

一种统一的鲁棒自适应控制方法及近期自适应控制结果综述

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摘要: 针对离散/连续时间情况下时不变/时变多种不同的鲁棒自适应控制系统, 基于归纳法提出一种统一的传统分析方法, 该方法为鲁棒自适应控制器的设计提供了一般性的指导原则. 在设计和实现鲁棒自适应控制器时要求具有与模型误差先验知识相关的假设, 通过所提出的算法可以将该假设消除, 使得已有的鲁棒自适应控制理论得到进一步发展. 此外, 通过稳定性分析证明了该算法对于更宽松边界的模型误差具有鲁棒性. 最后, 归纳总结了近期自适应控制方法在处理非光滑不确定性、执行器故障补偿、欠驱动非完整约束、分布式一致性和随机系统控制等问题上取得的代表性成果.

关键词: 自适应控制; 稳定性; 鲁棒性; 非光滑非线性; 执行器故障; 欠驱动系统; 一致性; 随机自适应控制
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A unified conventional approach for robust adaptive control systems and some reviews on recent developments in adaptive control

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Abstract: A unified approach is presented to analyzing various types of discrete and continuous time invariant/varying robust adaptive control systems. The approach provides a general guideline for the design of robust adaptive controllers. The essential tool of analysis is the inductive technique. Then we further develop the existing robust adaptive control theory. With the proposed unified approach, all the assumptions related to the a priori knowledge of modelling errors, which is required for the design and implementation of some existing robust adaptive controllers, can be removed. Also, the class of modelling errors including unmodelled dynamics and bounded external disturbances is enlarged in the sense that a looser bound can be allowed in the stability analysis. Some reviews on recent developments in adaptive control are provided, in which the issues including adaptive compensation for non-smooth nonlinearities and actuator failures, adaptive control of underactuated nonholonomic systems, distributed adaptive consensus control and stochastic adaptive control are discussed in detail.

Keywords: adaptive control; stability; robustness; non-smooth nonlinearities; actuator failures; underactuated systems; consensus; stochastic adaptive control

0 Introduction

0.1 Conventional adaptive control

Adaptive control is a design idea of self-tuning the control parameters based on performance error related

information to better fit the environment. Thus a variety of objectives such as system stability, desired output tracking with guaranteed steady-state accuracy and

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transient performance can be achieved. Since it was conceived in the early 1950s, it has been a research area of great theoretical and practical significance. The design of autopilots for high performance aircraft was one of the primary motivations for active research in adaptive control^[1]. During nearly seven decades of its development, a good number of adaptive control design approaches have been proposed for different classes of systems to solve various problems. Model reference adaptive control (MRAC)^[2-3], system and parameter identification based schemes^[4-5], adaptive pole placement control^[6-7] are some commonly used conventional adaptive control methods.

The prominent feature of adaptive control in handling systems with unknown parameters constitutes one of the reasons for the rapid development of this technique. An adaptive controller is normally designed by combining parameter update law and control law. The former is also known as a parameter estimator which provides the adaptation law for adjusting controller parameters at each time instant^[8].

Adaptive control techniques can be classified into direct and indirect ones according to the procedure of obtaining the controller parameters. The methods of computing the controller parameters based on the estimated system parameters are referred to as indirect adaptive control, whereas the methods of estimating the controller parameters directly without intermediate calculation are referred to as direct adaptive control. The common principle of conventional adaptive control techniques, no matter direct or indirect, is certainly equivalence principle. This means that the controller structure is designed as if all estimated parameters were true to achieve desired performances.

0.2 Robust adaptive control

The theory of robust adaptive control has been well established and understood. Essentially, the design of a robust adaptive controller involves appropriate modifications of the conventional adaptive laws. Various modification approaches have been proposed for both the direct model reference adaptive schemes in [9-19] and the indirect schemes in [11, 20-25]. These include normalization with parameter projection, σ -modification plus normalization, ϵ -modification, the use of dead-zones, and parameter projection alone. Although a general approach was presented for continuous time systems in [11], most approaches to the analysis of

the closed loop systems are different for the direct and indirect adaptive control schemes, as pointed out in [18]. While the stability result in [11] is independent of the structure of the controller, the structure of the estimator is fixed. Furthermore, for the indirect pole-placement control, the stability result was obtained only for limited choices of some design functions in the estimator. To establish the robustness properties of the adaptive controllers with various approaches, assumptions on the a priori knowledge related to the ignored unknown unmodelled dynamics of the plant are imposed, e.g., [9-18, 20-21]. These are then reflected in the design and implementation of the adaptive controllers, i.e., some parameters of the controllers have to be appropriately selected in order to ensure certain amount of robustness. In the case where a relative dead-zone is used, e.g., [13, 20-21], an upper bound on the gain of the unmodelled dynamics is needed to build a dead-zone in the estimator. The stability condition, on the other hand, requires this bound to be sufficiently small. Clearly, the need to choose such parameters makes it more complicated to implement these algorithms. When the general approach of [11] is applied to the indirect pole-placement design, the selection of some design functions is restricted to dead-zone, fixed σ -modification or ϵ -modification. Thus, as pointed out by the authors, it suffers similar drawbacks to the above. For the direct adaptive control schemes in [9-11], a priori knowledge on a lower bound of the stability margin of the unmodelled dynamics is required to generate a correct normalizing signal. The schemes in [14-16] also require such a lower bound to choose an appropriate filter and the robustness of the controller was established with the assumption that the zeros of the nominal model satisfy a boundary restriction related to the unmodelled dynamics.

0.3 Adaptive backstepping control

Backstepping is a recursive Lyapunov-based scheme for the class of "strict feedback" systems proposed at the beginning of 1990s. The technique was comprehensively addressed by Krstic, Kanellakopoulos and Kokotovic in [26]. The idea of backstepping is to design a controller recursively by considering some of the state variables as "virtual controls" and designing for them intermediate control laws. Backstepping achieves the goals of stabilization and tracking. The proof of these properties is a direct consequence of the recursive procedure, because a Lyapunov function is constructed for the entire system including the parameter estimates. In fact, when the controlled plant belongs to

the class of systems transformable into the parametric-strict feedback form, this approach guarantees global or regional regulation and tracking properties. An important advantage of the backstepping design method is that it provides a systematic procedure to design stabilizing controllers, following a step-by-step algorithm. With this method, the construction of feedback control laws and Lyapunov functions is systematic. Another advantage of backstepping is that it has the flexibility to tune design parameters to achieve certain types of transient performances.

0.4 Main objectives in this paper

Discussions in Section 0.2 motivate the need to develop a unified approach to the analysis of robust adaptive control systems, and a robust adaptive controller design which does not require a priori information on the unknown modelling errors for implementation. This is one of the main objectives of this paper. While the general approach in [11] establishes robust stability by using a general theory which relates the properties of signals in the mean sense over intervals of time, we shall adopt a simpler system theoretic approach. Some previous results focusing on a particular problem of indirect pole placement adaptive control with parameter projection^[22-25] have inspired the possibility of this work. We shall show that the unified approach can be applied to the analysis of various types of robust adaptive control systems, regardless of the estimation schemes and control strategies used. These include both the direct and indirect adaptive control of discrete / continuous time invariant / variant plants. With this approach, the class of unmodelled dynamics is enlarged in the sense that its effect can have infinite memory and thus a looser bound can be allowed in the stability analysis. This approach also gives a general guideline to designing robust adaptive controllers. That is, in the design of the adaptive controllers, all we need to do is to verify certain assumptions specified in the general approach. To illustrate the application of the approach, two example systems are considered. The first is an indirect discrete time adaptive predictive control system with parameter projection as the only modification in the estimator. The second is a direct continuous time model reference adaptive system using parameter projection and static normalizing signal.

The remaining part of this paper is devoted to providing some reviews on recent developments in adaptive control. Different issues including adaptive compensation for non-smooth nonlinearities and actuator

failures, adaptive control of underactuated systems, distributed adaptive consensus of uncertain multi-agent systems and stochastic adaptive control are discussed in detail.

1 A unified conventional approach for robust adaptive control systems

1.1 Notations

In this section, the following notations are used.

$c_i, k_i, i = 1, 2, \dots$, generic constants

u, y , input and output of the plant

$\omega(t)$, a vector related to the plant input and output

$1 + \omega^T(t)\omega(t)$, a normalizing signal

$x_c(t)$, a state vector of the closed loop adaptive system

$\eta(t)$, unmodelled dynamics

$d(t)$, external disturbance of the controlled plant

$m(t)$, system modelling error, i.e., $m(t) = \eta(t) + d(t)$

$m_i(t), i = 1, 2, \dots$, $W_i(s)m(t)$, where W_i is a vector containing proper stable transfer functions

$r(t)$, a reference trajectory for the plant output

M_0 , an intermediate sufficiently large positive constant

$\|\cdot\|$, the Euclidean norm of a finite dimensional vector

Definition 1 If a discrete function $f(t)$ satisfies

$$\sum_{\tau=s_i+1}^t |f(\tau)| \leq c_1 + c_2\epsilon(t - s_i - 1), \quad (1)$$

then

$$f(t) \in \sum_{s_i+1}^t (c_1, c_2\epsilon). \quad (2)$$

Definition 2 If a continuous function $f(t)$ satisfies

$$\int_{s_i}^t |f(\tau)| d\tau \leq c_1 + c_2\epsilon(t - s_i), \quad (3)$$

then

$$f(t) \in \mathcal{S}_{s_i}^t(c_1, c_2\epsilon). \quad (4)$$

1.2 The unified approach

The key points of the unified approach can be outlined as follows:

1) Express the closed loop adaptive system as

$$x_c(t+1) = A_c(x_c(t), t)x_c(t) + f_1(t+1) + f_2(t+1) + m_i(t+1) \quad (5)$$

for discrete-time systems or

$$\dot{x}_c(t) = A_c(x_c(t), t)x_c(t) + f_1(t) + f_2(t) + m_i(t) \quad (6)$$

for continuous-time systems, where A_c, f_1 and f_2 are appropriate dimensional matrix and vectors.

2) Establish that A_c is uniformly bounded and verify that its associated transition matrix, f_1, f_2 and m_i satisfy certain conditions. The details of these will be discussed in Sections 3 and 5 respectively for discrete and continuous time systems.

3) Identify an appropriate vector, $\omega(t)$, which is related to the plant input and output and used for the normalization in the estimator. Then study the trajectory $\|\omega(t)\|$.

4) Starting with $\tau_0 = 0$ and $i = 1, 2, \dots$, use an intermediate number M_0 to divide the time axis $[0, \infty)$ into the following two subsequences:

$$\mathfrak{R}_i^- = [\tau_{i-1}, s_i] \text{ and } \mathfrak{R}_i^+ = [s_i + 1, \tau_i - 1]$$

for discrete time systems or

$$\mathfrak{R}_i^- = [\tau_{i-1}, s_i] \text{ and } \mathfrak{R}_i^+ = (s_i, \tau_i)$$

for continuous time systems, where

$$\mathfrak{R}_i^1 = \{t \mid \|\omega(t)\| \leq M_0\}, \tag{7}$$

$$\mathfrak{R}_i^2 = \{t \mid \|\omega(t)\| > M_0\}, \tag{8}$$

i.e.

$$[0, \infty) = \left(\bigcup_{i=1}^{\infty} \mathfrak{R}_i^-\right) \cup \left(\bigcup_{i=1}^{\infty} \mathfrak{R}_i^+\right). \tag{9}$$

As will be shown in later sections, this division ensures that the ‘initial conditions’ $\|\omega(s_i)\| \leq M_0$ are true for the trajectory $\|\omega(t)\|$ over all \mathfrak{R}_i^+ .

5) Establish a uniform bound for $\|\omega(t)\|$ over \mathfrak{R}_i^+ through induction from $i = 1, 2, \dots$. This is possible from the division in (8).

With this unified approach, we can study the class of plants with the modelling error $m(t)$ satisfying the following assumption.

