

基于观测器的线性时变时滞多智能体系统一致性

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摘要: 针对系统状态不可测和具有通信时延的线性多智能体系统, 提出一种基于观测器的一致性控制算法. 设计观测器用于解决智能体状态不可测的问题, 在观测器的基础上, 提出一种控制协议来实现带时变时延的线性多智能体系统一致性. 利用模型转换的方式将原系统转换为新的模型系统, 在此基础上, 构造 Lyapunov-Krasovskii 函数并分析系统稳定性, 通过求解线性矩阵不等式获取控制器系数矩阵. 最后通过 Matlab 数值仿真验证所提出方法的有效性.

关键词: 多智能体系统; 线性多智能体系统; 一致性; 观测器; 时变时延

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Observer-based consensus of linear multi-agent systems with time-varying delays

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Abstract: A consensus control algorithm based on observers is proposed for linear multi-agent systems where the states of systems are unmeasurable and the communication time is delayed. The observer is designed to be used in solving unmeasurable states of agents. Then, a control protocol is proposed to realize consensus of linear multi-agent systems with time-varying delay. The formal system is transformed into a new system using model transformation. According to this system, the stability of the system is analyzed via constructing the Lyapunov-Krasovskii function, and the coefficient matrixes are obtained by solving linear matrix inequalities. Finally, Matlab simulation is conducted to demonstrate the effectiveness of the proposed method.

Keywords: multi-agent systems; linear multi-agent systems; consensus; observer; time-varying delay

0 引言

多智能体概念被提出以来,关于多智能体系统的协同控制研究已有很多,应用范围也很广泛,如一致性^[1-2]、机器人系统的跟踪控制^[3]、编队跟踪^[4-5]、无线网络^[6]、微型电网系统^[7]、交通运输^[8]、蜂拥^[9]等. 其中,多智能体系统一致性问题在很多领域(如航空航天、卫星、无人机等)因具有巨大的应用潜力而备受关注. 分布式一致性的研究目标就是如何设计控制器使其利用邻居信息使得系统个体共同关心的某(几)个状态量(如位置、速度、角度等)趋于相同.

在多智能体的研究中,系统模型的选取是非常重要的. 线性多智能体系统相比于其他系统,具有方便控制的优点. 因此,线性多智能体系统在国内被广

泛研究. 从拓扑结构的角度出发,文献[10]提出了固定拓扑下连续时间线性系统的分布式包容控制算法;文献[11]研究了在有向切换网络下的离散系统一致性问题;文献[12]提出了在动态拓扑结构下的离散线性系统的一致性控制算法;文献[13]考虑到真实系统会存在噪声干扰,提出了带噪声的线性多智能体系统的鲁棒一致性控制;文献[14]将拓扑结构与噪声综合考虑,研究了固定拓扑下的带有噪声的均方一致控制问题;文献[15]将自适应与博弈论相结合,研究了随机线性多智能体系统的自适应跟踪博弈;文献[16]考虑带有正负的无向加权拓扑结构,提出了一种新的非线性合并一致性控制协议,使得多智能体系统状态可以快速收敛至零.

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由于受到通讯带宽以及有限信息传播速度等限制,在实际系统中,信息交流会存在一定的延迟,故研究带时变时延的系统一致性问题具有重要意义^[17-24].文献[17]研究了在前馈、反馈通道下,网络控制系统的设计问题;基于此,文献[18]研究了离散时间网络多智能体系统,并提出了新的分布式控制算法;文献[21]研究了在固定拓扑和切换拓扑结构下非线性时延多智能体系统和同步振荡器的一致性;文献[22]研究了多智能体时滞网络的加权平均一致性问题;由于文献[23]不能同时消除颤动和收敛误差,文献[24]提出了一种新的固定时间非奇异终端滑模控制器,可以很好地解决固定时间下二阶系统的输入延迟问题.

除此之外,在一些实际系统中会存在系统状态信息不可测的情况.针对这一情况,一般采用引入观测器的方法解决这类问题^[25-27].文献[28]针对智能体速度不可测的多智能体系统,提出了一种基于滑模观测器的有限时间一致性控制算法;文献[29]考虑到多智能体系统会受到外部干扰的影响,提出了一种新型干扰观测器的设计思路:对由线性外系统 and 非线性外系统生成的干扰信号分别设计干扰观测器.

本文综合考虑时间延迟和系统状态不可测这两种因素,在文献[30]的基础上,将神经网络系统中关于时变时延的控制思想引入多智能体系统中,设计了分布式一致性控制算法.首先对原来的系统进行模型变换,然后构造Lyapunov-Krasovskii函数分析其稳定性,最后通过求解矩阵不等式得到系统控制器.相比于文献[21],本文的创新之处在于:1)研究的模型不同.文献[21]研究的是非线性网络模型和同步振荡器模型,而本文研究的是线性多智能体模型.2)研究的条件不同.文献[21]研究的是存在通讯时延的情况,而本文除考虑存在的通讯时延外,还考虑了系统状态不可测的情况.

下面对将要用到的一些符号进行说明:上标“ T ”表示矩阵的转置;上标“ -1 ”表示方阵的逆; $R^n, R^{n \times n}$ 表示 n 维实向量和 $n \times n$ 维实矩阵; $\mathbf{1}_N$ 表示所有元素为1的 N 维列向量; \otimes 表示矩阵的克罗内克积; $*$ 表示矩阵的对称部分.

