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具有不确定转移概率的马尔科夫复杂网络的聚类同步

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摘　要: 研究具有不确定转移概率的马尔科夫复杂网络系统的聚类同步问题,系统模型包含耦合的离散时变时滞和耦合的分布时变时滞. 通过充分考虑转移概率的性质和不确定区域的特性,用一个含有较少变量的有效技术代替传统的Young不等式来约束转移率中的不确定项. 同时,利用增广李雅普诺夫泛函和具有较小保守性的积分不等式,给出新的依赖时滞和时滞导数上下界的聚类同步准则. 最后通过数值仿真验证所提出方法的有效性.

关键词: 聚类同步；不确定转移率；马尔科夫复杂网络；增广李雅普诺夫泛函

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Cluster synchronization of Markovian complex networks with uncertain transition probabilities

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Abstract: This paper studies the cluster synchronization of Markovian complex networks with uncertain transition rates. This system model contains constant coupled discrete time varying delay and coupled distributed time varying delay. By fully considering the property of transition rates and the characteristic of uncertain domains, a more effective technique in stead of the traditional Young inequality is used to bind the uncertain terms in the transition rates. By applying the augmented Lyapunov-Krasovskii functional and a less conservative integral inequality, new cluster synchronization criteria are obtained, which contains the bounds of the delay and the derivative of delay. The example simulation demonstrates the effectiveness of the proposed method.

Keywords: cluster synchronization；uncertain transition probabilities；Markovian complex networks；augmented Lyapunov-Krasovskii functional

0 引　言

复杂网络是由多个节点和连接两个节点之间边所组成的网络,网络中的节点代表不同的个体,边表示个体间的关系. 近年来,随着现代科技的飞速发展和复杂性科学的深入探索,复杂网络的同步问题引起了人们的广泛关注[1-5].

作为一种特殊的同步现象,聚类同步要求复杂网络系统中每组节点之间达到同步,且不同组之间达不到同步[3]. 由于在生物科学与通信工程上的应用,聚类同步受到了广泛的研究[3-4,6-8]. 文献[4]研究了具有时变时滞复杂网络的指数同步问题,并给出了相应的聚类同步准则. 作为一种特殊的复杂网络,耦合神经网络的同步问题受到广泛关注[3-4,6-8]. 文献[6]研究了具有非相同节点的非线性耦合时滞网络的聚类同步问题,并给出了新的聚类同步准则.

在实际过程中,神经网络系统经常遇到信息闭锁现象,使得网络的状态具有有限的模态,并且有一个马尔科夫链支配这些模态[9]. 目前,对于马尔科夫神经网络的研究已经取得了很多成果. 文献[9]研究了具有马尔科夫跳变和混合时滞的复杂网络的指数同步问题. 然而,对于马尔科夫耦合神经网络的聚类同步的研究尚不多见. 文献[7]研究了混合时滞耦合神经网络的聚类同步问题. 此后,文献[8]研究了基于事件机制的耦合神经网络的牵引同步问题. 然而,文献[7-8]没有考虑信息闭锁情况,当系统出现马尔科夫跳变时,聚类同步准则不再有效. 为了克服以上问题,

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本文研究具有马尔科夫跳变的复杂网络的聚类同步问题. 由于在实际应用中,完全得到转移概率的全部信息十分困难,研究具有部分未知转移概率的马尔科夫系统是很有必要的. 文献[10] 研究了具有不完全转移率的连续Markov跳变奇异系统的H_∞控制问题. 文献[11]研究了具有不完全转移率的马尔科夫耦合神经网络的同步问题. 此外,在许多实际系统分析中,精确测得所有转移速率的成本十分昂贵,因此,具有不确定转移概率的马尔科夫系统引起了广泛研究[12-16]. 文献[15]研究了具有不确定和部分未知转移概率的马尔科夫复杂网络的指数同步问题. 文献[16]在研究具有不确定转移率的马尔科夫线性系统时,得到了具有较少变量的稳定准则. 然而,对于具有不确定转移概率的马尔科夫复杂网络中,在处理转移概率中不确定项时,大部分都利用Young不等式,从而增加了变量,使得计算比较复杂. 鉴于以上讨论,具有不确定转移概率的马尔科夫复杂网络的聚类同步问题较少见到报道.

本文以具有不确定转移概率的马尔科夫和非线性耦合的时变时滞复杂网络为研究对象,通过构造增广的Lyapunov泛函,利用具有较小保守性的积分不等式,同时,在处理转移概率中的不确定项时,采用可以减少变量的新技术,给出了新的聚类同步准则,所得结果与以往理论相比更具有一般性和实用性. 最后通过数值仿真验证了所提出方法的有效性.

1 问题描述

考虑如下马尔科夫复杂网络模型:

$$\begin{aligned}&\dot{x}_k(t)=\\&-C(\beta_t)x_k(t)+A(\beta_t)g(x_k(t))+B(\beta_t)g(x_k(t-\\&d(t)))+U_k(t)+a_1\sum_{l=1}^{N}G_{kl}^{(1)}(\beta_t)\mathit{\Gamma}_1(\beta_t)g(x_l(t))+\\&a_2\sum_{l=1}^{N}G_{kl}^{(2)}(\beta_t)\mathit{\Gamma}_2(\beta_t)g(x_l(t-d(t)))+\\&a_3\sum_{l=1}^{N}G_{kl}^{(3)}(\beta_t)\mathit{\Gamma}_3(\beta_t)\int_{t-d(t)}^{t}g(x_l(s))\mathrm{d}s,\\&\qquad k=1,2,\cdots,N.\end{aligned}\tag{1}$$

其中: $x_k(t)\in R^n$为第k个节点的状态向量; $C(\beta_t)$为正对角矩; $A(\beta_t)$、$B(\beta_t)$为连接权矩阵; $g(x_k(t))\in R^n$为第k个节点的输出; $U_k(t)$为外部输入; $a_1>0, a_2>0, a_3>0$为耦合强度; 时变时滞$d(t)$满足$0\leqslant d(t)\leqslant d, \mu_1\leqslant \dot{d}(t)\leqslant\mu_2, d$、$\mu_1$、$\mu_2$为常数; $\mathit{\Gamma}_{\alpha_1}(\beta_t)$ $(\alpha_1=1,2,3)$为节点之间的内部耦合矩阵; $G^{(\alpha_1)}(\beta_t)=G_{kl}^{(\alpha_1)}(\beta_t)_{N\times N}(\alpha_1=1,2,3)$为耦合结构矩阵,表示网络的拓扑结构,它们互不相等且满足条件

$$G_{kl}^{(\alpha_1)}(\beta_t)\geqslant 0,\ k\neq l,$$
$$G_{kk}^{(\alpha_1)}(\beta_t)=-\sum_{l=1,l\neq k}^{N}G_{kl}^{(\alpha_1)}(\beta_t).$$