Assumption 1 There exist nonnegative constants ϵ and d_0 such that the modelling error $m(t)$ satisfies

$$|m(t)| \leq \epsilon \max_{0 \leq \tau \leq t-1} \|\omega(\tau)\| + d_0 \tag{10}$$

for discrete-time systems or

$$|m(t)| \leq \epsilon \sup_{0 \leq \tau \leq t} \|\omega(\tau)\| + d_0 \tag{11}$$

for continuous systems.

Remark 1

1) The constant ϵ indicates the strength of the unmodelled dynamics $\eta(t)$ coupled to the nominal plant model and d_0 is a bound of the external disturbances. While their existence is noted, their knowledge is not required.

2) In terms of the bounding signals, the bound for the modelling error in (10) or (11), which allows the unmodelled dynamics to have infinite memory, is looser than those given in the existing literature such as [9-18, 20-21] on robust adaptive control. Thus, the class of modelling errors considered in this paper is wider and

can be enlarged to include any nonlinear unmodelled dynamics satisfying (10) or (11).

To study the trajectory $\|\omega(t)\|$ from the closed loop system, another assumption is made on the relationship between the vectors $\omega(t)$ and $x_c(t)$.

Assumption 2 The vector $\omega(t)$ is related to $x_c(t)$ as follows:

$$\|x_c(t)\| \leq c_1 \|\omega(t)\| + c_2 M_0 + \|m_1(t)\|, \tag{12}$$

$$\|\omega(t)\| \leq c_3 \|x_c(t)\| + c_4 M_0 + \|m_2(t)\|. \tag{13}$$

Remark 2

1) The effectiveness of (12) and (13) will be shown in the two examples in Sections 4 and 6.

2) From the definitions of $m_i, i = 1, 2, \dots$, they also satisfy similar bounding conditions to (10) or (11).

In the following sections, the approach outlined above will be used to establish the stability of various types of adaptive control systems. Although the essential approach is the same for both the discrete and continuous time systems, we shall consider them separately for clarity of presentation.

2 Stability analysis of discrete-time robust adaptive control systems

Let the resulting closed loop adaptive system be described by (5).

Let the intermediate constant M_0 satisfy $\frac{d_0}{M_0} \leq \delta$, where $0 < \delta < 1$. Also define $M^2 = k_1 M_0^2 + k_2$ where k_1 and k_2 are nonnegative constants with $k_1 \geq 1$ as specified later. Thus, if $\|\omega(\tau)\| \leq M, \forall \tau = 0, 1, \dots, t-1$, (10) can be rewritten as

$$|m(t)| \leq (1 + \alpha_1) M_0, \tag{14}$$

where $\alpha_1 = \epsilon(k_1^{1/2} + k_2^{1/2})$.

Now, suppose that the adaptive controller is designed such that A_c, f_1 and f_2 in (5) satisfy the following assumptions.

Assumption 3 A_c is uniformly bounded. Furthermore, if $\|\omega(\tau)\| \leq M$, for $\tau = 0, 1, \dots, t-1$, then for all $\epsilon \leq \epsilon_1^*, \delta \leq \delta_1^*$, the transition matrix $\Phi(t, \tau)$ solved from

$$\begin{aligned} \frac{d}{dt} \Phi(t, s_i + 1) &= A_c \Phi(t, s_i + 1), \\ \Phi(s_i + 1, s_i + 1) &= I \end{aligned} \tag{15}$$

satisfies

$$\|\Phi(t, \tau)\| \leq c\sigma^{t-\tau} \text{ for } t \geq \tau \text{ and } t, \tau \in \mathfrak{R}_i^+, \tag{16}$$

where $\sigma \in (0, 1), \epsilon_1^*$ and δ_1^* are sufficiently small positive numbers.

Assumption 4 If $\|\omega(\tau)\| \leq M$, for $\tau = 0, 1, \dots, t-1$, then

1) $\|f(s_i + 1)\| \leq (c_4 + c_5\alpha_1)M_0 + c_6$ and for $t \in \mathfrak{R}_i^+ \setminus (s_i + 1)$,

$$\|\tilde{f}_1(t)\| \leq c_7, \tag{17}$$

where

$$\tilde{f}_1(t) = \frac{f_1(t)}{(1 + \|\omega(t-1)\|^2)^{1/2}}. \tag{18}$$

2) For $t \in \mathfrak{R}_i^+$,

$$\|\tilde{f}_1(t)\|^2 \in \sum_{s_i+1}^{t-1} (c_9, c_{10}(\alpha_2 + \alpha_3)), \tag{19}$$

where

$$\alpha_2 = c_{11}[(k_1^{1/2} + k_2^{1/2}) + (k_1 + k_2)]\epsilon, \tag{20}$$

$$\alpha_3 = c_{12}\delta. \tag{21}$$

Assumption 5 $f_2(t)$ is uniformly bounded.

Remark 3 From Assumptions 3 and 4, we note that the properties of A_c and f_1 are imposed only for the case when $\|\omega(t)\|$ satisfies certain conditions. Comparing with some existing schemes, e.g., [9, 21], where these types of properties must be true unconditionally, this is less demanding from the design of adaptive controllers.

Remark 4 From Assumption 5, the intermediate number is also such that $\|f_2(t)\| \leq M_0, \forall t$. Clearly such an M_0 always exists.

Now, we can establish the stability of the adaptive system in the sense that all the signals are bounded.

Theorem 1 Consider the system described by (5). Under Assumptions 1 to 5, $\exists \epsilon^*$ such that $\epsilon \leq \epsilon^*$ ensures $\|\omega(t)\| \leq M$ bounded, $\forall t$.

Proof The result is proved through induction by assuming that $\|\omega(\tau)\| \leq M, \tau = 0, 1, \dots, t-1$. From the division of the time interval $[0, \infty)$ in (8), we note that it is sufficient to show that $\|\omega(t)\| \leq M$ for $t \in \mathfrak{R}_i^+$ with i starting from 1 to ∞ .

The solution of (5) is

$$x_c(t) = \Phi(t, s_i + 1)x_c(s_i + 1) + \sum_{\tau=s_i+1}^{t-1} \Phi(t, \tau) \times [f_1(\tau + 1) + f_2(\tau + 1) + m_3(\tau + 1)],$$

i.e.

$$x_c(t) = \Phi(t, s_i + 1)[A_c(s_i)x_c(s_i) + f_1(s_i + 1) + f_2(s_i + 1) + m_3(s_i + 1)] + \sum_{\tau=s_i+1}^{t-1} \Phi(t, \tau)[f_1(\tau + 1) + f_2(\tau + 1) + m_3(\tau + 1)].$$

Using (12) ~ (14), (16), boundedness of $\|A_c(s_i)\|$, Assumptions 4 and 5, and the inequality

$$\|f_1(\tau + 1)\| \leq (1 + \|\omega(\tau)\|)\|\tilde{f}_1(\tau + 1)\|,$$

we have

$$\|\omega(t)\| \leq c\sigma^{t-s_i-1}[(c_{12} + c_{13}\alpha_1)M_0 + c_{14}] + c_{14} \sum_{\tau=s_i+1}^{t-1} c\sigma^{t-\tau}[\|\tilde{f}_1(\tau + 1)\|\|\omega(\tau)\| + \|\tilde{f}_1(\tau + 1)\| + M_0]. \tag{22}$$

Following similar steps to those in [23], which involve the use of Schwartz inequality, Assumption 4, the Grownwall lemma and the theorem of the Arithmetic and Geometric Means, we can obtain

$$\|\omega(t)\|^2 \leq c_{16} + c_{16}M_0^2 + c_{17}\alpha_1^2M_0^2, \tag{23}$$

if $\alpha_2 \leq \alpha_2^*, \delta \leq \delta^*$ where α_2^* and δ^* are sufficiently small positive constants. From the definition of α_1 , (23) can be rewritten as

$$\|\omega(t)\|^2 \leq [c_{18} + c_{19}\epsilon^2(k_1 + k_2)]M_0^2 + c_{20}. \tag{24}$$

Then let $k_2 = c_{20}$ and $k_1 = \max\left\{1, \frac{c_{18} + c_{19}\epsilon^2k_2}{1 - c_{19}\epsilon^2}\right\}$ when $\epsilon \leq \bar{\epsilon}^*$ where $\bar{\epsilon}^*$ is a positive constant satisfying $1 - c_{19}(\bar{\epsilon}^*)^2 > 0$. Thus we can have

$$\|\omega_i(t)\|^2 \leq k_1M_0^2 + k_2 = M^2. \tag{25}$$

Finally, let $\epsilon^* = \min\{\epsilon_1^*, \bar{\epsilon}^*, \bar{\epsilon}^*\}$ where $\bar{\epsilon}^*$ is such that $\epsilon \leq \bar{\epsilon}^*$ gives $\alpha_2 \leq \alpha_2^*$. \square

Once ω is shown bounded, $m_i(t)$ is bounded from (10) and therefore $x_c(t)$ is bounded from (12). Hence, we have obtained a robustly stable adaptive system.

Thus, to design robust discrete-time adaptive controllers, regardless of the control schemes and estimation techniques, we just need to ensure that the resulting closed loop adaptive system satisfies Assumptions 1 to 5. In the next section, we shall use an example to illustrate how this is achieved.

3 Robust adaptive predictive control without the use of a dead-zone

To illustrate the application of the techniques developed in the previous section, the robustness of an indirect adaptive predictive controller, designed on the basis of [21], is analyzed. In the adaptive version of the predictive controller law proposed in [21], the estimator of the adaptive controller includes a parameter projection operation and a dead-zone. To implement such a dead-zone, some constants, i.e. those given in Assumption 1 of [21], should be known. Here, we shall show that with our approach, the inclusion of the dead-zone in the controller is unnecessary. By removing the dead-zone, the knowledge of the constants mentioned above is not required and Assumption 1 in [21] is relaxed.

3.1 Plant models

The same class of plants considered in [21] is to be controlled, i.e.,

$$\bar{A}(q^{-1})y(t) = \bar{B}(q^{-1})u(t) + \tilde{m}(t), \quad (26)$$

where \tilde{m} is the modelling error including unmodelled dynamics and external disturbances, $\bar{A}(q^{-1})$ and $\bar{B}(q^{-1})$ represent the nominal model of the system and are given by

$$\begin{aligned} \bar{A}(q^{-1}) &= 1 + \bar{a}_1q^{-1} + \dots + \bar{a}_nq^{-n}, \\ \bar{B}(q^{-1}) &= \bar{b}_1q^{-1} + \dots + \bar{b}_nq^{-n}. \end{aligned}$$

After some manipulations as in [21], the above model can be rewritten as

$$A(q^{-1})\tilde{y}(t) = B(q^{-1})\delta\tilde{u}(t) + m(t), \quad (27)$$

where $\tilde{y} = \frac{1}{T(q^{-1})}y(t)$, $\delta\tilde{u} = \frac{\Delta(q^{-1})}{T(q^{-1})}u(t)$, $m = \frac{\Delta(q^{-1})}{T(q^{-1})}\tilde{m}(t)$, $\Delta(q^{-1}) = 1 - q^{-1}$, $T(q^{-1})$ is a strictly stable design polynomial, and

$$\begin{aligned} A(q^{-1}) &= \bar{A}(q^{-1})\Delta(q^{-1}) = \\ &= 1 + a_1q^{-1} + \dots + a_nq^{-n}, \\ B(q^{-1}) &= \bar{B}(q^{-1}) = \\ &= b_1q^{-1} + \dots + b_{n-1}q^{-n+1}. \end{aligned}$$

Now, rewrite (27) in a regression form as

$$\tilde{y}(t) = \omega^T(t-1)v + m(t), \quad (28)$$

where $\omega(t-1)$ is a regression vector and v is a vector containing unknown parameters of the nominal system model, i.e.

$$\begin{aligned} \omega^T(t-1) &= [\tilde{y}(t-1), \dots, \tilde{y}(t-n), \delta\tilde{u}(t-2), \\ &\quad \dots, \delta\tilde{u}(t-n+1), \delta\tilde{u}(t-1)], \quad (29) \end{aligned}$$

$$v^T = [-a_1, \dots, -a_n, b_2, \dots, b_{n-1}, b_1]. \quad (30)$$

For the system given in (28), it can be verified that $m(t)$ satisfies (10) in Assumption 1 by following the analysis in [21]. And if $\omega(\tau)$ is bounded according to $\|\omega(\tau)\| \leq M$ for $\tau = 0, 1, \dots, t-1$, we can ensure that (14) is satisfied.

