第33卷 第12期
2018年 12月

控 制 与 决 策
Control and Decision

Vol.33 No.12
Dec. 2018

文章编号: 1001-0920(2018)12-2113-09 **DOI:** 10.13195/j.kzyjc.2017.0933

具有未建模动态和输出约束的耦合系统的分散自适应控制

张天平†, 王 敏

(扬州大学 信息工程学院，江苏 扬州 225127)

摘 要: 针对一类具有输入、状态未建模动态和非线性输入的耦合系统,提出一种自适应神经网络控制方案. 利用径向基函数神经网络逼近未知非线性连续函数;引入动态信号和正则化信号处理状态及输入未建模动态;通过引入非线性映射,将具有时变输出约束的严格反馈系统化为不含约束的严格反馈系统. 最后,通过理论分析验证闭环系统中所有信号是半全局一致最终有界的,仿真结果进一步验证了所提出控制方案的有效性.

关键词: 耦合系统；时变输出约束；非线性输入；未建模动态；非线性映射

中图分类号: TP13 **文献标志码:** A

Decentralized adaptive control for interconnected systems with unmodeled dynamics and output constraints

ZHANG Tian-ping†，*WANG Min*

(College of Information Engineering，Yangzhou University，Yangzhou 225127，China)

Abstract: An adaptive neural network control scheme is proposed for a class of interconnected systems with state and input unmodeled dynamics as well as input nonlinearity. The unknown nonlinear continuous functions are approximated by radial basis function neural networks(RBFNNs). State and input unmodeled dynamics are dealt with by introducing dynamic signal and normalization signal. The strict-feedback system with time-varying output constraint is transformed into a novel strict-feedback system without constraint by introducing one to one nonlinear mapping. By theoretical analysis, all the signals in the closed-loop control system are proved to be semi-globally uniformly ultimately bounded. Simulation results show the effectiveness of the proposed approach.

Keywords: interconnected systems；time-varying output constraint；input nonlinearity；unmodeled dynamics；nonlinear mapping

0 引 言

随着科技的快速发展,在实际的工业生产与社会生活中,所遇到的系统规模越来越大. 为了解决这些系统在实际应用中出现的诸多问题,从20世纪60年代起人们就开始了对“大规模系统”(简称大系统)进行研究,并取得了丰硕成果[1-7]. 文献[1-2]中研究了一类具有结构相似的非线性规范形大系统. 文献[3]针对一类具有未建模动态的输出反馈非线性耦合系统,提出两种自适应控制方案——集中控制与分散控制. 文献[4]针对一类具有随机切换的非线性耦合系统,基于后推设计的控制方法提出一种自适应控制方案. 文献[5]针对一类具有未建模动态的非线性强耦合系统,基于后推设计提出一种分散自适应模糊控制方案. 文献[6]针对一类具有未建模动态和死区输入的非线性互联系统,提出一种自适应模糊控制方案,利用模糊逻辑系统估计系统中不可量测的状态. 文献[7]针对一类具有动态输入输出和非线性耦合项的耦合系统,基于后推设计与MT滤波器设计自适应控制器,提出一种分散自适应控制方案.

由于传感器的精度限制,系统建模时总会出现各种各样的误差,比如测量噪声、建模误差等等,将这些误差统称为未建模动态. 文献[8-9]针对一类具有状态未建模的非线性系统,分别利用动态信号和Lyapunov函数去处理系统中未建模动态. 对于工程应用背景而言,系统不仅仅有状态未建模,同时也有可能存在输入未建模动态. 简单地说,输入未建模可

收稿日期: 2017-07-12；**修回日期:** 2017-11-03.

基金项目: 国家自然科学基金项目(61573307)；扬州大学高端人才支持计划项目(2016).

责任编委: 解学军.

作者简介: 张天平(1964−), 男, 教授, 博士生导师, 从事非线性系统控制、鲁棒自适应控制等研究；王敏(1992−), 男, 硕士生, 从事非线性系统的自适应控制的研究.

†通讯作者. E-mail: tpzhang@yzu.edu.cn

以理解为执行机构的未建模动态. 文献[10-14]给出了一类含有输入未建模动态系统的研究成果. 文献[12-14]针对一类具有输入未建模的输出反馈控制系统,提出一种自适应跟踪控制方案. 文献[10]针对一类具有输入及状态未建模的非线性切换系统,提出一种自适应有限时间跟踪控制方案. 文献[11]分析了具有输入未建模动态影响的严格反馈系统.

工业控制系统中往往需要对系统的输出进行约束,一旦系统输出违背约束条件,则系统的动态性能将会受到严重影响,甚至会导致系统不稳定. 为了解决输出约束问题,专家学者们提出许多方法,比如模型预测控制、定值调节器、误差转化函数. 但是,上述方法均需要进行复杂的数学计算. 为了方便设计,一种基于障碍李雅普诺夫函数(BLF)的方法被提出用于处理输出约束问题. 该方法最早被应用于解决具有Brunovsky形式[15]的系统输出约束问题. 文献[16]针对一类完全匹配的严格反馈非线性系统,分别基于BLF和ABLF(非对称障碍李雅普诺夫函数),提出两种自适应控制方案. 文献[17]针对一类具有有界扰动的输出反馈非线性系统,基于BLF提出一种自适应控制方案. 构造K滤波器与状态观测器对系统不可量测的状态进行重构和估计,文献[18]针对一类具有严格反馈形式的非线性系统,基于ABLF提出一种自适应控制方案. 文献[19]针对一类不确定高阶非线性切换系统,基于CUF、P-DUF并利用齐次控制方法,提出一种鲁棒自适应控制策略.

本文在文献[20-21]的基础上,针对一类具有输入及状态未建模动态和死区输入的耦合系统,提出一种自适应神经网络动态面控制方案. 本文主要创新点如下: 1) 与文献[20]中的定常全状态约束相比,本文放宽了约束条件,考虑了非对称时变输出约束问题; 2) 与文献[15-18]相比,本文通过引入非线性映射处理时变输出约束问题; 3) 与文献[5]相比,本文不仅考虑了状态未建模,同时也考虑了输入未建模动态与死区输入; 4) 将原有的控制增益$g_{i,m_i}(\bar{x}_{i,m_i})$与输入未建模子系统中的控制增益$d_{i,\Delta}$以及死区线性部分斜率$l_i$合并成一个控制增益$G_{i,m_i}(\bar{S}_{i,m_i})$,并用Nussbaum函数处理符号未知问题.

1 问题描述与预备知识

1.1 问题描述与基本假设

考虑如下一类具有未建模动态和非线性输入的N个子系统构成的耦合系统Σ,其中第i个子系统Σ_i如下:

$$\begin{cases}\dot{z}_i = q_i(z_i, x_i, t),\\ \dot{x}_{i,j} = f_{i,j}(\bar{x}_{i,j}) + g_{i,j}(\bar{x}_{i,j})x_{i,j+1} + \Delta_{i,j}(z_i, x_i, t),\\ \dot{x}_{i,m_i} = f_{i,m_i}(\bar{x}_{i,m_i}) + g_{i,m_i}(\bar{x}_{i,m_i})\Phi(\hbar_i) +\\ \qquad \Delta_{i,m_i}(z_i, x_i, t) + d_i(X, t),\\ y_i = x_{i,1},\ i = 1, 2, \cdots, N,\ j = 1, 2, \cdots, m_i - 1.\end{cases} \tag{1}$$

其中: $x_{i,j}$是系统的状态变量, $x_i = [x_{i,1}, \cdots, x_{i,m_i}]^{\mathrm{T}} \in R^{m_i}$, $\bar{x}_{i,j} = [x_{i,1}, \cdots, x_{i,j}]^{\mathrm{T}} \in R^j, 1 \leqslant j \leqslant m_i$; $X = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \cdots, x_N^{\mathrm{T}}]^{\mathrm{T}} \in R^m\Big(m = \sum\limits_{i=1}^{N} m_i\Big)$是耦合系统的全部状态变量; y_i是第i个子系统的输出; $f_{i,j}(\bar{x}_{i,j})$和$g_{i,j}(\bar{x}_{i,j})$是未知连续非线性函数; $\Delta_{i,j}(z_i, x_i, t)$是非线性动态扰动,且$\Delta_{i,j}(z_i, x_i, t)$和$q_i(z_i, x_i, t)$是满足Lispchitz条件的未知函数; $d_i(X, t)$是系统互联项; $\Phi_i(\hbar_i)$是受死区特性影响的输出,表达式如下:

$$\Phi_i(\hbar_i) = l_i\hbar_i + Q_i(\hbar_i). \tag{2}$$

$$Q_i(\hbar_i) = \begin{cases}-l_i b_{i,r},\ \hbar_i \geqslant b_{i,r};\\ -l_i\hbar_i,\ b_{i,l} < \hbar_i < b_{i,r};\\ -l_i b_{i,l},\ \hbar_i \leqslant b_{i,l}.\end{cases} \tag{3}$$

$b_{i,l}$和$b_{i,r}$是死区的左右截点; l_i是死区线性部分的斜率; $\hbar_i$是死区输入,同时也是输入未建模动态子系统的输出.

