Big\rangle;$$

2) 当$\theta\to+\infty$时,IVILFWA算子简化为传统的算术加权平均算子,即

$$\lim_{\theta\to+\infty}\text{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\Big\langle s_{\sum\limits_{i=1}^n w_i\cdot\tau(a_i)},\Big(\Big[\sum_{i=1}^n w_iu^L(a_i),\sum_{i=1}^n w_iu^U(a_i)\Big],\Big[\sum_{i=1}^n w_iv^L(a_i),\sum_{i=1}^n w_iv^U(a_i)\Big]\Big)\Big\rangle.$$

3.2 区间直觉语言Frank加权几何平均算子

定义8 设$\alpha_i(i=1,2,\cdots,n)$为一组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i(i=1,2,\cdots,n)$的权重向量,其中$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$且$\sum\limits_{i=1}^n w_i=1$,则区间直觉语言Frank加权几何平均算子(IVILFWG)定义如下:

$$\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\prod_{i=1}^n\alpha_i^{w_i}=\Big\langle s_{\prod\limits_{i=1}^n(\tau(a_i))^{w_i}},\Big(\Big[\log_\theta\Big(1+\prod_{j=1}^n(\theta^{u^L(a_i)}-1)^{w_i}\Big),\log_\theta\Big(1+\prod_{j=1}^n(\theta^{u^U(a_i)}-1)^{w_i}\Big)\Big],\Big[1-\log_\theta\Big(1+\prod_{i=1}^n(\theta^{1-v^L(a_i)}-1)^{w_i}\Big),1-\log_\theta\Big(1+\prod_{i=1}^n(\theta^{1-v^U(a_i)}-1)^{w_i}\Big)\Big]\Big)\Big\rangle.\quad(6)$$

其中:$\varOmega$为所有区间直觉语言变量集合,IVILFWG: $\varOmega^n\to\varOmega$.

定理6 设$\alpha_i(i=1,2,\cdots,n)$是一组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i(i=1,2,\cdots,n)$的权重向量,其中$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$且$\sum\limits_{i=1}^n w_i=1$,如果$r>0$,$\alpha$是一个区间直觉语言变量,则:

1) $\text{IVILFWG}((\alpha_1)^r,(\alpha_2)^r,(\alpha_3)^r,\cdots,(\alpha_n)^r)=(\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n))^r$;

2) $\text{IVILFWG}(\alpha_1\otimes\alpha,\alpha_2\otimes\alpha,\cdots,\alpha_n\otimes\alpha)=\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)\otimes\alpha$;

3) $\text{IVILFWG}((\alpha_1)^r\otimes\alpha,(\alpha_2)^r\otimes\alpha,\cdots,(\alpha_n)^r\otimes\alpha)=(\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n))^r\otimes\alpha$.

定理7 设$\alpha_i(i=1,2,\cdots,n)$和$\alpha_i'(i=1,2,\cdots,n)$是两组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i(i=1,2,\cdots,n)$的权重向量,其中$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$且$\sum\limits_{i=1}^n w_i=1$,则

$$\text{IVILFWG}(\alpha_1\otimes\alpha_1',\alpha_2\otimes\alpha_2',\cdots,\alpha_n\otimes\alpha_n')=\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)\otimes\text{IVILFWG}(\alpha_1',\alpha_2',\cdots,\alpha_n').$$

下面讨论IVILFWG算子的特殊情况.

1) 当$\theta\to 1$时,IVILFWG算子简化为IVILWG算子,即

$$\lim_{\theta\to 1}\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\text{IVILWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\Big\langle s_{\sum\limits_{i=1}^n w_i\cdot\tau(a_i)},\Big([(u^L(a_i))^{w_i},(u^U(a_i))^{w_i}],\Big[1-\prod_{i=1}^n(1-v^L(a_i))^{w_i},1-\prod_{i=1}^n(1-v^U(a_i))^{w_i}\Big]\Big)\Big\rangle;$$

