文章编号: 1001-0920(2018)06-1017-09 DOI: 10.13195/j.kzyjc.2017.0256

一类非匹配不确定非线性系统快速加幂积分控制

蒲 明†, 周 蓉, 熊 皑

(成都信息工程大学 控制工程学院，成都 610225)

摘 要: 针对传统加幂积分控制在远离原点区域收敛速度过慢的问题, 基于加幂积分控制和快速有限时间 Lyapunov 理论对一类非匹配不确定非线性系统提出一种快速化的加幂积分控制. 该方案保留加幂积分控制有限时间收敛、无抖振、无奇异性的优点, 能够加快传统加幂积分控制的收敛速度, 具有更强的鲁棒性、更小的稳态误差. 从理论上证明了所提出方案的所有优越性, 并通过仿真验证了所提出的所有结论.

关键词: 加幂积分；有限时间；李雅普诺夫理论；非线性控制系统

中图分类号: TP273 **文献标志码:** A

Fast adding one power integrator control for a class of nonlinear systems with mismatched uncertainties

PU Ming†, *ZHOU Rong*, *XIONG Ai*

(College of Control Engineering，Chengdu University of Information Technology，Chengdu 610225，China)

Abstract: Traditional adding one power integrator control(AOPIC) convergence speed is even slower than linear control if the states are far from origin. To solve this problem, based on the traditional AOPIC and fast finite time Lyapunov theorem(FFTLT), a fast adding one power integrator control(FAOPIC) method is proposed for a class of nonlinear systems with mismatched uncertainties and external disturbance, which has the advantages of the AOPIC and FFTLT. Then it is strictly proved that the FAOPIC can speed up the convergence process of AOPIC, which is just exponential convergent or non-fast finite time convergent at any point, and it has stronger robustness and smaller steady error. Simulations are conducted to verify that the proposed method has significant better performance than AOPIC, and is very suitable for engineering application.

Keywords: adding one power integrator；finite time；Lyapunov theorem；nonlinear control systems

0 引 言

近年来, 有限时间控制已成为非线性控制研究中的热点[1-7]. 因其更快的收敛速度、更强的鲁棒性、更小的稳态误差等优点, 有限时间控制已广泛应用于倒立摆[1]、船舶[6]、机械臂[7] 等对象, 且都得到了较为理想的应用效果. 有限时间控制包含 Terminal 滑模控制、加幂积分控制、齐次控制等几个主要分支. 首先发展并成熟起来的是 Terminal 滑模控制, 目前已发展为包含高精度、抗扰、观测器等多个方向并适用于大量工程对象的现代非线性控制方法[4,7]. 然而, Terminal 滑模最大的缺点是用于二阶及更高阶系统具有奇异性. 针对该问题, Feng 等[7] 提出了二阶系统非奇异 Terminal 滑模, 然而非奇异性即使在二维空间中也并非处处成立, 在过原点的一条直线上仍然存在着奇异性, 但能证明该区域为非吸引区. 基于该方案的其他改进非奇异 Terminal 滑模方法也存在类似问题. 另外, 该方案无法推广用于三阶及更高阶系统, 直接原因是非整数指数项求导产生的负指数项在滑模面上或状态收敛到原点时无穷大, 引起控制器奇异. 根本原因在于目前包括非奇异 Terminal 滑模控制在内的绝大多数控制方法都是基于“在控制器设计中精确删除不需要的系统动态项, 引入期望性能的动态项, 从而改变了被控系统动态性能”这一思路设计的, 例如, 系统存在发散项 $f(x)$, 控制器中对应设计 $-f(x)-x^{1/2}$ 以保证系统有限时间收敛. 因此高阶系统非奇异 Terminal 滑模有限时间控制没有理想的解

收稿日期: 2017-03-13；修回日期: 2017-06-25.

基金项目: 成都信息工程大学科研基金项目 (KYTZ201636).

责任编委: 张维海.

作者简介: 蒲明 (1981−), 男, 讲师, 从事滑模控制、有限时间控制等研究；周蓉 (1968−), 女, 副教授, 从事智能控制等研究.

†通讯作者. E-mail: msznuaa@163.com

决方案. 奇异性和有限时间稳定始终是不可避免的矛盾,国内外相关研究几乎停滞,较少见到重大突破.

AOPIC(Adding one power integrator control)是近年发展起来的一种新兴的有限时间控制方法[8-29]. 与Terminal滑模等其他有限时间控制的最大区别在于,AOPIC抛弃了设计精确抵消项的思路,而采用不等式放缩作为工具,直接以有限时间稳定或收敛为目标设计控制器,例如,若系统存在发散项$f(x)$,则采用AOPIC控制器(或AOPIC虚拟控制器)设计对应的收敛项为$\mathrm{sgn}(g(x)) = -\mathrm{sgn}(f(x)), |g(x)| > |f(x)|, \forall x$,因此既能保证系统有限时间稳定,又能避免控制器奇异性. 从设计步骤看,AOPIC可看作是一种改进的Backstepping控制,保留了在高阶系统设计中层层设计虚拟控制器,层层反推直至设计出最终系统控制器的步骤. 因为避免了高阶微分项,没有传统Backstepping控制"计算量爆炸"的问题,所以AOPIC是高阶非线性系统非奇异有限时间控制发展的重要方向.

在理论上,AOPIC从诞生以来就一直被改进和发展. Lin等[8]采用AOPIC构造光滑控制器使高阶非仿射下三角系统全局渐近稳定,这是常规的滑模控制或反馈线性化等方法无法实现的. 但光滑控制器所需条件过多,一般难以得到且大多时候并不必要,所以Qian等[9]基于齐次理论构造了一种连续的AOPIC,大大降低了控制器的设计难度,使得AOPIC真正从理论走向工程实用. 此后AOPIC成功应用于不确定非线性系统[10-11]、一类采样数据状态反馈控制[12]、随机系统[13-15]、p标准型切换非线性系统的H_∞鲁棒控制[16-17]等. 文献[18]将AOPIC与切换自适应控制相结合,得到了更简单且易于实现的线性反馈控制器. 文献[19]将AOPIC用于基于采样控制的一类本质非线性系统全局镇定,适于数字化控制. 文献[20]将AOPIC用于传统控制方法无法控制的一类高阶不确定非线性系统. 这些AOPIC作用下的系统可渐近稳定甚至全局渐近稳定. 随后的研究在AOPIC设计中引入有限时间Lyapunov理论,得到有限时间AOPIC(Finite time adding one power integrator control, FT-AOPIC),并用于一类下三角系统主导的不确定非线性系统. 这一类AOPIC比文献[8-9]设计的AOPIC具有更佳的性能,因为对应的李氏函数沿状态轨迹的导数不仅是负定的,而且小于$-cV^\alpha$. 类似的或改进的FT-AOPIC也纷纷被提出,但基本设计基础和思路并无变换[6,21-24]. 文献[25-26]进一步构造了C^1函数的FT-AOPIC,但这一系列文献以及文献[1-6,27-29]中,FT-AOPIC仍然是传统而非快速化的有限时间控制,其缺陷在于当状态远离原点时,收敛速度慢于指数收敛. 由于收敛速度正比于鲁棒性,在该区域,其鲁棒性将弱于线性控制.