1 预备知识和问题描述

1.1 代数图论

为了更好地描述问题,首先简要介绍一下图论的相关知识.对于一个由 n 个智能体组成的系统,假如不区分信息流动的方向,可以用无向图表示其拓扑结构.该无向图用 $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ 表示.其中: \mathcal{V} 表示由 n 个多智能体组成的节点集;边集 $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$

表示无向图 \mathcal{G} 中智能体之间的无向连接;矩阵 $\mathcal{A} = [a_{ij}] \in R^{n \times n}$ 表示无向图的邻接矩阵,若 $(j, i) \in \mathcal{E}$,则有 $a_{ij} > 0$,否则 $a_{ij} = 0$.通常情况下假设节点自身没有到自身的边,即 $a_{ii} = 0$.对于无向图而言, $(j, i) \in \mathcal{E} \leftrightarrow (i, j) \in \mathcal{E}$,也就是 $a_{ij} = a_{ji} > 0$,否则 $a_{ij} = 0$. $\mathcal{V}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ 表示多智能体 i 的所有邻居多智能体的集合. $L = [l_{ij}]_{n \times n} = D - \mathcal{A}$ 表示无向图的拉普拉斯矩阵.其中: $D = \text{diag}\{d_1, d_2, \dots, d_N\}$, $d_i = \sum_{j=1}^N a_{ij}$.若至少有一个节点到其他任何节点之间存在有向路径,则称该有向图包含有向生成树.若无向图的任意两节点存在路径,则称该无向图是连通的.那些所有连通的极大子图就是无向图的有向生成树.

1.2 问题描述

考虑下面由 n 个智能体组成的多智能体系统,智能体 i 的系统模型为

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \\ y_i(t) = Cx_i(t). \end{cases} \quad (1)$$

其中: $x_i(t) \in R^m$ 、 $u_i(t) \in R^r$ 、 $y_i(t) \in R^s$ 分别表示第 i 个智能体在 t 时刻的系统状态、控制输入、系统输出; $A \in R^{m \times m}$ 、 $B \in R^{m \times r}$ 、 $C \in R^{s \times m}$ 表示已知的系统矩阵、控制矩阵、输出矩阵.本文假设 C 是行满秩矩阵.

在一些实际系统中,由于成本限制等原因,常常无法直接得到智能体的全部状态.因此,本文对每个智能体设计如下观测器来观测系统状态:

$$\begin{cases} \dot{z}_i(t) = Az_i(t) + Bu_i(t) + G(y_i(t) - \xi_i(t)), \\ \xi_i(t) = Cz_i(t). \end{cases} \quad (2)$$

其中: $z_i(t)$ 、 $\xi_i(t)$ 分别表示第 i 个观测器在 t 时刻的状态和输出, $G \in R^{m \times s}$ 表示待定的反馈矩阵.

假设1 线性多智能体系统(1)中 (A, B) 可控, (A, C) 可观.

由于真实系统中存在时间延迟,令 $\tau(t)$ 表示在通讯交流过程中随时间变化的延迟时间,并且满足

$$\begin{cases} \tau_m < \tau(t) < \tau_M, \\ \dot{\tau}(t) \leq \mu. \end{cases} \quad (3)$$

其中: $0 \leq \tau_m < \tau_M$, μ 是正常数.

定义系统的初始状态为 $x(t) = \psi(t)$, $t \in [-\tau(t), 0]$,其中 $\psi(t)$ 为状态的初始函数.

注1 在文献[20]中也讨论了具有式(3)描述的时延特性的系统.相比于文献[20],本文研究的是线性多智能体模型,而不是神经网络模型;本文考虑系

统存在通讯时延和系统状态不可测两种情况,而文献[20]只是考虑了存在通讯时延的情况.

综上,对于第*i*个多智能体,基于上述观测器相对输出信息设计如下控制协议:

$$u_i(t) = K \sum_{j \in \mathcal{V}_i} a_{ij} [\xi_j(t - \tau(t)) - \xi_i(t - \tau(t))], \quad (4)$$

其中反馈矩阵 $K \in R^{r \times s}$.

由式(1)、(2)、(4)可知,多智能体系统可以写成

$$\dot{x}_i(t) = Ax_i(t) + BKC \sum_{j \in \mathcal{V}_i} a_{ij} [z_j(t - \tau(t)) - z_i(t - \tau(t))], \quad (5)$$

$$\dot{z}_i(t) = Az_i(t) + BKC \sum_{j \in \mathcal{V}_i} a_{ij} [z_j(t - \tau(t)) - z_i(t - \tau(t))] + GC[x_i(t) - z_i(t)]. \quad (6)$$

定义系统状态与观测器之间的误差 $h_i(t)$ 为

$$h_i(t) = x_i(t) - z_i(t), \quad (7)$$

则误差变化率为

$$\dot{h}_i(t) = (A - GC)h_i(t). \quad (8)$$

在给出本文主要定理之前,先给出几个相关定义、假设和引理如下.

定义1 线性多智能体系统(1)的一致性是指在任意初始条件下,控制器利用相邻智能体的信息使得 $\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j \in \mathcal{V}$.

假设2 无向图 \mathcal{G} 至少包含一棵无向生成树.

引理1^[1] 无向图 \mathcal{G} 的拉普拉斯矩阵 L 有且只有一个零特征值,其余特征值均大于零,并且零特征值对应的特征向量为 $\mathbf{1}_N$.