系统模态信号$\{\beta_t, t\geqslant 0\}$是一个在有限集合$\mathcal{S}=\{1,2,\cdots,s\}$里取值的右连续马尔科夫链,具有如下转移率:

$$\Pr\{\beta_{t+\Delta t}=j\mid \beta_t=i\}=\begin{cases}\hat{\pi}_{ij}\Delta t+o(\Delta t),\ i\neq j;\\1+\hat{\pi}_{ii}\Delta t+o(\Delta t), i=j.\end{cases}$$

其中: $\Delta t>0$, $\lim\limits_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0$; $\hat{\pi}_{ij}\geqslant 0(i\neq j)$为$t$时刻模态$i$到$t+\Delta t$时刻模态$j$的转移速率,且$\hat{\pi}_{ii}=-\sum\limits_{j=1,i\neq j}^{s}\hat{\pi}_{ij}$.

本文中,马尔科夫过程的转移率为不确定的. $\mathcal{D}_\pi=\{\hat{\mathit{\Pi}}=\mathit{\Pi}+\Delta\mathit{\Pi}:|\ \Delta\pi_{ij}\ |\leqslant\delta_{ij},\ \delta_{ij}\geqslant 0$,对于所有的$i,j\in\mathcal{S}, j\neq i\}$. 其中: $\mathit{\Pi}=(\pi_{ij})(i,j\in\mathcal{S})$为已知的常矩阵, $\Delta\mathit{\Pi}=(\Delta\pi_{ij})$为模态转移率矩阵中的不确定项. 对于所有的$i,j\in\mathcal{S}, j\neq i, \pi_{ij}>0$表示$\hat{\pi}_{ij}$的估计值. $\Delta\pi_{ij}=\hat{\pi}_{ij}-\pi_{ij}$为估计误差,属于区间$[-\delta_{ij},\delta_{ij}]$. 对于任意的$i\in\mathcal{S}$,有$\pi_{ii}=-\sum\limits_{j=1,j\neq i}^{s}\pi_{ij}$和$\Delta\pi_{ii}=-\sum\limits_{j=1,j\neq i}^{s}\Delta\pi_{ij}$成立, $\underline{\pi}_{ij}=\pi_{ij}-\delta_{ij}$为$\hat{\pi}_{ij}$的下界.

给出如下假设和引理.

假设1[17] 对于任意的$x_1,x_2\in R$,存在常数e_r^-和e_r^+,满足

$$e_r^-\leqslant\frac{g_r(x_1)-g_r(x_2)}{x_1-x_2}\leqslant e_r^+,\ r=1,2,\cdots,n.$$

记

$$E_1=\mathrm{diag}(e_1^+e_1^-,\cdots,e_n^+e_n^-),$$
$$E_2=\mathrm{diag}\Big(\frac{e_1^++e_1^-}{2},\cdots,\frac{e_n^++e_n^-}{2}\Big).$$

假设2 将β_t简化记作$i(i\in\mathcal{G})$,系统(1)中的耦合结构矩阵$G^{(\alpha_1)}(\beta_t)$记作$G_i^{(\alpha_1)}(\alpha_1=1,2,3)$,满足如下条件:

$$\begin{bmatrix}Z_{11,i}^{(\alpha_1)}&Z_{12,i}^{(\alpha_1)}&\cdots&Z_{1k,i}^{(\alpha_1)}\\Z_{21,i}^{(\alpha_1)}&Z_{22,i}^{(\alpha_1)}&\cdots&Z_{23,i}^{(\alpha_1)}\\\vdots&\vdots&\ddots&\vdots\\Z_{k1,i}^{(\alpha_1)}&Z_{k2,i}^{(\alpha_1)}&\cdots&Z_{kk,i}^{(\alpha_1)}\end{bmatrix}.$$

其中: $Z_{ii,i}^{(\alpha_1)}\in R^{m_i\times m_i}, Z_{ij,i}^{(\alpha_1)}\in R^{m_i\times m_j}, i,j=1,2,\cdots,k$. 在矩阵块$Z_{ij,i}^{(\alpha_1)}$中,每行向量分别相同, $Z_{ij,i}^{(\alpha_1)}=[v_i^{(\alpha_1)},v_i^{(\alpha_1)},\cdots,v_i^{(\alpha_1)}]^{\mathrm{T}}, v_i^{(\alpha_1)}=[v_{1,i}^{(\alpha_1)},v_{2,i}^{(\alpha_1)},\cdots,$

$v_{m_j,i}^{(\alpha_1)}]^{\mathrm{T}}$ 为向量.

定义1 复杂网络系统(1)的N个节点,分为m_k个组,即$\{(1,2,\cdots,m_1),(m_1+1,m_1+2,\cdots,m_1+m_2),\cdots,(m_1+m_2+\cdots+m_{k-1}+1,\ m_1+m_2+\cdots+m_{k-1}+2,\cdots,m_1+m_2+\cdots+m_{k-1}+m_k),m_1+m_2+\cdots+m_{k-1}+m_k=N\}$. 对于同一组内的任意两个节点$k$和$l$的状态$x_k(t)$和$x_l(t)$,满足条件$\lim\limits_{t\to\infty}\mathcal{E}\{\|x_k(t)-x_l(t)\|^2\}=0$,那么复杂网络系统为聚类同步的.