输入未建模动态子系统描述如下:

$$\begin{cases}\dot{\xi}_i = A_{i,\Delta}(\xi_i) + b_{i,\Delta}u_i,\\ \hbar_i = c_{i,\Delta}(\xi_i) + d_{i,\Delta}u_i.\end{cases} \tag{4}$$

其中: u_i是系统的实际控制输入, ξ_i是由输入u_i驱动的未建模动态的状态, $A_{i,\Delta}(\cdot)$和$b_{i,\Delta}$是未知向量, $c_{i,\Delta}(\cdot)$是未知函数, $d_{i,\Delta}$是未知常数.

控制目标是,针对每一个子系统设计一个自适应控制律u_i,使得每一个子系统的输出$y_i(t)$尽可能地跟踪给定的参考信号$y_{di}(t)$,且输出$y_i(t)$满足时变输出约束条件,即$-k_{i,b_1}(t) < y_i(t) < k_{i,a_1}(t)$,同时保证闭环系统所有信号有界. 其中$k_{i,a_1}(t)$和$k_{i,b_1}(t)$是已知正函数.

假设1[22] $g_{i,j}(\bar{x}_{i,j})(j = 1, 2, \cdots, m_i - 1)$的符号已知, $g_{i,m_i}(\bar{x}_{i,m_i})$的符号未知,且存在已知正常数$\underline{g}_{i,j}, \bar{g}_{i,j}, \bar{g}_{i,m_i}$和未知正常数$\underline{g}_{i,m_i}$,使得$\underline{g}_{i,j} \leqslant |g_{i,j}(\bar{x}_{i,j})| \leqslant \bar{g}_{i,j},\ j = 1, 2, \cdots, m_i$. 不失一般性,假设$\underline{g}_{i,j} \leqslant g_{i,j}(\bar{x}_{i,j}) \leqslant \bar{g}_{i,j},\ j = 1, 2, \cdots, m_i - 1, i = 1, 2, \cdots, N$.

假设2[23] 死区参数$b_{i,r}, b_{i,l}$和l_i是未知有界非零正常数,且符号已知.

注1 根据假设2,可以推出式(3)中$|Q_i(\hbar_i)| \leqslant$

D_i, D_i是一个未知正常数.

假设3[8] 未知非线性动态扰动$\Delta_{i,j}(z_i,x_i,t)(i=1,2,\cdots,N,\ j=1,2,\cdots,m_i)$满足如下不等式:

$$|\Delta_{i,j}(z_i,x_i,t)|\leqslant\varphi_{i,j,1}(|x_{i,j}|)+\varphi_{i,j,2}(\|z_i\|). \tag{5}$$

其中: $\varphi_{i,j,1}(\cdot)$是未知光滑函数, $\varphi_{i,j,2}(\cdot)$是未知非负的连续单调增函数, $\|\cdot\|$是向量的欧氏范数.

假设4[8] 未建模动态z_i是指数输入状态实用稳定的(exp-ISpS).

假设5[24] 参考输入$X_{di}=[y_{di},\dot{y}_{di},\ddot{y}_{di}]^{\mathrm{T}}\in\Pi_{i,0}$光滑可导. $\Pi_{i,0}=\{X_{di}: y_{di}^2+\dot{y}_{di}^2+\ddot{y}_{di}^2\leqslant B_{i,0}\}$, $|y_{di}(t)|<B_{i,1}(t)<\min\{k_{i,b_1}(t),k_{i,a_1}(t)\}$,其中$B_{i,0}$为已知正常数, $B_{i,1}(t)$为已知正函数.

假设6[25] 存在未知非负系数$\varsigma_{i,j}^k$,使得系统互联项满足如下不等式:

$$|d_i(X,t)|\leqslant\sum_{j=1}^{N}\varsigma_{i,j}^0+\sum_{j=1}^{N}\sum_{k=1}^{p}\varsigma_{i,j}^k\|\bar{x}_j\|^k, \tag{6}$$

其中p是一个已知正常数.

假设7[18] 存在已知正常数$\bar{k}_{i,b_1},\bar{k}_{i,a_1},\underline{k}_{i,b_1}$和$\underline{k}_{i,a_1}$,使$\bar{k}_{i,b_1}\geqslant k_{i,b_1}(t),\bar{k}_{i,a_1}\geqslant k_{i,a_1}(t),|\dot{k}_{i,b_1}|\leqslant\underline{k}_{i,b_1}$, $|\dot{k}_{i,a_1}|\leqslant\underline{k}_{i,a_1}$成立.

假设8[11] 对于输入未建模子系统(4),其相对阶数为零,即$d_{i,\Delta}\neq0$;并存在一个未知正常数$\bar{c}_{i,\Delta}>0$,使得$\|c_{i,\Delta}(\xi_i(t))\|\leqslant\bar{c}_{i,\Delta}\|\xi_i(t)\|$成立.

假设9[11] 存在Lyapunov函数$W_i(\xi_i)$满足

$$\begin{cases}\gamma_{i,1}\|\xi_i\|^2\leqslant W_i(\xi_i)\leqslant\gamma_{i,2}\|\xi_i\|^2,\\ \dfrac{\partial W_i}{\partial\xi_i}A_{i,\Delta}(\xi_i)\leqslant-2\delta_{i,0}W_i(\xi_i),\\ \left\|\dfrac{\partial W_i}{\partial\xi_i}\right\|\leqslant\gamma_{i,3}\|\xi_i\|.\end{cases} \tag{7}$$

其中: $\gamma_{i.1},\gamma_{i,2},\gamma_{i,3}$是正常数; $\delta_{i,0}$是已知正常数.

引理1[8] 如果V_i是子系统$\dot{z}_i=q_i(z_i,x_i,t)$的指数输入状态实用稳定(exp-ISpS)的Lyapunov函数,即不等式$\alpha_{i,1}(\|z_i\|)\leqslant V_i(z_i)\leqslant\alpha_{i,2}(\|z_i\|)$和

$$\frac{\partial V_i(z_i)}{\partial z_i}q_i(z_i,x_i,t)\leqslant-c_iV_i(z_i)+\gamma_i(|x_{i,1}|)+d_{i,0} \tag{8}$$

成立,则对于任意常数$\bar{c}_i\in(0,c_i)$,任意初始时刻$t_0>0$,任意初始状态$z(t_0)=z_{i,0},v_{i,0}>0$以及任意连续函数$\bar{\gamma}_i(\cdot)$满足$\bar{\gamma}_i(|x_{i,1}|)\geqslant\gamma_i(|x_{i,1}|)$,存在有限时间$T_{i,0}=\max\{0,\ln[V(z_{i,0})/v_{i,0}]/(c_i-\bar{c}_i)\}\geqslant0$,一个可量测的动态信号$v_i>0$. 其中$\alpha_{i,1}(\cdot),\alpha_{i,2}(\cdot),\gamma_i(\cdot)$是$K_\infty$类函数, $c_i,d_{i,0}$是已知正常数.定义动态信号$\dot{v}_i=-\bar{c}_iv_i+\bar{\gamma}_i(|x_{i,1}|)+d_{i,0},v_i(t_0)=v_{i,0}>0$,以及一个非负函数$D_i(t_0,t)$,当$t\geqslant t_0+T_{i,0}$时,有$D_i(t_0,t)=0$,使得$V_i(z)\leqslant v_i(t)+D_i(t_0,t)$. 不失一般性,取$\bar{\gamma}_i(|x_{i,1}|)=\gamma_i(|x_{i,1}|)$.

引理2[26] 对于任意连续实函数$f(x,y)$,存在光滑标量函数$\phi(x)\geqslant0$以及标量函数$\psi(y)\geqslant0$满足

$$|f(x,y)|\leqslant\phi(x)+\psi(y).$$

其中: $x\in R^m,y\in R^n$.