Remark 5 In [21], knowledge of certain constants similar to ϵ and d_0 in (10) is needed for the design of the estimator. In this paper, such a requirement is relaxed.

As in [21], we shall assume some knowledge on the range of unknown parameter vector v of the nominal system model. This is given in the following assumption.

Assumption 6 The parameter v in (30) lies in a known convex compact region \mathcal{C} and \mathcal{C} has the property that the polynomials $\hat{A}(q^{-1}), \hat{B}(q^{-1})$ induced by an arbitrary (nonzero) vector \hat{v} in \mathcal{C} are uniformly coprime.

Assumption 6 implies that

$$\|v_1 - v_2\| \leq k_v, \quad \forall v_1, v_2 \in \mathcal{C}, \quad (31)$$

where k_v is a constant and reflects the size of \mathcal{C} .

The control problem is to design an indirect adaptive predictive controller for the class of plants

under Assumption 6. No dead-zone shall be used in the parameter estimator. The adaptive controller designed should be robust such that all the signals in the adaptive system are bounded in the presence of modelling error $m(t)$.

3.2 Adaptive predictive control algorithm

In this section, the algorithm to design the adaptive predictive controller is presented. The parameter estimator employed is a basic one used to establish earlier global convergence results^[27] subject to parameter projection.

For simplicity of analysis, we use the following gradient estimation algorithm:

$$\hat{v}(t) = \mathcal{P} \left\{ \hat{v}(t-1) + \frac{\omega(t-1)e(t)}{1 + \omega^T(t-1)\omega(t-1)} \right\}, \quad (32)$$

where $\hat{v}(t)$ denotes the estimates of v at t and \mathcal{P} represents the projection operator necessary to ensure $\hat{v}(t) \in \mathcal{C}, \forall t$. $e(t)$ is the prediction error defined as

$$e(t) = y(t) - \omega^T(t-1)\hat{v}(t-1). \quad (33)$$

At every time instant, the prediction error $e(t)$ can be related to the estimates of polynomials $\hat{A}_{t-1}(q^{-1})$ and $\hat{B}_{t-1}(q^{-1})$ through the equation

$$\hat{A}_{t-1}(q^{-1})\tilde{y}(t) = \hat{B}_{t-1}(q^{-1})\delta\tilde{u}(t) + e(t). \quad (34)$$

This is equivalent to

$$x_c(t+1) = \hat{A}_t x_c(t) + \hat{B}_t \delta\tilde{u}(t) + L e(t+1), \quad (35)$$

where

$$\begin{aligned} x_c(t) &= [\tilde{y}(t), \dots, \tilde{y}(t-n+1), \\ &\quad \delta\tilde{u}(t-1), \dots, \delta\tilde{u}(t-n+2)], \quad (36) \end{aligned}$$

$\hat{A}_t =$

$$\begin{bmatrix} -\hat{a}_{1t} & -\hat{a}_{2t} & \dots & -\hat{a}_{nt} & \hat{b}_{2t} & \dots & \hat{b}_{n-2t} & \hat{b}_{n-1t} \\ 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix}, \quad (37)$$

$$\hat{B}_t^T = [\hat{b}_{1t}, 0, \dots, 0, 1, 0, \dots, 0], \quad (38)$$

$$L^T = [1, 0, \dots, 0, 0, 0, \dots, 0]. \quad (39)$$

Now the parameter estimates obtained from the estimator (32) and (33) are used to adjust the parameters of the predictive controller. The control $u(t)$ is generated through the control law

$$\delta\tilde{u}(t) = \hat{K}_t x_c(t) + \hat{H}_t Y^0(t), \quad (40)$$

where

$$\hat{\mathcal{K}}_t = [-\hat{p}_{0t}, \dots, -\hat{p}_{n-1t}, -\hat{l}_{1t}, \dots, -\hat{l}_{n-2t}]. \quad (41)$$

Following the certainty equivalence principle and based on (34) with $e(t) = 0$, $\hat{\mathcal{K}}_t$ is obtained from the estimated polynomials $\hat{A}_t(q^{-1})$ and $\hat{B}_t(q^{-1})$ using the proposed predictive algorithm as in [21]. From Assumption 6, we see that the coefficients in $\hat{\mathcal{K}}_t$ are bounded. In (40), \hat{H}_t is a suitable row vector and Y^0 is a suitable column vector containing reference signal y_r [21].

3.3 Stability of the adaptive predictive control system

We can now apply the approach in the last section to study the robustness and stability of the adaptive control algorithm (32), (33) and (40) when applied to system (26).

Firstly, an equation in the form of (5) is derived to describe the closed loop system by combining (35) and (40), where

$$A_c(x_c(t), t) = \hat{A}(t) + \hat{B}_t \hat{\mathcal{K}}_t, \quad (42)$$

$$f_1(t+1) = Le(t+1), \quad (43)$$

$$f_2(t) = \hat{B}_t \hat{H}_t Y^0(t), \quad (44)$$

$$m_3(t) = 0, \forall t. \quad (45)$$

To establish the stability result by applying the unified approach, we only need to show that the adaptive control system satisfies Assumptions 2 to 5.

From the definition of $\omega(t)$ in (29) and $x_c(t)$ in (36), we have

$$\|x_c(t)\| \leq \|\omega(t)\|.$$

Using (40), we get

$$\|\omega(t)\| = \|[x_c(t), \delta \tilde{u}(t)]\| \leq \|x(t)\| + M_0. \quad (46)$$

Thus Assumption 2 is verified. From the boundedness of \hat{B}_t , \hat{H}_t and $Y^0(t)$, $\|f_2(t)\|$ is ensured to be uniformly bounded and thus Assumption 5 holds. Define

$$\tilde{e} = \frac{e}{(1 + \omega^T(t-1)\omega(t-1))^{1/2}}. \quad (47)$$

Then to verify Assumptions 3 and 4, we need the following lemma which gives some useful properties of estimator (32) and (33).

Lemma 1 The estimators (32) and (33), when applied to system (26), have the following properties:

1) Assuming that $\|\omega(\tau)\| \leq M$, $\tau = 0, 1, \dots, t-1$, then

i) $|e(s_i+1)| \leq (k_v + 1 + \alpha_1)M_0$ and for $t \in \mathfrak{R}_i^+ \setminus (s_i+1)$,

$$|\tilde{e}(t)| \leq k_v + \alpha + 1, \quad (48)$$

ii) For $t \in \mathfrak{R}_i^+$,

$$\tilde{e}^2(t) \in \sum_{s_i+1}^{t-1} (k_v^2, (\alpha_2 + \alpha_3)). \quad (49)$$

$$2) \|\hat{v}(t) - \hat{v}(t-1)\| \leq |\tilde{e}(t)| \text{ for all } t \geq 0. \quad (50)$$

Proof From the hypotheses of the lemma and using (28), (14), (31)~(33), the results can be established. \square

Clearly, Lemma 1 ensures that Assumption 4 is satisfied from (43). From Lemma 1, Assumption 6 and properly chosen design parameters of the predictive controller as in [21], and by following the analysis in [23], we can verify Assumption 3.

Once all these assumptions hold, the result in Theorem 1 can be applied to the adaptive predictive control system and the stability result is formally stated as in the following theorem.

Theorem 2 Consider the adaptive system consisting of plant (26), estimators (32) to (33) and controller (40). Under Assumption 6, $\exists \epsilon^*$ such that for all $\epsilon \leq \epsilon^*$, the signals $y(t)$ and $u(t)$ are uniformly bounded $\forall t$ for all bounded initial conditions, reference signals and external disturbances.

Remark 6 A remark on the use of a relative dead-zone is given here. It can be shown, with our approach, that if an incorrect upper bound of ϵ at (10) is used to build a relative dead-zone function in the adaptive controller, the closed loop system is still BIBS stable for sufficiently small ϵ . Thus, the knowledge of constants in (10) is still not required even if a dead-zone is used.

4 Stability analysis of continuous-time robust adaptive control systems

From the previous two sections, we note that the inductive proof plays a key role in the stability analysis of discrete-time systems. Intuitively, the inductive technique cannot be applied to continuous time systems. However, the division of the time axis into a number of intervals in Section 2 makes the application of the technique possible. Of course, as expected, the analysis will be more involved.

Similar to the discrete-time case, we let the resulting closed loop adaptive system be described by (6). Now suppose the adaptive controller is designed in such a way that A_c , f_1 and f_2 in (6) satisfy the following assumptions.

Assumption 7 A_c is uniformly bounded. Furthermore, if $\|\omega(t_0)\| = M_0$, $\|\omega(t)\| > M_0$, and $\sup_{0 \leq \tau \leq t} \|\phi(\tau)\| = \|\phi(t)\|$, $\forall t \geq t_0$, then for all $\epsilon \leq \bar{\epsilon}_1^*$, $\delta \leq \delta_1^*$, the transition matrix $\Phi(t, \tau)$ solved from

$$\frac{d}{dt} \Phi(t, t_0) = A_c \Phi(t, t_0), \Phi(t_0, t_0) = I \quad (51)$$

satisfies

$$\|\Phi(t, \tau)\| \leq ce^{-\sigma(t-\tau)} \text{ for } t \geq \tau \geq t_0, \quad (52)$$

where $\sigma > 0$, and $\bar{\epsilon}_1^*, \delta_1^*$ are sufficiently small positive constants.

Assumption 8 If $\|\omega(t_0)\| = M_0, \|\omega(t)\| > M_0$, and $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|, \forall t \geq t_0$, then

$$\tilde{f}_1 = \frac{f_1}{(1 + \|\omega(t)\|^2)^{1/2}} \quad (53)$$

has the following properties:

1) $\|\tilde{f}_1(t)\| \leq c_{21}$ for $t \geq t_0$, (54)

2) $\|\tilde{f}_1\|^2 \in \mathcal{S}_{t_0}^t(c_{22}, (\alpha_4 + \alpha_5))$ for $t \geq t_0$, (55)

where

$$\alpha_4 = (c_{21} + \epsilon)\epsilon, \alpha_5 = (c_{21} + \delta)\delta. \quad (56)$$

Assumption 9 $f_2(t)$ is uniformly bounded, i.e.,

$$\|f_2(t)\| \leq M_0, \forall t. \quad (57)$$

Under Assumptions 7 ~ 9, the boundedness of $x_c(t)$ can be established as stated in the following lemma.

Lemma 2 Consider the system in (6). If $\|\omega(t_0)\| = M_0, \|\omega(t)\| > M_0$, and $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\phi(t)\|, \forall t \geq t_0$, then $\exists \epsilon_1^*$ such that $\forall \epsilon \leq \epsilon_1^*$,

$$\sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 \leq c_{23}M_0^2 + c_{24} \quad (58)$$

for all $t \geq 0$.