2) 当$\theta\to+\infty$时,IVILFWA算子简化为传统的算术加权平均算子,即

$$\lim_{\theta\to+\infty}\text{IVILFWG}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\Big\langle s_{\sum\limits_{i=1}^n w_i\cdot\tau(a_i)},\Big(\Big[\sum_{i=1}^n w_iu^L(a_i),\sum_{i=1}^n w_iu^U(a_i)\Big],\Big[\sum_{i=1}^n w_iv^L(a_i),\sum_{i=1}^n w_iv^U(a_i)\Big]\Big)\Big\rangle.$$

与区间直觉语言Frank加权算术平均算子(IVILFWA)类似,区间直觉语言Frank加权几何平均算子(IVILFWG)同样具有幂等性、有界性、单调性等性质.

3.3 区间直觉语言广义Frank加权算术平均算子

定义9 设$\alpha_i(i=1,2,\cdots,n)$为一组区间直觉语言变量,$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$为$\alpha_i$的权重向量,满足$0\leqslant w_i\leqslant 1(i=1,2,\cdots,n)$且$\sum\limits_{i=1}^n w_i=1$,则区间直觉语言广义Frank加权算术平均算子(IVILFGWA)定义如下(IVILFGWA : $\varOmega^n\to\varOmega$):

$$\mathrm{IVILFGWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\Big(\sum_{i=1}^{n}w_i\alpha_i^{\lambda}\Big)^{1/\lambda}=$$

$$\Big\langle s_{\big(\sum\limits_{i=1}^{n}w_i(\tau(a_i))^{\lambda}\big)^{1/\lambda}},\Big(\Big[\log_\theta\Big(1+\frac{\Big(\dfrac{\theta}{1+h^L}-1\Big)^{1/\lambda}}{(\theta-1)^{1/\lambda-1}}\Big),$$

$$\log_\theta\Big(1+\frac{\Big(\dfrac{\theta}{1+h^U}-1\Big)^{1/\lambda}}{(\theta-1)^{1/\lambda-1}}\Big)\Big],$$

$$\Big[1-\log_\theta\Big(1+\frac{\Big(\dfrac{\theta}{1+g^L}-1\Big)^{1/\lambda}}{(\theta-1)^{1/\lambda-1}}\Big),$$

$$1-\log_\theta\Big(1+\frac{\Big(\dfrac{\theta}{1+g^U}-1\Big)^{1/\lambda}}{(\theta-1)^{1/\lambda-1}}\Big)\Big]\Big)\Big\rangle.$$

其中

$$h^L=\prod_{i=1}^{n}\Big(\frac{(\theta-1)^{\lambda}-(\theta^{u_i^L}-1)^{\lambda}}{(\theta-1)^{\lambda-1}+(\theta^{u_i^L}-1)^{\lambda}}\Big)^{w_i},$$

$$h^U=\prod_{i=1}^{n}\Big(\frac{(\theta-1)^{\lambda}-(\theta^{u_i^U}-1)^{\lambda}}{(\theta-1)^{\lambda-1}+(\theta^{u_i^U}-1)^{\lambda}}\Big)^{w_i},$$

$$g^L=\prod_{i=1}^{n}\Big(\frac{(\theta-1)^{\lambda}-(\theta^{1-v_i^L}-1)^{\lambda}}{(\theta-1)^{\lambda-1}+(\theta^{1-v_i^L}-1)^{\lambda}}\Big)^{w_i},$$

$$g^U=\prod_{i=1}^{n}\Big(\frac{(\theta-1)^{\lambda}-(\theta^{1-v_i^U}-1)^{\lambda}}{(\theta-1)^{\lambda-1}+(\theta^{1-v_i^U}-1)^{\lambda}}\Big)^{w_i},$$

Ω为所有直觉不确定语言变量集合, $\lambda>0$.