引理3[12] 考虑由式(4)组成的系统以及如下一阶系统:

$$\dot{\bar{m}}_i=-\delta_{i,0}\bar{m}_i+|u_i|, \tag{9}$$

若假设9成立,则对于任意的$t\in[0,T],u_i(t)\in L_\infty[0,T]$,存在常数$\chi_{i,1},\chi_{i,2}>0$,使得如下不等式成立:

$$\|\xi_i(t)\|\leqslant$$
$$\chi_{i,1}(\|\xi_i(0)\|+|\bar{m}_i(0)|)\mathrm{e}^{-\delta_{i,0}t}+\chi_{i,2}|\bar{m}_i(t)|. \tag{10}$$

1.2 Nussbaum函数

定义1[27] 如果连续函数$N(\zeta)$满足如下性质:

$$\lim_{s\to\infty}\sup\frac{1}{s}\int_0^sN(\zeta)\mathrm{d}\zeta=+\infty,$$
$$\lim_{s\to\infty}\inf\frac{1}{s}\int_0^sN(\zeta)\mathrm{d}\zeta=-\infty, \tag{11}$$

则称$N(\zeta)$是具有Nussbaum形式的函数. 本文选用如下形式的Nussbaum型函数: $\mathrm{e}^{\zeta^2}\cos(\pi\zeta/2)$.

引理4[27] 设$V(\cdot)$和$\zeta(\cdot)$是定义在$[0,t_f)$上的光滑函数,满足$\forall t\in[0,t_f),V(t)\geqslant0,N(\zeta)$为一个光滑的Nussbaum型函数,若$\forall t\in[0,t_f)$,下列不等式:

$$0\leqslant V(t)\leqslant b_0+\mathrm{e}^{-b_1t}\int_0^t(g(x(\tau)N(\zeta)+1)\dot{\zeta}\mathrm{e}^{b_1\tau}\mathrm{d}\tau, \tag{12}$$

成立,则$V(t),\zeta(t),\int_0^tg(x(\tau))N(\zeta)\dot{\zeta}\mathrm{d}\tau$有界. 其中: b_0为适当的常数, b_1为正常数, $g(x(\tau))$为时变参数在闭区间$R=[I^-,I^+]$且$0\notin R$内取值.

1.3 非线性映射

为了处理系统中的时变输出约束问题,引入如下非线性映射:

$$\begin{cases}S_{i,1}=\log\Big[\dfrac{k_{i,b_1}(t)+x_{i,1}}{k_{i,a_1}(t)-x_{i,1}}\Big];\\ S_{i,j}=x_{i,j},\ j=2,3,\cdots,m_i;\\ Y_i=S_{i,1}.\end{cases} \tag{13}$$

则经过上式变换,原系统可转换成如下系统:

$$\begin{cases}\dot{z}_i=q_i(z_i,S_i,t),\\ \dot{S}_{i,1}=\mu_i[f_{i,1}(S_{i,1})+g_{i,1}(S_{i,1})S_{i,2}+\\ \qquad\Delta_{i,1}(z_i,S_i,t)]+\kappa_i,\\ \dot{S}_{i,2}=f_{i,1}(\bar{S}_{i,2})+g_{i,1}(\bar{S}_{i,2})S_{i,3}+\Delta_{i,2}(z_i,S_i,t),\\ \quad\vdots\\ \dot{S}_{i,m_i}=f_{i,1}(\bar{S}_{i,m_i})+g_{i,1}(\bar{S}_{i,m_i})\varPhi(\hbar_i)+d_i(S,t)+\\ \qquad\Delta_{i,m_i}(z_i,S_i,t),\\ Y_i=S_{i,1}.\end{cases} \tag{14}$$

其中

$$\mu_i=\frac{(\mathrm{e}^{S_{i,1}}+1)^2}{(k_{i,b_1}(t)+k_{i,a_1}(t))\mathrm{e}^{S_{i,1}}};$$

$$\kappa_i=\frac{(\mathrm{e}^{S_{i,1}}+1)(\dot{k}_{i,b_1}-\dot{k}_{i,a_1}\mathrm{e}^{S_{i,1}})}{(k_{i,b_1}(t)+k_{i,a_1}(t))\mathrm{e}^{S_{i,1}}};$$

$$S_i=[S_{i,1},S_{i,2},\cdots,S_{i,m_i}]^{\mathrm{T}}\in R^{m_i},\ i=1,2,\cdots,N;$$

$$\bar{S}_{i,j}=[S_{i,1},S_{i,2},\cdots,S_{i,j}]^{\mathrm{T}}\in R^{j},\ 1\leqslant j\leqslant m_i;$$

$$S=[S_1^{\mathrm{T}},S_2^{\mathrm{T}},\cdots,S_N^{\mathrm{T}}]^{\mathrm{T}}\in R^{m},\ m=\sum_{i=1}^{N}m_i.$$

以下设计过程均基于系统(14).

1.4 径向基函数神经网络(RBFNNs)

本文中使用RBFNNs在紧集$\Pi_{z_{i,j}}\subset R^{q_{i,j}}$上逼近一个未知非线性连续函数,即

$$H_{i,j}(Z_{i,j})=W_{i,j}^{\mathrm{T}}\Psi_{i,j}(Z_{i,j})+e_{i,j}(Z_{i,j}).$$

其中: $Z_{i,j}$是输入向量; $e_{i,j}(Z_{i,j})$是逼近误差, $W_{i,j}=[w_{i,j,1},w_{i,j,2},\cdots,w_{i,j,l_i}]^{\mathrm{T}}\in R^{l_i}$是未知权向量; 满足$|e_{i,j}(Z_{i,j})|\leqslant e_i^*$; $\Psi_{i,j}(Z_{i,j})=[\psi_{i,j,1}(Z_{i,j}),\psi_{i,j,2}(Z_{i,j}),\cdots,\psi_{i,j,l_i}(Z_{i,j})]^{\mathrm{T}}\in R^{l_i}$是基函数向量, $l_i>1$是第i个神经元节点数,通常取高斯函数$\psi_{i,j,k}(Z_{i,j})=\exp[-(Z_{i,j}-\mu_{i,j,k})^2/b_{i,j,k}]$.

$\mu_{i,j,k}$和$b_{i,j,k}$分别为基函数的中心与高斯函数的宽度, $1\leqslant i\leqslant N$, $1\leqslant j\leqslant m_i$, $1\leqslant k\leqslant l_i$; 理想权值$W_{i,j}^*$定义为

$$W_{i,j}^*=\arg\min_{W_{i,j}\in R^{l_i},Z_{i,j}\in\Pi_{z_{i,j}}}\{|H_{i,j}(Z_{i,j})-W_{i,j}^{\mathrm{T}}\psi_{i,j}(Z_{i,j})|\}.$$

2 自适应动态面控制器设计

定义$y_{i,j+1}=\omega_{i,j+1}-\alpha_{i,j}$, $S_{i,j+1}=y_{i,j+1}+s_{i,j+1}+\alpha_{i,j}$, $\theta_{i,j}=\|W_{i,j}\|^2$, 其中$\omega_{i,j+1}$, $\alpha_{i,j}$稍后给出.

Step 1: 定义第1个动态面误差

$$s_{i,1}=S_{i,1}-Y_{di}. \tag{15}$$

其中

$$Y_{di}=\log\Big[\frac{k_{i,b_1}(t)+y_{di}}{k_{i,a_1}(t)-y_{di}}\Big],$$

$$\dot{s}_{i,1}=\mu_i[f_{i,1}(S_{i,1})+g_{i,1}(S_{i,1})S_{i,2}+\Delta_{i,1}(z_i,S_i,t)]+\kappa_i-\dot{Y}_{di}. \tag{16}$$

取如下Lyapunov函数:

$$V_{i,s_1}=\frac{1}{2}s_{i,1}^2, \tag{17}$$

将V_{i,s_1}对时间t求导数,并将式(16)代入得

$$\dot{V}_{i,s_1}=s_{i,1}\{\mu_i[f_{i,1}(S_{i,1})+g_{i,1}(S_{i,1})S_{i,2}+\Delta_{i,1}(z_i,S_i,t)]+\kappa_i-\dot{Y}_{di}\}. \tag{18}$$

利用Young's不等式对式(18)进行化简,得

$$\mu_i g_{i,1}(S_{i,1})s_{i,1}s_{i,2}\leqslant\frac{1}{4}\bar{g}_{i,1}^2s_{i,2}^2+s_{i,1}^2\mu_i^2, \tag{19}$$

$$\mu_i g_{i,1}(S_{i,1})s_{i,1}y_{i,2}\leqslant\frac{1}{4}\bar{g}_{i,1}^2y_{i,2}^2+\mu_i^2s_{i,1}^2, \tag{20}$$

$$|s_{i,1}\mu_i\Delta_{i,1}(z_i,S_i,t)|\leqslant \mu_i|s_{i,1}|[\varphi_{i,1,1}(|S_{i,1}|)+\varphi_{i,1,2}(\|z_i\|)], \tag{21}$$

$$|s_{i,1}\mu_i\varphi_{i,1,1}(|x_{i,1}|)|\leqslant \frac{\mu_i^2s_{i,1}^2\varphi_{i,1,1}^2(|S_{i,1}|)}{a_{i,1,1}^2}+\frac{a_{i,1,1}^2}{4}, \tag{22}$$

其中$a_{i,1,1}$为已知正常数.

