

# 多性能指标下区间二型 T-S 模糊时滞系统的滤波器设计

赵 涛, 程冠鸿, 刘 凯<sup>†</sup>, 梁伟博

(四川大学 电气信息学院, 成都 610065)

**摘 要:** 区间二型 T-S 模糊时变延迟模型可以有效地处理具有参数不确定的非线性时变时滞系统. 针对多性能指标框架下区间二型 T-S 模糊时变延迟系统的滤波器设计仍然是有待解决的问题, 利用 Lyapunov-Krasovskii 泛函方法, 在多性能指标框架下提出前提隶属函数不匹配的区间二型模糊滤波器. 利用最新发展的积分不等式, 所设计的滤波误差系统可以同时满足耗散性、无源性、 $H_\infty$  和  $L_2-L_\infty$  性能. 通过矩阵解耦技术, 滤波器存在条件可以表示为线性矩阵不等式. 最后通过仿真实验验证了所提出方法的有效性.

**关键词:** 区间二型 T-S 模糊系统; 时变延迟; 滤波器设计; 多性能指标

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## Filter design of interval type-2 T-S fuzzy time-delay systems under multi-performance indexes

ZHAO Tao, CHENG Guan-hong, LIU Kai<sup>†</sup>, LIANG Wei-bo

(School of Electrical Engineering and Information, Sichuan University, Chengdu 610065, China)

**Abstract:** The nonlinear systems subjected to parameter uncertainties and time-varying delay can effectively handled by interval type-2(IT2) T-S fuzzy time-varying delay model. The filter design of IT2 T-S fuzzy time-varying delay systems under multi-performance indexes is still a problem that needs to be solved. Under multi-performance indexes, an IT2 fuzzy filter with unmatched premise membership functions is designed by employing a Lyapunov-Krasovskii function method. Using the recently developed integral inequality, the filtering error system can simultaneously satisfy the passive, dissipative,  $H_\infty$  and  $L_2-L_\infty$  performance indexes. Based on the matrix decoupling technology, the existing conditions of the IT2 fuzzy filter are presented in terms of linear matrix inequalities. Finally, numerical examples are provided to illustrate the effectiveness of the proposed approach.

**Keywords:** interval type-2 T-S fuzzy systems; time-varying delay; filter design; multi-performance indexes

## 0 引 言

近年来, T-S 模糊模型受到了广泛的关注. T-S 模糊模型是由非线性模糊权重将一系列线性子模型光滑连接而成的全局模型, 它可以在凸紧集内以任意精度逼近任意光滑非线性函数<sup>[1]</sup>. 在模糊控制领域, T-S 模糊模型在模糊系统的稳定性分析、控制器综合以及滤波器设计等方面都发挥着巨大的作用并取得了丰硕的理论成果<sup>[2-11]</sup>. 值得注意的是, 上述结果以一型模糊逻辑为基础, 这些传统的一型 T-S 模糊控制通常假定模糊权重不包含不确定性信息. 然而, 在实际中不仅具有非线性而且往往伴随着不确定性, 一旦所考虑的系统为具有参数不确定的非线性系统, 则基

于一型模糊逻辑的 T-S 模糊控制方法不能被直接应用. 因此, 如何更好地处理具有参数不确定的非线性系统成为研究的重点.

在高度不确定环境下, 一型模糊集<sup>[12]</sup>往往不能获得较好的效果, 为了提高系统处理不确定性的能力, Zedeh<sup>[13]</sup>提出了二型模糊集理论. 二型模糊集的隶属度不再是确定值, 而是表现为一型模糊集. 从空间维数来看, 一型模糊集可用二维空间描述, 而二型模糊集可用三维空间描述, 因此二型模糊集大大增加了设计的自由度, 使得二型模糊集在高度不确定的场合可以获得比一型模糊集更好的效果<sup>[1]</sup>. 区间二型 T-S 模糊模型由于计算简单而又不失广义二型 T-S 模糊

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作者简介: 赵涛(1988—), 男, 副研究员, 博士, 从事模糊控制及其应用等研究; 程冠鸿(1992—), 男, 硕士生, 从事模糊控制的研究.

<sup>†</sup>通讯作者. E-mail: kailiu@scu.edu.cn

模型的优点,更具有实际应用价值.近年来,区间二型模糊系统在稳定性分析、滤波以及故障检测问题方面得到了广泛的关注并取得了丰硕的成果<sup>[14-22]</sup>.文献[14]通过一种公共二次Lyapunov函数得出了区间二型模糊系统的状态反馈控制器的存在条件.文献[15]设计了一种在新的性能指标下的区间二型模糊输出反馈控制器.文献[16]在D稳定域约束下设计了一种新的区间二型模糊滤波器.文献[17]研究了区间二型模糊系统的传感器非线性故障检测.然而,上述文献没有考虑时变延迟下区间二型模糊系统的滤波器设计问题.

时变延迟现象广泛存在于实际系统中并可能使得系统性能降低甚至不稳定.目前,大量关于时滞系统的稳定性分析的研究得到了越来越多的关注,并在降低结论保守性方面取得了瞩目的成就<sup>[23-25]</sup>.近几年来,一型T-S模糊时滞系统的研究引发了研究人员的兴趣,同时也取得了一定的成果<sup>[26-28]</sup>.由于区间二型T-S模糊时变延迟系统结构复杂,它的滤波器设计仍然是有待解决的问题.在多性能指标框架下,可以有效地统一耗散性、无源性、 $H_\infty$ 和 $L_2-L_\infty$ 的性能分析,具有重要的理论意义.文献[29]对区间二型T-S模糊常时滞系统的滤波器设计进行了研究,然而在多性能指标统一框架下,区间二型T-S模糊时变延迟系统的滤波器设计仍然是比较有挑战的问题.与现有文献相比,本文主要创新点如下:1)通过Lyapunov-Krasovskii理论,在多性能指标一致框架下,提出了区间二型模糊时变延迟系统的模糊滤波器设计问题,并且能够同时保证耗散性、无源性、 $H_\infty$ 和 $L_2-L_\infty$ 的性能.2)所提出的模糊滤波器与模糊系统不必使用相同的隶属函数.3)利用最新发展的积分不等式,基于线性矩阵不等式的形式给出了滤波器存在的充分条件.最后通过Matlab仿真实验,验证了本文所提出方法的有效性.

### 1 系统描述

考虑如下的区间二型T-S模糊时变延迟系统.

系统规则  $i$ : 如果  $f_1(x(t))$  是  $\tilde{M}_{i1}$  且  $f_r(x(t))$  是  $\tilde{M}_{r1}$ , 则

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-d(t)) + D_{1i} w(t), \\ z(t) = C_i x(t) + C_{di} x(t-d(t)) + D_{2i} w(t), \\ y(t) = E_i x(t) + E_{di} x(t-d(t)) + D_{3i} w(t). \end{cases} \quad (1)$$

其中:  $\tilde{M}_{i1}$  为关于函数  $f_\alpha(x(t))$  的区间二型模糊集合,  $i = 1, 2, \dots, p, \alpha = 1, 2, \dots, r, r$  为正整数,  $x(t) \in R^n$  为状态向量;  $z(t) \in R^v$  为控制输出;  $y(t) \in R^m$

为测量输出;  $w(t) \in R^q$  为扰动输入;  $A_i, A_{di}, D_{1i}, C_i, C_{di}, D_{2i}, E_i, E_{di}$  和  $D_{3i}$  为已知的系统矩阵;  $d(t)$  为时变延迟, 满足

$$0 \leq d(t) \leq h, \quad \mu_1 \leq \dot{d}(t) \leq \mu_2, \quad (2)$$

$h, \mu_1, \mu_2$  是标量.