特殊地, 当$\lambda=1$时, IVILFGWA算子简化为IVILFWA算子, 与文献[28]类似, 即

$$\mathrm{IVILFWA}(\alpha_1,\alpha_2,\cdots,\alpha_n)=\sum_{i=1}^{n}w_i\alpha_i.$$

4　区间直觉语言模糊Frank信息集成算子在决策中的应用

设$A=\{A_1,A_2,\cdots,A_m\}$为一组方案集, $C=\{C_1,C_2,\cdots,C_n\}$为属性集合, 属性权重为$w=(w_1,w_2,\cdots,w_n)^{\mathrm T}$, 其中$w_j\in[0,1]$且$\sum\limits_{j=1}^{n}w_j=1(j=1,2,\cdots,n)$. 设专家给出的区间直觉语言决策矩阵为$R=[\alpha_{ij}]_{m\times n}$. 其中: $R_{ij}=\langle s_{\tau(a_{ij})},([u_{ij}^L,u_{ij}^U],[v_{ij}^L,v_{ij}^U])\rangle$ $(i=1,2,\cdots,m,\ j=1,2,\cdots,n)$为区间直觉语言变量集合, 表示对方案$A_i$在属性$C_j$下的评价值; $s_{\tau(a_{ij})}\in S,\ S=\{s_0,s_1,\cdots,s_g\}$为一个语言评价集合. 试选择最佳方案.

Step 1:　运用区间直觉语言Frank算术平均算子(IVILFWA)或区间直觉语言Frank几何平均算子(IVILFWG)对方案的属性值进行集成. 对矩阵$R=[\alpha_{ij}]_{m\times n}$中的第$i$行进行集成, 求方案$A_i$的属性值$\alpha_i$, 有

$$\alpha_i=\mathrm{IVILFWA}(\alpha_{i1},\alpha_{i2},\cdots,a_{in})=\sum_{j=1}^{n}w_j\alpha_{ij}=$$

$$\Big\langle s_{\sum\limits_{j=1}^{n}w_j\cdot\tau(a_{ij})},\Big(\Big[1-\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{1-u^L(a_{ij})}-1)^{w_j}\Big),1-\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{1-u^U(a_{ij})}-1)^{w_j}\Big)\Big],$$

$$\Big[\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{v^L(a_{ij})}-1)^{w_j}\Big),$$

$$\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{v^U(a_{ij})}-1)^{w_j}\Big)\Big]\Big)\Big\rangle,\tag{7}$$

或

$$\alpha_i=\mathrm{IVILFWG}(\alpha_{i1},\alpha_{i2},\cdots,\alpha_{in})=\prod_{j=1}^{n}(\alpha_{ij})^{w_j}=$$

$$\Big\langle s_{\prod\limits_{j=1}^{n}(\tau(a_{ij}))^{w_j}},\Big(\Big[\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{u^L(a_{ij})}-1)^{w_j}\Big),$$

$$\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{u^U(a_{ij})}-1)^{w_j}\Big)\Big],\Big[1-\log_\theta\Big(1+\prod_{j=1}^{n}(\theta^{1-v^L(a_{ij})}-1)^{w_j}\Big),1-\log_\theta\Big(1+$$

$$\prod_{j=1}^{n}(\theta^{1-v^U(a_{ij})}-1)^{w_j}\Big)\Big]\Big)\Big\rangle.\tag{8}$$

Step 2: 根据式(1)计算$A_i(i=1,2,\cdots,n)$方案的期望函数$\mathrm{E}(P_i)$, 根据式(2)计算$A_i(i=1,2,\cdots,n)$方案的精确函数$H(P_i)(i=1,2,\cdots,m)$.

Step 3: 根据定义3选出最佳方案.

5　示　例

某地区教育部需要对当地的4个学校$\{A_1,A_2,A_3,A_4\}$进行教学评估, 从以下4个方面进行考核: C_1——教学运行与监控; C_2——教育质量与成果; C_3——专业建设; C_3——社会评价及影响. 这4个方面的属性权重为$w=(0.27,0.33,0.22,0.18)$. 专家用区间直觉语言变量给出各个学校的评价值$R=[\alpha_{ij}]_{m\times n}$, 见表1. 专家采用的语言评价集$s=\{s_0,s_1,s_2,s_3,s_4,s_5,s_6\}$, 要求根据评估结果对上述4个学校进行优劣排序.