由假设4和引理1、引理2可得

$$\mu_i|s_{i,1}|\varphi_{i,1,2}(\|z_i\|)\leqslant \mu_i|s_{i,1}|\phi_{i,1}(v_i(t))+\mu_i|s_{i,1}|\vartheta_{i,1}(D_{i,1}(t_0,t)). \tag{23}$$

因此,利用Young's不等式化简式(23),可得

$$\mu_i|s_{i,1}|\phi_{i,1}(v_i(t))\leqslant\frac{\mu_i^2s_{i,1}^2\phi_{i,1}^2(v_i(t))}{a_{i,1}^2}+\frac{a_{i,1}^2}{4}, \tag{24}$$

$$\mu_i|s_{i,1}|\vartheta_{i,1}(D_{i,1}(t_0,t))\leqslant\mu_i^2s_{i,1}^2+\frac{\vartheta_{i,1}^2(D_{i,1}(t_0,t))}{4}, \tag{25}$$

其中$a_{i,1}$为已知正常数. 由于$D_{i,1}(t_0,t)$与$\vartheta_{i,1}(\cdot)$为连续函数,存在一个已知正常数$\vartheta_{i,1}^*$使得不等式$\vartheta_{i,1}^2(D_{i,1}(t_0,t))\leqslant{\vartheta_{i,1}^*}^2$成立.

将式(19)$\sim$(25)代入(18),同时令

$$H_{i,1}(Z_{i,1})=\mu_if_{i,1}(S_{i,1})-\dot{Y}_{di}+\frac{\mu_i^2s_{i,1}\varphi_{i,1,1}^2(|S_{i,1}|)}{a_{i,1,1}^2}+\frac{\mu_i^2s_{i,1}\phi_{i,1}^2(v_i(t))}{a_{i,1}^2}+3\mu_i^2s_{i,1}+\frac{s_{i,1}\varepsilon_{i,\bar{r}}^{-2}\gamma_{i,1}(|S_{i,1}|)}{\lambda_{i,0}}+\kappa_i, \tag{26}$$

其中

$$Z_{i,1}=[S_{i,1},s_{i,1},v_i,y_{di},k_{i,b_1},k_{i,a_1},\dot{k}_{i,b_1},\dot{k}_{i,a_1}]^{\mathrm{T}}.$$

设计虚拟控制律$\alpha_{i,1}$和参数$\hat{\theta}_{i,1}$的自适应律如下:

$$\alpha_{i,1}=\Big[-k_{i,1}s_{i,1}-\frac{s_{i,1}\hat{\theta}_{i,1}\|\Psi_{i,1}(Z_{i,1})\|^2}{2a_{i,0,1}^2}\Big]\frac{(\bar{k}_{i,b_1}+\bar{k}_{i,a_1})}{4\underline{g}_{i,1}}, \tag{27}$$

$$\dot{\hat{\theta}}_{i,1}=r_{i,1}\Big[\frac{s_{i,1}^2\|\Psi_{i,1}(Z_{i,1})\|^2}{2a_{i,0,1}^2}-\sigma_{i,1}\hat{\theta}_{i,1}\Big], \tag{28}$$

其中$\hat{\theta}_{i,j}$是$\theta_{i,j}$在t时刻的估计值.

取如下Lyapunov函数:

$$V_{i,sv}=V_{i,s_1}+\frac{v_i}{\lambda_{i,0}}, \tag{29}$$

其中

$$\dot{v}_i = -\bar{c}_i v_i + \bar{\gamma}_i(|S_{i,1}|) + d_{i,0}.$$

对式(29)求导,并将(26)和(27)代入,得

$$\dot{V}_{i,sv} \leqslant -(k_{i,1}-1)s_{i,1}^2 - \frac{s_{i,1}^2\tilde{\theta}_{i,1}\|\varPsi_{i,1}(Z_{i,1})\|^2}{2a_{i,0,1}^2} - \frac{\bar{c}_i v_i}{\lambda_{i,0}} + \frac{\bar{g}_{i,1}^2 y_{i,2}^2}{4} + \frac{\bar{g}_{i,1}^2 s_{i,2}^2}{4} + \Big(1-\frac{s_{i,1}^2}{\varepsilon_{i,\bar{r}}^2}\Big)\frac{\gamma_i(|S_{i,1}|)}{\lambda_{i,0}} + D_{i,1}, \tag{30}$$

其中

$$D_{i,1} = \frac{a_{i,1,1}^2}{4} + \frac{a_{i,1}^2}{4} + \frac{{\vartheta_{i,1}^*}^2}{4} + \frac{a_{i,0,1}^2}{2} + \frac{e_i^{*2}}{4} + \frac{d_{i,0}}{\lambda_{i,0}}.$$

构造如下一阶滤波器:

$$\tau_{i,2}\dot{\omega}_{i,2} + \omega_{i,2} = \alpha_{i,1},\ \omega_{i,2}(0) = \alpha_{i,1}(0), \tag{31}$$

其中$\tau_{i,2}$为时间常数. 又因$y_{i,2} = \omega_{i,2} - \alpha_{i,1}$,有

$$\dot{y}_{i,2} = \dot{\omega}_{i,2} - \dot{\alpha}_{i,1} = -\frac{y_{i,2}}{\tau_{i,2}} - \dot{\alpha}_{i,1}, \tag{32}$$

可得

$$\Big|\dot{y}_{i,2} + \frac{y_{i,2}}{\tau_{i,2}}\Big| \leqslant \eta_{i,2}(s_{i,1}, s_{i,2}, y_{i,2}, \hat{\theta}_{i,1}, v_i, y_{di}, \dot{y}_{di}, S_{i,1}, k_{i,b_1}, k_{i,a_1}, \dot{k}_{i,b_1}, \dot{k}_{i,a_1}), \tag{33}$$

于是有

$$y_{i,2}\dot{y}_{i,2} \leqslant -\frac{y_{i,2}^2}{\tau_{i,2}} + y_{i,2}^2 + \frac{\eta_{i,2}^2(\cdot)}{4}, \tag{34}$$

Step $\boldsymbol{j}$: 定义第$j(j=2,3,\cdots,m_i-1)$个动态面误差

$$s_{i,j} = S_{i,j} - \omega_{i,j}, \tag{35}$$

有

$$\dot{s}_{i,j} = f_{i,j}(\bar{S}_{i,j}) + g_{i,j}(\bar{S}_{i,j})S_{i,j+1} + \varDelta_{i,j}(z_i, S_i, t) - \dot{\omega}_{i,j}.$$

取如下Lyapunov函数:

$$V_{i,s_j} = \frac{1}{2}s_{i,j}^2. \tag{36}$$

将V_{i,s_j}对时间t求导,得

$$\dot{V}_{i,s_j} = s_{i,j}[f_{i,j}(\bar{S}_{i,j}) + g_{i,j}(\bar{S}_{i,j})(y_{i,j+1} + s_{i,j+1} + \alpha_{i,j}) + \varDelta_{i,j}(z_i, S_i, t) - \dot{\omega}_{i,j}]. \tag{37}$$

类似Step 1,利用Young's不等式化简式(37),同时令

$$H_{i,j}(Z_{i,j}) = f_{i,j}(\bar{S}_{i,j}) - \dot{\omega}_{i,j} + \frac{s_{i,j}\varphi_{i,j,1}^2(|\bar{S}_{i,j}|)}{a_{i,j,1}^2} + \frac{s_{i,j}\phi_{i,j}^2(v_i(t))}{a_{i,j}^2} + 3s_{i,j}, \tag{38}$$

其中$Z_{i,j} = [\bar{S}_{i,j}^{\rm T}, s_{i,j}, v_i, \dot{\omega}_{i,j}]^{\rm T}$.