Proof The solution of (6) is

$$x_c(t) = \Phi(t, t_0)x_c(t_0) + \int_{t_0}^t \Phi(t, \tau)[f_1(\tau) + f_2(\tau) + m_4(\tau)]d\tau.$$

Using (11) ~ (13) and (52) gives

$$\begin{aligned} \|\omega(t)\| &\leq ce^{-\sigma(t-t_0)}(\|\omega(t_0)\| + |\eta_2(t_0)| + M_0) + \\ &- [1pt]c_{25} \int_{t_0}^t ce^{-\sigma(t-\tau)}[\|\tilde{f}_1(\tau)\|\|\omega(\tau)\| + |\tilde{f}_1(\tau)| + \\ &|\tilde{f}_2(\tau)| + \|m_4(\tau)\|]d\tau \leq \\ &c_{26}M_0 + c_{27}\epsilon \sup_{0 \leq \tau \leq t} \|\omega(\tau)\| + \\ &c_{25} \int_{t_0}^t ce^{-\sigma(t-\tau)}|\tilde{f}_1(\tau)|\|\omega(\tau)\|d\tau + c_{28}. \end{aligned} \quad (59)$$

Now applying the Schwarz inequality and squaring both sides of (59), we get

$$\begin{aligned} \|\omega(t)\|^2 &\leq c_{29}M_0^2 + c_{30} \int_{t_0}^t e^{-\sigma(t-\tau)}\|\tilde{f}_1(\tau)\|^2 \times \\ &\|\omega(\tau)\|^2 d\tau + c_{31}\epsilon \sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 + c_{32}. \end{aligned} \quad (60)$$

Multiplying both sides of (60) by $e^{\sigma t}$ gives

$$e^{\sigma t}\|\omega(t)\|^2 \leq s^2(t) + c_{30} \int_{t_0}^t e^{\sigma\tau}\|\omega(\tau)\|^2\|\tilde{f}_1(\tau)\|^2, \quad (61)$$

where

$$s^2(t) = e^{\sigma t}[c_{29}M_0^2 + c_{31}\epsilon \sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 + c_{32}]. \quad (62)$$

Then applying the Bellman-Grownwall Lemma to (61) and using Assumption 8 yield

$$\begin{aligned} \|\omega(t)\|^2 &\leq e^{-\sigma t}s^2(t) + c_{30} \int_{t_0}^t e^{-\sigma t}\|\tilde{f}_1(\tau)\|^2s^2(\tau) \times \\ &e^{\int_{\tau}^t c_{30}\|\tilde{f}_1(\tau_1)\|^2d\tau_1}d\tau \leq \\ &c_{33}M_0^2 + c_{34} + c_{35}\epsilon \sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 \end{aligned} \quad (63)$$

for $\epsilon \leq \bar{\epsilon}_1^*$ and $\delta \leq \bar{\delta}^*$, where $\bar{\epsilon}_1^*$ and $\bar{\delta}^*$ are sufficiently small constants satisfying

$$c_{30}(\alpha_4^* + \alpha_5^*) < \sigma \quad (64)$$

with α_4^*, α_5^* depending on $\bar{\epsilon}_1^*$ and $\bar{\delta}^*$.

Note that the right side of (63) is nondecreasing.

Thus it can be rewritten as

$$\sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 \leq c_{33}M_0^2 + c_{34} + c_{35}\epsilon \sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2. \quad (65)$$

Then from (65), we get

$$\sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 \leq \frac{c_{33}}{1 - c_{35}\epsilon}M_0^2 + \frac{c_{34}}{1 - c_{35}\epsilon}, \quad (66)$$

if $\epsilon \leq \epsilon_2^*$ with the positive constant ϵ_2^* satisfying $c_{35}\epsilon_2^* < 1$.

Finally, the result is proved by letting $\epsilon_1^* = \min\{\bar{\epsilon}_1^*, \bar{\epsilon}_1^*, \epsilon_2^*\}$ and $c_{23} = \frac{c_{33}}{1 - c_{35}\epsilon_1^*}, c_{24} = \frac{c_{34}}{1 - c_{35}\epsilon_1^*}$. \square

Assumptions 7 and 8 require the properties of A_c and f_1 only for the case where $\|\omega(t)\|$ satisfies the condition $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for $t \geq t_0$. Clearly, it is difficult to design an adaptive controller to guarantee this condition. Thus we need more properties of A_c and f_1 when $\|\omega(t)\|$ does not satisfy this condition.

Assumption 10 A_c is uniformly bounded.

Furthermore, if $\|\omega(t)\|$ satisfies $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| \leq \sqrt{c_{23}M_0^2 + c_{24}}$ for $t \leq t_1$ and $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for all $t \geq t_1$, where $t_1 \in \mathfrak{R}_i^+$, then for all $\epsilon \leq \frac{\bar{\epsilon}_1^*}{\sqrt{c_{23} + \sqrt{c_{24}}}}$, $\delta \leq \delta_1^*$, the transition matrix $\Phi(t, \tau)$ satisfies

$$\|\Phi(t, \tau)\| \leq ce^{-\sigma(t-\tau)} \text{ for } t \geq \tau \geq s_i. \quad (67)$$

Assumption 11 If $\|\omega(t)\|$ satisfies $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\|$

$\leq \sqrt{c_{23}M_0^2 + c_{24}}$ for $t \leq t_1$ and $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for all $t \geq t_1$, where $t_1 \in \mathfrak{R}_i^+$, then $\tilde{f}_1(t)$ as defined in (53) has the following properties:

1) $|\tilde{f}_1(t)| \leq c_{21}$ for $t \geq s_i$, (68)

2) $\|\tilde{f}_1\|^2 \in \mathcal{S}_{s_i}^t(c_{22}, (\bar{\alpha}_4 + \alpha_5))$ for $t \geq s_i$, (69)

where

$$\bar{\alpha}_4 = [c_{21} + (\sqrt{c_{23}} + \sqrt{c_{24}})\epsilon] \times (\sqrt{c_{23}} + \sqrt{c_{24}})\epsilon. \tag{70}$$

Remark 7 Similar to Remark 3, it can be noted that the properties of A_c and f_1 are needed only for the case where $\|\omega(t)\|$ satisfies certain conditions for $t \in \mathfrak{R}_i^+$. In terms of adaptive controller design, this is less demanding than the schemes presented in [10-20].

With the above assumptions, we can have the following stability results for system (6).

Theorem 3 Under Assumptions 1, 2 and 7~11, the state vector $x_c(t)$ in the adaptive system (6) is bounded for all $t \geq 0$, if for all $\epsilon \leq \epsilon^*$, where $\epsilon^* = \frac{\epsilon_1^*}{\sqrt{c_{23}} + \sqrt{c_{24}}}$.

Proof We shall prove that $\omega(t)$ is bounded, which in turn implies the boundedness of x_c from (12) and (11). $\omega(t)$ can be ensured to be bounded if it is bounded in \mathfrak{R}_i^+ , $\forall i \geq 1$. This can be shown through induction. Thus we consider $t \in \mathfrak{R}_1^+$ first. From the continuity of $\omega(t)$, $\exists \bar{t}_1 \in \mathfrak{R}_1^+$ such that $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for $t \leq \bar{t}_1$. Then using Lemma 2 and noting that $\|\omega(s_1)\| = M_0$, we can show that for $t \leq \bar{t}_1$,

$$\sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 \leq c_{23}M_0^2 + c_{24} \tag{71}$$

for all $\epsilon \leq \epsilon_1^*$. Suppose that $\sup_{0 \leq \tau \leq t} \omega(\tau) \neq \omega(t)$ for $t \in [\bar{t}_1, t_1] \subset \mathfrak{R}_1^+$. Then (71) automatically holds for $t \leq t_1$. Now if $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for $t \geq t_1$, then $\|\omega(t)\|$ satisfies the conditions of Assumptions 10 and 11. Then following the same steps as in the proof of Lemma 7 and applying Assumptions 10 and 11 yield

$$\sup_{0 \leq \tau \leq t} \|\omega(\tau)\|^2 \leq c_{23}M_0^2 + c_{24} \tag{72}$$

for all $\epsilon \leq \epsilon^*$ where $\epsilon^* = \frac{\epsilon_1^*}{\sqrt{c_{23}} + \sqrt{c_{24}}}$. In this way, we can show that (72) holds for all $t \in \mathfrak{R}_1^+$.

Now assuming that (72) is true $\forall t \in \mathfrak{R}_i^+$, it can be shown that, by following the proof of Lemma 2 and the above argument, (72) is also true $\forall t \in \mathfrak{R}_{i+1}^+$ from Assumptions 10 and 11 with s_1 replaced by s_{i+1} and the fact that $\|\omega(s_{i+1})\| = M_0$. \square

From this section, we can conclude that if an adaptive controller is designed to satisfy Assumptions 2, 7~11, then it is robust with respect to the modelling error which satisfies Assumption 3.

5 A robust model reference adaptive controller

In this section, we apply the techniques in the last section to analyze a continuous time robust model

reference adaptive control system. The robust adaptive controller is obtained on the basis of [10] where the σ -modification and the dynamic normalization are replaced by parameter projection and static normalization here. Although the robustness of this controller was studied in [15] and [16], certain a priori knowledge on the unmodelled dynamics is required to choose an appropriate filter. Also, the robustness of the controller in [15] was established with the assumption that the zeros of the nominal model satisfy a boundary restriction related to the unmodelled dynamics. With our unified approach, we show that no a priori information is required from the unmodelled dynamics and the class of modelling errors is enlarged to include bounded external disturbances and unmodelled dynamics having infinity memory. In [19], a robust adaptive controller was proposed to remove the requirement on a priori knowledge and enlarge the class of unmodelled dynamics. Its results were obtained by using a normalizing signal having infinity memory and by modifying the estimation error signal proposed in [10]. The modified estimation error signal is generated from a differential equation involving system signals.

5.1 System models

To illustrate our approach, we first present the models of the plant to be controlled as follows:

$$y(t) = \{H(s)[1 + \epsilon_1 H_1(s)] + \epsilon_2 H_2(s)\}u(t) + d(t), \tag{73}$$

where $s = \frac{d}{dt}$, $H(s)$ is a nominal transfer function and defined by

$$H(s) = \frac{B(s)}{A(s)},$$

$$A(s) = s^n + a_{n-1}s^{n-1} + \dots + a_0,$$

$$B(s) = b_m s^m + b_{m-1}s^{m-1} + \dots + b_0,$$

with $m < n$, ϵ_1 and ϵ_2 are nonnegative constants, and $H_1(s)$ and $H_2(s)$ are respectively the multiplicative and additive unstructured uncertainties.

We assume that a user chosen reference model is given as

$$y_m(t) = W_m(s)r(t), \tag{74}$$

where $W_m(s)$ is a stable transfer function. The control problem is to design a controller for plant (73) such that the closed loop system is stable in the sense that all signals in the system are bounded for arbitrary bounded r and initial conditions, and the output $y(t)$ follows the output $y_m(t)$ of the model (74) as closely as possible. To solve the control problem, the following assumptions are made for the plant given in (73).

Assumption 12

A1: $H(s)$ is minimum phase, i.e., $B(s)$ is Hurwitz.

A2: An upper bound for n , the nominal plant relative degree $n^* = n - m$ and the sign of the high frequency gain, $\text{sgn}(b_m)$, are known.

A3: $H_1(s)$ and $H_2(s)$ are stable, and H, H_1 and H_2 are strictly proper.

A4: $d(t)$ is bounded.

A5: W_m has relative degree n^* and $r(t)$ is uniformly bounded piecewise continuous signals.

Remark 8 The above assumptions are the same as those in [19] except for the inclusion of external disturbances here. Note that apart from the Assumptions A3 and A4, no a priori knowledge is required from the modelling errors for the implementation of the adaptive controller proposed in the coming section.