方法1　利用IVILFWA集成算子进行集成.

决策步骤如下.

Step 1: 根据式(7)利用IVILFWA集成算子对矩阵$R=[\alpha_{ij}]_{m\times n}$中的第$i$行进行集成, 得到方案$A_i$的综合属性值$\alpha_i$, 即

表 1 4个学校的区间直觉语言评价值

	C_1	C_2	C_3	C_4
A_1	$\langle s_5$, [0.1, 0.2], [0.6, 0.7]$\rangle$	$\langle s_2$, [0.3, 0.4], [0.5, 0.6]$\rangle$	$\langle s_5$, [0.3, 0.4], [0.5, 0.55]$\rangle$	$\langle s_3$, [0.1, 0.2], [0.5, 0.6]$\rangle$
A_2	$\langle s_4$, [0.3, 0.4], [0.5, 0.6]$\rangle$	$\langle s_5$, [0.2, 0.3], [0.4, 0.5]$\rangle$	$\langle s_3$, [0.1, 0.15], [0.6, 0.8]$\rangle$	$\langle s_4$, [0.3, 0.4], [0.4, 0.5]$\rangle$
A_3	$\langle s_3$, [0.1, 0.2], [0.6, 0.7]$\rangle$	$\langle s_4$, [0.1, 0.2], [0.6, 0.7]$\rangle$	$\langle s_4$, [0.2, 0.3], [0.5, 0.7]$\rangle$	$\langle s_4$, [0.1, 0.2], [0.6, 0.7]$\rangle$
A_4	$\langle s_6$, [0.4, 0.5], [0.4, 0.6]$\rangle$	$\langle s_3$, [0.1, 0.2], [0.5, 0.6]$\rangle$	$\langle s_3$, [0.1, 0.2], [0.5, 0.6]$\rangle$	$\langle s_3$, [0.2, 0.3], [0.5, 0.6]$\rangle$

$$\alpha_1 = \langle s_{3.65}, [s_{0.215}, s_{0.316}], [s_{0.526}, s_{0.614}]\rangle,$$
$$\alpha_2 = \langle s_{3.12}, [s_{0.226}, s_{0.317}], [s_{0.465}, s_{0.584}]\rangle,$$
$$\alpha_3 = \langle s_{3.73}, [s_{0.123}, s_{0.223}], [s_{0.577}, s_{0.7}]\rangle,$$
$$\alpha_4 = \langle s_{3.81}, [s_{0.207}, s_{0.309}], [s_{0.471}, s_{0.6}]\rangle.$$

Step 2: 利用式(1)计算α_i的期望值,有

$$\mathrm{E}(\alpha_1) = s_{1.269},\ \mathrm{E}(\alpha_2) = s_{1.165},$$
$$\mathrm{E}(\alpha_3) = s_{0.997},\ \mathrm{E}(\alpha_4) = s_{1.377}.$$

Step 3: 对方案$A_i(i = 1,2,3,4)$进行排序,得$A_4 \succ A_1 \succ A_2 \succ A_3$,可知$A_4$为最佳选择方案.

方法2 利用IVILFWG算子进行集成.

与方法1类似,由于篇幅有限,计算过程省略. 由方法2得出的最终排序仍为$A_4 \succ A_1 \succ A_2 \succ A_3$,可知$A_4$为最佳选择方案.

为了更准确地得到例题的最优排序,本文研究随着θ取值的变化,学校的排序是否发生变化. 结果见表2.

表 2 根据不同的θ值对各方案进行优劣排序

θ	IVILFWA	IVILFWG
1.001	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
1.4	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
2	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
3	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
10	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
60	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
200	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$
1 000	$A_4 \succ A_1 \succ A_2 \succ A_3$	$A_4 \succ A_1 \succ A_2 \succ A_3$

研究发现,随着θ的变化,用两种不同的决策方法得出的优劣排序并没有变化,即$A_4 \succ A_1 \succ A_2 \succ A_3$,可知$A_4$评估分数最高.