引理1[18] 设$f_1,f_2,\cdots,f_N:R^m\mapsto R$在$R^m$的开子集$D$值非负,那么在$D$上$f_i$的反凸组合满足

$$\min_{\{\alpha_i|\alpha_i>0,\sum\limits_i\alpha_i=1\}}\sum_i\frac{1}{\alpha_i}f_i(t)=\sum_i f_i(t)+\max_{g_{i,j}(t)}\sum_{i\neq j}g_{i,j}(t),$$

$$\text{s.t.}\left\{g_{ij}:R^m\mapsto R,g_{j,i}(t)\triangleq g_{i,j}(t),\begin{bmatrix}f_i(t)&g_{i,j}(t)\\g_{i,j}(t)&f_j(t)\end{bmatrix}\geqslant 0\right\}.$$

引理2[19] 对于一个正定矩阵$R>0$和一个可微的函数$\{x(u)|u\in[a,b]\}$,如下不等式成立:

$$\int_a^b\dot{x}^{\mathrm{T}}(a)R\dot{x}(a)\mathrm{d}s\geqslant\frac{1}{b-a}\Theta_1^{\mathrm{T}}R\Theta_1+\frac{3}{b-a}\Theta_2^{\mathrm{T}}R\Theta_2+\frac{5}{b-a}\Theta_3^{\mathrm{T}}R\Theta_3.$$

其中

$$\Theta_1=x(b)-x(a),$$

$$\Theta_2=x(b)+x(a)-\frac{2}{b-a}\int_a^b x(\alpha)\mathrm{d}\alpha,$$

$$\Theta_2=x(b)-x(a)+\frac{6}{b-a}\int_a^b x(s)\mathrm{d}\alpha-\frac{12}{(b-a)^2}\int_a^b\int_\beta^b x(\alpha)\mathrm{d}\alpha\mathrm{d}\beta.$$

引理3[20] G是$N\times N$矩阵且属于集合$T(\hat{R},K)$,$(N-1)\times(N-1)$矩阵H满足$MG=HM$. 其中

$$H=MGJ,$$

$$M=\begin{bmatrix}1&-1&0&0&\cdots&0\\0&1&-1&0&\cdots&0\\\vdots&\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1&-1\end{bmatrix}_{(N-1)\times N},$$

$$J=\begin{bmatrix}1&1&1&1&\cdots&1\\0&1&1&1&\cdots&1\\\vdots&\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&0&1\\0&0&0&\cdots&0&0\end{bmatrix}_{N\times(N-1)},$$

1为环$\hat{R}$的乘法单位. 此外,对于所有的$i,j\in\{1,2,\cdots,N-1\}$,矩阵H可以写成如下形式:

$$H_{ij}=\sum_{k=1}^{j}(G_{(ik)}-G_{(i+1,k)}).$$

引理4[3] $(N-m)\times(N-m)$的矩阵$\tilde{H}_r$满足$\tilde{M}G^{(r)}=\tilde{H}_r\tilde{M}$,其中

$$\tilde{Z}^{(r)}=\begin{bmatrix}Z_{11}^{(r)}&&&\\&Z_{22}^{(r)}&&\\&&\ddots&\\&&&Z_{mm}^{(r)}\end{bmatrix}_{N\times N},$$

$$\tilde{M}=\begin{bmatrix}M_1&&&\\&M_2&&\\&&\ddots&\\&&&M_m\end{bmatrix}_{(N-k)\times N},$$

$$\tilde{J}=\begin{bmatrix}J_1&&&\\&J_2&&\\&&\ddots&\\&&&J_k\end{bmatrix}_{N\times(N-m)},$$

$$\tilde{H}_r=\tilde{M}\tilde{Z}^{(r)}\tilde{J},Z_{ii}^{(r)}\in R^{m_i\times m_i},$$

$$M_i\in R^{(m_i-1)\times m_i},J_i\in R^{m_i\times(m_i-1)},$$

$$r=1,2,3.$$

2 聚类同步准则

为方便起见,β_t简记为$i\ (i\in\mathcal{G})$,系统(1)可写为

$$\begin{aligned}&\dot{x}_k(t)=\\&-C_ix_k(t)+A_ig(x_k(t))+B_ig(x_k(t-d(t)))+\\&U_k(t)+a_1\sum_{l=1}^{N}G_{kl,i}^{(1)}\Gamma_{1,i}g(x_l(t))+a_2\sum_{l=1}^{N}G_{kl,i}^{(2)}\Gamma_{2,i}\times\\&g(x_l(t-d(t)))+a_3\sum_{l=1}^{N}G_{kl,i}^{(3)}\Gamma_{3,i}\int_{t-d(t)}^{t}g(x_l(s))\mathrm{d}s,\\&k=1,2,\cdots,N.\end{aligned}\qquad(2)$$

令$\boldsymbol{M}=\tilde{M}\otimes I_n$,$\tilde{M}$定义同引理4,且有

$$\bar{C}_i=I_N\otimes C_i,\ \bar{C}_i{}'=I_{N-m}\otimes C_i,$$

$$\bar{A}_i=I_N\otimes A_i,\ \bar{A}_i{}'=I_{N-m}\otimes A_i,$$

$$\bar{B}_i=I_N\otimes B_i,\ \bar{B}_i{}'=I_{N-m}\otimes B_i,$$

$$\bar{\Gamma}_{\alpha_1,i}=G_i^{(\alpha_1)}\otimes\Gamma_{\alpha_1,i},\ \alpha_1=1,2,3,$$

$$x(t)=[x_1^{\mathrm{T}}(t),x_2^{\mathrm{T}}(t),\cdots,x_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$g(x(t))=[g^{\mathrm{T}}(x_1(t)),g^{\mathrm{T}}(x_2(t)),\ldots,g^{\mathrm{T}}(x_N(t))]^{\mathrm{T}},$$

$$\bar{U}(t)=[U_1^{\mathrm{T}}(t),U_2^{\mathrm{T}}(t),\ldots,U_N^{\mathrm{T}}(t)]^{\mathrm{T}},$$

$$\bar{E}_1=I_{N-m}\otimes E_1,\ \bar{E}_2=I_{N-m}\otimes E_2,$$

$$e_\gamma=[0_{(N-m)n\times(\gamma-1)(N-m)n},$$

$I_{(N-m)n\times(N-m)n},\ 0_{(N-m)n\times(11-\gamma)(N-m)n}],$

$\gamma=1,2,\cdots,11,$

则系统(2)可写为

$$\dot{x}(t)=\\ -\bar{C}_i x(t)+\bar{A}_i g(x(t))+\bar{B}_i g(x(t-d(t)))+\bar{U}(t)+\\ a_1\bar{\varGamma}_{1,i}g(x(t))+a_2\bar{\varGamma}_{2,i}g(x(t-d(t)))+\\ a_3\bar{\varGamma}_{3,i}\int_{t-d(t)}^t g(x(s))\mathrm{d}s. \tag{3}$$