引理2^[2] 对于任意两个向量 $x \in R^n, y \in R^n$ 和任意方阵 $\Phi \in R^{n \times n}$ 且 $\Phi > 0$, 都有如下不等式成立: $\pm 2x^T y \leq x^T \Phi x + y^T \Phi^{-1} y$.

引理3^[3] 对于任意维数的常数矩阵 Ψ_1, Ψ_2 , 对称矩阵 $\Omega < 0$, 标量 $0 \leq \tau_m < \tau_M$ 和函数 $\tau(t) : R^+ \rightarrow [\tau_m, \tau_M], (\tau_M - \tau_m)\Psi_1 + \Omega < 0$ 且 $(\tau_M - \tau_m)\Psi_2 + \Omega < 0$ 成立的充要条件是 $(\tau(t) - \tau_m)\Psi_1 + (\tau_M - \tau(t))\Psi_2 + \Omega < 0$ 成立.

引理4 对于分块对称矩阵 $S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$, 当且仅当 $s_{11} < 0, s_{22} - s_{21}s_{11}^{-1}s_{12} < 0$ 成立, 或 $s_{22} < 0, s_{11} - s_{12}s_{22}^{-1}s_{21} < 0$ 成立时, 矩阵 $S < 0$ 成立. 其中: s_{11}, s_{22} 是对称矩阵, $s_{12}^T = s_{21}$.

2 一致性分析

由假设2可知,至少存在一个智能体能够直接或间接获得所有智能体的信息. 这样系统中不存在孤立个体,保证了系统状态有可能达到一致.

首先,由式(5)、(6)、(8)可知,多智能体系统、观测器和误差可写成下面的形式:

$$\dot{x}(t) = (I \otimes A)x(t) - (L \otimes BKC)z(t - \tau(t)), \quad (9)$$

$$\dot{z}(t) = (I \otimes A)z(t) - (L \otimes BKC)z(t - \tau(t)) + (I \otimes GC)h(t), \quad (10)$$

$$\dot{h}(t) = [I \otimes (A - GC)]h(t). \quad (11)$$

其中

$$x(t) = [x_1^T(t), x_2^T(t), \dots, x_n^T(t)]^T,$$

$$z(t) = [z_1^T(t), z_2^T(t), \dots, z_n^T(t)]^T,$$

$$h(t) = [h_1^T(t), h_2^T(t), \dots, h_n^T(t)]^T.$$

在系统状态、观测器状态和误差方面,定义新的变量 $\bar{x}_i(t), \bar{z}_i(t), \bar{h}_i(t)$. 其中

$$\bar{x}_i(t) = x_i(t) - \frac{1}{n} \sum_{j=1}^n x_j(t),$$

$$\bar{z}_i(t) = z_i(t) - \frac{1}{n} \sum_{j=1}^n z_j(t),$$

$$\bar{h}_i(t) = h_i(t) - \frac{1}{n} \sum_{j=1}^n h_j(t).$$

将上式写成矩阵的形式,即

$$\bar{x}(t) = (J \otimes I)x(t), \quad (12)$$

$$\bar{z}(t) = (J \otimes I)z(t), \quad (13)$$

$$\bar{h}(t) = (J \otimes I)h(t). \quad (14)$$

其中

$$\bar{x}(t) = [\bar{x}_1^T(t), \bar{x}_2^T(t), \dots, \bar{x}_n^T(t)]^T,$$

$$\bar{z}(t) = [\bar{z}_1^T(t), \bar{z}_2^T(t), \dots, \bar{z}_n^T(t)]^T,$$

$$\bar{h}(t) = [\bar{h}_1^T(t), \bar{h}_2^T(t), \dots, \bar{h}_n^T(t)]^T.$$

矩阵 $J = I - \frac{1}{n} \mathbf{1} \times \mathbf{1}^T$ 具有特性 $LJ = JL = L, J\mathbf{1}_n = 0$. 由定义1可知,若 $\lim_{t \rightarrow \infty} \bar{x}(t) = 0$, 则系统(1)达到一致. 若 $\lim_{t \rightarrow \infty} \bar{h}(t) = 0$, 则说明观测器能够准确观测出系统的状态信息.

对式(12)~(14)分别进行求导,并将式(9)~(11)代入得到下式:

$$\dot{\bar{x}}(t) = (I \otimes A)\bar{x}(t) - (L \otimes BKC)\bar{z}(t - \tau(t)), \quad (15)$$

$$\dot{\bar{z}}(t) = (I \otimes A)\bar{z}(t) - (L \otimes BKC)\bar{z}(t - \tau(t)) + (I \otimes GC)\bar{h}(t), \quad (16)$$

$$\dot{\bar{h}}(t) = [I \otimes (A - GC)]\bar{h}(t). \quad (17)$$

由引理1可知,存在这样一个正交矩阵 $U = \left[\frac{1}{\sqrt{n}}, U_1 \right]$, 可以将拉普拉斯矩阵对角化,即 $U^T L U = \Lambda = \text{diag}\{0, \lambda_2, \dots, \lambda_n\}$. 其中: $\lambda_1, \lambda_2, \dots, \lambda_n$ 是拉普拉斯矩阵 L 的特征值, $\lambda_1 = 0$.