第  $i$  条规则的激活强度定义为如下区间集:

$$W_i(x(t)) = [\underline{w}_i(x(t)), \bar{w}_i(x(t))], \quad i = 1, 2, \dots, p.$$

其中

$$\begin{aligned} \underline{w}_i(x(t)) &= \prod_{\alpha}^r \underline{\mu}_{\tilde{M}_{i\alpha}}(f_\alpha(x(t))) \geq 0, \\ \bar{w}_i(x(t)) &= \prod_{\alpha}^r \bar{\mu}_{\tilde{M}_{i\alpha}}(f_\alpha(x(t))) \geq 0, \end{aligned}$$

$\bar{\mu}_{\tilde{M}_{i\alpha}}(f_\alpha(x(t)))$  和  $\underline{\mu}_{\tilde{M}_{i\alpha}}(f_\alpha(x(t)))$  分别表示上、下隶属度.

全局的区间二型T-S模糊模型为

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^p \tilde{w}_i(x(t)) [A_i x(t) + A_{di} x(t-d(t)) + D_{1i} w(t)], \\ z(t) = \sum_{i=1}^p \tilde{w}_i(x(t)) [C_i x(t) + C_{di} x(t-d(t)) + D_{2i} w(t)], \\ y(t) = \sum_{i=1}^p \tilde{w}_i(x(t)) [E_i x(t) + E_{di} x(t-d(t)) + D_{3i} w(t)]. \end{cases} \quad (3)$$

其中  $\tilde{w}_i(x(t)) = \underline{a}_i(x(t)) \underline{w}_i(x(t)) + \bar{a}_i(x(t)) \bar{w}_i(x(t)) \geq 0, \forall i, \sum_{i=1}^p \tilde{w}_i(x(t)) = 1, \underline{a}_i(x(t)) \in [0, 1]$  和  $\bar{a}_i(x(t)) \in [0, 1]$  是两个非线性函数并且满足  $\underline{a}_i(x(t)) + \bar{a}_i(x(t)) = 1$ .

下面定义前提隶属函数不匹配的区间二型模糊滤波器.

滤波器系统规则  $j$ : 如果  $g_1(x(t))$  是  $\tilde{N}_{j1}$  且  $g_l(x(t))$  是  $\tilde{N}_{jl}$ , 则

$$\begin{cases} \dot{\hat{x}}(t) = A_{fj} \hat{x}(t) + B_{fj} y(t), \\ z_f(t) = C_{fj} \hat{x}(t). \end{cases} \quad (4)$$

其中:  $\tilde{N}_{j1}$  为关于函数  $g_\beta(x(t))$  的区间二型模糊集合,  $j = 1, 2, \dots, p, \beta = 1, 2, \dots, l, l$  为正整数;  $A_{fj}, B_{fj}$  和  $C_{fj}$  为滤波器参数. 第  $j$  条规则的激活强度定义为如下区间集:

$$M_j(x(t)) = [\underline{m}_j(x(t)), \bar{m}_j(x(t))], \quad j = 1, 2, \dots, p.$$

其中

$$\underline{m}_j(x(t)) = \prod_{\beta}^l \mu_{\tilde{N}_{j\beta}}(g_{\beta}(x(t))) \geq 0,$$

$$\bar{m}_j(x(t)) = \prod_{\beta}^l \bar{\mu}_{\tilde{N}_{j\beta}}(g_{\beta}(x(t))) \geq 0,$$

$\bar{\mu}_{\tilde{N}_{j\beta}}(g_{\beta}(x(t)))$ 和 $\mu_{\tilde{N}_{j\beta}}(g_{\beta}(x(t)))$ 分别表示上、下隶属度.全局的区间二型模糊滤波器定义为

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{j=1}^p \tilde{m}_j(x(t)) [A_{fj} \hat{x}(t) + B_{fj} y(t)], \\ z_f(t) = \sum_{j=1}^p \tilde{m}_j(x(t)) C_{fj} \hat{x}(t). \end{cases} \quad (5)$$

其中

$$\tilde{m}_j(x(t)) = \frac{\underline{\beta}_j(x(t)) \underline{m}_j(x(t)) + \bar{\beta}_j(x(t)) \bar{m}_j(x(t))}{\sum_{k=1}^p (\underline{\beta}_k(x(t)) \underline{m}_k(x(t)) + \bar{\beta}_k(x(t)) \bar{m}_k(x(t)))} \geq 0, \forall j,$$

$$\sum_{j=1}^p \tilde{m}_j(x(t)) = 1,$$

$\underline{\beta}_j(x(t)) \in [0, 1]$ 和 $\bar{\beta}_j(x(t)) \in [0, 1]$ 是两个非线性函数并且满足 $\underline{\beta}_j(x(t)) + \bar{\beta}_j(x(t)) = 1$ .

由式(3)和(5),滤波误差系统可定义为

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j [\bar{A}_{ij} \bar{x}(t) + \bar{A}_{dij} \bar{x}(t-d(t)) + \bar{D}_{1ij} w(t)], \\ e(t) = \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j [\bar{C}_{ij} \bar{x}(t) + \bar{C}_{dij} \bar{x}(t-d(t)) + \bar{D}_{2i} w(t)]. \end{cases} \quad (6)$$

其中

$$\bar{x}(t) = [x^T(t) \quad \hat{x}^T(t)], \quad e(t) = z(t) - z_f(t),$$

$$\bar{A}_{ij} = \begin{bmatrix} A_i & 0 \\ B_{fj} E_i & A_{fj} \end{bmatrix}, \quad \bar{A}_{dij} = \begin{bmatrix} A_{di} & 0 \\ B_{fj} E_{di} & 0 \end{bmatrix},$$

$$\bar{D}_{1ij} = \begin{bmatrix} D_{1i} \\ B_{fj} D_{3i} \end{bmatrix}, \quad \bar{C}_{ij} = [C_i \quad -C_{fj}],$$

$$\bar{C}_{di} = [C_{di} \quad 0], \quad \bar{D}_{2i} = D_{2i}.$$

为了得到本文的主要结果,先给出如下的假设、定义和引理.

**假设1**<sup>[16]</sup> 给定矩阵 $\Phi, \Psi_1, \Psi_2, \Psi_3$ 满足下列条件:

- 1)  $\Phi = \Phi^T \geq 0, \Psi_1 = \Psi_1^T \leq 0, \Psi_3 = \Psi_3^T$ ;
- 2)  $(\|D_{2i}\| + \|C_{di}\|)\|\Phi\| = 0, \forall i$ ;
- 3)  $(\|\Psi_1\| + \|\Psi_2\|)\|\Phi\| = 0$ ;
- 4)  $\bar{D}_{2i}^T \Psi_1 \bar{D}_{2i} + \bar{D}_{2i}^T \Psi_2 + \Psi_2^T \bar{D}_{2i} + \Psi_3 > 0, \forall i$ .