针对这一问题,受快速Lyapunov理论启发[30],本文的主要工作和创新点在于将FT-AOPIC快速化,在快速有限时间稳定Lyapunov理论框架下,通过设计新型虚拟控制器和最终控制器,使得AOPIC在全论域具有更快的收敛速度,以推进AOPIC性能的提升. 就目前所能查阅到的相关文献资料,本文在国内外相关研究中较早地提出快速化AOPIC,可看作AOPIC发展的第4个阶段. 4阶段分别为AOPIC[8]、渐近稳定或全局渐近稳定的AOPIC[9-20]、有限时间收敛的C^0或C^1 AOPIC[21-29]、本文提出的快速化有限时间AOPIC.

1 预备知识

定义1[13] 终端吸引子项记作$x^{\frac{q}{p}} \triangleq |x|^{\frac{q}{p}}\mathrm{sgn}(x)$, $0 < q < p$.

定义1使得终端吸引子项不会出现复数,同时将$|x|^{\frac{q}{p}}\mathrm{sgn}(x)$简写为$x^{\frac{q}{p}}$以简化本文的分析和证明.

下面给出引理1～引理4. 引理1～引理3是FAOPIC(Fast adding one power integrator control)设计中反复使用的数学工具;引理4是快速有限时间Lyapunov定理的衍生结论,也是本文控制器设计的基础.

引理1 对于任意实数$\alpha_1, \cdots, \alpha_n$和正数$\beta_1 > 0, \beta_2 > 0, \beta_3 \in (0,1)$,如下不等式成立[8]:

$$|\alpha_1|^{\beta_1}|\alpha_2|^{\beta_2} \leqslant \frac{\beta_1}{\beta_1+\beta_2}|\alpha_1|^{\beta_1+\beta_2} + \frac{\beta_2}{\beta_1+\beta_2}|\alpha_2|^{\beta_1+\beta_2}, \tag{1}$$

$$|\alpha_1^{\beta_3} - \alpha_2^{\beta_3}| \leqslant 2^{1-\beta_3}|\alpha_1 - \alpha_2|^{\beta_3}, \tag{2}$$

$$(|\alpha_1| + \cdots + |\alpha_n|)^{\beta_3} \leqslant |\alpha_1|^{\beta_3} + \cdots + |\alpha_n|^{\beta_3}. \tag{3}$$

引理2 已知函数

$$W(x_1, \cdots, x_k) = \int_{x_k^*}^{x_k} \left(s^{\beta_1} - x_k^{*\beta_1}\right)^{\beta_2} \mathrm{d}s.$$

其中:$\beta_i > 1 (i = 1, 2)$, x_k^*是关于$x_1, \cdots, x_{k-1}$的函数. 下列方程组成立[9]:

$$\begin{cases} \dfrac{\partial W}{\partial x_k} = \left(x_k^{\beta_1} - x_k^{*\beta_1}\right)^{\beta_2}; \\ \dfrac{\partial W}{\partial x_i} = -\beta_2 \dfrac{\partial x_k^{*\beta_1}}{\partial x_i} \displaystyle\int_{x_k^*}^{x_k} \left(s^{\beta_1} - x_k^{*\beta_1}\right)^{\beta_2 - 1} \mathrm{d}s, \\ \quad i = 1, 2, \cdots, k-1. \end{cases} \tag{4}$$

引理3 $x^{\beta_1}x^{\beta_2} = |x|^{\beta_1+\beta_2}$, $\beta_i > 0, i = 1, 2$.

证明 基于定义1,有

$$x^{\beta_1}x^{\beta_2} = |x|^{\beta_1}\mathrm{sgn}(x)|x|^{\beta_2}\mathrm{sgn}(x) = |x|^{\beta_1+\beta_2}. \qquad \square$$

引理4　若Lyapunov函数V_1, V_2分别满足$\dot{V}_1 + \alpha_1 V_1 + \alpha_2 V_1^{\beta} \leqslant 0, \alpha_1 > 0, \alpha_2 > 0, \beta \in (0,1)$和$\dot{V}_2 + \alpha_2 V_2^{\beta} \leqslant 0$,且$V_1(0) = V_2(0)$,则在任意点$V \in (0, V_1(0))$处$V_1$的收敛速度快于$V_2$, V_1收敛到原点的时间小于V_2收敛到原点的时间.

证明　$|\dot{V}| = |\alpha_1 V + \alpha_2 V^{\beta}| > |\alpha_2 V^{\beta}|, V \neq 0.$ □

本文以如下涵盖一大类工程对象的具有干扰的单输入单输出(Single input single output, SISO)2阶非匹配不确定非线性系统作为研究对象:

$$\begin{cases} \dot{x}_1 = x_2^{p_1} + f_1(x) + d_1(x,t), \\ \dot{x}_2 = u^{p_2} + f_2(x) + d_2(x,t). \end{cases} \tag{5}$$

其中: $x = [x_1, x_2]^{\mathrm{T}}$; p_1和p_2均为正数; u为待设计的控制器; $d_1(x,t)$和$d_2(x,t)$分别为各子系统受到的包括外干扰和不确定在内的复合项; $f_1(x)$, $f_2(x)$, $d_1(x,t)$, $d_2(x,t)$为C^0函数,且满足如下不等式:

$$\begin{cases} |f_1(x) + d_1(x,t)| \leqslant \tau_1, \\ |f_2(x) + d_2(x,t)| \leqslant \tau_2. \end{cases} \tag{6}$$

τ_1和τ_2为正数且有$f_1(0) = f_2(0) = 0$.