对式(12)~(14)作变换,得到新的变量 $\tilde{x}(t), \tilde{z}(t), \tilde{h}(t)$. 其中: $\tilde{x}(t) = (U^T \otimes I)\tilde{x}(t), \tilde{z}(t) = (U^T \otimes I)\tilde{z}(t), \tilde{h}(t) = (U^T \otimes I)\tilde{h}(t)$. 若 $\lim_{t \rightarrow \infty} \tilde{x}(t) = 0$, 则有 $\lim_{t \rightarrow \infty} \tilde{z}(t) = 0$. 从而线性多智能体系统(1)达到一致.

对上式整理可得

$$\dot{\tilde{x}}(t) = (I \otimes A)\tilde{x}(t) - (A \otimes BKC)\tilde{z}(t - \tau(t)), \quad (18)$$

$$\begin{aligned} \dot{\tilde{z}}(t) &= (I \otimes A)\tilde{z}(t) - (A \otimes BKC)\tilde{z}(t - \tau(t)) + \\ &\quad (I \otimes GC)\tilde{h}(t), \end{aligned} \quad (19)$$

$$\dot{\tilde{h}}(t) = [I \otimes (A - GC)]\tilde{h}(t). \quad (20)$$

定理1 对于多智能体系统(1), 满足假设1和假设2. 对于任意非负的实数 $\mu, 0 \leq \tau_m < \tau_M$, 延迟时间 $\tau(t)$ 满足式(3), 控制协议(4)能够使线性多智能体系统(1)达到一致, 即 $\lim_{t \rightarrow \infty} \tilde{x}(t) = 0$, 如果下面的条件满足: 存在正定矩阵 $P_1 \in R^{m \times m}, P_2 \in R^{m \times m}, Q_1 \in R^{m \times m}, R_1 \in R^{m \times m}$ 和矩阵 $K, G, N_i \in R^{m \times m} (i = 1, 2, \dots, 7), M_i \in R^{m \times m} (1, 2, \dots, 7)$, 使矩阵不等式

$$\Pi_1 = \begin{bmatrix} \Sigma & (\tau_M - \tau_m)\bar{N} & (\tau_M - \tau_m)\Omega^T \bar{R}_1 \\ * & -(\tau_M - \tau_m)\bar{R}_1^{-1} & 0 \\ * & * & -(\tau_M - \tau_m)\bar{R}_1^{-1} \end{bmatrix} < 0, \quad (21)$$

$$\Pi_2 = \begin{bmatrix} \Sigma & (\tau_M - \tau_m)\bar{M} & (\tau_M - \tau_m)\Omega^T \bar{R}_1 \\ * & -(\tau_M - \tau_m)\bar{R}_1^{-1} & 0 \\ * & * & -(\tau_M - \tau_m)\bar{R}_1^{-1} \end{bmatrix} < 0 \quad (22)$$

成立. 其中

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & -A \otimes P_1 BKC & 0 \\ * & \Sigma_{22} & -A \otimes P_1 BKC & I \otimes P_1 GC \\ * & * & -(1 - \mu)\bar{Q}_1 & 0 \\ * & * & * & \Sigma_{44} \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ \bar{N}_1 & \bar{M}_1 - \bar{N}_1 & -\bar{M}_1 & \\ \bar{N}_2 & \bar{M}_2 - \bar{N}_2 & -\bar{M}_2 & \\ \bar{N}_3 & \bar{M}_3 - \bar{N}_3 & -\bar{M}_3 & \\ \leftarrow \bar{N}_4 & \bar{M}_4 - \bar{N}_4 & -\bar{M}_4 & \\ \bar{N}_5 + \bar{N}_5^T & \bar{M}_5 - \bar{N}_5 + \bar{N}_6^T & \bar{N}_7^T - \bar{M}_5 & \\ * & \Sigma_{66} & \Sigma_{67} & \\ * & * & -\bar{M}_7 - \bar{M}_7^T & \end{bmatrix}.$$

这里

$$\begin{aligned} \Sigma_{11} &= I \otimes A^T P_1 + I \otimes P_1 A, \\ \Sigma_{22} &= I \otimes A^T P_1 + I \otimes P_1 A + \bar{Q}_1, \end{aligned}$$

$$\begin{aligned} \Sigma_{44} &= I \otimes (A^T P_2 + P_2 A - P_2 GC) - I \otimes (P_2 GC)^T, \\ \Sigma_{66} &= \bar{M}_6 + \bar{M}_6^T - \bar{N}_6 - \bar{N}_6^T, \\ \Sigma_{67} &= -\bar{N}_7^T - \bar{M}_6 + \bar{M}_7^T, \\ \bar{N} &= I \otimes N, N = [N_1, N_2, \dots, N_7]^T, \\ \bar{M} &= I \otimes M, M = [M_1, M_2, \dots, M_7]^T, \\ \bar{R}_1 &= I \otimes R_1, \bar{Q}_1 = I \otimes Q_1, \\ \Omega &= [0, 0, 0, I \otimes (A - GC), 0, 0, 0]. \end{aligned}$$

证明 首先针对系统(18)~(20), 构造Lyapunov-Krasovskii 函数

$$\begin{aligned} V(t, \delta(t)) &= \sum_{i=1}^5 V_i(t), \\ \delta(t) &= [\tilde{x}^T(t), \tilde{z}^T(t), \tilde{z}^T(t - \tau(t)), \tilde{h}^T(t), \\ &\quad \tilde{h}^T(t - \tau_m), \tilde{h}^T(t - \tau(t)), \tilde{h}^T(t - \tau_M)]^T. \end{aligned}$$