5.2 Design of robust adaptive controllers

As in [10], define the following filtered variables

$$\dot{\omega}_1 = \Lambda\omega_1 + qy, \quad \dot{\omega}_2 = \Lambda\omega_2 + qu, \quad (75)$$

where (Λ, q) is a controllable pair satisfying

$$(sI - \Lambda)^{-1}q = \frac{1}{F(s)}[s^{n-2}, \dots, 1]^T \quad (76)$$

with $F(s)$ as a Hurwitz polynomial of order $n - 1$. Both Λ and q are chosen by users. Let

$$\omega^T = [\omega_1^T, \omega_2^T, y]. \quad (77)$$

Then the control is given as

$$u = \omega^T\theta + c_0r, \quad (78)$$

where $\theta^T(t) = [\theta_1^T(t), \theta_2^T(t), \theta_3(t)]$ is a $(2n - 1)$ -dimensional control parameter vector and $c_0(t)$ is a feedforward parameter scalar. From [17] and [18], it can be shown that a desired parameter vector θ^* of θ and a desired parameter c_0^* of c_0 exist, and they can be obtained when the nominal transfer function $H(s)$ is known. When $H(s)$ is unknown, an adaptive law is required to update θ and c_0 . To achieve this and to ensure the robustness of the adaptive controller in the presence of modelling errors, parameter projection operation is introduced to the adaptive law. As in [10], the adaptive law to tune θ and c_0 is divided into the following two cases:

Case 1: $k_p = 1$. In this case, $c_0 = 1$.

$$\dot{\theta} = \mathcal{P}\left\{\theta, -\frac{\Gamma\bar{e}_1\zeta}{1 + \omega^T\omega}\right\}, \quad (79)$$

where $\Gamma = \Gamma^T > 0$ and

$$\begin{aligned} \bar{e}_1 &= y - y_m + \theta^T\zeta - v, \\ \zeta &= W_m(s)I\omega, \\ v &= W_m(s)\theta^T\omega, \end{aligned} \quad (80)$$

and \mathcal{P} denotes the projection operation defined in [15]

and [16] or [28].

Case 2: k_p is unknown. In this case, $c_0(t)$ is unknown and needs to be updated. The adaptive law is the same as that in [10] except that the σ -modification and the normalizing signal are replaced appropriately as in Case 1.

Remark 9 The results for the adaptive controller in Case 2 can be obtained by following the similar analyses as in Case 1 and [10]. Thus we just focus our attention on Case 1 without any further elaboration on Case 2.

5.3 Stability of the model reference adaptive systems

We now apply the technique in Section 4 to establish the robustness of the adaptive controller in the face of modelling errors. Before doing this, some preliminary analysis on the system is required.

As shown in [10], the plant (73) can be expressed as

$$y = W_m(s)[\omega^T\tilde{\theta} + r] + m(t), \quad (81)$$

where

$$\tilde{\theta} = \theta - \theta^*, \quad (82)$$

$$m(t) = \eta(t) + d, \quad (83)$$

$$\eta(t) = \Delta u, \quad (84)$$

$$\begin{aligned} \Delta(s) &= W_m(s)\epsilon_1 H_1(s)[1 - \theta_1^{*T}(sI - F)^{-1}q] + \\ &\quad \epsilon_2 H_2(s)[1 + W_m(s)\theta_3^* + \theta_2^{*T}(sI - F)^{-1}q]. \end{aligned} \quad (85)$$

Clearly, $\Delta(s)$ is strictly proper and stable from Assumption 12. Let $V(s)$ be an arbitrary Hurwitz polynomial and defined as

$$\begin{aligned} V(s) &= s^{n-2} + v_{n-3}s^{n-3} + \dots + v_0, \\ v^T &= [1, v_{n-3}, \dots, v_0]. \end{aligned}$$

Then

$$\eta(t) = \Delta \frac{F}{V} \frac{V}{F} u = \Delta \frac{F}{V} v^T \omega_2. \quad (86)$$

Thus from (86), the modelling error $m(t)$ in (83) can be verified to satisfy Assumption 1.

As shown in [10], the plant has the following state representation

$$\dot{x} = Ax + bu + m_5(t), \quad y = h^T x + m_6(t), \quad (87)$$

where (A, b, h^T) is a minimal state representation of $H(s)$. Then the closed loop adaptive system can be described as

$$\begin{aligned} \dot{x}_c &= A_c x_c + b_c \omega^T \tilde{\theta} + b_c c_0 r + m_5(t), \\ y &= h_c^T x_c + m_6(t), \end{aligned} \quad (88)$$

where A_c is a stable matrix satisfying $h_c^T (sI - A_c)^{-1} b_c = \frac{1}{W_m(s)}$ and $x_c^T = [x^T, \omega_1^T, \omega_2^T]$.

Clearly (88) is in the form of (6) with $f_1 = b_c \omega^T \tilde{\theta}$

and $f_2 = b_c c_0 r$. Thus the techniques in the previous sections can be applied to (88). We shall verify that all the assumptions stated in Theorem 3 are satisfied.

As r is bounded, so f_2 is uniformly bounded, i.e., Assumption 9 is satisfied.

Following the analysis in Appendix C of [17], we can show that

$$\|x(t)\| \leq c_{36}\|\omega(t)\| + \|m_7(t)\|. \quad (89)$$

Then

$$\|x_c(t)\| = \|[x^T(t), \omega_1^T(t), \omega_2^T(t)]\| \leq c_{37}\|\omega(t)\| + \|m_7(t)\|. \quad (90)$$

Also

$$\|\omega(t)\| = \|\omega_1^T(t), \omega_2^T(t), h_c^T x_c(t) + m_6(t)\| \leq c_{38}\|x_c(t)\| + \|m_8(t)\|. \quad (91)$$

Thus Assumption 2 is verified from (90) and (91).

As A_c is always stable, it satisfies Assumptions 7 and 10.

The properties of the estimator (79) and (80) in the following lemmas show that Assumptions 8 and 11 hold.

Lemma 3 The estimators (79) and (80), applied to the plant given in (73), have the following properties:

$$1) \left| \frac{\omega(t)^T \tilde{\theta}(t)}{(1 + \omega^T(t)\omega(t))^{1/2}} \right| \leq c_{39} \text{ for } t \geq 0 \quad (92)$$

and

$$2) \text{ If } \|\omega(t_0)\| = M_0, \|\omega(t)\| > M_0, \sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|, \forall t \geq t_0, \text{ then} \\ \frac{\omega(t)^T \tilde{\theta}(t)}{(1 + \omega^T(t)\omega(t))^{1/2}} \in \mathcal{S}_{t_0}^t(c_{40}/\epsilon_0, (\alpha_6 + \alpha_7)) \\ \text{for } t \geq t_0, \quad (93)$$

where

$$\alpha_6 = (c_{41}/\epsilon_0 + \epsilon)\epsilon, \alpha_7 = (c_{41}/\epsilon_0 + \delta)\delta + c_{42}\epsilon_0^\rho, \quad (94)$$

where $\rho > 0$ and $\epsilon_0 \in (0, 1]$.

Proof

$$1) \left| \frac{\omega(t)^T \tilde{\theta}(t)}{(1 + \omega^T(t)\omega(t))^{1/2}} \right| \leq \frac{\|\omega(t)\|}{(1 + \omega^T(t)\omega(t))^{1/2}} |\tilde{\theta}(t)| \leq c_{39}.$$

2) From (81), we have

$$\bar{e}_1 = \zeta^T \tilde{\theta} + m(t). \quad (95)$$

We consider the following positive definite function

$$V = \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}. \quad (96)$$

Then using (79) and (95) gives

$$\dot{V} \leq -\frac{1}{2} \times \frac{(\tilde{\theta}^T \zeta)^2}{1 + \omega^T \omega} + \frac{1}{2} \times \frac{m^2(t)}{1 + \omega^T \omega}. \quad (97)$$

Thus from the assumption of the lemma and (11), we get

$$\frac{(\zeta^T(\tau)\tilde{\theta}(\tau))^2}{1 + \omega^T \omega} \in \mathcal{S}_{s_i}^t(c_{43}, c_{44}(\epsilon + \delta)). \quad (98)$$

Then (93) can be obtained by following the same analysis given in the proof of Theorem C1 in [10]. \square

From the above lemma, we can readily get following lemma.

Lemma 4 If $\|\omega(t)\|$ satisfies $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| \leq \sqrt{c_{23}M_0^2 + c_{24}}$ for $t \leq t_1$ and $\sup_{0 \leq \tau \leq t} \|\omega(\tau)\| = \|\omega(t)\|$ for all $t \geq t_1$ where $t_1 \in \mathfrak{R}_i^+$, then

$$\frac{\omega(t)^T \tilde{\theta}(t)}{(1 + \omega^T(t)\omega(t))^{1/2}} \in \mathcal{S}_{t_0}^t(c_{40}/\epsilon_0, (\bar{\alpha}_6 + \alpha_7)) \\ \text{for } t \geq t_0, \quad (99)$$

where

$$\bar{\alpha}_6 = [c_{41}/\epsilon_0 + (\sqrt{c_{23}} + \sqrt{c_{24}})\epsilon]. \quad (100)$$

Proof Under the assumptions of the lemma, (11) becomes

$$|m(t)| \leq \epsilon(\sqrt{c_{23}} + \sqrt{c_{24}})M_0 \text{ for } t \leq t_1 \quad (101)$$

and

$$|m(t)| \leq \epsilon\|\omega(t)\| \leq \epsilon(\sqrt{c_{23}} + \sqrt{c_{24}})\|\omega(t)\| \text{ for } t \geq t_1. \quad (102)$$

Then substitute (101) and (102) to (97), the result follows by noting $\|\omega(t)\| > M_0$. \square

Once all the assumptions of Theorem 3 are verified, its result can be applied to the model reference adaptive system and this is stated in the following theorem.

Theorem 4 Consider the adaptive control system consisting of plant (73) with modelling errors satisfying (11) and controllers (78) ~ (80). There exists a positive constant ϵ^* such that for all $\epsilon \leq \epsilon^*$:

1) The closed loop system is globally stable in the sense that all signals remain bounded $\forall t$ for all finite initial states, any bounded r and arbitrarily bounded external disturbances.

$$2) \text{ The tracking error } e_1(t) = y(t) - y_m(t) \text{ satisfies} \\ \int_{t_0}^t e_1^2(\tau) d\tau \leq \beta_1 + \beta_2(\epsilon + d_0 + \epsilon_0)(t - t_0) \\ \text{for all } t_0 \geq 0, \quad (103)$$

where β_1 and β_2 are positive constants.

Proof

1) The stability is ensured by the direct application of Theorem 3. However, no squaring operation is needed on (59) since $\|f_1\|$, instead of $\|f_1\|^2$, satisfies (55) and (69).

2) Once the boundedness of all the signals in the adaptive system has been established, the tracking properties can be obtained by following similar analyses

used in [10]. □

Remark 10 If there is no modelling error, i.e., $\epsilon = 0, d = 0$, then $e_1 \rightarrow 0$ as $t \rightarrow \infty$ from proof of Corollary 4.1 in [10].

6 Some reviews on recent developments in adaptive control

6.1 Adaptive control with non-smooth nonlinearities

When dealing with practical control problems, the designer is inevitably led to face the difficulties tied to the presence of real physical components, which often contain non-smooth nonlinearities. In particular, actuators used in practice almost always contain static (e.g., dead-zone) or dynamic (e.g., backlash, hysteresis) nonlinearities, whose parameters are unknown and may vary with time. They are among the key factors limiting both static and dynamic performance of feedback control systems. As a matter of fact, these nonlinearities are particularly harmful and usually lead to a relevant deterioration of system performance. These non-smooth nonlinear characteristics are often neglected in control system design. Nevertheless, certain design methods based on different control objectives and system conditions have been developed and verified in theory and practice. Some of them are reviewed below.

In order to formulate the problem, the considered plant is supposed to be preceded by the actuating device $u(v)$ as in Figure 1, where u is the plant input which is not available for control design. The control objective is to force the output to asymptotically track the reference signal, to make all the signals in the closed-loop system bounded, and to compensate for the effects of non-smooth nonlinearities.

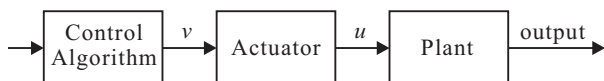


Figure 1 Block of a plant driven by the actuator

Adaptive control methods incorporated with other methodologies have been proposed to handle uncertain dynamic systems containing such non-smooth nonlinearities. The methods to compensate for the effects of non-smooth nonlinearities are divided into two groups.

Group 1: Handling the effects of non-smooth nonlinearities as a bounded “disturbance-like” term.