设计虚拟控制律$\alpha_{i,j}$和参数$\hat{\theta}_{i,j}$的自适应律如下:

$$\alpha_{i,j} = \frac{1}{g_{i,j}}\Big[-k_{i,j}s_{i,j} - \frac{s_{i,j}\hat{\theta}_{i,j}\|\varPsi_{i,j}(Z_{i,j})\|^2}{2a_{i,0,j}^2}\Big], \tag{39}$$

$$\dot{\hat{\theta}}_{i,j} = r_{i,j}\Big[\frac{s_{i,j}^2\|\varPsi_{i,j}(Z_{i,j})\|^2}{2a_{i,0,j}^2} - \sigma_{i,j}\hat{\theta}_{i,j}\Big]. \tag{40}$$

将式(38)和(39)代入(37),利用Young's不等式化简得

$$V_{i,s_j} \leqslant -(k_{i,j}-1)s_{i,j}^2 - \frac{s_{i,j}^2\tilde{\theta}_{i,j}\|\varPsi_{i,j}(Z_{i,j})\|^2}{2a_{i,0,j}^2} + \frac{\bar{g}_{i,j}^2 s_{i,j+1}^2}{4} + \frac{\bar{g}_{i,j}^2 y_{i,j+1}^2}{4} + D_{i,j}, \tag{41}$$

其中$D_{i,j} = \dfrac{a_{i,j,1}^2}{4} + \dfrac{{\vartheta_{i,j}^*}^2}{4} + \dfrac{a_{i,0,j}^2}{2} + \dfrac{e_i^{*2}}{4} + \dfrac{a_{i,j}^2}{4}$.

构造如下一阶滤波器:

$$\tau_{i,j+1}\dot{\omega}_{i,j+1} + \omega_{i,j+1} = \alpha_{i,j},\ \omega_{i,j+1}(0) = \alpha_{i,j}(0), \tag{42}$$

其中$\tau_{i,j+1}$为时间常数. 类似Step 1可得

$$y_{i,j+1}\dot{y}_{i,j+1} \leqslant -\frac{y_{i,j+1}^2}{\tau_{i,j+1}} + y_{i,j+1}^2 + \frac{\eta_{i,j+1}^2(\cdot)}{4}. \tag{43}$$

Step $\boldsymbol{m_i}$: 定义第m_i个动态面误差为

$$s_{i,m_i} = S_{i,m_i} - \omega_{i,m_i}, \tag{44}$$

有

$$\dot{s}_{i,m_i} = f_{i,m_i}(\bar{S}_{i,m_i}) + g_{i,m_i}(\bar{S}_{i,m_i})[l_i c_{i,\varDelta}(\xi_i) + Q_i(\hbar_i) + l_i d_{i,\varDelta}u_i] + d_i(S,t) + \varDelta_{i,m_i}(z_i, S_i, t) - \dot{\omega}_{i,m_i}. \tag{45}$$

取如下Lyapunov函数:

$$V_{i,s_{m_i}} = \frac{1}{2}s_{i,m_i}^2, \tag{46}$$

将$V_{i,s_{m_i}}$对时间t求导,并将式(45)代入,得

$$\dot{V}_{i,s_{m_i}} = s_{i,m_i}\{f_{i,m_i}(\bar{S}_{i,m_i}) + g_{i,m_i}(\bar{S}_{i,m_i})[l_i c_{i,\varDelta}(\xi_i) + l_i d_{i,\varDelta}u_i + Q_i(\hbar_i)] + \varDelta_{i,m_i}(z_i, S_i, t) + d_i(S,t) - \dot{\omega}_{i,m_i}\}. \tag{47}$$

由假设6可得

$$|s_{i,m_i}d_i(S,t)| \leqslant \Big\{\sum_{j=1}^N \varsigma_{i,j}^0 + \sum_{j=1}^N\sum_{k=1}^p \varsigma_{i,j}^k\|S_j\|^k\Big\}|s_{i,m_i}|. \tag{48}$$

令$L_i = \sum\limits_{j=1}^N \varsigma_{i,j}^0 d_{i,k} = \sum\limits_{j=1}^N (\varsigma_{i,j}^k)^2$,有

$$\sum_{i=1}^N |s_{i,m_i}d_i(S,t)| \leqslant \sum_{i=1}^N\Big[|s_{i,m_i}|L_i + \frac{Np}{2} + \sum_{k=1}^p d_{i,k}\|S_i\|^{2k}\frac{s_{i,m_i}^2}{2}\Big]. \tag{49}$$

由假设8和假设9以及引理3可以得到

$$\frac{|c_{i,\varDelta}(\xi_i(t))|}{1+\bar{m}_i(t)} \leqslant$$

$$\frac{\bar{c}_{i,\Delta}\chi_{i,1}(\|\xi_i(0)\|+\bar{m}_i(0)\mathrm{e}^{-\delta_{i,0}t})+\bar{c}_{i,\Delta}\chi_{i,2}\bar{m}_i(t)}{1+\bar{m}_i(t)}\leqslant$$
$$H_{i,\bar{m}_i}, \tag{50}$$

其中

$$H_{i,\bar{m}_i}=\max\{\bar{c}_{i,\Delta}\chi_{i,1}(\|\xi_i(0)\|+\bar{m}_i(0)),\bar{c}_{i,\Delta}\chi_{i,2}\}.$$

根据式(50)对(47)中项$s_{i,m_i}l_ic_{i,\Delta}(\xi_i)g_{i,m_i}(\bar{S}_{i,m_i})$进一步化简,可得

$$s_{i,m_i}l_ic_{i,\Delta}(\xi_i)g_{i,m_i}(\bar{S}_{i,m_i})\leqslant\frac{\bar{g}_{i,m_i}^2s_{i,m_i}^2}{4}+p_{i,\bar{m}_i}H_{i,c}. \tag{51}$$

其中: $H_{i,c}=l_{\max}^2H_{i,\bar{m}_i}^2$, $p_{i,\bar{m}_i}=[1+\bar{m}_i(t)]^2$.

令$H_i=H_{i,c}/\varepsilon_i^*$,则式(51)可进一步化简为

$$s_{i,m_i}l_ic_{i,\Delta}(\xi_i)g_{i,m_i}(\bar{S}_{i,m_i})\leqslant$$
$$\frac{\bar{g}_{i,m_i}^2s_{i,m_i}^2}{4}+p_{i,\bar{m}_i}H_is_{i,m_i}+\Big(1-\frac{s_{i,m_i}}{\varepsilon_i^*}\Big)p_{i,\bar{m}_i}H_{i,c}. \tag{52}$$

类似Step 1,利用Young's不等式化简式(51),令

$$H_{i,m_i}(Z_{i,m_i})=$$
$$f_{i,m_i}(\bar{S}_{i,m_i})-\dot{\omega}_{i,m_i}+\frac{s_{i,m_i}\varphi_{i,m_i,1}^2(|\bar{S}_{i,m_i}|)}{a_{i,m_i,1}^2}+$$
$$\frac{s_{i,m_i}\phi_{i,m_i}^2(v_i(t))}{a_{i,m_i}^2+2s_{i,m_i}}, \tag{53}$$

其中$Z_{i,m_i}=[\bar{S}_{i,m_i}^{\mathrm{T}},s_{i,m_i},v_i,\dot{\omega}_{i,m_i}]^{\mathrm{T}}$.

设计控制律u_i与参数$\hat{\theta}_i,\hat{L}_i,\hat{d}_{i,k},\hat{H}_i$的自适应律如下:

$$u_i=$$
$$N(\zeta_i)\Big[k_{i,m_i}s_{i,m_i}+\frac{s_{i,m_i}\hat{\theta}_{i,m_i}\|\Psi_{i,m_i}(Z_{i,m_i})\|^2}{2a_{i,0,m_i}^2}+$$
$$\hat{L}_i|s_{i,m_i}|+\frac{1}{2}\sum_{k=1}^{p}\hat{d}_{i,k}\|x_i\|^{2k}s_{i,m_i}+p_{i,\bar{m}_i}\hat{H}_i\Big], \tag{54}$$

$$\dot{\zeta}_i=$$
$$s_{i,m_i}\Big[k_{i,m_i}s_{i,m_i}+\frac{s_{i,m_i}\hat{\theta}_{i,m_i}\|\Psi_{i,m_i}(Z_{i,m_i})\|^2}{2a_{i,0,m_i}^2}+$$
$$\frac{1}{2}\sum_{k=1}^{p}\hat{d}_{i,k}\|S_i\|^{2k}s_{i,m_i}+\hat{L}_i|s_{i,m_i}|+p_{i,\bar{m}_i}\hat{H}_i\Big], \tag{55}$$

$$\dot{\hat{\theta}}_{i,m_i}=$$
$$r_{i,m_i}\Big(\frac{s_{i,m_i}^2\|\Psi_{i,m_i}(Z_{i,m_i})\|^2}{2a_{i,0,m_i}^2}-\sigma_{i,m_i}\hat{\theta}_{i,m_i}\Big), \tag{56}$$

$$\dot{\hat{L}}_i=r_{i,L}[|s_{i,m_i}|-\sigma_{i,L}\hat{L}_i], \tag{57}$$

$$\dot{\hat{d}}_{i,k}=r_{i,dk}\Big[\frac{\|S_i\|^{2k}s_{i,m_i}^2}{2}-\sigma_{i,dk}\hat{d}_{i,k}\Big], \tag{58}$$

$$\dot{\hat{H}}_i=r_{i,H}(p_{i,\bar{m}_i}s_{i,m_i}-\sigma_{i,H}\hat{H}_i). \tag{59}$$

其中: $\hat{H}_i$、$\hat{d}_{i,k}$、$\hat{L}_i$分别表示H_i、$d_{i,k}$及L_i在t时刻的估计值.