另外,为了进一步验证文中所提出方法的可行性,运用文献[29]中的区间Pythagorean模糊语言加权算术平均(IVPFLWA)算子解决此问题,经计算可得$\mathrm{E}(\alpha_1) = s_{1.591\,955}$,$\mathrm{E}(\alpha_2) = s_{1.488\,77}$,$\mathrm{E}(\alpha_3) = s_{1.431\,138}$,$\mathrm{E}(\alpha_4) = s_{1.760\,241}$,由此可知,学校优劣排序仍为$A_4 \succ A_1 \succ A_2 \succ A_3$,此排序与文中所提出的方法一致,从而验证了本文方法的有效性和实用性. 本文所用的方法涉及到参数θ,决策者可根据承担风险能力的强弱来适当地选择θ的数值. 本文方法比IVPFLWA算子[29]在应用时具有更大的灵活性.

6 结 论

针对Frank算子的研究目前主要集中在理论探讨部分,在多属性群决策的应用方面尚处于起步阶段. 本文研究Frank算子在区间直觉语言环境下的多属性群决策问题,具有较大的实用价值和理论意义.

本文首先在区间直觉语言变量和Frank算子的基础上,定义了Frank算子在区间直觉语言环境下的计算公式;然后提出了几种区间直觉语言Frank信息集成算子,例如区间直觉语言Frank加权算术平均算子、区间直觉语言Frank加权几何平均算子、区间直觉语言广义Frank加权算术平均算子,同时给出了算子所具有的幂等性、单调性、有界性等性质;最后,将这些算子应用于属性权重确知且属性值以区间直觉语言变量形式给出的多属性群决策问题中.

本文提出的方法定义清晰,是对直觉模糊决策方法的丰富,具有很强的实用性,可以解决大量的实际多属性群决策问题. 本文方法可以进一步应用到投资组合、风险管理、最优化理论、可靠性分析、绿色供应商评价等领域.

参考文献(References)

[1] Atanassov K. Intuitionistic fuzzy sets[J]. Fuzzy Sets and Systems, 1986, 20(1): 87-96.

[2] Zadeh L A. Fuzzy sets[J]. Information Control, 1965, 8(3): 338-353.

[3] Zadeh L A. The concept of a linguistic variable and its application to approximate reasoning[J]. Information Sciences, 1975, 8(3): 199-249.

[4] Xu Z S. Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment[J]. Information Sciences, 2004, 168(1/2/3/4): 171-184.

[5] Xu Z S. Group decision making with triangular fuzzy linguistic variables[C]. Int Conf on Intelligent Data Engineering and Automated Learning. Berlin Heidelberg: Springer, 2007, 4881: 17-26.

[6] Herrera F, Martinez L. A 2-tuple fuzzy linguistic representation model for computing with words[J]. IEEE

Trans on Fuzzy Systems, 2000, 8(6): 746-752.

[7] Ju Y, Liu X, Yang S. Trapezoid fuzzy 2-tuple linguistic aggregation operators and their application to multiple attribute decision making[J]. Soft Computing, 2014, 27(3): 1219-1232.

[8] 王坚强, 李寒波. 基于直觉语言集结算子的多准则决策方法[J]. 控制与决策, 2010, 25(10): 1571-1574.
(Wang J Q, Li H B. Multi-criteria decision-making method based on aggregation operators for intuitionistic linguistic fuzzy numbers[J]. Control and Decision, 2010, 25(10): 1571-1574.)

[9] Liu P D. Some generalized dependent aggregation operators with intuitionistic linguistic numbers and their application to group decision making[J]. J of Computer & System Sciences, 2013, 79(1): 131-143.

[10] Liu P D, Qi X. Some generalized dependent aggregation operators with 2-dimension linguistic information and their application to group decision making[J]. J of Intelligent & Fuzzy Systems, 2014, 27(4): 1761-1773.

[11] Liu P D, Jin F. Methods for aggregating intuitionistic uncertain linguistic variables and their application to group decision making[J]. Information Sciences, 2012, 205(1): 58-71.