定理1 在假设1和假设2成立的条件下,系统(3)为聚类渐近同步的,如果存在$(N-m)n\times(N-m)n$维正定矩阵P_i、$Y_{ij}(i,j\in\mathcal{S})$、$R$、$S$、$T$、$U_2$、$U_3$、$2(N-m)n\times 2(N-m)n$维正定矩阵$U_1$、$Q_{\alpha_2}(\alpha_2=1,2)$、$(N-m)n\times(N-m)n$维正定对角矩阵$R_{\alpha_1}$ $(\alpha_1=1,2,3)$、$3(N-m)n\times 3(N-m)n$维矩阵X,使得对于任意的$i\in\mathcal{S}$,有如下矩阵不等式成立:

$$\tilde{\varPhi}_i^\delta<0,\ \delta=1,2,3,4; \tag{4}$$

$$P_j-P_i-Y_{ij}\leqslant 0,\ \forall j\in\mathcal{S},\ j\neq i; \tag{5}$$

$$\begin{bmatrix}\bar{S} & X\\ * & \bar{S}\end{bmatrix}\geqslant 0. \tag{6}$$

其中

$$\tilde{\varPhi}_i^1=\tilde{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=0,\dot{d}(t)=\mu_1},$$

$$\tilde{\varPhi}_i^2=\tilde{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=0,\dot{d}(t)=\mu_2},$$

$$\tilde{\varPhi}_i^3=\tilde{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=d,\dot{d}(t)=\mu_1},$$

$$\tilde{\varPhi}_i^4=\tilde{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=d,\dot{d}(t)=\mu_2},$$

$$\tilde{\varPhi}_i(d(t),\dot{d}(t))=\\ \varPhi_{1i}+\tilde{\varPhi}_{2i}+\varPhi_3(d(t))+\varPhi_4(d(t),\dot{d}(t))+\varPhi_5(\dot{d}(t)),$$

$$\varPhi_{1i}=\mathrm{Sym}\{e_1^{\mathrm{T}}P_i\varUpsilon_i\varTheta_1\}+\varTheta_2^{\mathrm{T}}(Q_1+Q_2)\varTheta_2-\\ \varTheta_3^{\mathrm{T}}Q_2\varTheta_3+d(\varUpsilon_i\varTheta_1)^{\mathrm{T}}S\varUpsilon_i\varTheta_1-\\ \frac{1}{d}\begin{bmatrix}\varTheta_5\\ \varTheta_6\end{bmatrix}^{\mathrm{T}}\bar{\varXi}\begin{bmatrix}\varTheta_5\\ \varTheta_6\end{bmatrix}+de_4^{\mathrm{T}}Te_4-\frac{1}{d}e_7^{\mathrm{T}}Te_7+\\ \mathrm{Sym}\{e_8^{\mathrm{T}}U_3e_1-e_9^{\mathrm{T}}U_2e_3\}+\varTheta_2^{\mathrm{T}}\varXi_1\varTheta_2+\\ \varTheta_5^{\mathrm{T}}\varXi_2\varTheta_4+\varTheta_3^{\mathrm{T}}\varXi_3\varTheta_3,$$

$$\tilde{\varPhi}_{2i}=e_1^{\mathrm{T}}\sum_{j=1,j\neq i}^{s}\{\underline{\pi}_{ij}(P_j-P_i)+2\delta_{ij}Y_{ij}\}e_1,$$

$$\varPhi_3(d(t))=d(t)\mathrm{Sym}\{e_8^{\mathrm{T}}R(e_1-e_3)\}+\\ d(t)\varTheta_4^{\mathrm{T}}\Big(Z_1+\frac{1}{3}Z_3\Big)\varTheta_4+(d-\\ d(t))\mathrm{Sym}\{e_9^{\mathrm{T}}R(e_1-e_3)\}+\\ (d-d(t))\varTheta_6^{\mathrm{T}}\Big(Y_1+\frac{1}{3}Y_3\Big)\varTheta_6,$$

$$\varPhi_4(d(t),\dot{d}(t))=\mathrm{Sym}[d(t)e_8^{\mathrm{T}},(d-\\ d(t))e_9^{\mathrm{T}}]U_1[e_1^{\mathrm{T}}-(1-\dot{d}(t))e_2^{\mathrm{T}},\\ (1-\dot{d}(t))e_2^{\mathrm{T}}-e_3^{\mathrm{T}}]^{\mathrm{T}},$$

$$\varPhi_5(\dot{d}(t))=\dot{d}(t)\{e_8^{\mathrm{T}}U_3e_8-e_9^{\mathrm{T}}U_2e_9+\\ \mathrm{Sym}(e_9^{\mathrm{T}}U_2e_9-e_8^{\mathrm{T}}U_3e_8)\}+(1-\\ \dot{d}(t))\{-\varTheta_5^{\mathrm{T}}Q_1\varTheta_4+\\ \mathrm{Sym}(e_9^{\mathrm{T}}U_2e_2-e_8^{\mathrm{T}}U_3e_2)\},$$

$$\varUpsilon_i=[-\bar{C}_i',\bar{A}_i'+a_1\bar{H}_{1,i},\bar{B}_i'+a_2\bar{H}_{2,i},a_3\bar{H}_{3,i}],$$

$$\varTheta_1=[e_1^{\mathrm{T}},e_4^{\mathrm{T}},e_5^{\mathrm{T}},e_7^{\mathrm{T}}]^{\mathrm{T}},\ \varTheta_2=[e_1^{\mathrm{T}},e_4^{\mathrm{T}}]^{\mathrm{T}},$$

$$\varTheta_3=[e_3^{\mathrm{T}},e_6^{\mathrm{T}}]^{\mathrm{T}},\ \varTheta_4=[e_2^{\mathrm{T}},e_5^{\mathrm{T}}]^{\mathrm{T}},$$

$$\varTheta_5=\\ [e_1^{\mathrm{T}}-e_2^{\mathrm{T}},e_1^{\mathrm{T}}+e_2^{\mathrm{T}}-2e_8^{\mathrm{T}},e_1^{\mathrm{T}}-e_2^{\mathrm{T}}+6e_8^{\mathrm{T}}-6e_{10}^{\mathrm{T}}]^{\mathrm{T}},$$

$$\varTheta_6=\\ [e_2^{\mathrm{T}}-e_3^{\mathrm{T}},e_2^{\mathrm{T}}+e_3^{\mathrm{T}}-2e_9^{\mathrm{T}},e_2^{\mathrm{T}}-e_3^{\mathrm{T}}+6e_9^{\mathrm{T}}-6e_{11}^{\mathrm{T}}]^{\mathrm{T}}.$$

$$\bar{S}=\mathrm{diag}[S,3S,5S],$$

$$\bar{\varXi}=\begin{bmatrix}\bar{S} & X\\ * & \bar{S}\end{bmatrix},$$

$$\varXi_{\alpha_1}=\begin{bmatrix}-R_{\alpha_1}\bar{E}_1 & R_{\alpha_1}\bar{E}_2\\ * & -R_{\alpha_1}\end{bmatrix},\ \alpha_1=1,2,3,$$

$$\bar{H}_{\alpha_1,i}=\tilde{H}_{\alpha_1,i}\otimes\varGamma_{\alpha_1,i},\ \alpha_1=1,2,3,$$

$$\tilde{H}_{\alpha_1,i}=\tilde{M}Z_i^{(\alpha_1)}\tilde{J},\ \alpha_1=1,2,3.$$

$\tilde{M}$和$\tilde{J}$的定义同引理4.