本文的研究目标是: 设计全局非奇异的FAOPIC使得系统(5)中的x_1在扰动和不确定影响下,有限时间收敛到原点任意小的邻域内,且收敛速度快于传统的有限时间AOPIC.

2　快速Lyapunov函数构造及虚拟控制器设计

设计第1个子系统的虚拟控制器,选择李氏函数为$V_1 = \dfrac{1}{2}x_1^2$, V_1的一阶导数为

$$\dot{V}_1 = x_1\dot{x}_1 = x_1(x_2^{p_1} - x_2^{*p_1} + f_1(x) + d_1(x,t)) + x_1 x_2^{*p_1},$$

其中x_2^*为第1个子系统待设计的虚拟控制器. 设

$$\xi_1 = x_2^{k_1} - x_2^{*k_1},\ k_1 = p_1 + \gamma > 1,\ \gamma \in (0,1),$$
$$x_2^{*p_1} = -a x_1^{q_1},\ q_1 = p_1/k_1 \in (0,1),$$

其中a是待设计的正常数. 将ξ_1代入上式,根据引理1和引理3,有

$$\begin{aligned}
&\dot{V}_1 \leqslant \\
&|x_1||x_2^{k_1 \frac{p_1}{k_1}} - x_2^{*k_1 \frac{p_1}{k_1}}| - a|x_1|^{1+q_1} + \\
&|x_1||f_1(x) + d_1(x,t)| \leqslant \\
&2^{1-\frac{p_1}{k_1}}|x_1||x_2^{k_1} - x_2^{*k_1}|^{\frac{p_1}{k_1}} - a|x_1|^{1+q_1} + \tau_1|x_1| \leqslant \\
&\frac{2^{1-\frac{p_1}{k_1}}}{1+\dfrac{p_1}{k_1}}|x_1|^{1+\frac{p_1}{k_1}} + \frac{2^{1-\frac{p_1}{k_1}}\dfrac{p_1}{k_1}}{1+\dfrac{p_1}{k_1}}|\xi_1|^{1+\frac{p_1}{k_1}} - a|x_1|^{1+q_1} + \\
&\tau_1\Bigg(\frac{\dfrac{p_1}{k_1}}{1+\dfrac{p_1}{k_1}}1^{1+\frac{p_1}{k_1}} + \frac{1}{1+\dfrac{p_1}{k_1}}|x_1|^{1+\frac{p_1}{k_1}}\Bigg) = \\
&\frac{2^{1-q_1} + \tau_1}{1+q_1}|x_1|^{1+q_1} + \frac{2^{1-q_1}q_1}{1+q_1}|\xi_1|^{1+q_1} - \\
&a|x_1|^{1+q_1} + \tau_1\frac{q_1}{1+q_1} \triangleq \\
&C_1|x_1|^{1+q_1} + C_2|\xi_1|^{1+q_1} - a|x_1|^{1+q_1} + C_3.
\end{aligned} \tag{7}$$

其中

$$C_1 = \frac{2^{1-q_1} + \tau_1}{1+q_1},\ C_2 = \frac{2^{1-q_1}q_1}{1+q_1},\ C_3 = \frac{\tau_1 q_1}{1+q_1};$$

C_1, C_2, C_3均为正常数. 选择V_2为

$$V_2 = V_1 + W_2,$$
$$W_2 = \frac{1}{a^{\frac{k_1}{p_1}}\left(2 - \dfrac{1}{k_1}\right)}\int_{x_2^*}^{x_2}(s^{k_1} - x_2^{*k_1})^{2-\frac{1}{k_1}}\mathrm{d}s.$$

当$x_2 \geqslant x_2^*$时,有$x_2^* \leqslant s \leqslant x_2$,进一步使得$s^{k_1} - x_2^{*k_1} \geqslant 0$,所以积分式$W_2 \geqslant 0$. 当且仅当$x_2 = x_2^*$时等式$W_2 = 0$成立. 类似地,当$x_2 \leqslant x_2^*$时有$s^{k_1} - x_2^{*k_1} \leqslant 0$,从而有$W_2 > 0(x_2 \neq x_2^*)$. 因此所选$W_2$为一正定函数. 结合式(7),得到$V_2$的一阶导数为

$$\dot{V}_2 \leqslant C_1|x_1|^{1+q_1} + C_2|\xi_1|^{1+q_1} - a|x_1|^{1+q_1} + C_3 + \sum_{i=1}^{2}\frac{\partial W_2}{\partial x_i}\dot{x}_i. \tag{8}$$

其中$\dfrac{\partial W_2}{\partial x_1}\dot{x}_1$, $\dfrac{\partial W_2}{\partial x_2}\dot{x}_2$基于引理1和引理2推导如下:

$$\begin{aligned}
&\frac{\partial W_2}{\partial x_1}\dot{x}_1 = \\
&-\frac{1}{a^{\frac{k_1}{p_1}}\left(2 - \dfrac{1}{k_1}\right)}\left(2 - \frac{1}{k_1}\right)\times \\
&\frac{\partial x_2^{*k_1}}{\partial x_1}\int_{x_2^*}^{x_2}(s^{k_1} - x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s \times \dot{x}_1 = \\
&-\frac{1}{a^{\frac{p_1}{q_1}\frac{1}{p_1}}}\frac{\partial((-a^{\frac{1}{p_1}}x_1^{\frac{q_1}{p_1}})^{\frac{p_1}{q_1}})}{\partial x_1}\times \\
&\int_{x_2^*}^{x_2}(s^{k_1} - x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s\times \\
&(x_2^{p_1} + f_1(x) + d_1(x,t)) = \\
&\int_{x_2^*}^{x_2}(s^{k_1} - x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s\times \\
&(x_2^{p_1} + f_1(x) + d_1(x,t)),
\end{aligned}$$