[12] Liu P D. Some geometric aggregation operators based on interval intuitionistic uncertain linguistic variables and their application to group decision making[J]. Applied Mathematical Modelling, 2013, 37(4): 2430-2444.

[13] 王佩. 基于区间直觉语言信息的多准则群决策方法研究[D]. 长沙: 中南大学商学院, 2012.
(Wang P. Multi-criteria group decision making method based on interval intuitive language information[D]. Changsha: School of Business, Central South University, 2012.)

[14] Yager R R. On ordered weighted averaging aggregation operators in multi-criteria decision making[J]. IEEE Trans on Systems, Man, and Cybernetics, 1988, 18(1): 183-190.

[15] Yager R R. The power average operator[J]. IEEE Trans on Systems, Man, and Cybernetics — Part A: Systems and Humans, 2001, 31(6): 724-731.

[16] Yager R R. Prioritized aggregation operators[J]. Int J of Approximate Reasoning, 2008, 48(1): 263-274.

[17] Wang W, Liu X. Intuitionistic fuzzy information aggregation using Einstein operations[J]. IEEE Trans on Fuzzy Systems, 2012, 20(5): 923-938.

[18] Hamachar H. Uber logische verknunpfungenn unssharfer aussagen undderen zugenhorige bewertungsfunktione[C]. Progress in Cybernatics and Systems Research. New York: Hemisphere Pub Corp, 1975(3): 276−288.

[19] Frank M J. On the simultaneous associativity of $F(x,y)$ and $x+y-F(x,y)$[J]. Aequationes Mathematicae, 1979, 19(1): 194-226.

[20] Casasnovas J, Torrens J. An axiomatic approach to fuzzy cardinalities of finite fuzzy sets[J]. Fuzzy Sets and Systems, 2003, 133(2): 193-209.

[21] Yager R R. On some new classes of implication operators and their role in approximate reasoning[J]. Information Sciences, 2004, 167(1): 193-216.

[22] Sarkoci P. Domination in the families of Frank and Hamacher t-norms[J]. Kybernetika, 2005, 41(3): 349-360.

[23] Wang W S, Hua-Can H E. Research on flexible probability logic operator based on Frank T/S norms[J]. Acta Electronica Sinica, 2009, 37(5): 1141-1145.

[24] Deschrijver G. Generalized arithmetic operators and their relationship to t-norms in interval-valued fuzzy set theory[J]. Fuzzy Sets and Systems, 2009, 160(21): 3080-3102.

[25] Qin J, Liu X. Frank aggregation operators for triangular interval type-2 fuzzy set and its application in multiple attribute group decision making[J]. J of Applied Mathematics, 2014(3): 1-24.

[26] Qin J, Liu X, Pedrycz W. Frank aggregation operators and their application to hesitant fuzzy multiple attribute decision making[J]. Applied Soft Computing, 2016, 41(4): 428-452.

[27] Zhang Z M. Interval-valued intuitionistic fuzzy Frank aggregation operators and their applications to multiple attribute group decision making[J]. Neural Computing & Applications, 2017, 28(6): 1471−1501.

[28] Zhang X, Liu P, Wang Y. Multiple attribute group decision making methods based on intuitionistic fuzzy frank power aggregation operators[J]. J of Intelligent & Fuzzy Systems, 2015, 29(5): 2235-2246.

[29] Du Y Q, Hou F J, Wasif Zafar, et al. A novel method for multi-attribute decision making with interval-valued Pythagorean fuzzy linguistic information[J]. Int J of Intelligent Systems, 2017, 32(10): 1085-1112.

[30] Liang C, Zhao S, Zhang J. Multi-criteria group decision making method based on generalized intuitionistic trapezoidal fuzzy prioritized aggregation operators[J]. Int J of Machine Learning & Cybernetics, 2017, 8(2): 597-610.

[31] Xu Z S. On consistency of the weighted geometric mean complex judgement matrix in AHP 1[J]. European J of Operational Research, 2000, 126(3): 683-687.

（责任编辑：李君玲）