证 明 对于系统(3),考虑如下Lyapunov-Krasovskii泛函:

$$V(x(t),t,i)=\sum_{q=1}^{6}[V_q(x(t),t,i)]. \tag{7}$$

令

$$\phi_1(t)=\begin{bmatrix}\boldsymbol{M}x(t)\\ \boldsymbol{M}g(x(t))\end{bmatrix},\ \phi_2(s)=\begin{bmatrix}\int_{t-d(t)}^{t}\boldsymbol{M}x(s)\mathrm{d}s\\ \int_{t-d}^{t-d(t)}\boldsymbol{M}x(s)\mathrm{d}s\end{bmatrix},$$

$$V_1(x(t),t,i)=x^{\mathrm{T}}(t)\boldsymbol{M}^{\mathrm{T}}P_i\boldsymbol{M}x(t), \tag{8}$$

$$V_2(x(t),t,i)=\int_{t-d(t)}^{t}\phi_1^{\mathrm{T}}(s)Q_1\phi_1(s)\mathrm{d}s+\\ \int_{t-d}^{t}\phi_1^{\mathrm{T}}(s)Q_2\phi_1(s)\mathrm{d}s, \tag{9}$$

$$V_3(x(t),t,i)=\int_{t-d}^{t}(\boldsymbol{M}x(s))^{\mathrm{T}}\mathrm{d}sR\int_{t-d}^{t}\boldsymbol{M}x(s)\mathrm{d}s, \tag{10}$$

$$V_4(x(t),t,i)=\\ \int_{-d}^{0}\int_{t+\theta}^{t}(\boldsymbol{M}\dot{x}(s))^{\mathrm{T}}S\boldsymbol{M}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+$$

$$\int_{-d}^{0}\int_{t+\theta}^{t}(\boldsymbol{M}g(x(s)))^{\mathrm{T}}T\boldsymbol{M}g(x(s))\mathrm{d}s\mathrm{d}\theta, \tag{11}$$

$$V_5(x(t),t,i)=\phi_2^{\mathrm{T}}(s)U_1\phi_2(s), \tag{12}$$

$$V_6(x(t),t,i)=$$
$$(d-d(t))\Big(\frac{1}{d-d(t)}\int_{t-d}^{t-d(t)}\boldsymbol{M}x(s)\mathrm{d}s\Big)^{\mathrm{T}}\times$$
$$U_2\Big(\frac{1}{d-d(t)}\int_{t-d}^{t-d(t)}\boldsymbol{M}x(s)\mathrm{d}s\Big)+$$
$$d(t)\Big(\frac{1}{d(t)}\int_{t-d(t)}^{t}\boldsymbol{M}x(s)\mathrm{d}s\Big)^{\mathrm{T}}\times$$
$$U_3\Big(\frac{1}{d(t)}\int_{t-d(t)}^{t}\boldsymbol{M}x(s)\mathrm{d}s\Big), \tag{13}$$

$\mathcal{L}$为弱无穷小算子

$$\mathcal{L}V_1(x(t),t,i)=$$
$$2x^{\mathrm{T}}(t)\boldsymbol{M}^{\mathrm{T}}P_i\boldsymbol{M}\dot{x}(t)+\sum_{j=1}^{\mathcal{N}}\hat{\pi}_{ij}(\boldsymbol{M}x(t))^{\mathrm{T}}P_j\boldsymbol{M}x(t). \tag{14}$$

根据$\boldsymbol{M}$的结构,可得

$$\boldsymbol{M}\bar{C}_i=\bar{C}_i{}'\boldsymbol{M},\ \boldsymbol{M}\bar{A}_i=\bar{A}_i{}'\boldsymbol{M},$$
$$\boldsymbol{M}\bar{B}_i=\bar{B}_i{}'\boldsymbol{M},\ \boldsymbol{M}\bar{U}(t)=0,$$
$$\boldsymbol{M}\bar{\varGamma}_{\alpha_1,i}=(\tilde{M}\otimes I_n)(G_i^{(\alpha_1)}\otimes\varGamma_{\alpha_1,i})=$$
$$(\tilde{M}G_i^{(\alpha_1)})\otimes\varGamma_{\alpha_1,i}=(\tilde{H}_{\alpha_1,i}\tilde{M})\otimes\varGamma_{\alpha_1,i}=$$
$$(\tilde{H}_{\alpha_1,i}\otimes\varGamma_{\alpha_1,i})(\tilde{M}\otimes I_n)=\bar{H}_{\alpha_1,i}\boldsymbol{M},\ \alpha_1=1,2,3.$$

由式(14)可得

$$\mathcal{L}V_1(x(t),t,i)=$$
$$2\xi^{\mathrm{T}}(t)e_1^{\mathrm{T}}P_i\varUpsilon_i\varTheta_1\xi(t)+$$
$$\sum_{j=1}^{\mathcal{N}}(\pi_{ij}+\vartriangle\pi_{ij})(\boldsymbol{M}x(t))^{\mathrm{T}}P_j\boldsymbol{M}x(t), \tag{15}$$

$$\mathcal{L}V_2(x(t),t,i)=$$
$$\phi_1^{\mathrm{T}}(t)Q_1\phi_1(t)-(1-\dot{d}(t))\phi_1^{\mathrm{T}}(t-d(t))Q_1\phi_1(t-$$
$$d(t))+\phi_1^{\mathrm{T}}(t)Q_2\phi_1(t)-\phi_1^{\mathrm{T}}(t-d)Q_1\phi_1(t-d), \tag{16}$$

$$\mathcal{L}V_3(x(t),t,i)=$$
$$2\int_{t-d}^{t}(\boldsymbol{M}x(s))^{\mathrm{T}}\mathrm{d}sR(\boldsymbol{M}x(t)-\boldsymbol{M}x(t-d)), \tag{17}$$