$\int_{x_2^*}^{x_2}(s^{k_1} - x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s$为一正定函数. 由于

$$\mathrm{sgn}(x_2^{k_1} - x_2^{*k_1}) = \mathrm{sgn}(x_2 - x_2^*),$$

有

$$\int_{x_2^*}^{x_2}(s^{k_1} - x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s \leqslant$$

$$|x_2^{k_1}-x_2^{*k_1}|^{1-\frac{1}{k_1}}\mathrm{sgn}(x_2^{k_1}-x_2^{*k_1})\times$$
$$|x_2-x_2^*|\mathrm{sgn}(x_2-x_2^*)\leqslant$$
$$|\xi_1|^{1-\frac{1}{k_1}}\times 2^{1-\frac{1}{k_1}}|\xi_1|^{\frac{1}{k_1}}=$$
$$2^{1-\frac{1}{k_1}}|\xi_1|.$$

将上式代入 $\dfrac{\partial W_2}{\partial x_1}\dot{x}_1$,有

$$\frac{\partial W_2}{\partial x_1}\dot{x}_1\leqslant$$
$$|x_2^{p_1}-x_2^{*p_1}+x_2^{*p_1}|\int_{x_2^*}^{x_2}(s^{k_1}-x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s+$$
$$|f_1(x)+d_1(x,t)|\int_{x_2^*}^{x_2}(s^{k_1}-x_2^{*k_1})^{1-\frac{1}{k_1}}\mathrm{d}s\leqslant$$
$$2^{1-\frac{1}{k_1}}|\xi_1||x_2^{p_1}-x_2^{*p_1}|+$$
$$2^{1-\frac{1}{k_1}}|\xi_1||x_2^{*p_1}|+2^{1-\frac{1}{k_1}}|\xi_1|\tau_1\leqslant$$
$$2^{1-\frac{1}{k_1}}|\xi_1|2^{1-\frac{p_1}{k_1}}|x_2^{k_1}-x_2^{*k_1}|^{\frac{p_1}{k_1}}+$$
$$\Big(\frac{1}{1+q_1}|\xi_1|^{1+q_1}+\frac{q_1}{1+q_1}|x_1|^{1+q_1}\Big)\times$$
$$2^{1-\frac{1}{k_1}}a+2^{1-\frac{1}{k_1}}\tau_1\times$$
$$\Big(\frac{1}{1+q_1}|\xi_1|^{1+q_1}+\frac{q_1}{1+q_1}1^{1+q_1}\Big)=$$
$$\Big(2^{2-\frac{1}{k_1}-q_1}+2^{1-\frac{1}{k_1}}a\frac{1}{1+q_1}+$$
$$2^{1-\frac{1}{k_1}}\tau_1\frac{1}{1+q_1}\Big)|\xi_1|^{1+q_1}+$$
$$2^{1-\frac{1}{k_1}}a\frac{q_1}{1+q_1}|x_1|^{1+q_1}+2^{1-\frac{1}{k_1}}\tau_1\frac{q_1}{1+q_1}\triangleq$$
$$C_4|x_1|^{1+q_1}+C_5|\xi_1|^{1+q_1}+C_6. \tag{9}$$

各参数分别为

$$C_4=2^{1-\frac{1}{k_1}}\frac{aq_1}{1+q_1},$$
$$C_5=2^{2-\frac{1}{k_1}-q_1}+2^{1-\frac{1}{k_1}}a\frac{1}{1+q_1}+2^{1-\frac{1}{k_1}}\tau_1\frac{1}{1+q_1},$$
$$C_6=2^{1-\frac{1}{k_1}}\tau_1\frac{q_1}{1+q_1};$$

C_4, C_5, C_6 均为正常数. 根据引理2,有

$$\frac{\partial W_2}{\partial x_2}\dot{x}_2=$$
$$\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}(x_2^{k_1}-x_2^{*k_1})^{2-\frac{1}{k_1}}\times$$
$$(u^{p_2}+f_2(x)+d_2(x,t))\triangleq$$
$$C_7\xi_1^{2-\frac{1}{k_1}}u^{p_2}+C_7\xi_1^{2-\frac{1}{k_1}}(f_2(x)+d_2(x,t)), \tag{10}$$

其中 $C_7=\dfrac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}>0$. 将式(9)和(10)代入(8),得到

$$\dot{V}_2\leqslant$$
$$C_1|x_1|^{1+q_1}+C_2|\xi_1|^{1+q_1}-a|x_1|^{1+q_1}+$$
$$C_3+C_4|x_1|^{1+q_1}+C_5|\xi_1|^{1+q_1}+C_6+$$
$$C_7\xi_1^{2-\frac{1}{k_1}}u^{p_2}+C_7\xi_1^{2-\frac{1}{k_1}}(f_2(x)+d_2(x,t))=$$
$$(C_1+C_4)|x_1|^{1+q_1}+(C_2+C_5)|\xi_1|^{1+q_1}-$$
$$a|x_1|^{1+q_1}+C_7\xi_1^{2-\frac{1}{k_1}}u^{p_2}+C_3+C_6+$$
$$C_7\xi_1^{2-\frac{1}{k_1}}(f_2(x)+d_2(x,t)). \tag{11}$$

设 $\lambda_1>0$, $\lambda_2>0$, $\eta\in(0,1)$,根据引理1,有

$$\lambda_1V_2+\lambda_2V_2^\eta\leqslant$$
$$\lambda_1V_1+\lambda_1W_2+\lambda_2V_1^\eta+\lambda_2W_2^\eta=$$
$$\lambda_1\frac{1}{2}x_1^2+\lambda_1\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}\times$$
$$\int_{x_2^*}^{x_2}(s^{k_1}-x_2^{*k_1})^{2-\frac{1}{k_1}}\mathrm{d}s+\lambda_2\frac{1}{2^\eta}|x_1|^{2\eta}+$$
$$\lambda_2\Big(\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}\int_{x_2^*}^{x_2}(s^{k_1}-x_2^{*k_1})^{2-\frac{1}{k_1}}\mathrm{d}s\Big)^\eta\leqslant$$
$$\lambda_1\frac{1}{2}x_1^2+\lambda_1\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}|\xi_1|^{2-\frac{1}{k_1}}\times$$
$$|x_2^{k_1\frac{1}{k_1}}-x_2^{*k_1\frac{1}{k_1}}|+\lambda_2\frac{1}{2^\eta}|x_1|^{2\eta}+$$
$$\frac{\lambda_2}{a^{\frac{k_1}{p_1}\eta}\Big(2-\dfrac{1}{k_1}\Big)^\eta}|\xi_1|^{(2-\frac{1}{k_1})\eta}|x_2^{k_1\frac{1}{k_1}}-x_2^{*k_1\frac{1}{k_1}}|^\eta\leqslant$$
$$\lambda_1\frac{1}{2}x_1^2+\lambda_1\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}2^{1-\frac{1}{k_1}}|\xi_1|^2+$$
$$\lambda_2\frac{1}{2^\eta}|x_1|^{2\eta}+\frac{\lambda_2}{a^{\frac{k_1}{p_1}\eta}\Big(2-\dfrac{1}{k_1}\Big)^\eta}2^{(1-\frac{1}{k_1})\eta}|\xi_1|^{2\eta}\triangleq$$
$$C_8x_1^2+C_9|\xi_1|^2+C_{10}|x_1|^{2\eta}+C_{11}|\xi_1|^{2\eta}. \tag{12}$$

