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带有执行器饱和的变时滞Markovian跳变系统的DOBC控制

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摘 要: 针对一类转移概率部分未知的Markovian跳变系统,考虑系统中存在时变时滞以及执行器饱和的情况,研究此类系统基于干扰观测器的抗干扰控制 (Disturbance-observer-based-control, DOBC) 问题. 首先, 分析带有扰动估计误差的闭环系统的随机稳定性, 通过构建适当的模态依赖型Lyapunov-Krasovskii(L-K)泛函并引入自由权矩阵,给出闭环系统的随机稳定性判据;然后,将控制器增益以及观测器增益的求解问题转化为带有线性矩阵不等式约束的可行性问题,并通过迭代优化算法得到最大吸引域的估计值;最后,通过仿真算例,验证所提出方法的正确性和有效性.

关键词: 基于干扰观测器的控制；Markovian跳变系统；执行器饱和；时滞；抗干扰控制；线性矩阵不等式

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Disturbance-observer-based control for Markovian jump systems with time-varying delay and actuator saturation

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Abstract: This paper is concerned with the problem of the stochastic stability analysis and controller design of the time-delay Markovian jump system with disturbance and actuator saturation. The stochastic stability problem of the closed-loop system with disturbance estimation error is analyzed. By constructing the appropriate mode-dependent Lyapunov-Krasovskii functions and introducing the free-connection weighting matrices, the random stability criterions of the closed-loop system are given. By transforming it into a feasible problem with linear matrix inequalities, the gain matrices are acquired. And the estimation of maximized attractive domain is obtained by an iterative optimization algorithm. Finally, the simulation results show the effectiveness of the proposed method.

Keywords: disturbance-observer-based control (DOBC)；Markovian jump systems；actuator saturation；time-delay；anti-disturbance control；linear matrix inequality

0 引 言

在实际生产过程中,由于外部环境变化、子系统互联改变、内部元件失效与修复等原因,许多系统在运行时可能出现系统结构突变,例如网络控制系统[1]制造系统[2]、故障检测系统[3-6]等. Markovian跳变系统作为一类特殊的混杂切换系统,可以模拟这样的控制过程,其各模态之间的跳变遵循Markov过程,并受转移概率支配. 实际控制过程中的转移概率很难获取,并在很大程度上影响着跳变系统的稳定性. 因而,针对转移概率未知的研究成为近年来的研究热点.

扰动广泛存在于实际的控制器运行过程中,并会降低系统性能,严重的甚至会影响系统的稳定性. 因此,无论在理论研究还是实际应用中,如何抑制扰动,确保系统的稳定性并实现其性能成为了热点问题[7-10]. 目前,抗扰动控制方法主要包括鲁棒控制、自适应控制、输出调节理论、内模控制和基于干扰观测器的控制 (DOBC) 方法[11-20]等. DOBC方法于1987年由日本学者Nakao等[21]首次提出,并已经应用在数控机床[22]、转盘驱动[23]、硬盘[24]、导弹[25]和飞行控制[26]等系统中. DOBC的基本思想是通过设

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计干扰观测器来估计外部扰动,并基于观测器的输出在前馈通道中予以补偿,进而达到抵消干扰的目的,可以通过分析动态方程的误差来确保观测器的性能. Guo等[11]针对一类MIMO非线性系统,在DOBC框架下考虑了扰动的衰减和抑制问题,并假设未知的外部扰动由外源系统产生,简化了扰动问题的经典假设. 近几年,针对Markovian跳变系统的抗扰动问题,利用DOBC方法的研究成果时有报道[15,17,27-29].

在控制系统中,由于物理约束和安全性等原因,执行器饱和现象也经常发生. 近年来,已经出现许多针对带有执行器饱和的动态系统的稳定性分析和控制综合的研究. 时滞的存在也会影响系统性能,现有许多研究将DOBC方法应用于时滞系统中[7,19-20,30].

目前,带有执行器饱和与时滞的Markovian跳变系统的DOBC控制在理论上仍是富有挑战的问题. 首先,带有饱和的Markovian跳变系统的DOBC控制带来了非线性,增加了控制器系统分析和设计的难度;其次,不仅要考虑Markovian动态系统的镇定问题,还要考虑时滞因素带来的影响;再次,时滞和执行器饱和的互相作用影响Markov过程的动态行为.

本文的研究动机是解决带有执行器饱和的变时滞Markovian跳变系统的抗干扰控制问题. 为充分考虑扰动特性,本文将扰动作为系统状态的一部分,利用降阶观测器来估计扰动信号,并假设外部扰动满足一定的匹配条件,通过控制输入通道进入系统. 本文的主要贡献包括下面两个部分: 1) 通过构建模态依赖型L-K泛函和自由权矩阵方法,给出带有扰动误差估计的闭环系统随机稳定的充分条件; 2) 设计控制器与观测器,并通过迭代优化算法得到吸引域的最大估计值.

本文所用符号说明: $\boldsymbol{R}$表示实数集合; $\boldsymbol{R}^n$表示n维实向量空间; $\boldsymbol{R}^{n\times m}$表示$n\times m$阶实矩阵集合; $N^{\rm T}$表示矩阵N的转置;给定概率空间$(\varXi,\varUpsilon,\varTheta)$, $\varXi$表示取样空间, $\varUpsilon$表示事件代数, $\varTheta$表示定义在$\varUpsilon$上的事件概率; ${\rm E}\{\cdot\}$表示随机过程的数学期望; $\|x\|$表示向量x的Euclidean范数; $P>0(\geqslant 0)$表示矩阵P是正定矩阵(半正定矩阵);为表简化, $*$表示块矩阵中一个子块的对称部分;如果没有特别说明,则矩阵为适当维数.

1 问题描述和预备知识

假设系统状态完全可测,且外部扰动满足匹配条件,由控制输入通道进入系统. 在完全概率空间$(\varXi,\varUpsilon,\varTheta)$下,考虑带有外部扰动和执行器饱和的变时滞Markovian跳变系统

$$\begin{cases}\dot{x}(t)=A(g_t)x(t)+A_d(g_t)x(t-\tau(t)),\\ x(t+\theta)=\varphi_1(\theta),\ \forall\theta\in[-\tau,0].\end{cases}\tag{1}$$

其中: $x(t)\in\boldsymbol{R}^n$是状态向量; $u(t)\in\boldsymbol{R}^m$是控制输入; $d(t)\in\boldsymbol{R}^r$是未知扰动输入; x_0、g_0、t_0分别是系统的初始状态、初始模态和初始时间; $\tau(t)$是时变方程,满足$0<\tau(t)\leqslant\tau, \dot{\tau}(t)\leqslant h$, τ和h是已知的实常标量, $\varphi_1(\theta)$是初始连续的向量值函数,定义在$[-\tau,0]$区间内; $\sigma(\cdot):\boldsymbol{R}^m\to\boldsymbol{R}^m$是标准的向量饱和函数,定义为

$$\sigma(u)=[\sigma^{\rm T}(\cdot)_1,\cdots,\sigma^{\rm T}(\cdot)_m]^{\rm T},$$
$$\sigma(\cdot)_q={\rm sign}(\cdot)\cdot\min\{\cdot,1\},$$

${\rm sign}(\cdot)$为符号函数. Markov链$\{g_t,t\geqslant 0\}$表示定义在有限集$S=\{1,2,\cdots,N\}$上的一个右连续的Markov随机过程.