One way to deal with the non-smooth nonlinearities (dead-zone, backlash and hysteresis) is to represent the non-smooth nonlinearity as a linear term of input $v(t)$ and a bounded disturbance as follows:

$$u(v(t)) = m(t)v(t) + d(t), \tag{104}$$

where $m(t)$ is a time-varying or constant parameter and $d(t)$ is a bounded “disturbance-like” term. The effect of non-smooth nonlinearities will be treated as a bounded disturbance in the controller design.

Group 2: Inverse technique.

Another way to deal with the non-smooth nonlinearities is to employ an inverse to compensate for the effects of non-smooth nonlinearities. The compensation scheme is described in Figure 2, where the actual control $v(t)$ is a function of $u_d(t)$ as follows:

$$v(t) = \hat{N}^{-1}(u_d(t)), \tag{105}$$

where \hat{N}^{-1} represent an inverse of a non-smooth nonlinearity and u_d is the actual control input which will be designed using different control algorithms. The inverse can be a hard inverse or a smooth inverse based on the system conditions and control objectives.

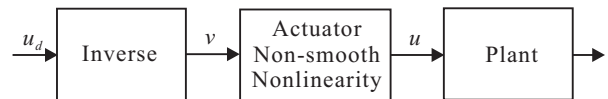


Figure 2 Compensation of non-smooth nonlinearity

In this section, we review the adaptive control methods incorporated with the above two methodologies to compensate for the effects of non-smooth nonlinearities containing dead-zone, backlash, hysteresis, or saturation in the actuator.

6.1.1 Dead-zone

Dead-zone, a static input-output characteristic, often appears in motors, valves and biomedical actuation systems. The study of adaptive control for systems with unknown dead-zone at the input was initiated in [29]. Sliding mode adaptive control was proposed in [30], where the effects of the dead-zone were compensated as a disturbance-like term. By using backstepping technique, an adaptive controller was developed for uncertain nonlinear systems with dead-zone input as in [31]. Unlike some existing control schemes for systems with dead-zone, the developed backstepping controllers do not require the uncertain parameters within known intervals. Also no knowledge is assumed on the bound of the ‘disturbance-like’ term, a combination of the external disturbances and a term separated from the dead-zone model. It was shown that the proposed controllers not only can guarantee stability, but also transient performance.

Another method for the control of dead-zone is to construct an adaptive dead-zone inverse. This approach was used in [8, 32-35], where the output of

a dead-zone was measurable and the inverse was a non-smooth inverse. Adaptive output feedback control of a class of nonlinear systems with unknown dead-zone input was investigated in [36] using backstepping approach, where a smooth inverse of the dead-zone was firstly introduced to compensate for the effects of the dead-zone. Such a smooth inverse can avoid chattering problems that may occur in the non-smooth inverse approach proposed. The specific treatment of the dead-zone may bring performance improvement. As system output feedback was employed, a state observer was required. To obtain such an observer, a new parametrization of the state observer for the plant was proposed to include two sets of parameters: one from the dead-zone nonlinearity and the other from the plant. Besides showing stability of the system, the transient performance in terms of L_2 norm of the tracking error was derived to be an explicit function of design parameters and thus our scheme allows designers to obtain the closed loop behavior by tuning design parameters in an explicit way.

6.1.2 Backlash

Backlash, a dynamic input-output characteristic, exists in a wide range of physical systems and devices, such as biology optics, electro-magnetism, mechanical actuators, electronic relay circuits and other areas. Backlash can be described as lines and segments^[32] or a differential equation^[37-38]. Several adaptive control schemes have been proposed for systems with backlash nonlinearity. Adaptive sliding mode control was proposed for systems with input backlash in [37], where the effect for the backlash was dealt with as a disturbance in the controller design. The backstepping-based adaptive control was proposed in [38-39]. A non-smooth inverse of backlash was employed to cancel the effects of nonlinearity in [32, 40-41], where the backlash was expressed as lines and segments. In [42], an efficient smooth adaptive inverse was firstly developed to compensate for the effects of the backlash with adaptive backstepping approach. This method was extended to output feedback control of nonlinear uncertain systems in [43] and decentralized adaptive control of interconnected systems with unknown interactions and backlash input in [44]. The smooth adaptive inverse in [42] is different from the inverse in [32], where the inverse indicator functions were non-smooth. The discontinuous case may cause chattering phenomenon in the recursive backstepping control. An efficient adaptive backlash inverse was developed when the backlash parameters

were unknown. In [38], a smooth sign-like function was firstly introduced to be used in the recursive backstepping approach. Besides, the chattering phenomena caused by discontinuous sign function can be avoided. The smooth control law can guarantee that the tracking error is still ensured to approach a prescribed bound.

6.1.3 Hysteresis

Hysteresis exists in a wide range of physical systems and devices. It can be described by two half-lines, two line segments, and the quadrilateral formed by those half-lines and segments^[32]. Research on adaptive control of systems with hysteresis nonlinearity has received great attention in recent years. In [45], the stability of systems with hysteresis was investigated, where a passive hysteresis operator was developed for Preisach hysteresis model. For lines-segments hysteresis nonlinearity, the inverse is easily obtained, e.g., [32, 46-47], where a non-smooth inverse nonlinearity model was constructed. In [48], another adaptive control approach was fused with the Prandtl-Ishlinskii model without constructing a hysteresis inverse, since the inverse was also difficult to be obtained. In [49], approximate inverse of the Preisach hysteresis operator with application to control of smart actuators was studied. An approximate implicit inverse approach was introduced for linear dynamical systems preceded with Prandtl-Ishlinskii hysteresis in [50-51]. In [52], the LuGre hysteresis model presented in [53] was investigated and the proposed hysteresis model with parametrization was integrated into a mechanical motion dynamics with lumped external load to completely represent the overall dynamics of a piezo-positioning mechanism. In [52], two simple adaptive backstepping control schemes were developed for the piezo-positioning mechanism. In the first scheme, the structure of the LuGre model was taken into account in the controller design when parameters of the model were known. In the second scheme, the parameters in the LuGre model can be allowed to be totally uncertain. In the second case, the LuGre model was divided into two parts. While the unknown parameters of one part was incorporated with unknown system parameters for estimation, the effect of the other part was treated as a bounded disturbance. An update law was developed to estimate the bound involving the partial hysteresis effect and the external load.

6.1.4 Bouc-Wen hysteresis

Bouc-Wen differential model is one of the most widely accepted phenomenological models of the hysteresis in mechanics^[54-57]. Actually it can be shown

that the backlash hysteresis model presented in [37-38] is a special case of the Bouc-Wen hysteresis model. Control of systems with the Bouc-Wen hysteresis has been reported in the works of [58-59, 60-61]. In [58], an adaptive controller was designed for a second-order structural system with unknown hysteresis to counteract the effect of an earthquake excitation, where the system's uncertain parameters must be within some known intervals and the effect of the hysteresis was treated in a similar way to that of handling a bounded disturbance. In [60], adaptive backstepping control was developed for the same class of systems as in [58] with Bouc-Wen hysteresis, where the structural parameters and isolation parameters were all uncertain parameters. Two schemes were proposed. In the first scheme, we used some available structure information in the design and the residual effect of the hysteresis was treated as a bounded disturbance. In the second scheme, we further took the structure of the Bouc-Wen model describing the hysteresis into account in the controller design, if a priori knowledge on some parameters of the model was available. It was shown that the proposed controller can guarantee stability and achieve tracking performance. Also with the proposed scheme, both transient and asymptotic performances were quantified as explicit functions of the design parameters so that designers can tune the design parameters in an explicit way to obtain the required closed loop behavior. Compared with the scheme in [58], system performance with the first scheme applied was still improved even though we needed much less knowledge from the system. When the second scheme was applied, the performance was significantly improved compared with the first scheme and the scheme in [58]. In [61], we took the hysteresis into account in backstepping-based controller design, instead of only considering its effect like bounded disturbances. To achieve this, a new hysteresis inverse was developed to efficiently compensate for the hysteresis effects when developing the control scheme with backstepping approach. Besides showing the stability of the close-loop system, output tracking error was also ensured to asymptotically approach to zero. In addition, transient performance in terms of L_2 norm of the tracking error was derived to be an explicit function of design parameters.

6.1.5 Saturation input

Saturation is always a potential problem for actuators of control systems as all actuators do saturate at some level. In many practical dynamic systems,

physical input saturation on hardware dictates that the magnitude of the control signal is always constrained. It often severely limits system performance, giving rise to undesirable inaccuracy or leading instability.

The development of control schemes for systems with input saturation has been a task of major practical interest as well as theoretical significance. When the system parameters are unknown, adaptive control schemes have been proposed, for example, model reference adaptive control in [62-63], predictive control in [64], discrete-time control approaches in [65], indirect adaptive regulator in [66-67], where uncertain parameters must be inside a known compact set. An adaptive force-balancing control scheme with actuator limits for a MEMS gyroscope was also presented in [68], where the plant was a stable second-order uncertain linear system. The system tracking error was shown to approach a signal generated by an artificially constructed system. [69] is the first paper to use adaptive backstepping approach to solve the saturation problem in uncertain nonlinear systems. To deal with saturation, a new system was constructed with the same order as that of the plant. With the error between the control input and saturated input as the input of the constructed system, a number of signals are generated to compensate the effect of saturation. With the proposed adaptive backstepping controller, the system tracking error was shown to approach a signal generated by the constructed system. The tracking error was also adjustable by an explicit choice of design parameters.

6.1.6 Quantization input

In quantized control systems, the control signal to the plant is a piece-wise constant function of time. There has been a great deal of attention to the study of quantized control systems, in which a control system is interacted with information quantization, due to its theoretical and practical importance in the study of digital control, hybrid systems, networked control systems. The main motivation for considering quantization in control systems comes from the observation that, for many control systems, quantization is not only inevitable, but also useful. An important aspect is to use quantization schemes that have sufficient precision and require low communication rate. The most practical quantizers are uniform quantizer, logarithmic quantizer, and hysteresis quantizer.

Quantized control of uncertain systems with known quantization parameters has been studied by using adaptive approaches in [70-77]. Adaptive control

schemes for uncertain systems with logarithmic or hysteresis input quantization have been reported for linear systems in [70] and nonlinear systems in [71], where the idea of constructing hysteresis type of input quantization was originally introduced. However the stability condition in [70] and [71] depends on the control signal, which is hard to be checked in advance as the control signal is only available after the controller is put into operation. In [72-73], a backstepping-based adaptive control scheme was presented for a class of uncertain strict-feedback nonlinear systems with hysteresis quantized input. Although the proposed method can avoid stability conditions depending on the control input, it requires the nonlinear functions to satisfy global Lipschitz conditions with known Lipschitz constants. This strict condition was relaxed recently in [74] by using an implicit adaptive controller. However in [74], the unknown parameters were only contained in the last differential equation of the system and the control signal was implicitly involved in the proposed control law. This means that the control signal needs to satisfy the equation resulted from the control law. It is nontrivial to solve the equation to obtain the control signal explicitly. In [75], a new quantizer was proposed based on a logarithmic (or hysteresis) quantizer and a uniform quantizer. To use such a quantizer, the resulting quantization error needs to be bounded. However, in our paper, the quantization error depends on the input control signal and cannot be assumed bounded. Clearly how to handle such a unbounded quantization error is difficult and challenging. [76, 78] addressed the issue of decentralized quantized control of interconnected systems with input quantization. In [79], a new adaptive controller was developed by introducing a hyperbolic tangent function. By proposing a new transformation of the final control signal and using the property of the quantizers, the effects from the quantization input were effectively compensated so that the global Lipschitz conditions required for the nonlinearities were relaxed. When quantized parameters were not known, new parameter updating laws were developed which do not require the knowledge on the bounds of such unknown parameters.