其中

$$C_8=\frac{\lambda_1}{2},\ C_9=\lambda_1\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}2^{1-\frac{1}{k_1}},$$
$$C_{10}=\lambda_2\frac{1}{2^\eta},\ C_{11}=\frac{\lambda_2}{a^{\frac{k_1}{p_1}\eta}\Big(2-\dfrac{1}{k_1}\Big)^\eta}2^{(1-\frac{1}{k_1})\eta};$$

C_8, C_9, C_{10}, C_{11} 均为正常数. 取 $2\eta=1+q_1$ 并考虑式(11)和(12),有

$$\dot{V}_2+\lambda_1V_2+\lambda_2V_2^\eta\leqslant$$
$$(C_1+C_4)|x_1|^{1+q_1}+(C_2+C_5)|\xi_1|^{1+q_1}-$$
$$a|x_1|^{1+q_1}+C_7\xi_1^{2-\frac{1}{k_1}}u^{p_2}+C_7\xi_1^{2-\frac{1}{k_1}}\times$$
$$(f_2(x)+d_2(x,t))+C_3+C_6+C_8x_1^2+$$
$$C_9|\xi_1|^2+C_{10}|x_1|^{1+q_1}+C_{11}|\xi_1|^{1+q_1}\triangleq$$
$$C_{12}|x_1|^{1+q_1}+C_{13}|\xi_1|^{1+q_1}-a|x_1|^{1+q_1}+$$

$$C_7\xi_1^{2-\frac{1}{k_1}}u^{p_2}+C_7\xi_1^{2-\frac{1}{k_1}}(f_2(x)+d_2(x,t))+$$
$$C_{14}+C_8x_1^2+C_9|\xi_1|^2.$$

其中

$$C_{12}=C_1+C_4+C_{10},\ C_{13}=C_2+C_5+C_{11},$$
$$C_{14}=C_3+C_6;$$

C_{12}, C_{13}和C_{14}均为正常数. 若参数a设计为

$$a=C_{12}+l_1=C_1+C_4+C_{10}+l_1=$$
$$C_1+2^{1-\frac{1}{k_1}}\frac{aq_1}{1+q_1}+C_{10}+l_1,$$
$$l_1>0,$$

则上式可以进一步简化为

$$\dot V_2+\lambda_1V_2+\lambda_2V_2^{\eta}\leqslant$$
$$-l_1|x_1|^{1+q_1}+C_{13}|\xi_1|^{1+q_1}+C_7\xi_1^{2-\frac{1}{k_1}}u^{p_2}+$$
$$C_7\xi_1^{2-\frac{1}{k_1}}(f_2(x)+d_2(x,t))+$$
$$C_{14}+C_8x_1^2+C_9|\xi_1|^2. \tag{13}$$

由上述分析可知C_4中含有a,即C_{12}含有a. 将a代入C_{12}可得a的具体表达式为

$$a=\frac{C_1+C_{10}+l_1}{1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}}=$$
$$\frac{\dfrac{2^{1-q_1}+\tau_1}{1+q_1}+\lambda_2\dfrac{1}{2^{\eta}}+l_1}{1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}}. \tag{14}$$

分母为

$$D(\gamma)=1-2^{1-\frac{1}{k_1}}\frac{q_1}{1+q_1}=$$
$$1-2^{1-\frac{1}{p_1+\gamma}}\frac{\dfrac{p_1}{k_1}}{1+\dfrac{p_1}{k_1}}=$$
$$1-2^{1-\frac{1}{p_1+\gamma}}\frac{p_1}{2p_1+\gamma}.$$

若$D(\gamma)=0$,则应有

$$\frac{2p_1+\gamma}{p_1}=2+\frac{\gamma}{p_1}=2^{1-\frac{1}{p_1+\gamma}}.$$

$2^{1-\frac{1}{p_1+\gamma}}$是$\gamma$的增函数,所以有

$$2^{1-\frac{1}{p_1+\gamma}}\in\Big(2^{1-\frac{1}{p_1}},2^{\frac{1}{\frac{1}{p_1}+1}}\Big)\subset\Big(2^{1-\frac{1}{p_1}},2\Big),$$

同时有$2+\gamma/p_1>2$. 所以$(2p_1+\gamma)/p_1=2^{1-\frac{1}{p_1+\gamma}}$对于任意选择的参数均不成立,有$D(\gamma)\neq0$. 即式(14)给出的$a$任意情况下均是有界的,这一有界性结论在非奇异AOPIC设计中极为重要.

在以上分析结论基础上,本文设计目标转化为设计控制器使得式(13)为负.