各模态之间的跳变转移概率为

$$\mathcal{P}\{g_{t+\Delta t}=j|g_t=i\}=$$
$$\begin{cases}\pi_{ij}\Delta t+o(\Delta t),\ i\neq j;\\ 1+\pi_{ij}\Delta t+o(\Delta t),\ i=j.\end{cases}$$

其中: $\Delta t\geqslant 0$, $\lim\limits_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0$; π_{ij}表示由t时刻的模态i到$t+\triangle t$时刻的模态j的跳变转移速率,且满足$i\neq j$时, $\pi_{ij}\geqslant 0, \pi_{ii}=-\sum\limits_{j=1,j\neq i}^{N}\pi_{ij}, i,j\in S$.

本文假设Markovian跳变系统的转移速率部分未知. 为了表述方便,作如下定义.

定义1 $\forall i\in S, S^i=S_k^i\bigcup S_{uk}^i$,其中

$$S_k^i\triangleq\{j:\pi_{ij}已知,\ j\in S\},$$
$$S_{uk}^i\triangleq\{j:\pi_{ij}未知,\ j\in S\}.$$

若$S^i\neq\varnothing$,则可以进一步表示为

$$S_k^i\triangleq\{k_1^i,k_2^i,\cdots,k_m^i\},\ 1\leqslant m\leqslant N,$$

其中$k_m^i\in S$表示集合S_k^i中的第m个元素,同时是转移速率矩阵$\varPi$第i行中序号为k_m^i的第m个已知的转移速率.

假设1[1] 扰动输入由如下外源动力系统生成:

$$\dot{w}(t)=W(g_t)w(t),\ d(t)=V(g_t)w(t).\tag{2}$$

其中: $w(t)\in\boldsymbol{R}^{m\times 1}$是外源系统的状态, $W(g_t)\in\boldsymbol{R}^{m\times m}$和$V(g_t)\in\boldsymbol{R}^{1\times m}$是已知的适维常数矩阵.

注1 工程中的多种扰动可以描述为模型(2),如未知常数扰动和未知幅值、相位的谐波扰动[11,31]. 这类扰动具有一定的规律,经常重复性地出现于系统中,可视作影响系统性能的主要扰动,如由执行机构带来的高频振动等. 当$W(g_t)$选取为$W(g_t)=$

$\begin{bmatrix} 0 & c \\ -c & 0 \end{bmatrix}$, $c>0$代表谐波扰动的频率,则$d(t)$代表已知频率但是未知幅值和相位信息的谐波扰动. 这类谐波扰动可以描述实际系统中的扰动[32-36].

为了便于描述,定义$g_t = i, i \in S$,并将$A(g_t)$、$A_d(g_t)$、$B(g_t)$、$W(g_t)$、$V(g_t)$、$L(g_t)$、$K(g_t)$、$F(g_t)$、$P(g_t)$分别定义为A_i、A_{di}、B_i、W_i、V_i、L_i、K_i、F_i、P_i.

假设2[37]　(A_i, B_i)是可控的,(W_i, B_iV_i)是可观的.

令F_{iq}是矩阵$F_i \in i^{m\times n}$的第q行,定义如下对称多面体:

$$\Psi(F_i) = \{x(t) \in \boldsymbol{R}^n : |F_{iq}x(t)| \leqslant 1, q = 1, 2, \cdots, m\}. \tag{3}$$

定义2[38]　对于任意的初始模态$g_0 \in S$,在初始状态$x_0 \in \psi, \psi \subset \boldsymbol{R}^n$下,存在一个正的标量参数$T(x_0, g_0)$使得

$$\lim_{T_f\to\infty} \mathrm{E}\Big\{\int_0^{T_f} x^{\mathrm{T}}(t, x_0, g_0)x(t, x_0, g_0)\mathrm{d}t\Big|x_0, g_0\Big\} \leqslant T(x_0, g_0),$$

则集合$\psi \subset \boldsymbol{R}^n$被称为Markovian跳变系统均方意义下的吸引域.

定义3　如果每条起始于G中某点的系统轨迹在任何时间里都保持在G内,则称G为系统的一个不变集.

对于任意矩阵$P_i > 0$,定义椭圆

$$\eta(P_i) = \{x(t) \in \boldsymbol{R}^n : x^{\mathrm{T}}(t)P_ix(t) \leqslant 1\}. \tag{4}$$

令$\wp$是一个对角线上的元素是0或1的$m\times m$对称矩阵集合. 假设$\wp$的每一个元素标记为U_v,其中$v = 1, 2, \cdots, 2^m$, $U_v^- = I - U_v$. 显然,如果$U_v \in \wp$,可得$U_v^- \in \wp$.

引理1[37]　设$K_i, F_i \in \boldsymbol{R}^{m\times n}$为给定的,如果$x(t) \in \Psi(F_i)$,则$\sigma(K_ix(t))$可以写为

$$\sigma(K_ix(t)) = \sum_{v=1}^{2^m} \eta_v(U_vK_i + U_v^-F_i)x(t), \tag{5}$$

其中标量η_v满足$0 \leqslant \eta_v \leqslant 1$和$\sum_{v=1}^{2^m}\eta_v = 1, v = 1, 2, \cdots, 2^m$.

扰动观测器设计为

$$\begin{cases} \hat{d}(t) = V_i\hat{w}(t), \\ \hat{w}(t) = v(t) - L_ix(t), \\ \dot{v}(t) = (W_i + L_iB_iV_i)(v(t) - L_ix(t)) + \\ \qquad L_i(A_ix(t) + A_{di}x(t-\tau) + B_iu(t)). \end{cases} \tag{6}$$

在此基础上, DOBC设计为

$$u(t) = -\hat{d}(t) + K_ix(t). \tag{7}$$

其中: $\hat{d}(t)$是$d(t)$的估计值,用于补偿$d(t)$; $v(t)$是由式(6)中第3个等式给出的辅助变量,可视为观测器状态; K_i和L_i分别是待求的控制器和观测器的增益.

定义扰动估计误差$e_w(t) = w(t) - \hat{w}(t)$,并定义扩展向量$\xi(t) = [x^{\mathrm{T}}(t) \quad e_w^{\mathrm{T}}(t)]^{\mathrm{T}}$. 结合引理1和式(3)、(4)、(6), $\forall\xi(t) \in \Psi(F_i)$,且$F_i = [F_{1i}, V_i] \in \boldsymbol{R}^{m\times(n+r)}$,系统饱和项$\sigma(u(t) + d(t))$可以表述为

$$\sigma(u(t) + d(t)) = \sum_{v=1}^{2^m}\eta_v[(U_vK_i + U_v^-F_{1i})x(t) + V_ie_w(t)]. \tag{8}$$

因此,对于任意的$\xi(t) \in \Psi(F_i)$,扰动估计误差的动态方程可以写为

$$\dot{e}_w(t) = \sum_{v=1}^{2^m}\eta_v[(W_i + L_iB_iV_i)e_w(t) + L_iB_iU_v^-(F_{1i} - K_i)x(t)]. \tag{9}$$

进一步,闭环系统方程可以重新描述为

$$\dot{\xi}(t) = \sum_{v=1}^{2^m}\eta_v[\tilde{A}_i\xi(t) + \tilde{A}_{di}\xi(t - \tau(t))],$$
$$\xi(t+\theta) = \varphi(\theta),\ \forall\theta \in [-\tau, 0]. \tag{10}$$