其中

$$\begin{cases} V_1(t) = \tilde{x}^T(t)\bar{P}_1\tilde{x}(t), \\ V_2(t) = \int_{t-\tau(t)}^t \tilde{z}^T(s)\bar{Q}_1\tilde{z}(s)ds, \\ V_3(t) = \int_{-\tau_M}^{-\tau_m} \int_{t+s}^t \dot{\tilde{h}}^T(v)\bar{R}_1\dot{\tilde{h}}(v)dvds, \\ V_4(t) = \tilde{z}^T(t)\bar{P}_1\tilde{z}(t), \\ V_5(t) = \tilde{h}^T(t)\bar{P}_2\tilde{h}(t). \end{cases}$$

并且

$$\begin{aligned} \bar{P}_1 &= I_n \otimes P_1, \bar{Q}_1 = I_n \otimes Q_1, \\ \bar{R}_1 &= I_n \otimes R_1, \bar{P}_2 = I_n \otimes P_2. \end{aligned}$$

分别对 $V_i(t) (i = 1, 2, \dots, 5)$ 求导, 并将式(18)~(20)代入, 得到

$$\begin{aligned} \dot{V}_1(t) &= \tilde{x}^T(t)[I_n \otimes A^T P_1 + I_n \otimes P_1 A]\tilde{x}(t) - \\ &\quad 2\tilde{x}^T(t)(A \otimes P_1 BKC)\tilde{z}(t - \tau(t)), \\ \dot{V}_2(t) &\leq \tilde{z}^T(t)\bar{Q}_1\tilde{z}(t) - (1 - \mu)\tilde{z}^T(t - \tau(t))\bar{Q}_1 \times \\ &\quad \tilde{z}(t - \tau(t)), \\ \dot{V}_3(t) &= (\tau_M - \tau_m)\tilde{h}^T(t)[I \otimes ((A - GC)^T R_1 \times \\ &\quad (A - GC))]\tilde{h}(t) + \int_{t-\tau_m}^{t-\tau_M} \dot{\tilde{h}}^T(s)\bar{R}_1\dot{\tilde{h}}(s)ds, \\ \dot{V}_4(t) &= \tilde{z}^T(t)[I \otimes (A^T P_1 + P_1 A)]\tilde{z}(t) - \\ &\quad 2\tilde{z}^T(t - \tau(t)) \times [A \otimes (BKC)^T P_1]\tilde{z}(t) + \\ &\quad 2\tilde{h}^T(t)[I \otimes (GC)^T P_1]\tilde{z}(t). \\ \dot{V}_5(t) &= \tilde{h}^T(t)[I \otimes (A - GC)^T P_2 + I \otimes P_2 \times \\ &\quad (A - GC)]\tilde{h}(t). \end{aligned}$$

于是有

$$\dot{V}(t, \delta(t)) \leq \delta^T(t) \prod \delta(t) + \int_{t-\tau_m}^{t-\tau_M} \dot{\tilde{h}}^T(s)\bar{R}_1\dot{\tilde{h}}(s)ds +$$

$$\begin{aligned}
 & (\tau_M - \tau_m)\tilde{h}^T(t)[I \otimes ((A - GC)^T R_1 \times \\
 & (A - GC))] \tilde{h}(t) = \\
 & \delta^T(t) \prod \delta(t) + \int_{t-\tau_m}^{t-\tau_M} \dot{h}^T(s) \bar{R}_1 \dot{h}(s) ds + \\
 & (\tau_M - \tau_m)\tilde{h}^T(t)[I \otimes ((A - GC)^T R_1 \times \\
 & (A - GC))] \tilde{h}(t) + 2\delta^T(t) \bar{N} [\tilde{h}(t - \tau_m) - \\
 & \tilde{h}(t - \tau(t)) - \int_{t-\tau(t)}^{t-\tau_m} \dot{h}(s) ds] + 2\delta^T(t) \times \\
 & \bar{M} [\tilde{h}(t - \tau(t)) - \tilde{h}(t - \tau_M) - \\
 & \int_{t-\tau_M}^{t-\tau(t)} \dot{h}(s) ds]. \tag{23}
 \end{aligned}$$

其中

$$\Pi = \begin{bmatrix} \Pi_{11} & 0 & -\Lambda \otimes P_1 BKC & 0 & 0 & 0 & 0 \\ * & \Pi_{22} & -\Lambda \otimes P_1 BKC & I \otimes P_1 GC & 0 & 0 & 0 \\ * & * & -(1 - \mu)\bar{Q}_1 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix},$$

$$\begin{aligned}
 \Pi_{11} &= I \otimes A^T P_1 + I \otimes P_1 A, \\
 \Pi_{22} &= I \otimes A^T P_1 + I \otimes P_1 A + \bar{Q}_1, \\
 \Pi_{44} &= I \otimes (A^T P_2 + P_2 A - P_2 GC - (P_2 GC)^T).
 \end{aligned}$$