将式(48)$\sim$(55)代入(47),利用Young's不等式化简,得

$$V_{i,s_{m_i}}\leqslant$$
$$G_i(\bar{S}_{i,m_i})(N(\zeta_i)+1)\dot{\zeta}_i-\Big(k_{i,m_i}-\frac{\bar{g}_{i,m_i}^2}{4}\Big)s_{i,m_i}^2-$$
$$\tilde{L}_i|s_{i,m_i}|-\frac{1}{2}\sum_{k=1}^{p}\tilde{d}_{i,k}\|S_i\|^{2k}s_{i,m_i}^2+p_{i,\bar{m}_i}s_{i,m_i}\tilde{H}_i-$$
$$\frac{s_{i,m_i}^2\tilde{\theta}_{i,m_i}\|\Psi_{i,m_i}(Z_{i,m_i})\|^2}{2a_{i,0,m_i}^2}+D_{i,m_i}. \tag{60}$$

其中

$$D_{i,m_i}=\frac{a_{i,m_i,1}^2}{4}+\frac{a_{i,m_i}^2}{4}+\frac{{\vartheta_{i,m_i}^*}^2}{4}+\frac{a_{i,0,m_i}^2}{2}+$$
$$\frac{e_i^{*2}}{4}+\frac{\bar{g}_{i,m_i}^2D_i^2}{4}+\frac{Np}{2},$$
$$G_{i,m_i}(\bar{S}_{i,m_i})=g_{i,m_i}(\bar{S}_{i,m_i})d_{i,\Delta}l_i.$$

3 稳定性分析

定义有界闭集

$$\Pi_{i,j}=\{[s_{i,1},\cdots,s_{i,j},y_{i,2},\cdots,y_{i,j},v_i,\hat{\theta}_{i,j},\hat{L}_i,\hat{d}_{i,k},$$
$$\hat{H}_i]:V_{i,m_i}\leqslant p\}\in R^{p_{i,j}},$$
$$j=1,2,\cdots,m_i,\ i=1,2,\cdots,N,$$

其中$p>0$是一个已知的设计常数.

假设连续函数$\eta_{i,j+1}(\cdot)$在紧集$\Pi_{i,j}\times\Pi_{i,0}$上的最大值为$M_{i,j+1}$,函数$Q_i(s_{i,1},S_{i,1})=(1-s_{i,1}^2/\varepsilon_{i,\bar{r}}^2)\bar{\gamma}_i(|S_{i,1}|)$在紧集$\Pi_{i,1}\times\Pi_{i,0}$上的最大值为$\mu_{i,1}$,函数$P_i(\bar{m}_i(t),\zeta_i(t),s_{i,m_i})=p_{i,\bar{m}_i}H_{i,c}$, $(1-s_{i,m_i}/\varepsilon_i^*)$在紧集$\Pi_{i,m_i}$上的最大值为$\mu_{i,0}$.

定义如下Lyapunov函数:

$$V=\sum_{i=1}^{N}V_i=\sum_{i=1}^{N}\sum_{j=1}^{m_i}V_{i,j}. \tag{61}$$

其中

$$\begin{cases}V_{i,1}=V_{i,sv}+\dfrac{y_{i,2}^2}{2}+\dfrac{\tilde{\theta}_{i,1}^2}{2r_{i,1}},\\ V_{i,j}=V_{i,s_j}+\dfrac{y_{i,j+1}^2}{2}+\dfrac{\tilde{\theta}_{i,j}^2}{2r_{i,j}},\\ V_{i,m_i}=V_{i,s_{m_i}}+\dfrac{\tilde{L}_i^2}{2r_{i,L}}+\dfrac{1}{2r_{i,dk}}\displaystyle\sum_{k=1}^{p}\tilde{d}_{i,k}^2+\\ \qquad\dfrac{\tilde{H}_i^2}{2r_{i,H}}+\dfrac{\tilde{\theta}_{i,m_i}^2}{2r_{i,m_i}};\end{cases} \tag{62}$$

$\tilde{L}_i=\hat{L}_i-L_i$, $\tilde{H}_i=\hat{H}_i-H_i$, $\tilde{\theta}_{i,j}=\hat{\theta}_{i,j}-\theta_{i,j}$, $\tilde{d}_{i,k}=\hat{d}_{i,k}-d_{i,k}$, $j=2,3,\cdots,m_i-1$.

定理1 考虑由系统(1)、虚拟控制律(27)、(39)、控制律(54)以及参数自适应律(28)、(40)、(55)$\sim$(59)组成的闭环系统,如果假设1$\sim$假设9成立,输出初值$y_i(0)$满足$-k_{i,b_1}(0)<y_i(0)<k_{i,a_1}(0)$,则对于任意给

定的正常数p以及初始条件$V(0)\leqslant p$和满足

$$\begin{cases}k_{i,j}\geqslant 1+\dfrac{\bar g_{i,j}^2}{4}+\dfrac{c_0}{2},\ j=1,2,\cdots,m_i-1;\\ k_{i,m_i}\geqslant\dfrac{\bar g_{i,m_i}^2}{2}+\dfrac{c_0}{2},\ i=1,2,\cdots,N;\\ \dfrac{1}{\tau_{i,j+1}}\geqslant 1+\dfrac{\bar g_{i,j}^2}{4}+\dfrac{c_0}{2},\ j=1,\cdots,m_i-1;\\ c_{i,0}=\min\limits_{1\leqslant i\leqslant N,1\leqslant j\leqslant m_i}\{r_{i,j}\sigma_{i,j},\ r_{i,L}\sigma_{i,L},r_{i,dk}\sigma_{i,dk},\\ \qquad r_{i,H}\sigma_{i,H},\ \bar c_i\}\end{cases}\tag{63}$$

的正常数$k_{i,j},\tau_{i,j},c_{i,0}$,该闭环系统有如下性质: 1) 闭环系统所有信号是有界的; 2) 系统输出$y_i(t)$满足如下时变输出约束条件$-k_{i,b_1}(t)<y_i(t)<k_{i,a_1}(t),\forall t>0$.

证明　1) 对式(62)中第1个子式求导,可得

$$\dot V_{i,1}\leqslant -(k_{i,1}-1)s_{i,1}^2-\Big(\frac{1}{\tau_{i,2}}-\frac{\bar g_{i,1}^2}{4}-1\Big)y_{i,2}^2-\frac{\sigma_{i,1}\tilde\theta_{i,1}^2}{2}-\frac{\bar c_i v_i}{\lambda_{i,0}}+\mu_{i,1}+D'_{i,1}+\frac{\bar g_{i,1}^2 s_{i,2}^2}{4},\tag{64}$$

其中

$$D'_{i,1}=D_{i,1}+\frac{M_{i,2}^2}{4}+\frac{\sigma_{i,1}\theta_{i,1}^2}{2}.$$

同理可得

$$\dot V_{i,j}\leqslant -(k_{i,j}-1)s_{i,j}^2-\Big(\frac{1}{\tau_{i,j+1}}-\frac{\bar g_{i,j}^2}{4}-1\Big)y_{i,j+1}^2-\frac{\sigma_{i,j}\tilde\theta_{i,j}^2}{2}+D'_{i,j}+\frac{\bar g_{i,j}^2 s_{i,j+1}^2}{4},\tag{65}$$

$$\dot V_{i,m_i}\leqslant -\Big(k_{i,m_i}-\frac{\bar g_{i,m_i}^2}{4}\Big)s_{i,m_i}^2+G_i(\bar S_{i,m_i})[N(\xi_i)+1]\dot\xi_i-\frac{\sigma_{i,\rho_i}\tilde L_i^2}{2}-\frac{\sigma_{i,m_i}\tilde\theta_{i,m_i}^2}{2}-\frac{\sigma_{i,dk}\tilde d_{ik}^2}{2}+\frac{\bar g_{i,m_i}^2 s_{i,m_i+1}^2}{4}-\frac{\sigma_{i,H}\tilde H_i^2}{2}+D'_{i,m_i}.\tag{66}$$