**定义1**<sup>[16]</sup> 给定矩阵 $\Phi, \Psi_1, \Psi_2, \Psi_3$ 满足假设1,如

果对于任意的 $t > 0$ 并有 $w(t) \in L_2[0, \infty)$ ,总存在标量 $\delta$ 使得不等式

$$\int_0^t J(t) dt - e^T(t) \Phi e(t) \geq \delta \quad (7)$$

成立,则系统(6)是扩展耗散的.其中

$$J(t) = e^T(t) \Psi_1 e(t) + 2e^T(t) \Psi_2 w(t) + w^T(t) \Psi_3 w(t).$$

**注1** 定义1中,在不同 $\Phi, \Psi_1, \Psi_2$ 及 $\Psi_3$ 的取值下,此定义涵盖了包括耗散性、无源性、 $H_{\infty}$ 和 $L_2$ - $L_{\infty}$ 在内的不同性能指标.

**引理1**<sup>[30]</sup> 令 $x$ 是 $[\alpha, \beta] \rightarrow \mathbf{R}$ 上的可微函数.对称矩阵 $R \in \mathbf{R}^{n \times n}, Z_1, Z_3 \in \mathbf{R}^{3n \times 3n}$ ;任意矩阵 $Z_2 \in \mathbf{R}^{3n \times 3n}, N_1, N_2 \in \mathbf{R}^{3n \times n}$ ,满足

$$\bar{\Phi} = \begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0,$$

则下列不等式成立:

$$-\int_{\alpha}^{\beta} \dot{x}^T(s) R \dot{x}(s) ds \leq \varpi^T \bar{\Omega} \varpi. \quad (8)$$

其中

$$\varpi = \left[ x^T(\beta) \quad x^T(\alpha) \quad \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} x^T(s) ds \right]^T,$$

$$\bar{\Omega} = (\beta - \alpha) \left( Z_1 + \frac{1}{3} Z_3 \right) + \text{Sym} \{ N_1 \bar{\Pi}_1 + N_2 \bar{\Pi}_2 \},$$

$$\bar{\Pi}_1 = \bar{e}_1 - \bar{e}_2, \quad \bar{\Pi}_2 = 2\bar{e}_3 - \bar{e}_1 - \bar{e}_2,$$

$$\bar{e}_1 = [I \quad 0 \quad 0], \quad \bar{e}_2 = [0 \quad I \quad 0], \quad \bar{e}_3 = [0 \quad 0 \quad I].$$

**引理2**<sup>[31]</sup> 令 $\varsigma \in \mathbf{R}^n, \Phi = \Phi^T (\in \mathbf{R}^{n \times n}), B \in \mathbf{R}^{m \times n}$ 满足 $\text{rank}(B) < n$ ,下列各式是等价的:

- 1)  $\varsigma^T \Phi \varsigma < 0, \forall B \varsigma = 0$ ,其中 $\varsigma \neq 0$ ;
- 2)  $(B^{\perp})^T \Phi B^{\perp} < 0$ ;
- 3)  $\exists L \in \mathbf{R}^{n \times m}, \Phi + LB + B^T L^T < 0$ .

## 2 主要结论

本节利用Lyapunov-Krasovskii方法给出区间二型滤波器的存在条件.为了简化证明过程,引入下列中间变量:

$$\eta_1(t) = \left[ \eta_3^T(t) \quad \int_{t-d(t)}^t \bar{x}^T(s) ds \quad \int_{t-h}^{t-d(t)} \bar{x}^T(s) ds \right]^T,$$

$$\eta_2(t) = [\bar{x}^T(t) \quad \dot{\hat{x}}^T(t)]^T,$$

$$\eta_3(t) = [\bar{x}^T(t) \quad \bar{x}^T(t-d(t)) \quad \bar{x}^T(t-h)]^T,$$

$$\eta_4(t) =$$

$$\left[ \frac{1}{d(t)} \int_{t-d(t)}^t \bar{x}^T(s) ds \quad \frac{1}{h-d(t)} \int_{t-h}^{t-d(t)} \bar{x}^T(s) ds \right]^T,$$

$$\eta_5(t) = [\dot{\hat{x}}^T(t) \quad \dot{\hat{x}}^T(t-d(t)) \quad \dot{\hat{x}}^T(t-h)]^T,$$

$$\xi(t) = [\eta_3^T(t) \quad \eta_5^T(t) \quad \eta_4^T(t)]^T,$$

$$e_i = [0_{2n \times 2(i-1)n} \quad I_{2n} \quad 0_{2n \times 2(8-i)n}], \quad i = 1, 2, \dots, 8.$$

**定理1** 假定增益矩阵 $A_{fj}, B_{fj}, C_{fj}$ 已知.对于给定的标量 $h, \mu_1 < \mu_2 < 1$ ,有 $d(t) \in [0, h], \dot{d}(t) \in$

$[\mu_1, \mu_2]$ , 如果存在矩阵  $P_1 > 0, P_2 > 0, Q_1 > 0, Q_2 > 0, R > 0$ , 对称矩阵  $X_1, X_3, Y_1, Y_3$  和任意矩阵  $X_2, Y_2, N_1, N_2, M_1, M_2, L$  满足如下线性矩阵不等式:

$$\bar{C}_{ij}^T \Phi \bar{C}_{ij} - P_1 < 0, \quad i, j = 1, 2, \dots, p; \quad (9)$$

$$\begin{bmatrix} \Xi_1 + \Xi_2 - \Xi_{3ij} & -\Xi_{4ij} \\ * & -\Xi_{5i} \end{bmatrix} + L\Gamma_{ij} + \Gamma_{ij}^T L^T < 0, \quad i, j = 1, 2, \dots, p; \quad (10)$$

$$\Phi_1 = \begin{bmatrix} X_1 & X_2 & N_1 \\ * & X_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0; \quad (11)$$

$$\Phi_2 = \begin{bmatrix} Y_1 & Y_2 & M_1 \\ * & Y_3 & M_2 \\ * & * & R \end{bmatrix} \geq 0. \quad (12)$$

则滤波误差系统渐近稳定并满足性能指标(7). 其中

$$\begin{aligned} \Xi_1 &= \text{Sym}\{\Pi_1^T P \Pi_2\} + \Pi_3^T Q_1 \Pi_3 - \tilde{d}(t) \Pi_4^T (Q_1 - Q_2) \Pi_4 - \Pi_5^T Q_2 \Pi_5 + h e_4^T R e_4, \\ \Xi_2 &= d(t) \Pi_6^T \left( X_1 + \frac{1}{3} X_3 \right) \Pi_6 + (h - d(t)) \Pi_9^T \left( Y_1 + \frac{1}{3} Y_3 \right) \Pi_9 + \text{Sym}\{\Pi_6^T N_1 \Pi_7 + \Pi_6^T N_2 \Pi_8 + \Pi_9^T M_1 \Pi_{10} + \Pi_9^T M_2 \Pi_{11}\}, \\ \Xi_{3ij} &= e_1^T \bar{C}_{ij}^T \Psi_1 \bar{C}_{ij} e_1 + e_1^T \bar{C}_{ij}^T \Psi_1 \bar{C}_{di} e_2 + e_2^T \bar{C}_{di}^T \Psi_1 \bar{C}_{ij} e_1 + e_2^T \bar{C}_{di}^T \Psi_1 \bar{C}_{di} e_2, \\ \Xi_{4ij} &= e_1^T \bar{C}_{ij}^T \Psi_1 \bar{D}_{2i} + e_2^T \bar{C}_{di}^T \Psi_1 \bar{D}_{2i} + e_1^T \bar{C}_{ij}^T \Psi_2 + e_2^T \bar{C}_{di}^T \Psi_2, \\ \Xi_{5i} &= \bar{D}_{2i}^T \Psi_1 \bar{D}_{2i} + \bar{D}_{2i}^T \Psi_2 + \Psi_2^T \bar{D}_{2i} + \Psi_3, \\ \Gamma_{ij} &= [\bar{A}_{ij} e_1 + \bar{A}_{dij} e_3 - e_4 \quad \bar{D}_{1ij}], \\ \Pi_1 &= [e_1^T \quad e_2^T \quad e_3^T \quad d(t) e_7^T \quad (h - d(t)) \quad e_8^T]^T, \\ \Pi_2 &= [e_4^T \quad e_5^T \quad e_6^T \quad e_1^T - \tilde{d}(t) e_2^T \quad \tilde{d}(t) e_2^T - e_3^T]^T, \\ \Pi_3 &= [e_1^T \quad e_4^T]^T, \quad \Pi_4 = [e_2^T \quad e_3^T]^T, \\ \Pi_5 &= [e_3^T \quad e_6^T]^T, \quad \Pi_6 = [e_1^T \quad e_2^T \quad e_7^T]^T, \\ \Pi_7 &= e_1 - e_2, \quad \Pi_8 = 2e_7 - e_1 - e_2, \\ \Pi_9 &= [e_2^T \quad e_3^T \quad e_8^T]^T, \quad \Pi_{10} = e_2 - e_3, \\ \Pi_{11} &= 2e_8 - e_2 - e_3, \quad \tilde{d}(t) = 1 - \dot{d}(t), \\ P &= \begin{bmatrix} P_1 & 0_{2n \times 8n} \\ * & P_2 \end{bmatrix}. \end{aligned}$$