由引理2可得

$$\begin{aligned}
 & -2\delta^T(t) \bar{N} \int_{t-\tau(t)}^{t-\tau_m} \dot{h}(s) ds \leq \\
 & (\tau(t) - \tau_m)\delta^T(t) \bar{N} \times \\
 & \bar{R}_1^{-1} \bar{N}^T \delta(t) + \int_{t-\tau(t)}^{t-\tau_m} \dot{h}^T(s) \bar{R}_1 \dot{h}(s) ds, \tag{24}
 \end{aligned}$$

$$\begin{aligned}
 & -2\delta^T(t) \bar{M} \int_{t-\tau_M}^{t-\tau(t)} \tilde{h}(s) ds \leq \\
 & (\tau_M - \tau(t))\delta^T(t) \bar{M} \bar{R}_1^{-1} \times \\
 & \bar{M}^T \delta(t) + \int_{t-\tau_M}^{t-\tau(t)} \dot{h}^T(s) \bar{R}_1 \dot{h}(s) ds. \tag{25}
 \end{aligned}$$

将式(24)、(25)代入(23),可得

$$\begin{aligned}
 \dot{V}(t) &\leq \delta^T(t) \Sigma \delta(t) + (\tau_M - \tau_m)\delta^T(t) \Omega^T \bar{R}_1 \Omega \delta(t) + \\
 & (\tau(t) - \tau_m)\delta^T(t) \bar{N} \bar{R}_1^{-1} \bar{N}^T \delta(t) + \\
 & (\tau_M - \tau(t))\delta^T(t) \bar{M} \bar{R}_1^{-1} \bar{M}^T \delta(t) = \\
 & \delta^T(t) [\Sigma + (\tau_M - \tau_m)\Omega^T \bar{R}_1 \Omega + (\tau(t) - \tau_m) \times \\
 & \bar{N} \bar{R}_1^{-1} \bar{N}^T + (\tau_M - \tau(t))\bar{M} \bar{R}_1^{-1} \bar{M}^T] \delta(t), \tag{26}
 \end{aligned}$$

其中 Σ 具体值已在上面定理中给出,不再赘述.

若式(21)、(22)成立,则根据引理4可得 $\Sigma + (\tau_M - \tau_m)\Omega^T \bar{R}_1 \Omega + \bar{N} \bar{R}_1^{-1} \bar{N}^T < 0$ 且 $\Sigma + (\tau_M - \tau_m) \times \Omega^T \bar{R}_1 \Omega + \bar{M} \bar{R}_1^{-1} \bar{M}^T < 0$ 成立. 此时, $\Sigma + (\tau_M - \tau_m)$

$\times \Omega^T \times \bar{R}_1 \Omega + (\tau(t) - \tau_m)\bar{N} \bar{R}_1^{-1} \bar{N}^T + (\tau_M - \tau(t))\bar{M} \bar{R}_1^{-1} \bar{M}^T < 0$ 成立. 因此可得 $\dot{V}(t, \delta(t)) < 0$ 成立,即在控制协议(4)下,多智能体系统(1)能够达到一致. \square

当系数矩阵未知时,式(21)、(22)不能通过 Matlab 的 LMI 工具箱进行求解. 因此,采用变量替换的方法,现给出如下推论.

推论1 对于多智能体系统(1),若假设1、假设2成立,且延迟时间 $\tau(t)$ 满足式(3),则对于任意给定实数 η ,非负实数 μ 和正常数 τ_m, τ_M ,控制协议(4)能够使线性多智能体系统(1)达到一致的充分条件如下:存在正定矩阵 $\hat{P}_1 \in R^{m \times m}, \hat{P}_2 \in R^{m \times m}, \hat{Q}_1 \in R^{m \times m}$ 和矩阵 $S_1 \in R^{m \times m}, \hat{N} \in R^{m \times m}, \hat{M} \in R^{m \times m}$,使得线性矩阵不等式

$$\begin{aligned}
 \Xi_1 &= \begin{bmatrix} \hat{\Sigma} & (\tau_M - \tau_m)\hat{N} & \eta(\tau_M - \tau_m)\hat{\Omega}^T(\hat{P}_2 A^T - S_1^T) \\ * & -\eta(\tau_M - \tau_m)\hat{P}_2 & 0 \\ * & * & -\eta(\tau_M - \tau_m)\hat{P}_2^{-1} \end{bmatrix} \\
 &< 0,
 \end{aligned}$$

$$\begin{aligned}
 \Xi_2 &= \begin{bmatrix} \hat{\Sigma} & (\tau_M - \tau_m)\hat{M} & \eta(\tau_M - \tau_m)\hat{\Omega}^T(\hat{P}_2 A^T - S_1^T) \\ * & -\eta(\tau_M - \tau_m)\hat{P}_2 & 0 \\ * & * & -\eta(\tau_M - \tau_m)\hat{P}_2^{-1} \end{bmatrix} \\
 &< 0
 \end{aligned}$$