$$\mathcal{L}V_4(x(t),t,i)\leqslant$$
$$d(\boldsymbol{M}\dot{x}(t))^{\mathrm{T}}S\boldsymbol{M}\dot{x}(t)-\int_{t-d}^{t}(\boldsymbol{M}\dot{x}(s))^{\mathrm{T}}S\boldsymbol{M}\dot{x}(s)\mathrm{d}s+$$
$$d(\boldsymbol{M}g(x(t)))^{\mathrm{T}}T\boldsymbol{M}g(x(t))-$$
$$\frac{1}{d}\int_{t-d(t)}^{t}(\boldsymbol{M}g(x(s)))^{\mathrm{T}}\mathrm{d}sT\int_{t-d(t)}^{t}\boldsymbol{M}g(x(s))\mathrm{d}s, \tag{18}$$

$$\mathcal{L}V_5(x(t),t,i)=$$
$$2\xi^{\mathrm{T}}(t)\begin{bmatrix}d(t)e_8\\(d-d(t))e_9\end{bmatrix}^{\mathrm{T}}U_1\begin{bmatrix}e_1-(1-\dot{d}(t))e_2\\(1-\dot{d}(t))e_2-e_3\end{bmatrix}\xi(t), \tag{19}$$

$$\mathcal{L}V_6(x(t),t,i)=$$
$$-\dot{d}(t)\xi^{\mathrm{T}}(t)e_9^{\mathrm{T}}U_2e_9\xi(t)+2\xi^{\mathrm{T}}(t)e_9^{\mathrm{T}}U_2[(1-\dot{d}(t))e_2-$$
$$e_3+\dot{d}(t)e_9]\xi(t)+\dot{d}(t)\xi^{\mathrm{T}}(t)e_8^{\mathrm{T}}U_3e_8\xi(t)+$$
$$2\xi^{\mathrm{T}}(t)e_8^{\mathrm{T}}U_3[e_1-(1-\dot{d}(t))e_2-\dot{d}(t)e_8]\xi(t). \tag{20}$$

根据引理1和引理2,可得

$$-\int_{t-d}^{t}(\boldsymbol{M}\dot{x}(s))^{\mathrm{T}}S\boldsymbol{M}\dot{x}(s)\mathrm{d}s=$$
$$-\int_{t-d(t)}^{t}(\boldsymbol{M}\dot{x}(s))^{\mathrm{T}}S\boldsymbol{M}\dot{x}(s)\mathrm{d}s-$$
$$\int_{t-d}^{t-d(t)}(\boldsymbol{M}\dot{x}(s))^{\mathrm{T}}S\boldsymbol{M}\dot{x}(s)\mathrm{d}s\leqslant$$
$$-\frac{1}{d(t)}\zeta^{\mathrm{T}}(t)\{(e_1^{\mathrm{T}}-e_2^{\mathrm{T}})S(e_1-e_2)+3(e_1^{\mathrm{T}}+e_2^{\mathrm{T}}-$$
$$2e_8^{\mathrm{T}})S(e_1+e_2-2e_8)+5(e_1^{\mathrm{T}}-e_2^{\mathrm{T}}+6e_8^{\mathrm{T}}-6e_{10}^{\mathrm{T}})\times$$
$$S(e_1-e_2+6e_8-6e_{10})\}\zeta(t)-$$
$$\rho_2\frac{1}{d-d(t)}\zeta^{\mathrm{T}}(t)\{(e_2^{\mathrm{T}}-e_3^{\mathrm{T}})S(e_2-e_3)+$$
$$3(e_2^{\mathrm{T}}+e_3^{\mathrm{T}}-2e_9^{\mathrm{T}})S(e_2+e_3-2e_9)+5(e_2^{\mathrm{T}}-e_3^{\mathrm{T}}+$$
$$6e_9^{\mathrm{T}}-6e_{11}^{\mathrm{T}})S(e_2-e_3+6e_9-6e_{11})\}\zeta(t)=$$
$$-\frac{1}{d(t)}\zeta^{\mathrm{T}}(t)\varTheta_5^{\mathrm{T}}\bar{S}\varTheta_5\zeta(t)-$$
$$\frac{1}{d-d(t)}\zeta^{\mathrm{T}}(t)\varTheta_6^{\mathrm{T}}\bar{S}\varTheta_6\zeta(t)\leqslant$$
$$-\frac{1}{d}\zeta^{\mathrm{T}}(t)\begin{bmatrix}\varTheta_5\\\varTheta_6\end{bmatrix}^{\mathrm{T}}\bar{\varXi}\begin{bmatrix}\varTheta_5\\\varTheta_6\end{bmatrix}\zeta(t), \tag{21}$$

其中$\bar{\varXi}\geqslant 0$.

根据假设1,可得

$$\phi_1^{\mathrm{T}}(t)\begin{bmatrix}-R_1\bar{E}_1 & R_1\bar{E}_2\\ * & -R_1\end{bmatrix}\phi_1(t)\geqslant 0, \tag{22}$$

$$\phi_1^{\mathrm{T}}(t-d(t))\begin{bmatrix}-R_2\bar{E}_1 & R_2\bar{E}_2\\ * & -R_2\end{bmatrix}\phi_1(t-d(t))\geqslant 0, \tag{23}$$

$$\phi_1^{\mathrm{T}}(t-d)\begin{bmatrix}-R_3\bar{E}_1 & R_3\bar{E}_2\\ * & -R_3\end{bmatrix}\phi_1(t-d)\geqslant 0. \tag{24}$$

由式(8)～(24),可得

$$\mathcal{E}\{\mathcal{L}V(x(t),t,i)\}\leqslant\mathcal{E}\{\xi^{\mathrm{T}}(t)\varPhi_i(d(t),\dot{d}(t))\xi(t)\}.$$

其中

$$\xi(t)=$$
$$\Big\{(\boldsymbol{M}x(t))^{\mathrm{T}},(\boldsymbol{M}x(t-d(t)))^{\mathrm{T}},(\boldsymbol{M}x(t-d))^{\mathrm{T}},$$

$$(\boldsymbol{M}g(x(t)))^{\mathrm{T}}, (\boldsymbol{M}g(x(t-d(t))))^{\mathrm{T}}, (\boldsymbol{M}g(x(t-d)))^{\mathrm{T}}, \Big(\int_{t-d(t)}^{t}\boldsymbol{M}g(x(s))\mathrm{d}s\Big)^{\mathrm{T}}, \frac{1}{d(t)}\Big(\int_{t-d(t)}^{t}\boldsymbol{M}x(s)\mathrm{d}s\Big)^{\mathrm{T}}, \frac{1}{d-d(t)}\Big(\int_{t-d}^{t-d(t)}\boldsymbol{M}x(s)\mathrm{d}s\Big)^{\mathrm{T}}, \frac{2}{(d(t))^2}\Big(\int_{-d(t)}^{0}\int_{t+\alpha}^{t}\boldsymbol{M}x(s)\mathrm{d}s\mathrm{d}\alpha\Big)^{\mathrm{T}}, \frac{2}{(d-d(t))^2}\Big(\int_{-d}^{-d(t)}\int_{t+\alpha}^{t-d(t)}\boldsymbol{M}x(s)\mathrm{d}s\mathrm{d}\alpha\Big)^{\mathrm{T}}\Big\}^{\mathrm{T}}.$$