其中

$$\tilde{A}_i = \begin{bmatrix} A_i + B_i(U_vK_i + U_v^-F_{1i}) & B_iV_i \\ L_iB_iU_v^-(F_{1i} - K_i) & W_i + L_iB_iV_i \end{bmatrix},$$
$$\tilde{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}.$$

定义4[34]　如果对于任意的初始模态$g_0 \in S$,初始状态$\varphi \in \Lambda[-\tau, 0]$和闭环系统的解$\xi(t)$,闭环系统均方意义下的吸引域为

$$\Im \triangleq \Big\{\varphi \in \Lambda[-\tau, 0] : \lim_{T_f\to\infty}\mathrm{E}\Big\{\int_o^{T_f}\|\xi(t)\|^2\mathrm{d}t\Big\} < \infty\Big\},$$

则$X_\delta \subset \Im$是在初始条件下的吸引域估计值. 其中

$$X_\delta = \{\varphi \in \Lambda[-\tau, 0] : \max|\varphi| \leqslant \delta_1, \max|\dot{\varphi}| \leqslant \delta_2\}, \tag{11}$$

标量$\delta_1, \delta_2 > 0$的最大值将在下文中求得.

2　主要结果

2.1　随机稳定性分析

定理1　如果存在对称正定矩阵P_i、Q_{si}、Q_s,对称矩阵R_i、R_{si},适当维数的矩阵M_{pi},使得不等式

$$\begin{bmatrix} \Pi_{1i}^{11} & \Pi_{1i}^{12} & \Pi_{1i}^{13} & \Pi_{1i}^{14} & \tau M_{1i}^{\mathrm{T}} & \tau M_{5i}^{\mathrm{T}} \\ * & \Pi_{1i}^{22} & \Pi_{1i}^{23} & \Pi_{1i}^{24} & \tau M_{2i}^{\mathrm{T}} & \tau M_{6i}^{\mathrm{T}} \\ * & * & \Pi_{1i}^{33} & \Pi_{1i}^{34} & \tau M_{3i}^{\mathrm{T}} & \tau M_{7i}^{\mathrm{T}} \\ * & * & * & \Pi_{1i}^{44} & \tau M_{4i}^{\mathrm{T}} & \tau M_{8i}^{\mathrm{T}} \\ * & * & * & * & -\tau Q_{3i} & 0 \\ * & * & * & * & * & -\tau Q_{3i} \end{bmatrix} < 0; \quad (12)$$

$$\sum_{j\in S_k^i} \pi_{ij}(Q_{sj} - R_{si}) - Q_s \leqslant 0; \quad (13)$$

$$P_j - R_i \leqslant 0, Q_{sj} - R_{si} \leqslant 0, j \in S_{uk}^i,\ j \neq i; \quad (14)$$

$$P_j - R_i \geqslant 0, Q_{sj} - R_{si} \geqslant 0, j \in S_{uk}^i,\ j = i; \quad (15)$$

$$\eta(P_i) \subset \Psi(F_i); \quad (16)$$

对于$i \in S, p = 1, 2, \cdots, 8, s = 1, 2, 3$成立,则闭环系统(10)是随机稳定的. 其中

$$\Pi_{1i}^{11} = P_i\tilde{A}_i + \tilde{A}_i^{\mathrm{T}}P_i + Q_{1i} + Q_{2i} + \tau Q_1 + \tau Q_2 + \frac{\tau^2}{2}Q_2 + M_{1i} + M_{1i}^{\mathrm{T}} + \sum_{j\in S_k^i}\pi_{ij}(P_j - R_i),$$

$$\Pi_{1i}^{12} = P_i\tilde{A}_{di} - M_{1i} + M_{2i}^{\mathrm{T}} + M_{5i}^{\mathrm{T}},$$

$$\Pi_{1i}^{13} = M_{3i} - M_{5i}^{\mathrm{T}},$$

$$\Pi_{1i}^{14} = \tilde{A}_i^{\mathrm{T}}P_i,$$

$$\Pi_{1i}^{22} = -(1-h)Q_{1i} - M_{2i} - M_{2i}^{\mathrm{T}} + M_{6i} + M_{6i}^{\mathrm{T}},$$

$$\Pi_{1i}^{23} = -M_{3i}^{\mathrm{T}} + M_{7i},$$

$$\Pi_{1i}^{24} = \tilde{A}_{di}^{\mathrm{T}}P_i,$$

$$\Pi_{1i}^{33} = -Q_{2i} - M_{7i} - M_{7i}^{\mathrm{T}},$$

$$\Pi_{1i}^{34} = 0,$$

$$\Pi_{1i}^{44} = -2P_i.$$

此外,集合

$$\eta_1(P_i, Q_{1i}, Q_1, Q_{2i}, Q_2, Q_{3i}, Q_3) = \Big\{\varphi \in \Lambda[-\tau, 0] : \varphi^{\mathrm{T}}(0)P_i\varphi(0) + \int_{-\tau(t)}^{0}\varphi^{\mathrm{T}}(s)Q_{1i}\varphi(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{\theta}^{0}\varphi^{\mathrm{T}}(s)Q_1\varphi(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\varphi^{\mathrm{T}}(s)Q_{2i}\varphi(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{\theta}^{0}\varphi^{\mathrm{T}}(s)Q_2\varphi(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{\theta}^{0}\dot{\varphi}^{\mathrm{T}}(s)Q_{3i}\dot{\varphi}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{\theta}^{0}\int_{\beta}^{0}\dot{\varphi}^{\mathrm{T}}(s)Q_3\dot{\varphi}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta \leqslant 1\Big\} \quad (17)$$

被包含在闭环系统均方意义下的吸引域内.

证明 为使结果有更低的保守性,对闭环系统(10)选取模态依赖型L-K泛函

$$V(\xi(t), i) = \xi^{\mathrm{T}}(t)P_i\xi(t) + \int_{t-\tau(t)}^{t}\xi^{\mathrm{T}}(s)Q_{1i}\xi(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t}\xi^{\mathrm{T}}(s)Q_1\xi(s)\mathrm{d}s\mathrm{d}\theta + \int_{t-\tau}^{t}\xi^{\mathrm{T}}(s)Q_{2i}\xi(s)\mathrm{d}s + \int_{-\tau}^{0}\int_{t+\theta}^{t}\xi^{\mathrm{T}}(s)Q_2\xi(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\xi}^{\mathrm{T}}(s)Q_{3i}\dot{\xi}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{\xi}^{\mathrm{T}}(s)Q_3\dot{\xi}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta. \quad (18)$$