3 FAOPIC设计及误差分析

设计FAOPIC控制器为

$$u^{p_2}=-\frac{2-\dfrac{1}{k_1}}{2-\dfrac{1}{k_1}+\varphi}\tau_2\xi_1^{\varphi}-$$
$$\frac{C_{13}+l_2}{C_7}\xi_1^{q_1-1+\frac{1}{k_1}}-\frac{C_9}{C_7}\xi_1^{\frac{1}{k_1}}. \tag{15}$$

其中:$l_2>0,\varphi>0$. 控制器(15)是C^0函数,是无抖振的. 将式(14)和(15)代入(13),可得

$$\dot V_2+\lambda_1V_2+\lambda_2V_2^{\eta}\leqslant$$
$$-l_1|x_1|^{1+q_1}+C_{13}|\xi_1|^{1+q_1}-\frac{2-\dfrac{1}{k_1}}{2-\dfrac{1}{k_1}+\varphi}\times$$
$$\tau_2C_7|\xi_1|^{2-\frac{1}{k_1}+\varphi}-\frac{C_7}{C_7}(C_{13}+l_2)\times$$
$$|\xi_1|^{q_1-1+\frac{1}{k_1}+2-\frac{1}{k_1}}-\frac{C_7}{C_7}C_9|\xi_1|^2+$$
$$\tau_2C_7|\xi_1|^{2-\frac{1}{k_1}}+C_{14}+C_8x_1^2+C_9|\xi_1|^2=$$
$$-\frac{2-\dfrac{1}{k_1}}{2-\dfrac{1}{k_1}+\varphi}\tau_2C_7|\xi_1|^{2-\frac{1}{k_1}+\varphi}+\tau_2C_7|\xi_1|^{2-\frac{1}{k_1}}\times$$
$$1^{\varphi}-l_1|x_1|^{1+q_1}-l_2|\xi_1|^{1+q_1}+C_{14}+C_8x_1^2\leqslant$$
$$-\frac{2-\dfrac{1}{k_1}}{2-\dfrac{1}{k_1}+\varphi}\tau_2C_7|\xi_1|^{2-\frac{1}{k_1}+\varphi}+\frac{2-\dfrac{1}{k_1}}{2-\dfrac{1}{k_1}+\varphi}\times$$
$$\tau_2C_7|\xi_1|^{2-\frac{1}{k_1}+\varphi}+\tau_2C_7\frac{\varphi}{2-\dfrac{1}{k_1}+\varphi}\times1^{2-\frac{1}{k_1}+\varphi}-$$
$$l_1|x_1|^{1+q_1}-l_2|\xi_1|^{1+q_1}+C_{14}+C_8x_1^2\leqslant$$
$$\tau_2C_7\frac{\varphi}{2-\dfrac{1}{k_1}+\varphi}-l_1|x_1|^{1+q_1}-$$
$$l_2|\xi_1|^{1+q_1}+C_{14}+C_8x_1^2. \tag{16}$$

当$|x_1|\geqslant1$时,取

$$l_1>C_8|x_1|^{1-q_1}=\frac{\lambda_1}{2}|x_1|^{1-q_1},$$

则有

$$-l_1|x_1|^{1+q_1}+C_8x_1^2<0;$$

当$|x_1|<1$时,取$l_1>C_8=\lambda_1/2$,则有

$$-l_1|x_1|^{1+q_1}+C_8x_1^2<0.$$

即总存在足够大的参数

$$l_1>\max\Big\{\frac{\lambda_1}{2},\frac{\lambda_1}{2}|x_1(0)|^{1-q_1}\Big\},$$

使得$-l_1|x_1|^{1+q_1}+C_8x_1^2<0$对于任意$x_1\neq0$点成立.

再考虑

$$-l_2|\xi_1|^{1+q_1}+C_{14}+\tau_2C_7\frac{\varphi}{2-\dfrac{1}{k_1}+\varphi},$$

容易发现,若满足

$$|\xi_1|^{1+q_1}>C_{15}\triangleq\frac{C_{14}+\tau_2C_7\dfrac{\varphi}{2-\dfrac{1}{k_1}+\varphi}}{l_2},$$

则有

$$-l_2|\xi_1|^{1+q_1}+C_{14}+\tau_2C_7\frac{\varphi}{2-\dfrac{1}{k_1}+\varphi}<0.$$

C_{15}的分子均为有限正数,所以总是存在足够大的l_2使C_{15}可任意小. V_2从任意初始点出发总可以在有限时间内收敛到原点的任意小闭球内,被控系统在控制器(15)作用下是有限时间收敛的稳定系统.

根据式(7)和(14)可得$|x_1|$的误差上界为

$$\begin{aligned}&\dot{V}_1\leqslant\\&C_1|x_1|^{1+q_1}+C_2C_{15}-a|x_1|^{1+q_1}+C_3=\\&C_1|x_1|^{1+q_1}+C_2\frac{C_{14}}{l_2}+\frac{\varphi}{l_2\Big(2-\dfrac{1}{k_1}+\varphi\Big)}\times\\&C_2\tau_2C_7-\frac{C_1+\lambda_2\dfrac{1}{2^\eta}+l_1}{1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}}|x_1|^{1+q_1}+C_3=\\&-\frac{C_1\Big(1-1+2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}\Big)+\lambda_2\dfrac{1}{2^\eta}+l_1}{1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}}\times\\&|x_1|^{1+q_1}+C_3+C_2\frac{C_3+C_6}{l_2}+C_2\tau_2\varphi\times\\&\frac{\Big(1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}\Big)^{\frac{k_1}{p_1}}}{l_2\Big(C_1+\lambda_2\dfrac{1}{2^\eta}+l_1\Big)^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)\Big(2-\dfrac{1}{k_1}+\varphi\Big)}.\end{aligned}$$

若如下不等式成立,则可知$\dot{V}_1<0$:

$$\begin{aligned}&|x_1|>\\&C_{16}\triangleq\Big(\frac{C_2C_{15}+C_3}{a-C_1}\Big)^{\frac{1}{1+q_1}}=\\&\Big(\frac{\Big(1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}\Big)\Big(C_2\dfrac{C_3+C_6}{l_2}+C_3\Big)}{C_12^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}+\lambda_2\dfrac{1}{2^\eta}+l_1}+\\&\frac{C_2\tau_2\varphi\Big(1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}\Big)}{C_12^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}+\lambda_2\dfrac{1}{2^\eta}+l_1}\times\\&\frac{\Big(1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}\Big)^{\frac{k_1}{p_1}}}{l_2\Big(C_1+\lambda_2\dfrac{1}{2^\eta}+l_1\Big)^{\frac{k_1}{p_1}}}\times\\&\frac{1}{\Big(2-\dfrac{1}{k_1}\Big)\Big(2-\dfrac{1}{k_1}+\varphi\Big)}\Big)^{\frac{1}{1+q_1}}.\end{aligned}$$

C_1, C_2, C_3和C_6均是独立于l_1, l_2, λ_2的常数,对于任意给定的p_1,容易选择γ和φ使得$2-1/k_1\neq 0$和$2-1/k_1+\varphi\neq 0$成立. 例如,当$p_1=0.5$时,选择$\gamma=0.5$使得$2-1/k_1=2-1=1\neq 0$,所以总是存在足够大的参数l_1, l_2, λ_2使得C_{16}足够小.