6.2 Adaptive control of time-delay systems

Time-delay phenomenon is commonly found in chemical processes, biological reactors, rolling mills, communication networks, etc. Since the existence of delay usually deteriorates the performance of closed-loop system, the stabilization and control of time-

delay systems is a topic of great importance and has received increasing attention. The difficulty to control systems with time-delay in control input is that such systems belong to non-minimum phase systems and the standard adaptive backstepping technique has only been shown applicable to minimum phase systems. Adaptive control of systems with unknown input time-delay was considered by [80], but the technique used was based on conventional pole placement adaptive scheme instead of backspetting approach and thus no transient performance was established. In [81], we re-visited the standard backstepping design approach without any modification and applied the designed adaptive controller to systems with modeled dynamics including unknown constant input time-delay. This result showed that the standard backstepping controllers possess certain robustness against modeled input time-delay and, as an additional significance, the result revealed that the class of systems stabilized by these controllers can be extended to non-minimum systems. Two key techniques were used in our analysis. Firstly, we transformed the unmodeled dynamics with delayed system input to another dynamics with a delayed system state as its input. We then found a state-space realization associated with the new dynamics. In this way, the effects of unmodeled dynamics with unknown input time-delay were bounded by static functions of state variables of the overall system. Such techniques were extended to deal with time-varying input delay in [82]. In addition two techniques used in [81], a new smooth function was proposed to compensate for the effects of reference inputs and two novel composite Lyapunov-Krasovskii functions were developed by introducing an exponential term, which is different from the general Lyapunov-Krasovskii functions.

6.3 Adaptive control of time-varying systems

Time-variations in dynamical systems occur in many physical systems. In [83], output feedback control was considered for linear time-varying systems when the sign of high-frequency gain was known. In [84], the problem of adaptive control with unknown sign of high-frequency gain for linear time invariant systems was studied by introducing the Nussbaum gain method. In [85], Nussbaum gain incorporating with the backstepping technique was used to design adaptive output stabilizer for high order uncertain time invariant nonlinear systems with unknown sign of high-frequency gain in the absence of external disturbances, where the nonlinearities considered should satisfy sector conditions. In [86], disturbance decoupling

was addressed for nonlinear time invariant systems with known sign of the high frequency gain. The result obtained was critically depending on a function of the system output y and the reference trajectory y_r . In [87], a flat zone was used to handle the problem of nonlinear time invariant systems with unknown sign of high frequency gain in the presence of disturbances. The bound of the disturbance and all the unknown parameters need to be estimated at every step in the backstepping process. This results in the problem of over-parametrization and makes the implementation complicated. In [88], state-feedback control was considered for a class of uncertain time-varying nonlinear systems in the presence of disturbances. In [89], adaptive output feedback control was designed for a class of uncertain time-varying nonlinear systems. The nonlinear functions considered are not required to satisfy the sector type of conditions like [85]. To handle the disturbances, well defined functions are introduced to eliminate their effects in the Lyapunov functions employed in the recursive design steps. To deal with the time variation problem, an estimator was used to estimate the bound of the variation rates. Furthermore, the over-parametrization problem was also solved by using the concept of tuning functions. As projection operation was used, the design of tuning functions were different from existing schemes. With the proposed controller, system stability can be ensured.

6.4 Adaptive control with actuator failures

An actuator, as appeared in Figure 1, cannot always be guaranteed to work normally in practice. Note the occurrence of certain failures may prevent an actuator from executing the control command effectively, then causes deteriorated performance or even instability of the system. Accommodating actuator failures is important to ensure safety of the systems, especially for life-critical systems including aircrafts, spacecrafts, nuclear power plants. Moreover, the increasing demands for safety and reliability in modern industrial systems with great complexity stimulate the development of more actuator fault tolerant control techniques.

Several effective control design approaches have been developed to address the actuator failure accommodation problem for both linear^[90-95] and nonlinear systems^[96-104]. They can be roughly classified into two categories, i.e., passive and active approaches. Typical passive approaches aim at achieving insensitivity of the system to certain presumed failures

by adopting robust control techniques. Since fixed controllers are used throughout failure/failure-free cases, and failure detection/diagnostic (FDD) is not required in these results, the design methods are computationally attractive. However, they are often conservative for large failure pattern changes since the achieved system performance based on worst-case failures may not be satisfactory for each failure scenario. In contrast to the passive methods, the structures and/or the parameters of the controllers are adjustable in real time when active design approaches are utilized. Furthermore, FDD is often required in active approaches and provide the estimated failure information to the controller design. The adverse effects brought by the actuator failures, even if large failure pattern changes are involved, can, therefore, be compensated for and the system stability is maintained. A number of active schemes have been presented, such as pseudo-inverse method^[105], eigenstructure assignment^[106-107], multiple model^[90-91, 104, 108], model predictive control^[109], and sliding mode control based scheme^[95]. Different from the ideas of redesigning the nominal controllers for the post-failure plants in these schemes, virtual actuator method^[110] hides the effects of the failures from the nominal controller to preserve the nominal controller in the loop.

Apart from these, adaptive control is also an active method well suited for actuator failure compensation^[97, 111-112] because of its prominent ability to adapt to the structural, parametric uncertainties and variations in the systems. As opposed to most of the active approaches, many adaptive control design schemes can be applied with neither control restructuring nor FDD processing. Moreover, not only the uncertainties caused by the failures, but also the unknown system parameters are estimated online for updating the controller parameters adaptively. In [92-93], Tao et al. proposed a class of adaptive control methods for linear systems with total loss of effectiveness (TLOE) type of actuator failures. As discussed in Section 0.3, backstepping technique^[26] has been widely used to design adaptive controllers for uncertain nonlinear systems due to its prominent advantages in relaxing relative degree limitation and improving transient performance. The results in [92-93] have been successfully extended to nonlinear systems in [99-101, 113] by adopting the backstepping technique. In [114], a robust adaptive output feedback controller was designed based on the backstepping technique to

stabilize nonlinear systems with uncertain TLOE failures involving parameterizable and unparameterizable time varying terms. In fact, adaptive control also serves as an assisting tool for other methods as in [90-91, 96, 98, 102, 104, 115].

It is well known that the backstepping technique can provide a promising way to improve the transient performance of adaptive systems in terms of L_2 and L_∞ norms of tracking errors in failure-free cases if certain trajectory initializations can be performed. However, since the actuator failures are normally uncertain in occurrence time, pattern and value, such trajectory initializations involving state-resetting actions are difficult at the time instants when actuator failures occur. To handle this issue, a new adaptive actuator failure compensation control scheme was proposed in [116] by incorporating a prescribed performance bound technique presented in [117]. By defining a monotonically increasing smooth function with bounded range, the tracking error can be guaranteed within the prescribed error bound all the time, no matter whether actuator failures occur or not. Thus the transient performance in terms of the convergence rate and overshoot of the tracking error can be improved by adjusting certain design parameters. Besides, asymptotical tracking can be achieved with the smooth function designed to pass through the origin.

Note that in all the aforementioned results on adaptive control of systems with actuator failures, only the cases with finite number of failures are considered. It is assumed that one actuator may only fail once and the failure mode does not change afterwards. This implies that there exists a finite time T_r such that no further failure occurs on the system after T_r . However, it is possible that some actuator failures occur intermittently in practice. Specifically, the actuators may unawarely change from a failure mode to a normally working mode or another different failure mode infinitely many times. The idea of stability analysis based on the Lyapunov function for the case with finite number of failures cannot be directly extended to the case with infinite number of failures, because the possible increase of the Lyapunov function cannot be ensured boundedness automatically when the parameters may experience an infinite number of jumps. In [118], a modular design based adaptive compensation approach was presented for the case with the intermittent actuator failures. Nonlinear damping terms were introduced

in the control module to establish an input-to-state property of the system with parameter estimation errors considered as the inputs. Then projection operation was involved in the parameter update laws to ensure the boundedness of estimation errors. It was shown that the boundedness of all the closed-loop signals can be ensured provided that the time interval between two successive changes of failure pattern is bounded below by an arbitrary positive number. And the tracking error can be small in the mean square sense if the changes of failure pattern are infrequent. The modular-based method was then extended to controlling a general class of uncertain systems with piecewise jumping parameters^[119]. Alternative solutions to treat intermittent actuator failures were provided by estimating the bounds of uncertain jumping parameters in [120-121] and utilizing switched control technique in [122].

Some more recent results on adaptive actuator failure compensation can be found in [123-126], where distributed consensus and event-triggered control issues were considered. In [127], a new supervisory switching based adaptive fault tolerant control scheme was proposed, such that fault detection and actuator switching can be performed simultaneously without the need for fault detection estimators.

6.5 Adaptive control of nonholonomic underactuated systems

Nonholonomic underactuated mechanical systems are special systems full of research interest and practical relevance. Referring to those mechanical systems whose number of control variables is less than the degrees of freedom, the underactuated systems are abundant in real life, ranging from landing vehicles, surface ships, and underwater vehicles to spacecrafts. Mobile robots and surface vessels are two typical nonholonomic underactuated systems that receive tremendous consideration in literature. For the tracking and stabilization control of underactuated mechanical systems, many methodologies have been proposed for controlling these systems by researchers.

Typical examples of underactuated mechanical systems are nonholonomic mobile robots and underactuated ships, which have received lots of attention from the control community. In control theory, stabilization is usually regarded as a special case of the tracking problem. However for controlling underactuated mechanical systems with nonholonomic constraints, such as nonholonomic mobile robots, the stabilization and tracking problems are totally different, and thus they

are normally considered separately.

For nonholonomic mobile robots, due to the Brockett's necessary condition, there exists no continuous time-invariant state-feedback controller that can asymptotically stabilize the mobile robot. For tracking control, a time-varying state-feedback tracking controller^[128] was proposed for kinematics and simplified dynamics of mobile robots based on backstepping^[26]. In [129], an adaptive tracking controller was designed for nonholonomic mobile robots with parametric uncertainties. In [130], by using coordinate transformation, an adaptive controller was designed for nonholonomic mobile robots such that the tracking and stabilization problems can be solved with only one controller. In [131], a global time-varying output feedback controller that can solve tracking and stabilization problems for nonholonomic mobile robots was proposed based on a coordinate transformation. In [132], a transverse function approach was proposed for the stabilization of arbitrary reference trajectories for nonholonomic mobile robots based on a change of coordinates. In [133], the global tracking and stabilization control of internally damped mobile robots with unknown parameters, and subject to input torque saturation and external disturbances, was considered. A new adaptive scheme was proposed to ensure that the bounds of the control torques are functions of only design parameters and reference trajectories, and thus are computable in advance. In [134], an adaptive output feedback tracking controller for nonholonomic mobile robots was proposed to guarantee that the tracking errors were confined to an arbitrarily small sphere. A new adaptive control scheme was developed, which included designing a new adaptive state feedback controller and two high-gain observers to estimate the unknown linear and angular velocities respectively. In [135] finite-time leaderless consensus control of multi-agent systems consisting of a group of nonlinear mechanical systems with parametric uncertainties was considered. New adaptive finite time continuous distributed control algorithms were proposed for the multi-agent systems.

For the underactuated ships, in [136], the method developed for chained-form systems was applied for tracking control of underactuated ships through a coordinate transformation. Exponential stability of the closed-loop system was ensured and the position and the angle of the ship were steered to track the reference trajectory. In [137], two tracking control schemes were proposed using Lyapunov's direct and passivity

approaches for underactuated ships. In [138], a tracking controller was developed for underactuated surface ships to globally asymptotically track a reference which was allowed to be a curve including a straight line. In [139], an universal controller was proposed for underactuated ships to achieve stabilization and tracking control simultaneously based on Lyapunov's direct method and backstepping technique. In [140], a continuous time-varying tracking controller was proposed to yield globally bounded tracking by transforming the ship error system into a skew-symmetric form and generating a time-varying dynamic oscillator. In [141], a global robust adaptive controller was proposed for an underactuated ship to follow a reference path under both constant and time-varying disturbances and parametric uncertainties. In [142], a global smooth controller was proposed for underactuated ships that achieves the practical stabilization of arbitrary reference trajectories including fixed points and non-admissible trajectories based on the transverse function approach developed in [132]. In [143], a nonlinear model-based adaptive output feedback controller was proposed for global asymptotic tracking in the presence of parametric uncertainties associated with nonlinear ship dynamics. In [144], by using nonlinear coordinate changes to transform the ship dynamics to a system affine in the ship's velocities, observers were designed to estimate unmeasured velocities, and tracking controllers were then proposed based on Lyapunov's direct method and backstepping. In [145], a robust adaptive output feedback controller was designed for dynamic position of a surface vessel. In [146], a continuous adaptive time-varying tracking controller was designed to yield globally uniformly ultimately bounded tracking by transforming the ship tracking system into a skew-symmetric form and designing a time-varying dynamic oscillator.