在这种情形下, 定义1中的  $\delta$  满足

$$\delta = -V(0). \quad (13)$$

**证明** 选择如下 Lyapunov 函数:

$$\begin{aligned} V(x_t) &= \eta_1^T(t) P \eta_1(t) + \int_{t-d(t)}^t \eta_2^T(t) Q_1 \eta_2(t) ds + \\ &\quad \int_{t-h}^{t-d(t)} \eta_2^T(t) Q_2 \eta_2(t) ds + \\ &\quad \int_{-h}^0 \int_{t+\theta}^t \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds d\theta, \end{aligned} \quad (14)$$

求得

$$\begin{aligned} \dot{V}(x_t) &= \eta_1^T(t) P \eta_1(t) + \eta_1^T(t) P \dot{\eta}_1(t) + \eta_2^T(t) Q_1 \eta_2(t) - \\ &\quad \eta_2^T(t - d(t)) Q_1 \eta_2(t - d(t)) (1 - \dot{d}(t)) + \\ &\quad \eta_2^T(t - d(t)) Q_2 \eta_2(t - d(t)) (1 - \dot{d}(t)) - \\ &\quad \eta_2^T(t - h) Q_1 \eta_2(t - h) + h \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) - \\ &\quad \int_{t-h}^t \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds. \end{aligned} \quad (15)$$

将  $\eta_1^T(t) = \xi^T(t) \Pi_1^T, \dot{\eta}_1(t) = \Pi_2 \xi(t), \eta_2(t) = \Pi_3 \xi(t), \eta_2(t - d(t)) = \Pi_4 \xi(t), \eta_2(t - h) = \Pi_5 \xi(t), \dot{\hat{x}}(t) = e_4 \xi(t)$  及  $\tilde{d}(t) = 1 - \dot{d}(t)$  代入式(15), 并化简得

$$\begin{aligned} \dot{V}(t) &= \xi^T(t) \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \Xi_1 \right) \xi(t) - \\ &\quad \int_{t-h}^t \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds. \end{aligned} \quad (16)$$

其中  $\Xi_1 = \text{Sym}\{\Pi_1^T P \Pi_2\} + \Pi_3^T Q_1 \Pi_3 - d(t) \Pi_4^T (Q_1 - Q_2) \Pi_4 - \Pi_5^T Q_2 \Pi_5 + h e_4^T R e_4$ .

基于式(6), 展开(7)中的  $J(t)$  得

$$\begin{aligned} J(t) &= \xi^T(t) \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \Xi_{3ij} \right) \xi(t) + \\ &\quad \xi^T(t) \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \Xi_{4ij} \right) w(t) + \\ &\quad w^T(t) \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \Xi_{4ij}^T \right) \xi(t) + \\ &\quad w^T(t) \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \Xi_{5i} \right) w(t); \end{aligned} \quad (17)$$

$$\begin{aligned} \Xi_{3ij} &= e_1^T \bar{C}_{ij}^T \Psi_1 \bar{C}_{ij} e_1 + e_1^T \bar{C}_{ij}^T \Psi_1 \bar{C}_{di} e_2 + \\ &\quad e_2^T \bar{C}_{di}^T \Psi_1 \bar{C}_{ij} e_1 + e_2^T \bar{C}_{di}^T \Psi_1 \bar{C}_{di} e_2, \\ \Xi_{4ij} &= e_1^T \bar{C}_{ij}^T \Psi_1 \bar{D}_{2i} + e_2^T \bar{C}_{di}^T \Psi_1 \bar{D}_{2i} + e_1^T \bar{C}_{ij}^T \Psi_2 + \\ &\quad e_2^T \bar{C}_{di}^T \Psi_2, \\ \Xi_{5i} &= \bar{D}_{2i}^T \Psi_1 \bar{D}_{2i} + \bar{D}_{2i}^T \Psi_2 + \Psi_2^T \bar{D}_{2i} + \Psi_3. \end{aligned}$$

由式(16)和(17)可以得到

$$\begin{aligned} \dot{V}(x_t) - J(t) &= \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix}^T \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \begin{bmatrix} \Xi_1 - \Xi_{3ij} & -\Xi_{4ij} \\ * & -\Xi_{5i} \end{bmatrix} \right) \times \\ &\quad \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix} - \int_{t-h}^t \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds. \end{aligned} \quad (18)$$

由式(11)、(12)及引理1可得

$$\begin{aligned} & - \int_{t-h}^t \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds = \\ & - \int_{t-d(t)}^t \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds - \int_{t-h}^{t-d(t)} \dot{\hat{x}}^T(s) R \dot{\hat{x}}(s) ds \leq \end{aligned}$$

$$\begin{aligned}
 & d(t)\xi^T(t)\Pi_6^T\left(X_1 + \frac{1}{3}X_3\right)\Pi_6\xi(t) + \\
 & \xi^T(t)\text{Sym}\{\Pi_6^T N_1(\bar{e}_1 - \bar{e}_2)\Pi_6 + \\
 & \Pi_6^T N_2(2\bar{e}_3 - \bar{e}_1 - \bar{e}_2)\Pi_6\xi(t)\} + \\
 & (h - d(t))\xi^T(t)\Pi_9^T\left(Y_1 + \frac{1}{3}Y_3\right)\Pi_9\xi(t) + \\
 & \xi^T(t)\text{Sym}\{\Pi_9^T M_1(\bar{e}_1 - \bar{e}_2)\Pi_9 + \\
 & \Pi_9^T M_2(2\bar{e}_3 - \bar{e}_1 - \bar{e}_2)\Pi_9\xi(t)\} = \\
 & \xi^T(t)\Xi_2\xi(t). \tag{19}
 \end{aligned}$$

其中

$$\begin{aligned}
 \Xi_2 = & d(t)\Pi_6^T\left(X_1 + \frac{1}{3}X_3\right)\Pi_6 + (h - d(t))\Pi_9^T\left(Y_1 + \right. \\
 & \left. \frac{1}{3}Y_3\right)\Pi_9 + \text{Sym}\{\Pi_6^T N_1\Pi_7 + \Pi_6^T N_2\Pi_8 + \\
 & \Pi_9^T M_1\Pi_{10} + \Pi_9^T M_2\Pi_{11}\}.
 \end{aligned}$$

由式(18)和(19)可得

$$\begin{aligned}
 \dot{V}(x_t) - J(t) \leq & \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix}^T \left( \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \begin{bmatrix} \Xi_1 + \Xi_2 - \Xi_{3ij} & -\Xi_{4ij} \\ * & -\Xi_{5i} \end{bmatrix} \right) \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix}. \tag{20}
 \end{aligned}$$

由式(6)可以等价转换得到  $\Gamma_{ij} \begin{bmatrix} \xi(t) \\ w(t) \end{bmatrix} = 0$ , 其中