将以上所有分析、证明和结论总结为定理1.

定理1 对于给定的任意小常数$C_{16}>0$,若非匹配不确定非线性系统(5)满足式(6),则总存在一组控制器参数,使得系统状态x_1在FAOPIC控制器(17)作用下在有限时间内收敛到$|x_1|\leqslant C_{16}$以内,有

$$\begin{aligned}&u=\Big(-\frac{2-\dfrac{1}{k_1}}{2-\dfrac{1}{k_1}+\varphi}\tau_2\xi_1^{\varphi}-\frac{C_{13}+l_2}{C_7}\times\\&\quad\xi_1^{q_1-1+\frac{1}{k_1}}-\frac{C_9}{C_7}\xi_1^{\frac{1}{k_1}}\Big)^{\frac{1}{p_2}},\end{aligned}$$

$$l_1>0, l_2>0,\ \lambda_1>0,\ \lambda_2>0,\ \tau_1>0,\ \tau_2>0,$$

$$p_1>0,\ \varphi>0,\ p_2>0,\ k_1=p_1+\gamma,\ \gamma\in(0,1),$$

$$q_1=\frac{p_1}{k_1},\ \eta=\frac{1+q_1}{2},\ \xi_1=x_2^{k_1}-x_2^{*k_1},$$

$$x_2^{*p_1}=-ax_1^{q_1},\ a=\frac{C_1+C_{10}+l_1}{1-2^{1-\frac{1}{k_1}}\dfrac{q_1}{1+q_1}},$$

$$C_1=\frac{2^{1-q_1}+\tau_1}{1+q_1},\ C_2=\frac{2^{1-q_1}q_1}{1+q_1},$$

$$C_3=\frac{\tau_1q_1}{1+q_1},\ C_4=2^{1-\frac{1}{k_1}}\frac{aq_1}{1+q_1},$$

$$C_5=\frac{2^{1-\frac{1}{k_1}}}{1+q_1}(2^{1-q_1}(1+q_1)+a+\tau_1),$$

$$C_6=2^{1-\frac{1}{k_1}}\tau_1\frac{q_1}{1+q_1},\ C_7=\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)},$$

$$C_8=\frac{\lambda_1}{2},\ C_9=\lambda_1\frac{1}{a^{\frac{k_1}{p_1}}\Big(2-\dfrac{1}{k_1}\Big)}2^{1-\frac{1}{k_1}},$$

$$C_{10}=\lambda_2\frac{1}{2^\eta},\ C_{11}=\frac{\lambda_2}{a^{\frac{k_1}{p_1}\eta}\Big(2-\dfrac{1}{k_1}\Big)^\eta}2^{(1-\frac{1}{k_1})\eta},$$

$$C_{12}=C_1+C_4+C_{10},\ C_{13}=C_2+C_5+C_{11},$$

$$C_{14} = C_3 + C_6,\ C_{15} = \frac{C_{14} + \tau_2 C_7 \dfrac{\varphi}{2 - \dfrac{1}{k_1} + \varphi}}{l_2},$$

$$C_{16} = \left(\frac{C_2 C_{15} + C_3}{a - C_1}\right)^{\frac{1}{1+q_1}}. \tag{17}$$

注1　在传统的非线性控制中，系统(5)的f_1, d_1至少应为C^1函数以构造连续控制器. 但在本文控制方法中，因为不含有f_1和d_1的导数，所以放宽了该条件.

注2　本文方法只要求系统(5)的指数是正数，不再要求$p_1 \geqslant p_2$成立.

4　仿真验证

以文献[29]的二阶非线性系统为仿真对象，有

$$\begin{cases} \dot{x}_1 = x_2^{\frac{5}{3}} + x_1^{\frac{10}{11}}, \\ \dot{x}_2 = u^3 + x_1^2 \sin x_2. \end{cases}$$

其中：$p_1 = 5/3$, $p_2 = 3$. 初始值设为$x_1(0) = -2$, $x_2(0) = 4$. 相关控制参数选择为$l_1 = 1$, $l_2 = 1$, $\gamma = 1/11$, $\tau_1 = 1$, $\tau_2 = 0.5$, $\lambda_1 = 1$, $\lambda_2 = 1$. 根据定理1可得如下参数值：

$$k_1 = 1.7576,\ q_1 = 0.9483,\ \eta = 0.9741,$$
$$C_1 = 1.0453,\ C_2 = 0.5044,\ C_3 = 0.4867,$$
$$C_4 = 4.8754,\ C_5 = 3.5037,\ C_6 = 4.3833,$$
$$C_7 = 0.0843,\ C_8 = 0.5,\ C_9 = 0.2328,$$
$$C_{10} = 0.5090,\ C_{11} = 0.1202,\ C_{12} = 6.4298,$$
$$C_{13} = 4.1284,\ C_{14} = 4.8700,$$
$$a = 7.4298,\ \varphi = 0.5.$$

仿真步长固定为0.001 s. FAOPIC控制器结构如式(17).