其中$P_i, Q_{1i}, Q_1, Q_{2i}, Q_2, Q_{3i}, Q_3 > 0$,并且

$$\sum_{j=1}^{N}\pi_{ij}Q_{sj} \leqslant Q_s,\ s = 1, 2, 3. \quad (19)$$

因此,函数$V(\cdot)$的弱无穷小算子为

$$LV(\xi(t), i) = \sum_{v=1}^{2^m}\eta_v\xi^{\mathrm{T}}(t)P_i(\tilde{A}_i\xi(t) + \tilde{A}_{di}\xi(t-\tau(t))) + \sum_{v=1}^{2^m}\eta_v(\tilde{A}_i\xi(t) + \tilde{A}_{di}\xi(t-\tau(t)))^{\mathrm{T}}P_i\xi(t) + \xi^{\mathrm{T}}(t)\sum_{j=1}^{N}\pi_{ij}P_j\xi(t) + \xi^{\mathrm{T}}(t)Q_{1i}\xi(t) + \sum_{j=1}^{N}\pi_{ij}\int_{t-\tau(t)}^{t}\xi^{\mathrm{T}}(s)Q_{1j}\xi(s)\mathrm{d}s - (1-\dot{\tau}(t))\xi^{\mathrm{T}}(t-\tau(t))Q_{1i}\xi(t-\tau(t)) + \tau\xi^{\mathrm{T}}(t)Q_1\xi(t) - \int_{t-\tau}^{t}\xi^{\mathrm{T}}(s)Q_1\xi(s)\mathrm{d}s + \xi^{\mathrm{T}}(t)Q_{2i}\xi(t) - \xi^{\mathrm{T}}(t-\tau)Q_{2i}\xi(t-\tau) + \int_{t-\tau}^{t}\xi^{\mathrm{T}}(s)\Big(\sum_{j=1}^{N}\pi_{ij}Q_{2j} - Q_2\Big)\xi(s)\mathrm{d}s + \tau\xi^{\mathrm{T}}(t)Q_2\xi(t) + \tau\dot{\xi}^{\mathrm{T}}(t)Q_{3i}\dot{\xi}(t) - \int_{t-\tau}^{t}\dot{\xi}^{\mathrm{T}}(s)Q_{3i}\dot{\xi}(s)\mathrm{d}s + \frac{\tau^2}{2}\dot{\xi}^{\mathrm{T}}(t)Q_3\dot{\xi}(t) + \int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\xi}^{\mathrm{T}}(s)\Big(\sum_{j=1}^{N}\pi_{ij}Q_{3j} - Q_3\Big)\dot{\xi}(s)\mathrm{d}s\mathrm{d}\theta. \quad (20)$$

注意到下面的等式对于任意适当维数的矩阵M_{pi} $(p = 1, 2, \cdots, 8)$都成立:

$$2\varepsilon^{\mathrm{T}}(t)M_{11i}\Big[\xi(t) - \xi(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{\xi}(s)\mathrm{d}s\Big] = 0,$$

$$2\varepsilon^{\mathrm{T}}(t)M_{21i}\Big[\xi(t-\tau(t)) - \xi(t-\tau) - \int_{t-\tau}^{t-\tau(t)}\dot{\xi}(s)\mathrm{d}s\Big] = 0,$$

$$2[\xi^{\mathrm T}(t)P_i+\dot\xi^{\mathrm T}(t)P_i][-\dot\xi(t)+\tilde A_i\xi(t)+\tilde A_{di}\xi(t-\tau(t))]=0. \tag{21}$$

其中

$$\varepsilon(t)=[\xi^{\mathrm T}(t)\ \ \xi^{\mathrm T}(t-\tau(t))\ \ \xi^{\mathrm T}(t-\tau)\ \ \dot\xi^{\mathrm T}(t)]^{\mathrm T},$$
$$M_{11i}=[M_{1i}^{\mathrm T}\ \ M_{2i}^{\mathrm T}\ \ M_{3i}^{\mathrm T}\ \ M_{4i}^{\mathrm T}]^{\mathrm T},$$
$$M_{21i}=[M_{5i}^{\mathrm T}\ \ M_{6i}^{\mathrm T}\ \ M_{7i}^{\mathrm T}\ \ M_{8i}^{\mathrm T}]^{\mathrm T}.$$

由于$\sum_{j=1}^{N}\pi_{ij}=0$,可知存在对称正定矩阵R_i、$R_{si}(s=1,2,3)$使得

$$\sum_{j=1}^{N}\pi_{ij}R_i=\sum_{j=1}^{N}\pi_{ij}R_{si}=0. \tag{22}$$

由式(18)～(22)可得

$$LV(\xi(t),i)\leqslant \sum_{v=1}^{2^m}\eta_v\varepsilon^{\mathrm T}(t)\times[\varPi_{2i}+\tau M_{11i}Q_{3i}^{-1}M_{11i}^{\mathrm T}+\tau M_{21i}Q_{3i}^{-1}M_{21i}^{\mathrm T}]\varepsilon(t). \tag{23}$$

其中

$$\varPi_{2i}=\begin{bmatrix}\varPi_{2i}^{11}&\varPi_{1i}^{12}&\varPi_{1i}^{13}&\varPi_{1i}^{14}&\tau M_{1i}^{\mathrm T}&\tau M_{5i}^{\mathrm T}\\ *&\varPi_{1i}^{22}&\varPi_{1i}^{23}&\varPi_{1i}^{24}&\tau M_{2i}^{\mathrm T}&\tau M_{6i}^{\mathrm T}\\ *&*&\varPi_{1i}^{33}&\varPi_{1i}^{34}&\tau M_{3i}^{\mathrm T}&\tau M_{7i}^{\mathrm T}\\ *&*&*&\varPi_{1i}^{44}&\tau M_{4i}^{\mathrm T}&\tau M_{8i}^{\mathrm T}\\ *&*&*&*&-\tau Q_{3i}&0\\ *&*&*&*&*&-\tau Q_{3i}\end{bmatrix},$$

$$\varPi_{2i}^{11}=P_i\tilde A_i+\tilde A_i^{\mathrm T}P_i+Q_{1i}+Q_{2i}+\tau Q_1+\tau Q_2+\frac{\tau^2}{2}Q_3+M_{1i}+M_{1i}^{\mathrm T}+\sum_{j\in s_k^i}\pi_{ij}(P_j-R_i)+\sum_{j\in s_{uk}^i}\pi_{ij}(P_j-R_i),$$

$\varPi_{1i}^{12}$、$\varPi_{1i}^{13}$、$\varPi_{1i}^{14}$、$\varPi_{1i}^{22}$、$\varPi_{1i}^{23}$、$\varPi_{1i}^{24}$、$\varPi_{1i}^{33}$、$\varPi_{1i}^{34}$、$\varPi_{1i}^{44}$如定理1中所述,同时

$$\sum_{j=1}^{N}\pi_{ij}Q_{sj}-\sum_{j=1}^{N}\pi_{ij}R_{si}-Q_s=\sum_{j\in S_k^i}\pi_{ij}(Q_{sj}-R_{si})+\sum_{j\in S_{uk}^i}\pi_{ij}(Q_{sj}-R_{si})-Q_s\leqslant 0. \tag{24}$$

注意到,如果$\forall i\in S_k^i$,由不等式(12)～(14)和$\pi_{ij}\geqslant 0(\forall i,j\in S,i\neq j)$可以得到$\varPi_{2i}<0$. 如果$\forall i\in S_{uk}^i$,由不等式(12)～(15)和$\pi_{ii}=-\sum_{j=1,j\neq i}^{N}\pi_{ij}<0$也可以得到$\varPi_{2i}<0$. 因此

$$LV(\xi(t),i)<0, \tag{25}$$

这意味着

$$\lim_{T_f\to\infty}\mathrm E\Big\{\int_o^{T_f}\|\xi(t)\|^2\mathrm dt\Big\}<\infty.$$