$\Gamma_{ij} = [\bar{A}_{ij}e_1 + \bar{A}_{dij}e_3 - e_4 \quad \bar{D}_{1ij}]$ . 由于式(10)成立, 再结合式(20), 基于引理2可得

$$\sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \begin{bmatrix} \Xi_1 + \Xi_2 - \Xi_{3ij} & -\Xi_{4ij} \\ * & -\Xi_{5i} \end{bmatrix} \leq 0, \tag{21}$$

因此

$$\dot{V}(x_t) - J(t) \leq 0. \tag{22}$$

由式(22)可得

$$\int_0^t J(s)ds \geq V(x_t) - V(0). \tag{23}$$

式(23)又可写为

$$\begin{aligned}
 & \int_0^t J(s)ds \geq \\
 & \eta_1^T(t)P\eta_1(t) + \int_{t-d(t)}^t \eta_2^T(t)Q_1\eta_2(t)ds + \\
 & \int_{t-h}^{t-d(t)} \eta_2^T(t)Q_2\eta_2(t)ds + \\
 & \int_{-h}^0 \int_{t+\theta}^t \dot{\tilde{x}}^T(s)R\dot{\tilde{x}}(s)dsd\theta + \delta. \tag{24}
 \end{aligned}$$

要证明滤波误差系统满足定义1中的性能指标  $\int_0^t J(t)dt - e^T(t)\Phi e(t) \geq \delta$ , 需要考虑  $\|\Phi\| = 0$  和  $\|\Phi\| \neq 0$  两种情况:

1)  $\|\Phi\| = 0$  时, 由式(24)可得到  $\int_0^t J(t)dt - e^T(t)\Phi e(t) \geq \delta$  恒成立;

2)  $\|\Phi\| \neq 0$  时, 由假设1可得  $\|D_{2i}\| + \|C_{di}\| = 0, \|\Psi_1\| + \|\Psi_2\| = 0$ , 即  $\|D_{2i}\| = \|C_{di}\| = 0, \|\Psi_1\| = \|\Psi_2\| = 0, \Psi_3 > 0$ . 所以  $J(t) = w^T(t)\Psi_3 w(t) > 0, e(t) = \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \bar{C}_{ij} \bar{x}(t)$ . 则由  $P_1 > \bar{C}_{ij}^T \Phi \bar{C}_{ij}$  可得  $\int_0^t J(s)ds - e^T(t)\Phi e(t) = \int_0^t J(s)ds - \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \bar{x}^T(t) \bar{C}_{ij}^T \Phi \bar{C}_{ij} \bar{x}(t) = \int_0^t J(s)ds - \sum_{i=1}^p \sum_{j=1}^p \tilde{w}_i \tilde{m}_j \xi^T(t) e_1^T \bar{C}_{ij}^T \Phi \bar{C}_{ij} e_1 \xi(t) > \int_0^t J(s)ds - \xi^T(t) e_1^T P_1 e_1 \xi(t) > \int_0^t J(s)ds - \xi^T(t) \Pi_1^T P \Pi_1(t) = \int_0^t J(s)ds - \eta_1^T(t) P \eta_1(t) > \int_{t-d(t)}^t \eta_2^T(t) Q_1 \eta_2(t) ds + \int_{t-h}^{t-d(t)} \eta_2^T(t) Q_2 \eta_2(t) ds + \int_{-h}^0 \int_{t+\theta}^t \dot{\tilde{x}}^T(s) R \dot{\tilde{x}}(s) ds d\theta + \delta > \delta. \tag{25}$

综上, 滤波误差系统满足定义1中的性能指标  $\int_0^t J(t)dt - e^T(t)\Phi e(t) \geq \delta$ . 当扰动  $w(t) \equiv 0$  时, 由  $\Psi_1 < 0$  可得

$$\dot{V}(x_t) \leq J(t) < 0, \tag{26}$$

即滤波误差系统是渐近稳定的.  $\square$

由假设1可知  $\Phi \geq 0, \Psi_1 \leq 0$ , 总有矩阵  $\tilde{\Phi}, \tilde{\Psi}_1$  满足

$$\Phi = \tilde{\Phi}^T \tilde{\Phi}, \Psi_1 = -\tilde{\Psi}_1^T \tilde{\Psi}_1 \leq 0. \tag{27}$$

定理1在增益矩阵已知的情况下, 给出了性能分析条件. 在定理1的基础上, 下面给出区间二型模糊滤波器的存在条件.

**定理2** 给定标量  $\alpha, \beta, h, \mu_1 < \mu_2 < 1$ , 有  $d(t) \in [0, h], \dot{d}(t) \in [\mu_1, \mu_2]$ , 如果存在矩阵  $P_1 > 0, P_2 > 0, Q_1 > 0, Q_2 > 0, R > 0$ , 对称矩阵  $X_1, X_3, Y_1, Y_3$  和任意矩阵  $X_2, Y_2, N_1, N_2, M_1, M_2, L_{11}, L_{12}, L_{21}, \bar{A}_{fj}, \bar{B}_{fj}, \bar{C}_{fj}$  满足如下线性矩阵不等式, 则滤波误差系统渐近稳定并满足所要求的性能指标.

$$\begin{bmatrix} -P_1 & \bar{C}_{ij}^T \tilde{\Phi}^T \\ * & -I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, p; \tag{28}$$

$$\begin{bmatrix} \Xi_1 + \Xi_2 + \Xi_{6ij} + \Xi_{6ij}^T & -\Xi'_{4ij} + \Xi_{7ij} \\ * & -\Xi'_{5i} \\ e_1^T \bar{C}_{ij}^T \tilde{\Psi}_1^T + e_2^T \bar{C}_{di}^T \tilde{\Psi}_1^T \\ \leftarrow \bar{D}_{2i}^T \tilde{\Psi}_1^T \\ -I \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, p; \tag{29}$$

$$\Phi_1 = \begin{bmatrix} X_1 & X_2 & N_1 \\ * & X_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0; \tag{30}$$

$$\Phi_2 = \begin{bmatrix} Y_1 & Y_2 & M_1 \\ * & Y_3 & M_2 \\ * & * & R \end{bmatrix} \geq 0. \tag{31}$$

其中

$$\Xi_1 = \text{Sym}\{\Pi_1^T P \Pi_2\} + \Pi_3^T Q_1 \Pi_3 - \tilde{d}(t) \Pi_4^T (Q_1 - Q_2) \Pi_4 - \Pi_5^T Q_2 \Pi_5 + h e_4^T R e_4,$$

$$\begin{aligned} \Xi_2 = & d(t) \Pi_6^T \left( X_1 + \frac{1}{3} X_3 \right) \Pi_6 + \\ & (h - d(t)) \Pi_9^T \left( Y_1 + \frac{1}{3} Y_3 \right) \Pi_9 + \\ & \text{Sym}\{\Pi_6^T N_1 \Pi_7 + \Pi_6^T N_2 \Pi_8 + \\ & \Pi_9^T M_1 \Pi_{10} + \Pi_9^T M_2 \Pi_{11}\}, \end{aligned}$$

$$\Xi'_{4ij} = e_1^T \bar{C}_{ij}^T \Psi_2 + e_2^T \bar{C}_{di}^T \Psi_2,$$

$$\Xi'_{5i} = \bar{D}_{2i}^T \Psi_2 + \Psi_2^T \bar{D}_{2i} + \Psi_3,$$

$$\Xi_{6ij} = e_1^T \alpha \begin{bmatrix} L_{11} A_i + \bar{B}_{fj} E_i & \bar{A}_{fj} \\ L_{21} A_i + \beta \bar{B}_{fj} E_i & \beta \bar{A}_{fj} \end{bmatrix} e_1 +$$