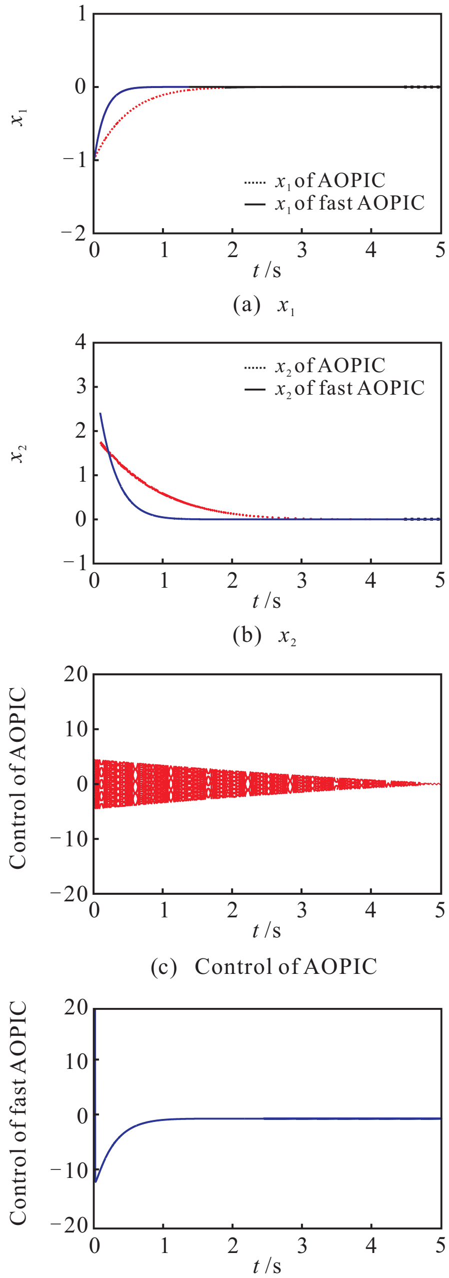
文献[29]设计的AOPIC控制器如下：

$$u = -(1 + l_{21} + l_{22} + l_{23})^{\frac{1}{3}} \times \left(x_2^{\frac{11}{6}} - \left(-\beta_1^{\frac{6}{11}} x_1^{\frac{6}{11}}\right)^{\frac{11}{6}}\right)^{\frac{5}{33}}.$$

其中参数为

$$l_{21} = \frac{5}{11}\left(\frac{18}{11}\right)^{\frac{6}{5}} 2^{\frac{1}{5}} = 0.9428,$$
$$\beta_1 = 3^{\frac{11}{10}},$$
$$l_{22} = \frac{17}{22}\left(\frac{15}{22}\right)^{\frac{5}{17}} (1 + x_1^2)^{\frac{22}{17}} = 0.6904(1 + x_1^2)^{\frac{22}{17}},$$
$$l_{23} = \frac{6}{11}\left(\frac{30}{11}\right)^{\frac{5}{6}} \left(\frac{17}{11}\right)^{\frac{11}{6}} (3^{\frac{121}{60}} + 3^{\frac{121}{30}}) = 260.5114.$$

无干扰下仿真结果如图1所示.



(a)　x_1

(b)　x_2

(c)　Control of AOPIC

(d)　Control of fast AOPIC

图1　无干扰时仿真结果

在图1(a)中，AOPIC对应的x_1大约在1.8 s后收敛到0，而本文提出的FAOPIC对应的x_1大约在0.6 s后收敛到0. 图1(b)中，AOPIC对应的x_2在2.9 s后收敛到0，而FAOPIC对应的x_2在1.1 s后收敛到0. 所以FAOPIC对应的状态收敛速度明显快于AOPIC. 图1(c)是AOPIC控制器曲线图. 为了使控制量足够大，以保证被控状态收敛速度足够大，选取足够大的参数，如$l_{23} = 260.5114$. 但过大的参数使得控制器曲线出现明显的抖振. 该控制器上的抖振直接作用在第二阶子系统上，引起x_2的轨迹也有微小抖振，影响了稳态误差的精度. 图1(d)是FAOPIC的控制器曲线，由于控制器结构不同，无需很大的参数便可保证被控系

统状态快速收敛,控制器不会有抖振产生. AOPIC和FAOPIC的差距在收敛速度和稳态精度两方面都有所体现.

在外干扰作用下,对AOPIC和FAOPIC控制效果进行对比仿真,假定初值为$x_1(0) = -2$, $x_2(0) = 4$. 外干扰为$d(t) = \sin t$,两种方案下控制器参数同上,仿真结果如图2所示.

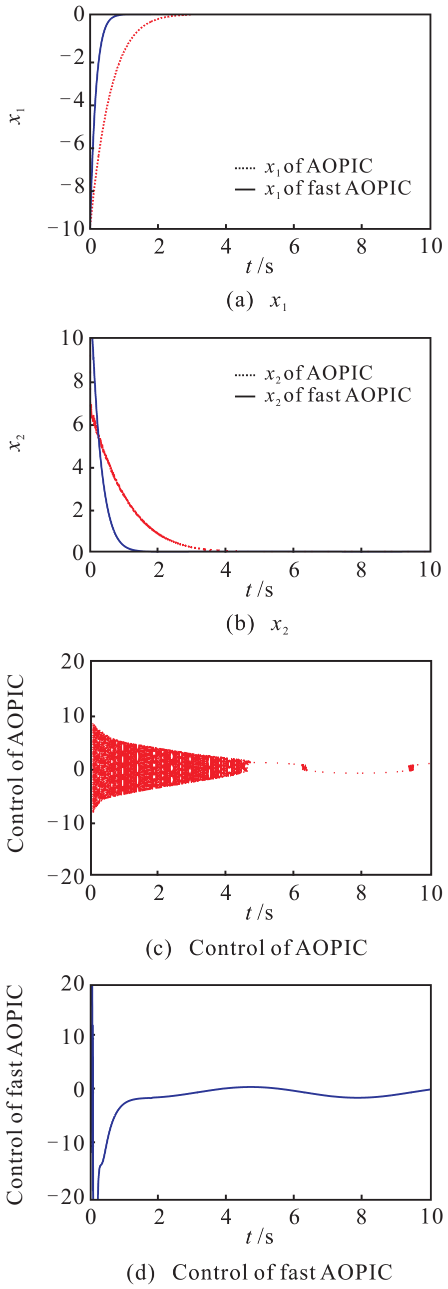


图2 考虑干扰及大初始误差时的仿真结果

由图2可见,虽然两种控制方案均具有鲁棒性,但本文所提出的方案仍然在收敛速度、稳态精度、控制器无抖振等方面具有明显的优势.

5 结 论

本文针对一类非匹配不确定的二阶非线性系统提出的FAOPIC,是一种结合AOPIC和快速Lyapunov理论得到的非线性控制方案,改善了系统状态远离原点阶段的收敛速度和鲁棒性,同时保留了AOPIC用于二阶系统非奇异性、有限时间收敛性、无抖振的优点. 比传统AOPIC收敛速度更快、稳态精度更高、动态过程更短,对参数选择的限制也被放宽. 所以,FAOPIC能够使得AOPIC性能在全论域内被进一步提升.

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