因此,得到$\varPhi_i(d(t),\dot{d}(t))<0$,当且仅当$\varPhi_i^{\delta}<0$, $\delta=1,2,3,4$. 其中

$$\varPhi_i^{\delta}(d(t),\dot{d}(t))=\bar{\varPhi}_i^{\delta}(d(t),\dot{d}(t))+\varPhi_{2i},$$

$$\bar{\varPhi}_i(d(t),\dot{d}(t))=\varPhi_{1i}+\varPhi_3(d(t))+\varPhi_4(d(t),\dot{d}(t))+\varPhi_5(\dot{d}(t)),$$

$$\varPhi_{2i}=e_1^{\mathrm{T}}\sum_{j=1}^{\mathcal{N}}(\pi_{ij}+\Delta\pi_{ij})P_je_1,\ \varPhi_i^{\delta}=\bar{\varPhi}_i^{\delta}+\varPhi_{2i},$$

$$\bar{\varPhi}_i^1=\bar{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=0,\dot{d}(t)=\mu_1},$$

$$\bar{\varPhi}_i^2=\bar{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=0,\dot{d}(t)=\mu_2},$$

$$\bar{\varPhi}_i^3=\bar{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=d,\dot{d}(t)=\mu_1},$$

$$\bar{\varPhi}_i^4=\bar{\varPhi}_i(d(t),\dot{d}(t))_{d(t)=d,\dot{d}(t)=\mu_2}.$$

应用Dynkin's公式,可得

$$\mathcal{E}\{V(x(t),t,i)\}-\mathcal{E}\{V(x(0),0,i_0)\}=\mathcal{E}\Big\{\int_0^t\mathcal{L}V(x(s),s,r_s)\mathrm{d}s\Big\}. \tag{25}$$

存在正的常数β,有下式成立:

$$\beta\mathcal{E}\{\|\boldsymbol{M}x(t)\|^2\}\leqslant \mathcal{E}\{V(x(t),t,i)\}= \mathcal{E}\{V(x(0),0,i_0)\}+\mathcal{E}\Big\{\int_0^t\mathcal{L}V(x(s),s,r_s)\mathrm{d}s\Big\}\leqslant \mathcal{E}\{V(x(0),0,i_0)\}-\gamma\int_0^t\mathcal{E}\{\|\boldsymbol{M}x(s)\|^2\}\mathrm{d}s. \tag{26}$$

则系统(3)为均方聚类渐近同步的.

下面证明当式(4)$\sim$(6)成立时,保证$\varPhi_i^{\delta}<0(\delta=1,2,3,4)$成立. 令$\varTheta_i^{\delta}=e_1\bar{\varPhi}_i^{\delta}e_1^{\mathrm{T}}$, $\delta=1,2,3,4$,有

$$e_1\varPhi_i^{\delta}e_1^{\mathrm{T}}=\varTheta_i^{\delta}+\sum_{j=1}^{s}(\pi_{ij}+\Delta\pi_{ij})P_j=$$

$$\varTheta_i^{\delta}+\sum_{j=1,j\neq i}^{s}\{(\pi_{ij}+\Delta\pi_{ij})P_j+(\pi_{ii}+\Delta\pi_{ii})P_i\}=$$

$$\varTheta_i^{\delta}+\sum_{j=1,j\neq i}^{s}(\pi_{ij}+\Delta\pi_{ij})P_j-\sum_{j=1,j\neq i}^{s}(\pi_{ij}+\Delta\pi_{ij})P_i=$$

$$\varTheta_i^{\delta}+\sum_{j=1,j\neq i}^{s}\{(\pi_{ij}-\delta_{ij}+\delta_{ij}+\Delta\pi_{ij})(P_j-P_i)\}=$$

$$\varTheta_i^{\delta}+\sum_{j=1,j\neq i}^{s}\{\underline{\pi}_{ij}(P_j-P_i)+(\delta_{ij}+\Delta\pi_{ij})(P_j-P_i)\},$$

根据式(5)和$Y_{ij},\Delta\pi_{ij}\in[-\delta_{ij},\,\delta_{ij}]$, $i,j\in\mathcal{S}$, $j\neq i$,可得

$$(\delta_{ij}+\Delta\pi_{ij})(P_j-P_i)\leqslant(\delta_{ij}+\Delta\pi_{ij})Y_{ij}\leqslant 2\delta_{ij}Y_{ij},\quad i,j\in\mathcal{S},\ j\neq i.$$

因此可得$\varPhi_i^{\delta}<0$, $\delta=1,2,3,4$. □

注1 在式(21)中,采用具有较小保守性的不等式引理2,相比Jensen不等式和反凸组合技术产生了更紧的下界,从而具有较小的保守性.

注2 文献[15]采用两个松弛矩阵$G_{ij},T_{ij}(i,j\in\mathcal{S},j\neq i)$来处理转移概率矩阵中的不确定项,从而增加了决策变量的个数,使得计算较为复杂. 本文通过充分考虑转移概率和不确定项的性质,只采用了一组松弛变量$Y_{ij}(i,j\in\mathcal{S},\ j\neq i)$处理不确定项,从而大大减少了计算复杂度.

注3 文献[3-4,6-8]分别研究了参数为常矩阵的复杂网络的聚类同步问题,由于随机故障和外部环境扰动,复杂网络系统中的参数经常出现马尔科夫跳变,本文首次研究了具有不确定转移率的马尔科夫复杂网络的聚类同步问题,并给出了相应的聚类同步准则,从而该同步准则更具有一般性和实用性.