其中

$$D'_{i,j}=\frac{M_{i,j+1}^2}{4}+\frac{\sigma_{i,j}\theta_{i,j}^2}{2},$$
$$D'_{i,m_i}=\frac{\sigma_{i,L_i}L_i^2}{2}+D_{i,m_i}+\frac{\sigma_{i,dk}d_{ik}^2}{2}+\frac{M_{i,m_i+1}^2}{4}+\frac{\sigma_{i,H}H_i^2}{2}+\frac{\sigma_{i,m_i}\theta_{i,m_i}^2}{2}.$$

对V求导,得

$$\dot V=\sum_{i=1}^N\dot V_i=\sum_{i=1}^N\sum_{j=1}^{m_i}\dot V_{i,j}.\tag{67}$$

将式(63)～(66)代入(67),可得

$$\dot V_i\leqslant -c_{i,0}V_i+[G_{i,m_i}(\bar S_{i,m_i})N(\zeta_i)+1]\dot\zeta_i+\mu_{i,0}+\mu_{i,1}+\bar\mu_i,\tag{68}$$

其中$\bar\mu_i=\sum\limits_{j=1}^{m_i}D'_{i,j}$.

令$\mu_{i,2}=\bar\mu_i+\mu_{i,0}+\mu_{i,1}$, 对式(68)两边积分,可得

$$V_i(t)\leqslant\int_0^t[G_{i,m_i}(\bar S_{i,m_i})N(\zeta_i)+1]\dot\zeta_i\mathrm{e}^{c_{i,0}(\tau-t)}\mathrm{d}\tau+\frac{\mu_{i,2}}{c_{i,0}}+V_i(0).\tag{69}$$

由引理(4)可知$V_i(t),\zeta_i(t),\int_0^t G_{i,m_i}(\bar S_{i,m_i})N(\zeta_i)\dot\zeta_i\mathrm{d}\tau$在$[0,t_f)$上是有界的. 又因为$V(t)=\sum\limits_{i=1}^N V_i(t)$,所以$V(t)$也是有界的,因此闭环系统中所有信号均是有界的. 由于两个系统之间是一一对应的关系,原系统中所有信号也是有界的.

2) 因系统中所有信号是有界的,所以$s_{i,1}\in L_\infty$. 又由$Y_{di}\in L_\infty$,所以根据式(15)得到$S_{i,1}\in L_\infty$, 进一步, 根据式(13)可得$-k_{i,b1}(t)<x_{i,1}(t)<k_{i,a1}(t),\forall t>0$. □

4　仿真结果

考虑具有未建模动态的小车上通过弹簧连接的双倒立摆系统,如图1所示.

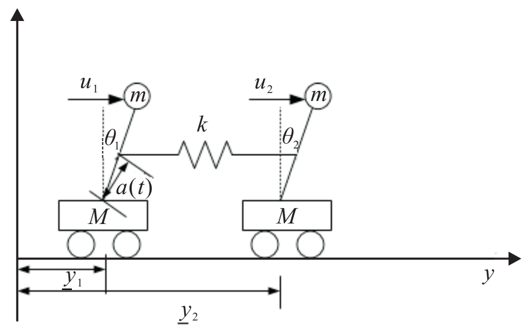

图1　双倒立摆系统模型

u_1, u_2分别是每个倒立摆的输入,定义状态向量$X_1=[x_{11},x_{12}]^{\mathrm T}=[\theta_1,\dot\theta_1]^{\mathrm T}$,$X_2=[x_{21},x_{22}]^{\mathrm T}=[\theta_2,\dot\theta_2]^{\mathrm T}$,则双倒立摆系统的动态方程[28-29]如下:

$$\dot z_1=q_1(z_1,X_1,t),$$
$$\dot X_1=\begin{bmatrix}0 & 1\\ \dfrac{g}{cl}-\dfrac{ka(t)(a(t)-cl)}{cml^2} & 0\end{bmatrix}X_1+\begin{bmatrix}0\\ \dfrac{1}{cml^2}\end{bmatrix}\varPhi(\hbar_1)+\begin{bmatrix}0 & 0\\ \dfrac{ka(t)(a(t)-cl)}{cml^2} & 0\end{bmatrix}X_2-\begin{bmatrix}0\\ d_1(X_1,t)\end{bmatrix}+\begin{bmatrix}\varDelta_{11}(z_1,X_1,t)\\ \varDelta_{12}(z_1,X_1,t)\end{bmatrix},$$
$$\dot z_2=q_2(z_2,X_2,t),$$
$$\dot X_2=$$

$$\begin{bmatrix} 0 & 1 \\ \dfrac{g}{cl}-\dfrac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} X_2+\begin{bmatrix} 0 \\ \dfrac{1}{cml^2} \end{bmatrix} \varPhi(\hbar_2)+$$

$$\begin{bmatrix} 0 & 0 \\ \dfrac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} X_1-\begin{bmatrix} 0 \\ d_2(X_2,t) \end{bmatrix}+$$

$$\begin{bmatrix} \varDelta_{21}(z_2,X_2,t) \\ \varDelta_{22}(z_2,X_2,t) \end{bmatrix}.$$

其中: m 为摆的质量, M 为小车的质量, l 为杆的长度, L 为连接弹簧的自然长度, k 为弹簧的弹性系数, g 为重力加速度;

$$\varPhi_i(\hbar_i)=\begin{cases} 1.5(\hbar_i-2.5),\ \hbar_i\geqslant 2.5; \\ 0,\ \ -1.5<\hbar_i<2.5; \\ 1.5(\hbar_i+1.5),\ \hbar_i\leqslant -1.5; \end{cases}$$

$$\begin{cases} \dot{\xi}_{i,1}=-2\xi_{i,1}-\xi_{i,1}^2+\xi_{i,2}, \\ \dot{\xi}_{i,2}=-2\xi_{i,2}+u_i, i=1,2; \\ \hbar_i=\xi_{i,1}+\dfrac{(2\xi_{i,2}^2-\xi_{i,2})}{(1+\xi_{i,2}^2)}+u_i. \end{cases}$$

$$d_1(X_1,t)=\beta_1 x_{12}^2+\left[\frac{k(a(t)-cl)}{cml^2}\right](\underline{y}_1-\underline{y}_2),$$

$$d_2(X_2,t)=\beta_2 x_{22}^2+\left[\frac{k(a(t)-cl)}{cml^2}\right](\underline{y}_2-\underline{y}_1);$$

$$\beta_1=\frac{m\sin(x_{11})}{M},\ \beta_2=\frac{m\sin(x_{21})}{M};$$

$$\underline{y}_1=\sin(2t),\ \underline{y}_2=L+\sin(3t);$$

$$a(t)=\sin(5t);\ c=\frac{m}{(m+M)};$$

$$\varDelta_{11}=0.1\sin t+z_1,\ \varDelta_{12}=2z_1^2;$$

$$\varDelta_{21}=2z_2+0.1\cos t,\ \varDelta_{22}=3z_2^2.$$

$$\dot{z}_1=-z_1+x_{12}^2+0.125\sin t,$$

$$\dot{z}_2=-z_2+0.125x_{21}^2\cos t.$$

仿真中取 $m=M=10\,\mathrm{kg}$; $l=1\,\mathrm{m}$; $L=2\,\mathrm{m}$; $g=10\,\mathrm{m/s^2}$; $k=1\,\mathrm{N/m}$. 给定的参考信号为 $y_{d1}(t)=0$, $y_{d2}(t)=0$, 仿真结果如图2～图5所示.