$$e_1^T \alpha \begin{bmatrix} L_{11} A_{di} + \bar{B}_{fj} E_{di} & 0 \\ L_{21} A_{di} + \beta \bar{B}_{fj} E_{di} & 0 \end{bmatrix} e_3 +$$

$$e_4^T \begin{bmatrix} L_{11} A_i + \bar{B}_{fj} E_i & \bar{A}_{fj} \\ L_{21} A_i + \beta \bar{B}_{fj} E_i & \beta \bar{A}_{fj} \end{bmatrix} e_1 +$$

$$e_4^T \begin{bmatrix} L_{11} A_{di} + \bar{B}_{fj} E_{di} & 0 \\ L_{21} A_{di} + \beta \bar{B}_{fj} E_{di} & 0 \end{bmatrix} e_3 -$$

$$e_4^T L_1 e_4 - e_1^T \alpha L_1 e_4,$$

$$\Xi_{7ij} = e_1^T \alpha \begin{bmatrix} L_{11} D_{1i} + \bar{B}_{fj} D_{3i} \\ L_{21} D_{1i} + \beta \bar{B}_{fj} D_{3i} \end{bmatrix} +$$

$$e_4^T \begin{bmatrix} L_{11} D_{1i} + \bar{B}_{fj} D_{3i} \\ L_{21} D_{1i} + \beta \bar{B}_{fj} D_{3i} \end{bmatrix},$$

$$L_1 = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & \beta L_{12} \end{bmatrix},$$

$$P = \begin{bmatrix} P_1 & 0_{2n \times 8n} \\ * & P_2 \end{bmatrix}.$$

滤波器增益矩阵可计算为

$$A_{fj} = L_{12}^{-1} \bar{A}_{fj}, B_{fj} = L_{12}^{-1} \bar{B}_{fj}, C_{fj} = \bar{C}_{fj}. \tag{32}$$

**证明** 由Schur补引理,  $\bar{C}_{ij}^T \Phi \bar{C}_{ij} - P_1 < 0$ 可表示为

$$\begin{bmatrix} -P_1 & \bar{C}_{ij}^T \Phi \\ * & -I \end{bmatrix} < 0, \tag{33}$$

即式(28)成立.

取任意矩阵  $L = \begin{bmatrix} e_1^T \alpha L_1 + e_4^T L_1 \\ 0_{q \times 2n} \end{bmatrix}$ , 则

$$\begin{aligned} L \Gamma_{ij} = & \begin{bmatrix} e_1^T \alpha L_1 + e_4^T L_1 \\ 0_{q \times 2n} \end{bmatrix} [\bar{A}_{ij} e_1 + \bar{A}_{dij} e_3 - e_4 \quad \bar{D}_{1ij}] = \\ & \begin{bmatrix} \Xi_{6ij} & \Xi_{7ij} \\ 0_{q \times 2n} & 0_{q \times 2n} \end{bmatrix}. \end{aligned} \tag{34}$$

其中

$$\begin{aligned} \Xi_{6ij} = & e_1^T \alpha L_1 \bar{A}_{ij} e_1 + e_1^T \alpha L_1 \bar{A}_{dij} e_3 - e_1^T \alpha L_1 e_4 + \\ & e_4^T L_1 \bar{A}_{ij} e_1 + e_4^T L_1 \bar{A}_{dij} e_3 - e_4^T L_1 e_4, \\ \Xi_{7ij} = & e_1^T \alpha L_1 \bar{D}_{1ij} + e_4^T L_1 \bar{D}_{1ij}. \end{aligned}$$

有

$$\begin{bmatrix} \Xi_1 + \Xi_2 - \Xi_{3ij} & -\Xi_{4ij} \\ * & -\Xi_{5i} \end{bmatrix} + L \Gamma_{ij} + \Gamma_{ij}^T L^T < 0,$$

可合并为

$$\begin{aligned} & \begin{bmatrix} \Xi_1 + \Xi_2 - \Xi_{3ij} & -\Xi_{4ij} \\ * & -\Xi_{5i} \end{bmatrix} + \begin{bmatrix} \Xi_{6ij} & \Xi_{7ij} \\ 0_{q \times 2n} & 0_{q \times 2n} \end{bmatrix} + \\ & \begin{bmatrix} \Xi_{6ij} & \Xi_{7ij} \\ 0_{q \times 2n} & 0_{q \times 2n} \end{bmatrix}^T < 0. \end{aligned} \tag{35}$$

式(35)可等价于

$$\begin{bmatrix} \Xi_1 + \Xi_2 - \Xi_{3ij} + \Xi_{6ij} + \Xi_{6ij}^T & -\Xi_{4ij} + \Xi_{7ij} \\ * & -\Xi_{5i} \end{bmatrix} < 0. \tag{36}$$

由Schur补引理, 有

$$\begin{aligned} & \begin{bmatrix} \Xi_1 + \Xi_2 + \Xi_{6ij} + \Xi_{6ij}^T & -\Xi_{4ij} + \Xi_{7ij} \\ * & -\Xi'_{5i} \\ * & * \end{bmatrix} \rightarrow \\ & \begin{bmatrix} e_1^T \bar{C}_{ij}^T \tilde{\Psi}_1^T + e_2^T \bar{C}_{di}^T \tilde{\Psi}_1^T \\ \bar{D}_{2i}^T \tilde{\Psi}_1^T \\ -I \end{bmatrix} < 0, \end{aligned} \tag{37}$$

即式(29)成立. 其中

$$\Xi'_{4ij} = e_1^T \bar{C}_{ij}^T \Psi_2 + e_2^T \bar{C}_{di}^T \Psi_2,$$

$$\Xi'_{5i} = \bar{D}_{2i}^T \Psi_2 + \Psi_2^T \bar{D}_{2i} + \Psi_3.$$

取  $L_1 = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & \beta L_{12} \end{bmatrix}$  并令  $L_{12} A_{fj} = \bar{A}_{fj}, L_{12} B_{fj} = \bar{B}_{fj}$  和  $C_{fj} = \bar{C}_{fj}$ , 则有

$$L_1 \bar{A}_{ij} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & \beta L_{12} \end{bmatrix} \begin{bmatrix} A_i & 0 \\ B_{fj} E_i & A_{fj} \end{bmatrix} =$$

$$\begin{bmatrix} L_{11} A_i + \bar{B}_{fj} E_i & \bar{A}_{fj} \\ L_{21} A_i + \beta \bar{B}_{fj} E_i & \beta \bar{A}_{fj} \end{bmatrix},$$

$$L_1 \bar{A}_{dij} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & \beta L_{12} \end{bmatrix} \begin{bmatrix} A_{di} & 0 \\ B_{fj} E_{di} & 0 \end{bmatrix} =$$

$$L_1 \bar{D}_{1ij} = \begin{bmatrix} L_{11}A_{di} + \bar{B}_{fj}E_{di} & 0 \\ L_{21}A_{di} + \beta\bar{B}_{fj}E_{di} & 0 \end{bmatrix},$$

$$L_1 \bar{D}_{1ij} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & \beta L_{12} \end{bmatrix} \begin{bmatrix} D_{1i} \\ B_{fj}D_{3i} \end{bmatrix} = \begin{bmatrix} L_{11}D_{1i} + \bar{B}_{fj}D_{3i} \\ L_{21}D_{1i} + \beta\bar{B}_{fj}D_{3i} \end{bmatrix}.$$