成立. 其中

$$\begin{aligned}
 \hat{\Sigma} &= \begin{bmatrix} \hat{\Sigma}_{11} & 0 & -\Lambda \otimes BKC & 0 & \hat{N}_1 & \hat{M}_1 - \hat{N}_1 - \hat{M}_1 \\ * & \hat{\Sigma}_{22} & -\Lambda \otimes BKC & I_n \otimes S_1 & \hat{N}_2 & \hat{M}_2 - \hat{N}_2 - \hat{M}_2 \\ * & * & -(1 - \mu)\hat{Q}_1 & 0 & \bar{N}_3 & \bar{M}_3 - \bar{N}_3 - \bar{M}_3 \\ * & * & * & \hat{\Sigma}_{44} & \hat{N}_4 & \hat{M}_4 - \bar{N}_4 - \hat{M}_4 \\ * & * & * & * & \hat{\Sigma}_{55} & \hat{\Sigma}_{56} & \hat{\Sigma}_{57} \\ * & * & * & * & * & \hat{\Sigma}_{66} & \hat{\Sigma}_{67} \\ * & * & * & * & * & * & \hat{\Sigma}_{77} \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
 \hat{\Sigma}_{11} &= I_n \otimes (\hat{P}_1 A^T + A \hat{P}_1), \\
 \hat{\Sigma}_{22} &= I_n \otimes (\hat{P}_1 A^T + A \hat{P}_1 + \hat{Q}_1), \\
 \hat{\Sigma}_{44} &= I_n \otimes (\hat{P}_2 A^T + A \hat{P}_2 - S_1 - S_1^T), \\
 \hat{\Sigma}_{55} &= \bar{N}_5 + \bar{N}_5^T, \hat{\Sigma}_{56} = \bar{M}_5 - \bar{N}_5 + \bar{N}_6^T, \\
 \hat{\Sigma}_{57} &= \bar{N}_7^T - \bar{M}_5, \hat{\Sigma}_{66} = \bar{M}_6 + \bar{M}_6^T - \bar{N}_6 - \bar{N}_6^T, \\
 \hat{\Sigma}_{67} &= -\bar{N}_7^T - \bar{M}_6 + \bar{M}_7^T, \hat{\Sigma}_{77} = -\bar{M}_7 - \bar{M}_7^T, \\
 \hat{P}_2 &= I_n \otimes P_2, \hat{Q}_1 = I_n \otimes \bar{Q}_1, S_1 = GC \hat{P}_2, \\
 \hat{N} &= [\hat{N}_1^T, \hat{N}_2^T, \bar{N}_3^T, \hat{N}_4^T, \bar{N}_5^T, \bar{N}_6^T, \bar{N}_7^T]^T, \\
 \hat{M} &= [\hat{M}_1^T, \hat{M}_2^T, \bar{M}_3^T, \hat{M}_4^T, \bar{M}_5^T, \bar{M}_6^T, \bar{M}_7^T]^T, \\
 \hat{\Omega}^T &= [0, 0, 0, 1, 0, 0, 0]^T.
 \end{aligned}$$

证明 首先,令 $P_0 = \text{diag}\{I_n \otimes P_1^{-1}, I_n \otimes P_1^{-1}, I_{nm}, I_n \otimes P_2^{-1}, I_{nm}, I_{nm}, I_{nm}, I_n \otimes P_2^{-1}\}$, 将 $\Pi_1 < 0, \Pi_2 < 0$ 两端分别乘 P_0 , 然后作变量替换, 令

$$\begin{aligned} \hat{P}_1 &= P_1^{-1}, \hat{P}_2 = P_2^{-1}, S_1 = GCP_2^{-1}, \\ \hat{N}_1 &= P_1^{-1}N_1, \hat{N}_2 = P_1^{-1}N_2, \hat{N}_4 = P_2^{-1}N_4, \\ \hat{N}_1 &= I_n \otimes \hat{N}_1, \hat{N}_2 = I_n \otimes \hat{N}_2, \hat{N}_4 = I_n \otimes \hat{N}_4, \\ \hat{M}_1 &= P_1^{-1}M_1, \hat{M}_2 = P_1^{-1}M_2, \hat{M}_4 = P_2^{-1}M_4, \\ \hat{M}_1 &= I_n \otimes \hat{M}_1, \hat{M}_2 = I_n \otimes \hat{M}_2, \hat{M}_4 = I_n \otimes \hat{M}_4, \\ \hat{M}_4 &= I_n \otimes \hat{M}_4, R_1 = \eta \hat{P}_2^{-1}. \end{aligned}$$

由此可得矩阵 Ξ_1, Ξ_2 .

显然, 不等式 $\Pi_1 < 0, \Pi_2 < 0$ 等价于 $\Xi_1 < 0, \Xi_2 < 0$, 则 $\lim_{t \rightarrow \infty} x_i(t) = 0 (i = 1, 2, \dots, N)$, 即多智能体系统(1)达到一致. \square

注2 通过上述变换可以直接利用 Matlab 中的 LMI 工具箱求解, 无需调整参数, 这给控制器的设计带来了极大方便. 若 C 是行满秩矩阵, 则可得 $G = S_1 P_2 C^T (C C^T)^{-1}$; 若 C 是可逆矩阵, 则有 $G = S_1 P_2 C^{-1}$.

注3 本文中的时延 $\tau(t)$ 有上下界, 且在变化率不超过给定值 μ 的条件下考虑线性系统(1)的一致性. 在特殊情况下, 当时延 $\tau(t)$ 为固定值 τ_0 时, 本文所设计的控制器(4)对线性系统(1)一致性问题同样适用.

注4 当系统参数不可测时, 本文用观测器的输出状态 $\xi_i(t)$ 代替系统的输出状态 $y_i(t)$ 设计控制器. 若系统状态可测, 则本文设计的控制器(4)也能解决线性系统(1)的一致性问题.

注5 本文设定控制器的增益矩阵 K 为待定矩阵. 若增益矩阵 K 给定, 则可以分析矩阵 K 的取值对系统的影响, 如收敛时间长短、消耗能量多少等.