3 数值仿真

考虑具有6个节点的耦合神经网络模型(2),参数如下:

$$x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix},\ g(x(t))=\begin{bmatrix}\tanh(x_1(t))\\\tanh(x_2(t))\end{bmatrix},$$

$$d(t)=0.8+0.2\sin t,\ C_1=\begin{bmatrix}1.0&0\\0&1.0\end{bmatrix},$$

$$A_1=\begin{bmatrix}0.3&-0.3\\-0.5&0.3\end{bmatrix},\ B_1=\begin{bmatrix}-4.2&-5.0\\-2.3&-5.2\end{bmatrix},$$

$$C_2=\begin{bmatrix}1.1&0\\0&1.0\end{bmatrix},\ A_2=\begin{bmatrix}0.3&-0.2\\-0.49&0.31\end{bmatrix},$$

$$B_2=\begin{bmatrix}-3.8&-4.9\\-2.1&-4.8\end{bmatrix},$$

$$G_1^{(\alpha_1)}=G_2^{(\alpha_1)}=\begin{bmatrix}-15&14.9&0.05&0.05\\14.9&-15&0.05&0.05\\0.02&0.02&-12&11.96\\0.02&0.02&11.96&-12\end{bmatrix},$$

$$\Gamma_{1,i}=\begin{bmatrix}4.1&0\\0&4.1\end{bmatrix},\ \Gamma_{2,i}=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix},$$

$$\Gamma_{3,i}=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},\ i=1,2,\ U(t)=\begin{bmatrix}1.0\\-0.6\end{bmatrix}.$$

不确定转移概率矩阵为

$$\hat{\Pi}=\begin{bmatrix}-0.9+\Delta\pi_{11}&0.9+\Delta\pi_{12}\\1.0+\Delta\pi_{21}&-1.0+\Delta\pi_{22}\end{bmatrix},$$

$$|\ \Delta\pi_{ij}\ |\leqslant 0.5\pi_{ij},\ i,\ j\in\mathcal{S},j\neq i.$$

根据假设可知

$$E_1=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ E_2=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix}.$$

图1和图2分别给出了前两个节点和后两个节点的同步误差,计算公式如下:

$$e_1(t)=\sqrt{\sum_{i=1}^{2}(x_{1i}-x_{2i})^2},$$

$$e_2(t)=\sqrt{\sum_{i=1}^{2}(x_{3i}-x_{4i})^2}.$$

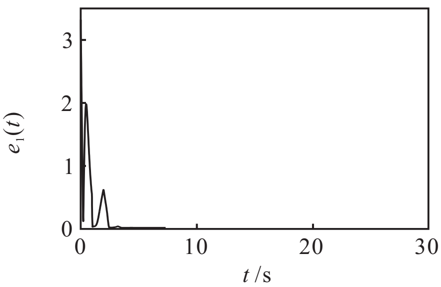

图1　系统(2)前两个节点的同步误差

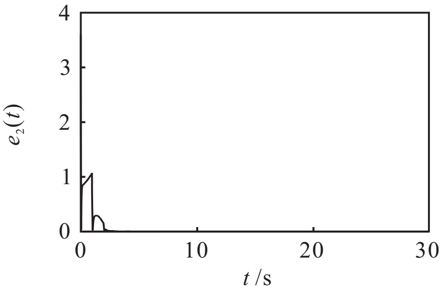

图2　系统(2)后两个节点的同步误差

图3给出了系统(2)的全局误差,计算公式如下:

$$e(t)=\sum_{i=1}^{2}\sqrt{\sum_{j=2}^{6}(x_{1i}-x_{ji})^2}.$$

由定理1和图1～图3可知,马尔科夫复杂网络系统(2)为聚类同步的. 此外,系统(2)的模态跳变如图4所示.

注4　在本文方法中,只采用了一组松弛变量Y_{ij} $(i,j\in\mathcal{S},\ j\neq i)$来处理不确定项,与文献[15]处理转移概率矩阵中不确定项的方法相比,减少了20个决策变量.

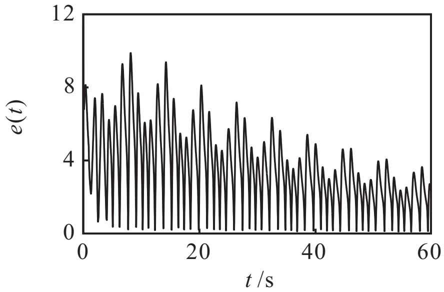

图3　系统(2)全局误差

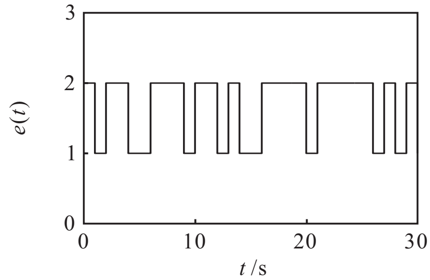

图4　系统(2)模态跳变图

当复杂网络系统(1)不含马尔科夫跳变时,系统简化为

$$\begin{aligned}&\dot{x}_k(t)=\\&-Cx_k(t)+Ag(x_k(t))+Bg(x_k(t-\\&d(t)))+U_k(t)+a_1\sum_{l=1}^{N}G_{kl}^{(1)}\Gamma_1 g(x_l(t))+\\&a_2\sum_{l=1}^{N}G_{kl}^{(2)}\Gamma_2 g(x_l(t-d(t)))+\\&a_3\sum_{l=1}^{N}G_{kl}^{(3)}\Gamma_3\int_{t-d(t)}^{t}g(x_l(s))\mathrm{d}s,\\&\qquad k=1,2,\cdots,N,\end{aligned}\tag{27}$$

系统参数参见文献[7],其中

$$C=\begin{bmatrix}1.0&0\\0&1.0\end{bmatrix},\ A=\begin{bmatrix}2&-0.1\\-5&3\end{bmatrix},$$

$$B=\begin{bmatrix}-1.5&-0.1\\-0.2&-2.5\end{bmatrix},\ U(t)=\begin{bmatrix}-0.02\\0.04\end{bmatrix}.$$

通过求解线性矩阵不等式,可以得到本文方法的最大时滞上界为$d_{\max}=1.1306$. 然而,文献[3,7,21]的最大时滞上界分别为$d_{\max}=0.9753$, $d_{\max}=1.0098$和$d_{\max}=0.7931$. 因此,本文方法具有较小的保守性. 此外,当模型中含有马尔科夫跳变时,文献[3,7,21]的方法是不可行的,本文方法更具有普遍性.

4　结　论

本文通过建立增广李雅普诺夫泛函和应用较小保守性的积分不等式,得到了含有时滞和时滞导数上

下界依赖的聚类同步准则. 在处理转移概率中的不确定项时,充分考虑转移概率和不确定项的性质,采用了只含有一组松弛变量的新技术,从而大大减少了计算负担. 所提出方法增加了系统在实际中的运用范围,更加接近实际存在的系统模型. 数值仿真表明了所提出方法的可行性和有效性.

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