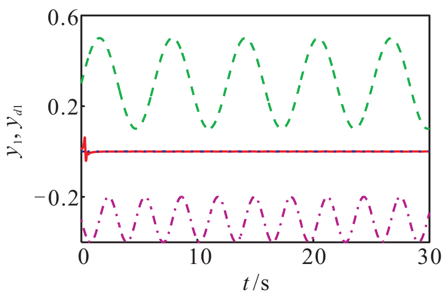

图 2 输出 y_1(实线)和期望信号 y_{d1}(点线)以及约束上界 k_{1,a_1}(点画线)与下界 $-k_{1,b_1}$(虚线)

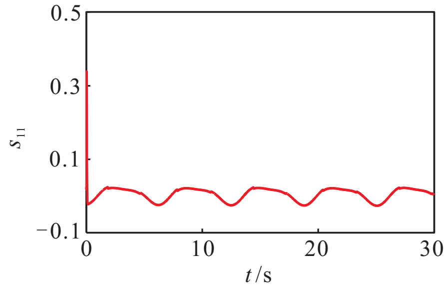

图 3 跟踪误差 s_{11}

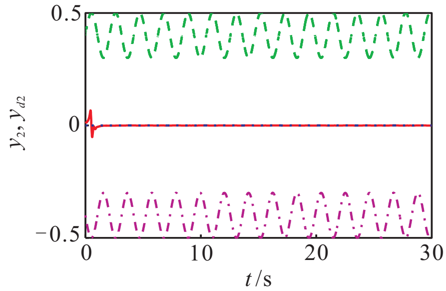

图 4 输出 y_2(实线)和期望信号 y_{d2}(点线)以及约束上界 k_{2,a_1}(点画线)与下界 $-k_{2,b_1}$(虚线)

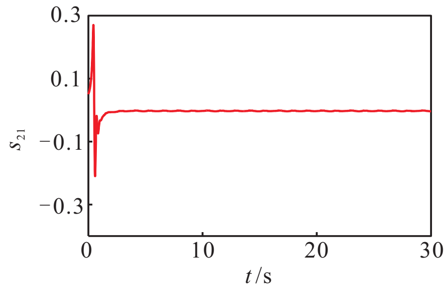

图 5 跟踪误差 s_{21}

5 结 论

本文针对一类具有未建模动态和时变输出约束以及非线性输入的耦合系统,提出了一种自适应神经网络动态面控制方案. 通过非线性映射变换达到输出约束的目的;通过引入动态信号和正则化信号处理状态和输入未建模动态. 理论分析证明了闭环系统中所有信号是半全局一致最终有界的,同时输出也满足非对称时变输出约束条件. 仿真结果也进一步验证了所提出控制方案的有效性.

参考文献(References)

[1] Zhang T P, Feng C B. Decentralized adaptive fuzzy control for large-scale nonlinear systems[J]. Fuzzy Sets and Systems, 1997, 92(1): 61-70.

[2] Zhang T P. Stable direct adaptive fuzzy control for a class of MIMO nonlinear systems[J]. Int J of Systems Science, 2003, 92(1): 375-388.

[3] Xia X N, Zhang T P, Wang Q. Decentralized adaptive output feedback dynamic surface control of interconnected nonlinear systems with unmodeled dynamics[J]. J of The Franklin Institute, 2015, 352(3): 1031-1055.

[4] Li H, Niu B, Zhang L. Adaptive backstepping based neural decentralized control for stochastic switched systems[C]. Proc of the 31st Youth Academic Annual Conf on Chinese Association of Automation(YAC). Wuhan, 2016: 261-266.

[5] Wang H Q, Liu W X, Qiu J B, et al. Adaptive fuzzy decentralized control for a class of strong interconnected nonlinear systems with unmodeled dynamics[J]. IEEE Trans on Fuzzy Systems, 2017, 99: 1-11, DOI: 10.1109/TFUZZ.2017.2694799.

[6] Wang H Q, Liu W X, Liu X P, et al. Adaptive fuzzy decentralized control for a class of interconnected nonlinear system with unmodeled dynamics and dead zones[J]. Neurocomputing, 2016, 214: 972-980.

[7] Liu L, Xie X J. Decentralized adaptive stabilization for interconnected systems with dynamic input-output and nonlinear interaction[J]. Automatica, 2010, 46(6): 1060-1067.

[8] Jiang Z P, Praly L. Design of robust adaptive controllers for nonlinear systems with dynamic uncertainties[J]. Automatica, 1998, 34(7): 825-840.

[9] Jiang Z P, Hill D J. A robust adaptive backstepping scheme for nonlinear systems with unmodeled dynamics[J]. IEEE Trans on Automatic Control, 1999, 44(9): 1705-1711.

[10] Mao J, Xiang Z R, Huang S P. Adaptive witched nonlinear systems with unmodeled dynamics[J]. Neurocomputing, 2016, 196: 42-52.

[11] Krstic M, Kokotovic P V. Robust control of nonlinear systems with input unmodeled dynamics[J]. Systems and Control Letters, 2000, 41(2): 115-122.

[12] Krstic M, Sun J, Kokotovic P V. Robust control of nonlinear systems with input unmodeled dynamics[J]. IEEE Trans on Automatic Control, 1996, 41(6): 913-920.

[13] Xia X N, Zhang T P, Yi Y, et al. Adaptive prescribed performance control of output feedback systems including input unmodeled dynamics[J]. Neurocomputing, 2016, 190: 226-236.

[14] Xia X N, Zhang T P. Adaptive reduced-order K-filters based DSC of uncertain systems with input unmodeled dynamics and dead zone[C]. Proc of the 35th Chinese Control Conf. Piscataway: Institute of Electrical and Electronics Engineers Inc, 2016: 463-468.

[15] Ngo K B, Mahony R, Jiang Z P. Integrator backstepping using barrier functions for systems with multiple state constraints[C]. Proc of the 44th IEEE Conf on Decision and Control. Piscataway: Institute of Electrical and Electronics Engineers Inc, 2005: 8306-8312.

[16] Tee K P, Ge S S, Tay E H. Barrier Lyapunov functions for the control of output-constrained nonlinear systems[J]. Automatica, 2009, 45(4): 918-927.

[17] Ren B B, Ge S S, Tee K P, et al. Adaptive neural control for output feedback nonlinear systems using a barrier Lyapunov function[J]. IEEE Trans on Neural Networks, 2010, 21(8): 1339-1345.

[18] Tee K P, Ren B B, Ge S S. Control of nonlinear systems with time-varying output constraints[J]. Automatica, 2011, 47(11): 2511-2516.

[19] Liu L, Yang X B. Robust adaptive state constraint control for uncertain switched high-order nonlinear systems[J]. IEEE Trans on Industrial Electronics, 2017, DOI: 10.1109/TIE.2017.2701774.

[20] Zhang T P, Xia M Z, Yi Y. Adaptive neural dynamic surface control of strict-feedback nonlinear systems with full state constraints and unmodeled dynamics[J]. Automatica, 2017, 81: 232-239.

[21] Meng W C, Yang Q M, Sun Y X. Adaptive neural control of nonlinear MIMO systems with time-varying output constraints[J]. IEEE Trans on Neural Network and Learning Systems, 2015, 26(5): 1074-1085.

[22] Ge S S, Wang C. Direct adaptive NN control of class of nonlinear systems[J]. IEEE Trans on Neural Networks, 2002, 13(1): 214-221.

[23] Xia X N, Zhang T P. Adaptive output feedback control of nonlinear systems with unmodeled dynamics and prescribed tracking performance[C]. Proc of the 34th Chinese Control Conf. Piscataway: Institute of Electrical and Electronics Engineers Inc, 2015: 2983-2988.

[24] Zhang T P, Zhu Q, Yi Yang, Adaptive neural control nonaffine pure-feedback nonlinear systems with input nonlinearity and perturbed uncertainties[J]. Int J of Systems Science, 2012, 34(4): 375-388.

[25] Mehraeen S, Jagannathan S, Crow M L. Decentralized dynamic surface control of large-scale interconnected systems in strict-feedback form using neural networks with asymptotic stabilization[J]. IEEE Trans on Neural Networks, 2011, 22(11): 1709-1722.

[26] Lin W, Qian C J. Adaptive control of nonlinearly parameterized systems: The smooth feedback case[J]. IEEE Trans on Automatic Control, 2002, 47(8): 1249-1266.

[27] Ye X D, Jiang Z P. Adaptive nonlinear design without a priori knowledge of control directions[J]. IEEE Trans on Automatic Control, 1998, 43(11): 1617-1621.

[28] Liu Y J, Tong S C, Li T S. Observer-based adaptive fuzzy tracking control for a class of uncertain nonlinear MIMO systems[J]. Fuzzy Sets and Systems, 2011, 164(1): 25-44.

[29] Huang Y S, Wu M. Robust decentralized direct adaptive output feedback fuzzy control for a class of large-scale non-affine nonlinear systems[J]. Information Sciences, 2014, 181(11): 2392-2404.

(责任编辑：孙艺红)