A vital problem in the control of underactuated ships is that failure occurrence on the actuators may cause control failure and even lead to disasters. Such failures are often uncertain in time, value and pattern, and may cause instability or even catastrophic accidents. For the sake of reliability and safety, actuator failure compensation has received an increasing amount of attention. In [147], an intelligent self-repairing adaptive control scheme was proposed for a class of nonlinear MIMO system. A direct self-repairing controller of a nonlinear SISO system was firstly designed, and then the control scheme was promoted to a nonlinear MIMO system. The error signals were replaced by the state variables to deal with the

high derivative problems of the desired signals and a nonlinear regulating function was brought in to improve the performances of the sliding mode.

6.6 Distributed adaptive consensus control

Distributed coordination of multiple dynamic subsystems (also known as multi-agent systems) has achieved rapid development during the past decades. This is mainly due to its widespread potential applications in various fields such as mobile robot networks, intelligent transportation management, surveillance, and monitoring. Consensus, which is aimed at achieving the agreement of all subsystem states, has received the most significant attention in this field as it often serves as the basic requirement of other collective control objectives. A large number of effective control approaches has been proposed to solve the consensus problems, see [148-153] for instance.

According to whether the desired consensus values are determined by exogenous inputs, which are sometimes regarded as virtual leaders, these approaches are often classified as leaderless consensus and leader-following consensus solutions. Besides, many of the early works were established for systems with first-order dynamics, whereas more results have been reported in recent years for systems with second or higher-order dynamics. A comprehensive overview of the state-of-the-art in consensus control can be found in [154], in which the results on some other interesting topics including finite-time consensus and consensus under limited communication conditions including time delays, asynchronization and quantization were also discussed.

All the aforementioned results are obtained by assuming that the internal dynamics of each subsystem is exactly known. The assumption restricts these results from being applied to the scenarios with uncertain subsystem dynamics. Although some well-known techniques including robust control and adaptive control have been shown as promising tools to handle parametric and structural uncertainties in centralized control of single systems, it is still non-trivial to solve distributed consensus problems by adopting these techniques in the presence of intrinsic subsystem uncertainties. In [155], adaptive consensus tracking control of multiple first-order nonlinear subsystems with uncertain but bounded dynamics was investigated. The main challenges are attributed to the following two factors: 1) the time-varying reference state is only accessible to a subset of the subsystems, and 2) the information transmission condition is described by a

directed graph, which is associated with asymmetric Laplacian matrix. Then if the Lyapunov function is defined in terms of local neighborhood consensus errors, computing its derivative inevitably results in cross-coupling terms which are difficult to be counteracted by designing distributed parameter estimators. By assuming that all nonlinearities involved in each subsystem's dynamics are bounded, uniformly ultimate boundedness of the consensus errors is shown if the local control gains are sufficiently large. The results were extended to a more general case with higher-order subsystem dynamics in [156]. In [157], distributed adaptive control of first-order systems with similar structures to those in [155] was investigated. By introducing extra information exchange of local consensus errors among the linked agents, the assumptions on boundedness of inherent nonlinear functions can be relaxed. Apart from these, there were also some other results on distributed adaptive control of multi-agent systems, for instance [158-160]. In [161], output consensus tracking problem for nonlinear subsystems in the presence of mismatched unknown parameters was investigated under directed graph condition. By designing an estimator whose dynamics is governed by a chain of n integrators for the desired trajectory in each subsystem, bounded output consensus tracking for the overall system can be achieved. However, it is not easy to check whether the derived sufficient condition in the form of LMI is satisfied by choosing the design parameters properly. Moreover, transmissions of online parameter estimates among the neighbors are required, which may increase communication burden and also cause some other potential problems such as those related to network security.

In [162], the problem in [161] was revisited and a backstepping based distributed adaptive control solution was provided to achieve asymptotically consensus tracking. By assuming that the desired reference signal is linear parameterized with basis functions known by all the subsystems, one can introduce a local estimator for the uncertain reference in each subsystem. Based on the local estimation errors, a new Lyapunov technique was presented such that all the designed control laws and parameter update laws can be designed in a totally distributed manner. Recently, a new smooth function based distributed adaptive tracking control scheme was proposed in [163] for nonlinear multi-agent systems with unknown parameters and uncertain disturbances. Based on the scheme, asymptotically output consensus tracking can be achieved without

the assumption on linearly parameterized reference trajectory and known basis function. Nevertheless, the information transmission condition was assumed to be represented by an undirected graph. In [164], the results in [163] were extended to the case of directed graph, by considering multiple Euler-Lagrange systems with parametric uncertainties.

6.7 Stochastic adaptive control

For most practical systems, one or more parts may associate with randomness, such as random disturbances (or noises), stochastic inputs, random time-delay, and even stochastically switching dynamic mode. Due to this fact, the related mathematical models contain stochastic elements in nature, and have been named as stochastic systems^[165-168]. The controller design of stochastic systems is, therefore, one of the elementary problems in the control theory.

Since the stochastic integral was first introduced, there have been a great many researchers concentrating on the stochastic differential equations (SDE) modelled as in the form of [169]:

$$dx_t = f(x_t, t)dt + g(x_t, t)d\omega_t, \quad (106)$$

where $x_t \in R^n$ are the system states, $f : (R^n, R) \rightarrow R^n$ and $g : (R^n, R) \rightarrow R^{n \times r}$ are continuous in t and locally Lipschitz functions in x_t with $f(0, t) = 0$ and $g(0, t) = 0$, ω_t is the r -dimensional independent standard Wiener process defined on the complete probability space (Ω, F, P) . Much attentions have been paid to the solutions, stability analyses, as well as control strategies of stochastic systems (106), e.g., [170-176] and the references therein.

On the other hand, in the adaptive control field, most works have been focused on deterministic systems, while the results for stochastic systems remain limited compared to their deterministic counterpart due to some distinctive features^[176-179]. 1) Unlike a deterministic system, even for a given deterministic input, a stochastic system may not produce the same output. 2) The structure and the fundamental theory for stochastic systems are quite different which becomes much more involved due to the coupling of the randomness, and in fact much more complex than those in deterministic case. 3) When applying the Lyapunov theory, there exists a second-order derivative term, namely the Hessian term, which brings a key obstacle to the generalization of the methodologies previously developed for deterministic cases to stochastic ones. 4) Especially, for systems with a stochastically switching dynamic mode, the supporting

algebra results are still lacking and there are still many challenging problems to be solved.

Most research on adaptive control of the stochastic systems was concentrated on linear control systems described by an autoregressive moving average with exogenous input (ARMAX) models or a linear stochastic differential equation with uncertain parameters and random disturbances, see in [177, 180-184] and the references therein. An identification problem was firstly considered and the stochastic control problem was split into a linear filtering problem and a corresponding deterministic control problem. Various kinds of identification algorithms have been employed and explored, e.g., least square algorithm, maximum likelihood algorithm, weighted least algorithm, Kalman filtering or projection algorithm, and so on, to construct the estimates of the uncertainties.

In the 1990s, a nonlinear design method for nonlinear systems was established in a recursive way, which is known as the backstepping technique^[26]. Later, a backstepping control technique was established successfully for systems with stochastic disturbance by Krstic and Deng^[171], in which a quartic Lyapunov function, instead of a quadratic one, was employed because of its advantage in dealing with the difficulties brought by the Hessian term during the design stage. Various control schemes were prosperously developed for stochastic nonlinear systems, e.g., [172, 176, 179, 185-193], and the concept of the adaptive control has also been extended to the stochastic nonlinear systems for its capability to deal with uncertainties^[176, 179, 189, 194]. In [176, 194], tracking problems for two different classes of stochastic nonlinear systems in strict-feedback form and with unknown parametric nonlinearities were considered. By employing the stochastic Lyapunov-like theorem and the backstepping design technique, adaptive state feedback and output feedback control were both developed to assure the tracking performances. Then the problem was considered for their interconnected stochastic counterparts in [179, 189-190] with parametric and nonlinear uncertainties.

Another more specified stochastic system is Markovian jump system, which has been established to describe the systems subject to frequent unpredictable structural changes, which may be caused by random failures, sudden environment disturbances, dynamic mode switching, etc. In essence, a Markovian jump system concerns with two components: the state $X(t)$

and the mode $r(t)$. For the linear case, a Markovian jump system can be expressed as the following classical form:

$$dX(t) = A(r(t))X(t)dt, \quad (107)$$

where $r(t)$ is a Markov chain taking values in $S = \{1, 2, \dots, N\}$. During the operation, the system switches from one mode to another in a Markovian way. In 2006, Mao^[195] investigated the exponential stability of a nonlinear stochastic differential equations with Markovian switching formed as

$$dX(t) = f(X(t), t, r(t))dt + g(X(t), t, r(t))d\omega_t. \quad (108)$$

Though the stability and control problem of Markovian jump systems has received a lot of attention^[156, 175, 196-200], few results have been achieved for the corresponding adaptive strategy. Recently, an adaptive failure compensation problem was studied for a class of nonlinear uncertain systems subject to stochastic actuator failures and unknown parameters^[198-199], in which the actuator failures were modeled by a Markovian sequence, and an adaptive backstepping-based failure compensation strategy was proposed, which ensured the boundedness in probability of all the closed-loop signals. Most recently, in [201], a class of stochastic nonlinear switched systems possessing unmeasured states, completely unknown nonlinear system functions, unmodeled dynamics, and arbitrary switchings, was examined, and an adaptive fuzzy output feedback controller was proposed. It is noted that the switching mode of the system is stochastic yet not limited to Markovian.

Based on those aforementioned achievements, adaptive control problems were further investigated with respect to various design objectives (e.g., finite-time stabilization^[156, 202]), or system structure (e.g., high-order system^[203], time-delay system^[204]).

Although the adaptive control problem of stochastic systems is attractive and practical which has drawn lots of attentions over the past few decades, how to deal with the couplings among uncertainties and the stochastic factors remains an interesting yet challenging topic. Moreover, more powerful mathematical tools are further expected to enrich the methodologies and theories on stochastic differential equations, especially for the nonlinear ones.

7 Conclusion

In this paper, we have presented a unified conventional approach for the analysis of robust discrete and continuous time adaptive control systems, including both direct and indirect schemes. This approach also

provides a general guideline to the design of robust adaptive controllers. That is, an adaptive controller can be designed to meet some proposed properties given in the assumptions. With this approach, the a priori knowledge of modelling errors for the design and implementation of adaptive controllers is no longer required. Furthermore, the effect of the modelling error can be allowed to have infinite memory.

An indirect discrete time predictive adaptive control system and a direct continuous time model reference adaptive control system, developed based on [21] and [10] respectively, are used to illustrate our approach. It is shown that, without the use of the dead-zone in the predictive controller and without the use of a special dynamic normalizing signal, the same results as in [21] and [10] can still be obtained. This then relaxes the assumption on the knowledge of an upper bound related to the gain of the plant unmodelled dynamics in [21] and removes the assumption on the knowledge of a stability boundary of the unmodelled dynamics in [10, 15-16].

The approach presented can also be applied to analyze and design decentralized adaptive controllers using either the indirect scheme as in [25] and [205], or the direct scheme as in [206].

Besides, recent developments on the issues of adaptive compensation for non-smooth nonlinearities and actuator failures, adaptive control of underactuated nonholonomic systems, distributed adaptive consensus control, and stochastic adaptive control are discussed in this paper.

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