于是可得

$$\Xi_{6ij} = e_1^T \alpha \begin{bmatrix} L_{11}A_i + \bar{B}_{fj}E_i & \bar{A}_{fj} \\ L_{21}A_i + \beta\bar{B}_{fj}E_i & \beta\bar{A}_{fj} \end{bmatrix} e_1 + e_1^T \alpha \begin{bmatrix} L_{11}A_{di} + \bar{B}_{fj}E_{di} & 0 \\ L_{21}A_{di} + \beta\bar{B}_{fj}E_{di} & 0 \end{bmatrix} e_3 + e_4^T \begin{bmatrix} L_{11}A_i + \bar{B}_{fj}E_i & \bar{A}_{fj} \\ L_{21}A_i + \beta\bar{B}_{fj}E_i & \beta\bar{A}_{fj} \end{bmatrix} e_1 + e_4^T \begin{bmatrix} L_{11}A_{di} + \bar{B}_{fj}E_{di} & 0 \\ L_{21}A_{di} + \beta\bar{B}_{fj}E_{di} & 0 \end{bmatrix} e_3 - e_4^T L_1 e_4 - e_1^T \alpha L_1 e_4,$$

$$\Xi_{7ij} = e_1^T \alpha \begin{bmatrix} L_{11}D_{1i} + \bar{B}_{fj}D_{3i} \\ L_{21}D_{1i} + \beta\bar{B}_{fj}D_{3i} \end{bmatrix} + e_4^T \begin{bmatrix} L_{11}D_{1i} + \bar{B}_{fj}D_{3i} \\ L_{21}D_{1i} + \beta\bar{B}_{fj}D_{3i} \end{bmatrix}.$$

由不等式(11)和(12),可得式(30)和(31)成立;前述证明过程中已经证得不等式(28)和(29)成立,则区间二型模糊滤波器增益矩阵可计算为

$$A_{fj} = L_{12}^{-1} \bar{A}_{fj}, B_{fj} = L_{12}^{-1} \bar{B}_{fj}, C_{fj} = \bar{C}_{fj}. \quad \square$$

### 3 数值算例

本节将通过Matlab仿真说明本文方法的有效性.

**例1**<sup>[32]</sup> 考虑区间二型模糊时变延迟系统(3),其中模型参数为

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},$$

$$C_1 = [1 \ 0], C_2 = [0.5 \ -0.6],$$

$$C_{d1} = [-0.8 \ 0.6], C_{d2} = [-0.2 \ 1],$$

$$D_{21} = 0.3, D_{22} = -0.6,$$

$$E_1 = [1 \ -0.5], E_2 = [-0.2 \ 0.3],$$

$$E_{d1} = [0.1 \ 0], E_{d2} = [0 \ 0.2],$$

$$D_{31} = 0, D_{32} = 0.$$

隶属函数参数为

$$\tilde{w}_1(x_1) = 1 - \frac{1}{1 + e^{-x_1+k}}.$$

其中: $k \in [1, 5]$ 代表不确定性参数, $\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1)$ . 接下来,设计形如式(5)的区间二型模糊滤波器,相关隶属函数和参数如下:

$$\bar{\beta}_j = \underline{\beta}_j = 0.5,$$

$$\underline{m}_1(x_1) = 1 - \frac{1}{1 + e^{\frac{-x_1+0.25}{2}}},$$

$$\bar{m}_1(x_1) = 1 - \frac{1}{1 + e^{\frac{-x_1-0.25}{2}}},$$

$$\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1),$$

$$\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1).$$

限于篇幅,本例只给出了 $L_2$ - $L_\infty$ 性能指标下的滤波器设计.令 $\Phi = I, \Psi_1 = \Psi_2 = 0, \Psi_3 = \gamma^2 I$ .当取 $\alpha = 10, \beta = 100, h = 0.5, \mu = \mu_2 = -\mu_1 = 0.2$ 时,得到最小的 $L_2$ - $L_\infty$ 性能指标 $\gamma = 1.7591$ ,并求得滤波器增益矩阵为

$$A_{f1} = \begin{bmatrix} -0.3304 & 0.1170 \\ -0.1174 & -0.3307 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0009 \\ -0.0044 \end{bmatrix},$$

$$C_{f1} = [-0.0174 \ 0.0074],$$

$$A_{f2} = \begin{bmatrix} -0.3304 & 0.1170 \\ -0.1174 & -0.3307 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.0009 \\ -0.0044 \end{bmatrix},$$

$$C_{f2} = [-0.0174 \ 0.0074].$$

图1描述了在初始条件 $x_1 = 1, x_2 = -2, \hat{x}_1 = 1, \hat{x}_2 = -2$ 下 $x(t)$ 和 $\hat{x}(t)$ 的状态回复曲线.可以看出所设计的区间二型模糊滤波器与原始区间二型模糊时变延迟系统组成的闭环系统可保证渐近稳定.

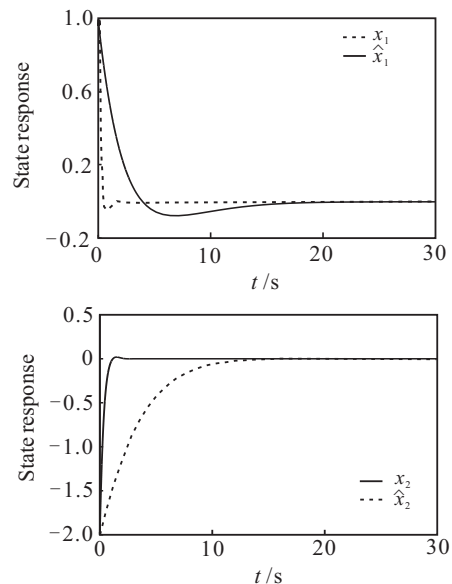


图1  $x_1(t), \hat{x}_1(t), x_2(t), \hat{x}_2(t)$  的状态回复曲线

假设扰动为

$$w(t) = \begin{cases} e^{-t} \cos t, & t \leq 55; \\ 0, & t > 55; \end{cases}$$

图2描述了在零初始条件下系统的误差回复曲线.

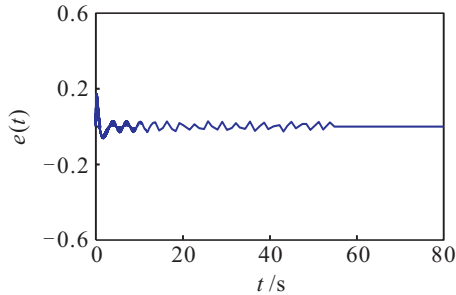


图2 零初始条件下的误差回复曲线

通过图1和图2可以证实本文设计的区间二型模糊滤波器可以有效地处理区间二型模糊时变延迟系统;在仿真中总是假定不确定参数随机变化,因此本文提出的方法可以较好地处理模糊时变延迟系统中隶属函数中的不确定参数.

**例2** 在如图3所示的实际弹簧阻尼系统<sup>[33]</sup>中,由牛顿运动定律可得

$$m\ddot{x} + F_f + F_s = u(t). \quad (38)$$

其中: $m$ 为物体质量, $F_f$ 和 $F_s$ 分别为物体所受摩擦力和弹簧的弹力, $u(t)$ 为水平方向合力, $x$ 为位移.