由等式(18)和不等式(25)可知,如果$\varphi(\theta)\in\eta_1(P_i,Q_{1i},Q_1,Q_{2i},Q_2,Q_3,Q_{3i}),\forall\theta\in[-\tau,0]$,则

$$\begin{aligned}&\xi^{\mathrm T}(t)P_i\xi(t)\leqslant V(\xi(t),i)\leqslant\\ &\varphi^{\mathrm T}(0)P_i\varphi(0)+\int_{-\tau(t)}^{0}\varphi^{\mathrm T}(s)Q_{1i}\varphi(s)\mathrm ds+\\ &\int_{-\tau}^{0}\int_{\theta}^{0}\varphi^{\mathrm T}(s)Q_1\varphi(s)\mathrm ds\mathrm d\theta+\\ &\int_{-\tau}^{0}\varphi^{\mathrm T}(s)Q_{2i}\varphi(s)\mathrm ds+\\ &\int_{-\tau}^{0}\int_{\theta}^{0}\varphi^{\mathrm T}(s)Q_2\varphi(s)\mathrm ds\mathrm d\theta+\\ &\int_{-\tau}^{0}\int_{\theta}^{0}\dot\varphi^{\mathrm T}(s)Q_3\dot\varphi(s)\mathrm ds\mathrm d\theta+\\ &\int_{-\tau}^{0}\int_{\theta}^{0}\int_{\beta}^{0}\dot\varphi^{\mathrm T}(s)Q_3\dot\varphi(s)\mathrm ds\mathrm d\theta\mathrm d\beta\leqslant 1.\end{aligned} \tag{26}$$

因此,对于任意初始条件在集合(17)中的闭环系统,其状态轨迹保持在集合$\xi^{\mathrm T}(t)P_i\xi(t)\leqslant 1$中,也就是$\xi(t)\in\eta(P_i)$中. 由于$\eta(P_i)\subset\varPsi(F_i)$,可以得到$\xi(t)\in\varPsi(F_i)$. 综上所述,部分已知转移速率的闭环系统(10)是随机稳定的,集合(16)被包含在闭环系统均方意义下的吸引域中. □

2.2 扰动观测器求解和吸引域估计

定理2 如果存在对称正定矩阵X_{1i}、P_{2i}、U_{1si}、U_{2si}、U_{1sij}、U_{2sij},对称矩阵V_{1i}、V_{2i}、V_{1si}、V_{2si},适当维数的矩阵$\tilde M_{1pi}$、$\tilde M_{2pi}$、Y_i、H_{1i}、H_{2i},使得不等式

$$\begin{bmatrix}\varPi_{3i}^{11}&\varPi_{3i}^{12}&\varPi_{3i}^{13}&\varPi_{3i}^{14}&\varPi_{3i}^{15}&\varPi_{3i}^{16}&\varPi_{3i}^{17}\\ *&\varPi_{3i}^{22}&\varPi_{3i}^{23}&\varPi_{3i}^{24}&\varPi_{3i}^{25}&\varPi_{3i}^{26}&0\\ *&*&\varPi_{3i}^{33}&\varPi_{3i}^{34}&\varPi_{3i}^{35}&\varPi_{3i}^{36}&0\\ *&*&*&\varPi_{3i}^{44}&\varPi_{3i}^{45}&\varPi_{3i}^{46}&0\\ *&*&*&*&\varPi_{3i}^{55}&\varPi_{3i}^{56}&0\\ *&*&*&*&*&\varPi_{3i}^{66}&0\\ *&*&*&*&*&*&\varPi_{3i}^{77}\end{bmatrix}<0,\ i\in S_k^i; \tag{27}$$

$$\begin{bmatrix}\tilde\varPi_{3i}^{11}&\varPi_{3i}^{12}&\varPi_{3i}^{13}&\varPi_{3i}^{14}&\varPi_{3i}^{15}&\varPi_{3i}^{16}&\tilde\varPi_{3i}^{17}\\ *&\varPi_{3i}^{22}&\varPi_{3i}^{23}&\varPi_{3i}^{24}&\varPi_{3i}^{25}&\varPi_{3i}^{26}&0\\ *&*&\varPi_{3i}^{33}&\varPi_{3i}^{34}&\varPi_{3i}^{35}&\varPi_{3i}^{36}&0\\ *&*&*&\varPi_{3i}^{44}&\varPi_{3i}^{45}&\varPi_{3i}^{46}&0\\ *&*&*&*&\varPi_{3i}^{55}&\varPi_{3i}^{56}&0\\ *&*&*&*&*&\varPi_{3i}^{66}&0\\ *&*&*&*&*&*&\tilde\varPi_{3i}^{77}\end{bmatrix}<0,\ i\notin S_k^i; \tag{28}$$

$$\mathrm{diag}\Big\{\sum_{j\in S_k^i}\pi_{ij}(U_{1sij}-V_{1si})-U_{1si},$$

$$\sum_{j\in S_k^i}\pi_{ij}(U_{2sij}-V_{2si})-U_{2si}\Big\}\leqslant 0; \tag{29}$$

$$\begin{bmatrix}-V_{1i} & 0 & X_{1i}\\ * & P_{2j}-V_{1i} & 0\\ * & * & -X_{1j}\end{bmatrix}\leqslant 0,$$

$$\mathrm{diag}\{U_{1sij}-V_{1si},U_{2sij}-V_{2si}\}\leqslant 0,\ j\in S_{uk}^i,\ j\neq i; \tag{30}$$

$$\mathrm{diag}\{X_{1i}-V_{1i},P_{2i}-V_{2i}\}\geqslant 0,$$

$$\mathrm{diag}\{U_{1sii}-V_{1si},U_{2sii}-V_{2si}\}\geqslant 0,\ j\in S_{uk}^i,\ j=i; \tag{31}$$

$$\begin{bmatrix}-X_{1i} & 0 & V_{iq}^{\mathrm T}\\ * & -P_{2i} & H_{1iq}^{\mathrm T}\\ * & * & -I\end{bmatrix}<0,\ q=1,2,\cdots,m; \tag{32}$$

对于$\forall i\in S, P=1,2,\cdots,8, S=1,2,3$成立,则闭环系统(10)是随机稳定的. 其中

$$\varPi_{3i}^{11}=\begin{bmatrix}\varPi_{3i}^{1111} & \varPi_{3i}^{1112}\\ * & \varPi_{3i}^{1122}\end{bmatrix},$$

$$\tilde\varPi_{3i}^{11}=\begin{bmatrix}\tilde\varPi_{3i}^{1111} & \varPi_{3i}^{1112}\\ * & \varPi_{3i}^{1122}\end{bmatrix},$$

$$\varPi_{3i}^{1111}=A_iX_{1i}+X_{1i}A_i^{\mathrm T}+B_iU_vY_i+ B_iU_v^-H_{1i}+Y_i^{\mathrm T}U_vB_i^{\mathrm T}+H_{1i}^{\mathrm T}U_v^-B_i^{\mathrm T}+ U_{11ii}+U_{12ii}+\tau U_{11i}\tau U_{12i}+ \tilde M_{11i}+\tilde M_{11i}^{\mathrm T}+\pi_{ii}X_{1i}-\sum_{j\in S_k^i}\pi_{ij}V_{1i},$$

$$\tilde\varPi_{3i}^{1111}=A_iX_{1i}+X_{1i}A_i^{\mathrm T}+B_iU_vY_i+ B_iU_v^-H_{1i}+Y_i^{\mathrm T}U_vB_i^{\mathrm T}+H_{1i}^{\mathrm T}U_v^-B_i^{\mathrm T}+ U_{11ii}+U_{12ii}+\tau U_{11i}+\tau U_{12i}+ \tilde M_{11i}+\tilde M_{11i}^{\mathrm T}-\sum_{j\in S_k^i}\pi_{ij}V_{1i},$$

$$\varPi_{3i}^{1112}=B_iV_i+[H_{2i}B_iU_v^-(H_{1i}-Y_i)]^{\mathrm T},$$

$$\varPi_{3i}^{1122}= P_{2i}W_i+H_{2i}B_iV_i+W_i^{\mathrm T}P_{2i}+ V_i^{\mathrm T}B_i^{\mathrm T}H_{2i}^{\mathrm T}+U_{21ii}+U_{22ii}+\tau U_{21i}+ \tau U_{22i}+\tilde M_{11i}+\tilde M_{11i}^{\mathrm T}+\sum_{j\in S_k^i}\pi_{ij}(P_{2j}-V_{2i}),$$

$$\varPi_{3i}^{12}=\mathrm{diag}\{A_{di}X_{1i}-\tilde M_{11i}+\tilde M_{12i}^{\mathrm T}+\tilde M_{15i}^{\mathrm T}, -\tilde M_{21i}+\tilde M_{22i}^{\mathrm T}+\tilde M_{25i}^{\mathrm T}\},$$

$$\varPi_{3i}^{13}=\mathrm{diag}\{\tilde M_{13i}-\tilde M_{15i}^{\mathrm T},\tilde M_{23i}-\tilde M_{25i}^{\mathrm T}\},$$

$$\varPi_{3i}^{14}=\begin{bmatrix}\varPi_{3i}^{1411} & \varPi_{3i}^{1412}\\ \varPi_{3i}^{1421} & \varPi_{3i}^{1422}\end{bmatrix},$$

$$\varPi_{3i}^{1411}=X_{1i}A_i^{\mathrm T}+Y_i^{\mathrm T}U_vB_i^{\mathrm T}+H_{1i}^{\mathrm T}U_v^-B_i^{\mathrm T},$$

$$\varPi_{3i}^{1412}=B_iV_i+[H_{2i}B_iU_v^-(H_{1i}-Y_i)]^{\mathrm T},$$

$$\varPi_{3i}^{1421}=V_i^{\mathrm T}B_i^{\mathrm T},$$

$$\varPi_{3i}^{1422}=W_i^{\mathrm T}P_{2i}+V_i^{\mathrm T}B_i^{\mathrm T}H_{2i}^{\mathrm T},$$

$$\varPi_{3i}^{15}=\mathrm{diag}\{\tau\tilde M_{11i}^{\mathrm T},\tau\tilde M_{21i}^{\mathrm T}\},$$

$$\varPi_{3i}^{16}=\mathrm{diag}\{\tau\tilde M_{15i}^{\mathrm T},\tau\tilde M_{25i}^{\mathrm T}\},$$

$$\varPi_{3i}^{17}=[\sqrt{\pi_{ik_1^i}}X_{1i},\cdots,\sqrt{\pi_{ik_{r-1}^i}}X_{1i}, \sqrt{\pi_{ik_{r+1}^i}}X_{1i},\cdots,\sqrt{\pi_{ik_m^i}}X_{1i}],$$

$$\tilde\varPi_{3i}^{17}=[\sqrt{\pi_{ik_1^i}}X_{1i},\cdots,\sqrt{\pi_{ik_m^i}}X_{1i}],$$

$$\varPi_{3i}^{22}= \mathrm{diag}\{-(1-h)U_{11ii}-\tilde M_{12i}-\tilde M_{12i}^{\mathrm T}+\tilde M_{16i}+\tilde M_{16i}^{\mathrm T}, -(1-h)U_{21ii}-\tilde M_{22i}-\tilde M_{22i}^{\mathrm T}+\tilde M_{26i}+\tilde M_{26i}^{\mathrm T}\},$$

$$\varPi_{3i}^{23}=\mathrm{diag}\{-\tilde M_{13i}^{\mathrm T}+\tilde M_{17i},-\tilde M_{23i}^{\mathrm T}+\tilde M_{27i}\},$$

$$\varPi_{3i}^{24}=\mathrm{diag}\{X_{1i}A_{di}^{\mathrm T},0\},$$

$$\varPi_{3i}^{25}=\mathrm{diag}\{\tau\tilde M_{12i}^{\mathrm T},\tau\tilde M_{22i}^{\mathrm T}\},$$

$$\varPi_{3i}^{26}=\mathrm{diag}\{\tau\tilde M_{16i}^{\mathrm T},\tau\tilde M_{26i}^{\mathrm T}\},$$

$$\varPi_{3i}^{33}=\mathrm{diag}\{-U_{12ii}-\tilde M_{17i}-\tilde M_{17i}^{\mathrm T}, -U_{22ii}-\tilde M_{27i}-\tilde M_{27i}^{\mathrm T}\},$$

$$\varPi_{3i}^{34}=0,$$

$$\varPi_{3i}^{35}=\mathrm{diag}\{\tau\tilde M_{13i}^{\mathrm T},\tau\tilde M_{23i}^{\mathrm T}\},$$

$$\varPi_{3i}^{36}=\mathrm{diag}\{\tau\tilde M_{17i}^{\mathrm T},\tau\tilde M_{27i}^{\mathrm T}\},$$

$$\varPi_{3i}^{44}=\mathrm{diag}\{-2X_{1i},-2P_{2i}\},$$

$$\varPi_{3i}^{45}=\mathrm{diag}\{\tau\tilde M_{14i}^{\mathrm T},\tau\tilde M_{24i}^{\mathrm T}\},$$

$$\varPi_{3i}^{46}=\mathrm{diag}\{\tau\tilde M_{18i}^{\mathrm T},\tau\tilde M_{28i}^{\mathrm T}\},$$

$$\varPi_{3i}^{55}=\mathrm{diag}\{-\tau U_{13ii},-\tau U_{23ii}\},$$

$$\varPi_{3i}^{66}=\mathrm{diag}\{-\tau U_{13ii},-\tau U_{23ii}\},$$

$$\varPi_{3i}^{77}=\mathrm{diag}\{-X_{1k_1^i},\cdots,-X_{1k_{r-1}^i}, -X_{1k_{r+1}^i},\cdots,-X_{1k_m^i}\},$$

$$\tilde\varPi_{3i}^{77}=\mathrm{diag}\{-X_{1k_1^i},\cdots,-X_{1k_m^i}\},$$

$$F_{1i}=H_{1i}X_{1i}^{-1},$$

观测器增益矩阵为$L_i=P_{2i}^{-1}H_{2i}$,状态反馈控制器增益矩阵$K_i=Y_iX_{1i}^{-1}$.