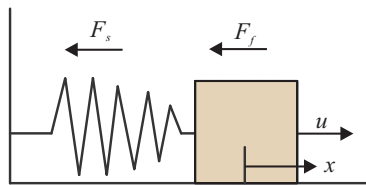


图3 弹簧阻尼系统

由于 $F_f = c\dot{x}$  ( $c > 0$ ),  $F_s = \hat{k}(1 + a^2x^2)x$ , 其中 $\hat{k}$ 和 $a$ 为常数,则式(38)可表示为

$$m\ddot{x} + c\dot{x} + \hat{k}x + \hat{k}a^2x^3 = u(t).$$

定义

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix},$$

$$\bar{f}(t) = \frac{-\hat{k} - \hat{k}a^2x_1^2(t)}{m}.$$

令 $x_1(t) \in [-2, 2]$ ,  $m = 1 \text{ kg}$ ,  $c = 2 \text{ (N}\cdot\text{m)/s}$ ,  $a = 0.3 \text{ m}^{-1}$ . 假设 $\hat{k} \in [5, 8] \text{ N/m}$ , 则当 $\hat{k} = 5$ ,  $x_1(t) = 0$ 时,  $\bar{f}(t)$ 取最大值为 $\bar{f}_{\max} = -5$ , 当 $\hat{k} = 8$ ,  $x_1(t) = \pm 2$ 时,  $\bar{f}(t)$ 取最小值为 $\bar{f}_{\min} = -10.88$ . 由于 $\hat{k}$ 为不确定参数,存在的一型模糊滤波方法不能被应用. 按照文献<sup>[33]</sup>中的建模方法,原始的非线性系统可以近似转化为模型(3),其中相关参数为

$$A_1 = \begin{bmatrix} 0 & 1 \\ \bar{f}_{\min} & -\frac{c}{m} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \bar{f}_{\max} & -\frac{c}{m} \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{11} = \begin{bmatrix} -0.03 \\ -0.02 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix},$$

$$C_1 = [0.1 \ 0.1], C_2 = [0.1 \ 0.1],$$

$$C_{d1} = [-0.8 \ 0.6], C_{d2} = [-0.2 \ 1],$$

$$D_{21} = -0.7, D_{22} = 0.2,$$

$$E_1 = [1 \ 0], E_2 = [1 \ 0],$$

$$E_{d1} = [0.1 \ 0], E_{d2} = [0 \ 0.2],$$

$$D_{31} = 0, D_{32} = 0.$$

由于不确定参数 $\hat{k}$ 取值的不同,区间二型T-S模糊模型的上、下隶属函数如下:

$$\underline{w}_1(x_1) = \frac{-\bar{f} + \bar{f}_{\max}}{\bar{f}_{\max} - \bar{f}_{\min}}, \hat{k} = 5;$$

$$\bar{w}_1(x_1) = \frac{-\bar{f} + \bar{f}_{\max}}{\bar{f}_{\max} - \bar{f}_{\min}}, \hat{k} = 8;$$

$$\underline{w}_2(x_1) = \frac{-\bar{f} - \bar{f}_{\max}}{\bar{f}_{\max} - \bar{f}_{\min}}, \hat{k} = 8;$$

$$\bar{w}_2(x_1) = \frac{-\bar{f} - \bar{f}_{\max}}{\bar{f}_{\max} - \bar{f}_{\min}}, \hat{k} = 5.$$

$\hat{w}_i(x_1) = \zeta(x_i)\underline{w}_i(x_i) + \bar{\zeta}(x_i)\bar{w}_i(x_i)$ ,  $\zeta(x_i) \in [0, 1]$  是不确定参数,  $\tilde{w}_i(x_1) = \hat{w}_i(x_1) / \sum_{s=1}^p \hat{w}_s(x_1)$ .

为了方便,本例设计的滤波器形式与例1相同. 接下来,给出 $L_2$ - $L_\infty$ 性能指标下的滤波器设计. 令 $\Phi = I, \Psi_1 = \Psi_2 = 0, \Psi_3 = \gamma^2 I$ . 当取 $\alpha = 1, \beta = 10, h = 0.5, \mu = \mu_2 = -\mu_1 = 0.2$ 时,得到最小的 $L_2$ - $L_\infty$ 性能指标 $\gamma = 1.0302$ ,并求得滤波器增益矩阵为

$$A_{f1} = \begin{bmatrix} -1.4257 & 0.0550 \\ -0.0207 & -1.4097 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0748 \\ -0.3739 \end{bmatrix},$$

$$C_{f1} = [-0.0009 \ 0.0270],$$

$$A_{f2} = \begin{bmatrix} -1.4257 & 0.0550 \\ -0.0207 & -1.4097 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.0748 \\ -0.3739 \end{bmatrix},$$

$$C_{f2} = [-0.0009 \ 0.0271].$$

图4描述了弹簧阻尼系统在初始条件 $x_1 = 1, x_2 = -2, \hat{x}_1 = 1, \hat{x}_2 = -2$ 下 $x(t)$ 和 $\hat{x}(t)$ 的状态回复曲线. 可以看出在本文设计的区间二型模糊滤波器作用下,弹簧阻尼系统的状态可以被较好地估计.

假设扰动为

$$w(t) = \begin{cases} e^{-t} \cos t, & t \leq 55; \\ 0, & t > 55; \end{cases}$$

在零初始条件下弹簧阻尼系统的误差回复曲线 $e(t)$ 如图5所示。

由图4和图5可以看出,本文提出的区间二型模糊滤波方法对参数不确定的弹簧阻尼系统是有效的。

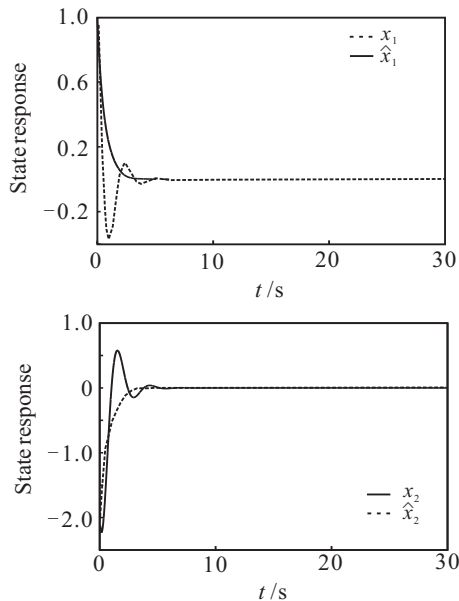


图4 弹簧阻尼系统 $x_1(t), \hat{x}_1(t), x_2(t), \hat{x}_2(t)$ 的状态回复曲线

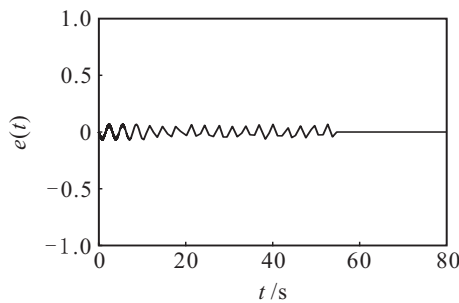


图5 弹簧阻尼系统零初始条件下误差回复曲线 $e(t)$

**注2** 区间二型T-S模糊模型在处理具有参数不确定的非线性系统方面具有一定的优势。由于现有的一型模糊模型不能有效表示具有不确定参数的非线性系统,文献[32]所提出的一型模糊方法无法直接用于例1和例2中的参数不确定性非线性系统。另外,本文所提出方法涵盖了包括耗散性、无源性、 $H_\infty$ 和在 $L_2-L_\infty$ 内的不同性能指标。相较于文献[16, 29],时变延迟首次被考虑在区间二型T-S模糊系统滤波器的设计之中。

## 4 结论

本文研究了多性能指标下区间二型T-S模糊时变延迟系统的滤波问题。所设计的区间二型模糊滤波器不需要与原始的区间二型T-S模糊时变延迟系统分享相同的隶属函数,增加了设计的灵活性。通过建立Lyapunov-Krasovskii泛函,滤波误差系统可以同时满足耗散性、无源性、 $H_\infty$ 和 $L_2-L_\infty$ 性能。利用积分不等式技术,给出了滤波器存在的充分条件。通过

Matlab仿真实验,验证了本文所提出方法的有效性。

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