此外,吸引域估计值为$\varGamma_\delta\leqslant 1$,其中

$$\varGamma_\delta=$$

$$\delta_1^2\Big[\lambda_{\max}(X_i^{-1})+\tau\lambda_{\max}(X_i^{-1}U_{1ii}X_i^{-1})+\frac{1}{2}\tau^2\lambda_{\max}(X_i^{-1}U_{1i}X_i^{-1})+\tau\lambda_{\max}(X_i^{-1}U_{2ii}X_i^{-1})+\frac{1}{2}\tau^2\lambda_{\max}(X_i^{-1}U_{2i}X_i^{-1})\Big]+\delta_2^2\Big[\frac{1}{2}\tau^2\lambda_{\max}\times(X_i^{-1}U_{3ii}X_i^{-1})+\frac{1}{6}\tau^3\lambda_{\max}(X_i^{-1}U_{3i}X_i^{-1})\Big].\tag{33}$$

证明　令

$$P_i=\begin{bmatrix}P_{1i}&0\\0&P_{2i}\end{bmatrix},\quad Q_{si}=\begin{bmatrix}Q_{1si}&0\\0&Q_{2si}\end{bmatrix},\quad Q_s=\begin{bmatrix}Q_{1s}&0\\0&Q_{2s}\end{bmatrix},\quad M_{pi}=\begin{bmatrix}M_{1pi}&0\\0&M_{2pi}\end{bmatrix},$$
$$X_{1i}=P_{1i}^{-1},\ X_i=\begin{bmatrix}X_{1i}&0\\0&I\end{bmatrix},$$
$$Y_i=K_iX_{1i},$$
$$H_{1i}=F_{1i}X_{1i},\ H_{2i}=P_{2i}L_i,$$
$$U_{sij}=X_iQ_{sj}X_i=\begin{bmatrix}U_{1sij}&0\\0&U_{2sij}\end{bmatrix},\quad U_{si}=X_iQ_sX_i=\begin{bmatrix}U_{1si}&0\\0&U_{2si}\end{bmatrix},$$
$$V_i=X_iR_iX_i=\begin{bmatrix}V_{1i}&0\\0&V_{2i}\end{bmatrix},\quad V_{si}=X_iR_{si}X_i=\begin{bmatrix}V_{1si}&0\\0&V_{2si}\end{bmatrix},$$
$$\tilde{M}_{pi}=X_iM_{pi}X_i=\begin{bmatrix}\tilde{M}_{1pi}&0\\0&\tilde{M}_{2pi}\end{bmatrix}.\tag{34}$$

不等式(13)两边同时乘以$\mathrm{diag}\{X_i,X_i,X_i,X_i,X_i,X_i\}$,可得到

$$\begin{bmatrix}\Pi_{4i}^{11}&\Pi_{4i}^{12}&\Pi_{3i}^{13}&\Pi_{3i}^{14}&\Pi_{3i}^{15}&\Pi_{3i}^{16}\\ *&\Pi_{4i}^{22}&\Pi_{3i}^{23}&\Pi_{3i}^{24}&\Pi_{3i}^{25}&\Pi_{3i}^{26}\\ *&*&\Pi_{3i}^{33}&\Pi_{3i}^{34}&\Pi_{3i}^{35}&\Pi_{3i}^{36}\\ *&*&*&\Pi_{3i}^{44}&\Pi_{3i}^{45}&\Pi_{3i}^{46}\\ *&*&*&*&\Pi_{3i}^{55}&0\\ *&*&*&*&*&\Pi_{3i}^{66}\end{bmatrix}<0.\tag{35}$$

其中

$$\Pi_{4i}^{11}=\begin{bmatrix}\Pi_{4i}^{1111}&\Pi_{3i}^{1112}\\ *&\Pi_{3i}^{1122}\end{bmatrix},$$
$$\Pi_{4i}^{1111}=A_iX_{1i}+X_{1i}A_i^{\mathrm T}+B_iU_vY_i+B_iU_v^-H_{1i}+Y_i^{\mathrm T}U_vB_i^{\mathrm T}+H_{1i}^{\mathrm T}U_v^-B_i^{\mathrm T}+U_{11ii}+U_{12ii}+\tau U_{11i}+\tau U_{12i}+\tilde{M}_{11i}+\tilde{M}_{11i}^{\mathrm T}+\sum_{j\in S_k^i}\pi_{ij}(X_{1i}X_{1j}^{-1}X_{1i}-V_{1i}),$$
$$\Pi_{4i}^{22}=\mathrm{diag}\{-(1-h)U_{11ii}-\tilde{M}_{12i}-\tilde{M}_{12i}^{\mathrm T}+\tilde{M}_{16i}+\tilde{M}_{16i}^{\mathrm T},\ -(1-h)U_{21ii}-\tilde{M}_{22i}-\tilde{M}_{22i}^{\mathrm T}+\tilde{M}_{26i}+\tilde{M}_{26i}^{\mathrm T}\},$$

Π_{3i}^{1112}、Π_{3i}^{1122}、Π_{3i}^{12}、Π_{3i}^{13}、Π_{3i}^{14}、Π_{3i}^{15}、Π_{3i}^{16}、Π_{3i}^{23}、Π_{3i}^{24}、Π_{3i}^{25}、Π_{3i}^{26}、Π_{3i}^{33}、Π_{3i}^{34}、Π_{3i}^{35}、Π_{3i}^{36}、Π_{3i}^{44}、Π_{3i}^{45}、Π_{3i}^{46}、Π_{3i}^{55}、Π_{3i}^{66}已在定理2中给出.

由于$\pi_{ii}<0,\forall i\in S$,对不等式(35)分以下两种情况处理:

1) 当$i\in S_k^i$时,应用Schur补引理,不等式(35)等价于(27);

2) 当$i\notin S_k^i$时,应用Schur补引理,不等式(35)等价于(28).

不等式(13)两边同时乘以X_i,可得到不等式(29). 不等式(14)两边同时乘以X_i,并应用Schur补引理,可得到不等式(30). 不等式(15)两边同时乘以X_i,可得到不等式(31). 显然

$$\xi^{\mathrm T}(t)P_i\xi(t)\leqslant V(\xi(t),i)\leqslant V(\xi_0,g_0)\leqslant\max_{\theta\in[-\tau,0]}|\varphi(\theta)|^2\Big[\lambda_{\max}(P_i)+\tau\lambda_{\max}(Q_{1i})+\frac{1}{2}\tau^2\lambda_{\max}(Q_1)+\tau\lambda_{\max}(Q_{2i})+\frac{1}{2}\tau^2\lambda_{\max}(Q_2)\Big]+\max_{\theta\in[-\tau,0]}|\dot\varphi(\theta)|^2\Big[\frac{1}{2}\tau^2\lambda_{\max}(Q_{3i})+\frac{1}{6}\tau^3\lambda_{\max}(Q_3)\Big]=\Gamma(\varphi,\dot\varphi).$$

如果集合$\Gamma(\varphi,\dot\varphi)\leqslant 1$,容易得到$\xi^{\mathrm T}(t)P_i\xi(t)\leqslant 1$,闭环系统状态向量$\xi(t)$的轨迹由$T(\varphi,\dot\varphi)\leqslant 1$开始,并保持在集合$\Gamma(\varphi,\dot\varphi)\leqslant 1$中. 因此,由式(32)控制约束满足$|F_{iq}\xi(t)\leqslant 1$. 吸引域的估计值可由$\Gamma(\varphi,\dot\varphi)\leqslant 1$得到,即$\Gamma_\delta\leqslant 1$. □

为了求得最大化吸引域的估计值,引入迭代优化算法进行求解. 定理2的条件是双线性矩阵不等式(BMIs),无法直接通过线性矩阵不等式求解. 可以通

过事先确定的值,解出BMIs. 由如下迭代算法[38]得到最大化初始状态域的估计值.

Step 1: 选取适当的非零初始矩阵H_{2i}.

Step 2: 求解定理2得到H_{1i}和Y_i.

Step 3: 由Step 2得到的H_{1i}和Y_i求解如下的优化问题得到H_{2i}:

$$\min \quad r;$$

$$\text{s.t. 不等式}(27)\sim(32),$$

$$\begin{bmatrix}\beta_1 I & 0 & I\\ * & \beta_1 I-P_{2i} & 0\\ * & * & X_{1i}\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}\beta_2 I-U_{11ii} & 0\\ * & \beta_2 I-U_{21ii}\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}\beta_3 I-U_{11i} & 0\\ * & \beta_3 I-U_{21i}\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}\beta_4 I-U_{12ii} & 0\\ * & \beta_4 I-U_{22ii}\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}\beta_5 I-U_{12i} & 0\\ * & \beta_5 I-U_{22i}\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}\beta_6 I-U_{13ii} & 0\\ * & \beta_6 I-U_{23ii}\end{bmatrix}\geqslant 0,$$

$$\begin{bmatrix}\beta_7 I-U_{13i} & 0\\ * & \beta_7 I-U_{23i}\end{bmatrix}\geqslant 0.$$

其中

$$r=\beta_1+\tau\beta_2+\frac{1}{2}\tau^2\beta_3+\tau\beta_4+\frac{1}{2}\tau^2\beta_5+\frac{1}{2}\tau^2\beta_6+\frac{1}{6}\tau^3\beta_7.$$

可求得最大化的吸引域估计值为$\delta_{\max}=\dfrac{1}{\sqrt{r}}$.

Step 4: 如果没有r的进一步改进(例如$|r_{\text{new}}-r_{\text{old}}|<\varepsilon$,其中$\varepsilon$是一个非常小的正数),则停止,否则转入Step 5.

Step 5: 由Step 3得到H_{2i},返回Step 2.

3 仿真算例

在实际的系统中,机械臂系统应用在许多领域中. 考虑单连机械臂[39]如下:

$$\ddot\theta(t)=-\frac{MGL}{J}\sin(\theta(t))-\frac{D(t)}{J}\dot\theta(t)+\frac{1}{J}\sigma(u(t)+d(t)).$$

其中:$\theta(t)$是臂的角位置,$u(t)$是控制输入,$d(t)$是未知的扰动输入,M是有效载荷的质量,J是惯性矩,$G=9.80$是重力加速度,$L=0.5$是机械臂的长度,$D(t)=2$是粘滞摩擦. 假设M和J有两个不同模态. 设$x_1(t)=\theta(t),x_2(t)=\dot\theta(t)$线性化的两个模态的系统为

$$\dot x(t)=\begin{bmatrix}0 & 1\\ -\dfrac{4.90M(g_t)}{J(g_t)} & -\dfrac{2}{J(g_t)}\end{bmatrix}x(t)+\begin{bmatrix}0\\ \dfrac{1}{J(g_t)}\end{bmatrix}\sigma(u(t)+d(t)).$$

其中:$x(t)=[x_1^{\mathrm T}(t)\quad x_2^{\mathrm T}(t)]^{\mathrm T}$;$J(g_t)$和$M(g_t)$依赖于跳变模态$g_t,g_t=1,2,J(1)=1,J(2)=5;M(1)=1,M(2)=5$.

假设扰动$d(t)$为式(2)所描述的初态未知扰动,且

$$W_1=W_2=\begin{bmatrix}0 & 0.5\\ -0.5 & 0\end{bmatrix},$$

$$V_1=[2.5\quad 0],$$

$$V_2=[1.5\quad 0].$$

令$\tau(t)=0.3(1-\sin t)$,则$\tau=0.6,h=0.3,\dot\tau(t)=-0.3\cos t,A_{d1}=A_{d2}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}$,部分未知转移速率矩阵为$\begin{bmatrix}-1.0 & 1.0\\ ? & ?\end{bmatrix}$,这里“?”代表未知元素. 求解定理2,可得

$$K_1=[-0.2097\quad 0.0688],$$

$$K_2=[-0.0141\quad -0.0005],$$

$$L_1=\begin{bmatrix}9.8689 & 0.0201\\ 0.0401 & 4.9241\end{bmatrix},$$

$$L_2=\begin{bmatrix}3.9831 & 0.0231\\ 0.0115 & 9.1879\end{bmatrix}.$$

直接求解吸引域估计值,得到$r=72.3777,\delta_{\max}=0.1175$;通过迭代优化算法,得到$r=0.8228,\delta_{\max}=1.1025$. 初始模态$g_t=1$,系统的初始状态$x(0)=[0.3\quad -0.3]^{\mathrm T}$.

系统模态为Markov过程的随机切换规律,见图1. 图2为开环系统的状态轨迹,由图2可知系统是不稳定的. 图3为闭环系统的状态轨迹,由图3看出系统状态可以收敛到原点,表示系统是随机稳定的. 图4描述了扰动的估计误差,表明所给扰动观测器的有效性. 这4幅图说明了DOBC方法对带有执行器饱和的

变时滞Markovian跳变系统的抗干扰控制是有效的.

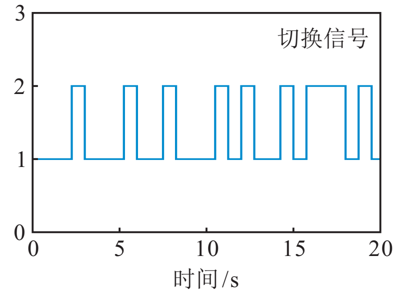

图1　系统模态

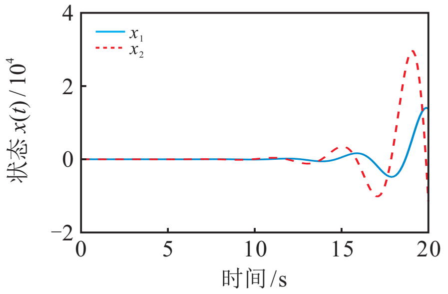

图2　开环系统状态轨迹

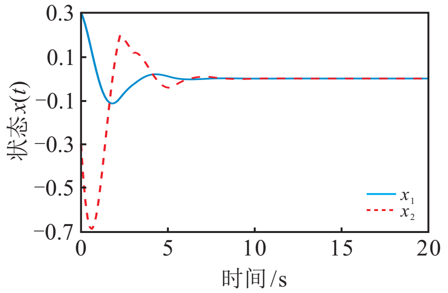

图3　闭环系统状态轨迹

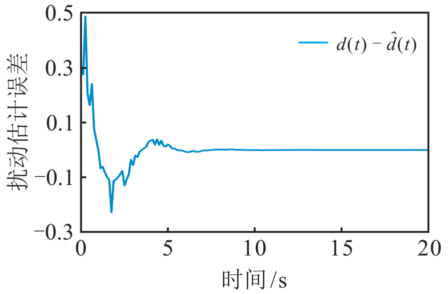

图4　输入扰动的估计误差

4　结　论

本文基于干扰观测器研究了带有执行器饱和的变时滞Markovian跳变系统的抗干扰控制问题. 首先,建立适当的模态依赖型L-K泛函,并引入自由权矩阵方法,得到了带有扰动估计误差的闭环系统随机稳定的充分条件;然后,利用线性矩阵不等式技术,得到了求解控制器和观测器增益的线性矩阵不等式条件,并使用迭代优化算法得到吸引域的最大估计值;最后,通过仿真验证了所得结果的正确性和所提方法的有效性,当系统同时存在扰动、时滞和执行器饱和时,该方法有效确保了系统的稳定性和控制精度.

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