注6 虽然本文研究的是线性系统一致性问题, 但其控制思想可以应用在编队控制: 控制每个智能体与中心点的相对位置收敛到某一固定数值. 这一部分可以作为之后的扩展工作继续做下去.

3 仿真实例

本小节给出一个仿真实例来验证本文结论的正确性. 如图1所示, 考虑一个由4个智能体组成的线性多智能体系统.

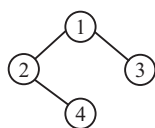


图1 由4个节点组成的无向图

本文假设 $a_{ij} = 1$, 系统的拉普拉斯矩阵

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

假设每个智能体系统(1)中的参数矩阵如下:

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1.25 \\ 1.25 \end{bmatrix}, C = [16, 1].$$

令 $\tau_M = 0.02, \tau_m = 0, \mu = 0.1, \eta = 1$, 通过 Matlab 工具箱 feasp 求解器求解推论1中线性矩阵不等式 $\Xi_1 < 0, \Xi_2 < 0$, 得到不等式成立的一组解

$$\begin{aligned} P_1 &= 10^7 \begin{bmatrix} 1.5519 & -0.018 \\ -0.018 & 1.5612 \end{bmatrix}, \\ P_2 &= 10^7 \begin{bmatrix} 1.5943 & -0.0052 \\ -0.0052 & 1.5953 \end{bmatrix}, \\ Q_1 &= 10^7 \begin{bmatrix} 1.6212 & -0.0079 \\ -0.0079 & 1.6497 \end{bmatrix}, \\ S_1 &= \begin{bmatrix} 1.7283 & -0.0094 \\ -0.0094 & 1.7313 \end{bmatrix}, K = 1, \end{aligned}$$

$$\text{则 } G = \begin{bmatrix} 0.0059 \\ 0.0004 \end{bmatrix}.$$

设系统时延 $\tau(t)$ 在 $[0, 0.02]$ 之间变化, 取 $\tau(t) = 0.01 \times \sin t + 0.01$, 当 $t \in [-\tau(t), 0]$ 时, 取初始条件 $x_1(t) = [10, 8]^T, x_2(t) = [5, 3]^T, x_3(t) = [-5, -3]^T, x_4(t) = [-10, -8]^T$.

为了验证上述线性系统在控制协议(4)下的收敛性, 给出如下仿真: 图2和图3给出了智能体随时间变化的运动轨迹, 表明了系统(1)最终能够达到一致.

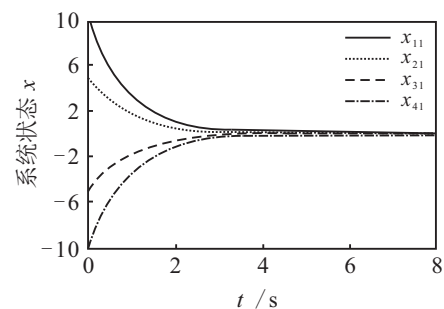


图2 系统状态轨迹 $x_{i1}(t) (i = 1, 2, 3, 4)$

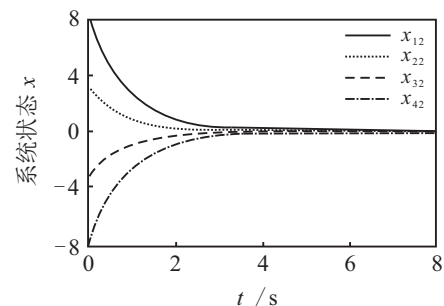


图3 系统状态轨迹 $x_{i2}(t) (i = 1, 2, 3, 4)$

图4和图5给出了观测器状态随时间变化的轨迹. 在图2~图5中, $\mu = 0.01$.

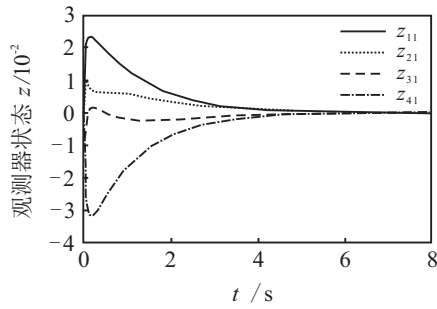


图4 观测器状态轨迹 $z_{i1}(t)(i = 1, 2, 3, 4)$

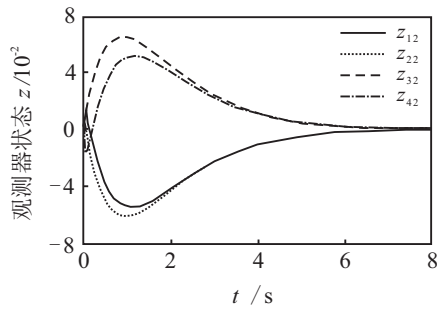


图5 观测器状态轨迹 $z_{i2}(t)(i = 1, 2, 3, 4)$

由图2~图5可以看出,智能体状态的收敛时间为4.5 s,观测器状态的收敛时间为5.5 s,表明在控制协议(4)的作用下,智能体状态最终能够达到一致.

图6和图7表示状态估计误差随时间变化的曲线,其中 $\mu = 0.01$. 从图6和图7中可以看出,系统和观测器的状态误差在5 s内收敛到0. 该仿真实验验证了算法的有效性.

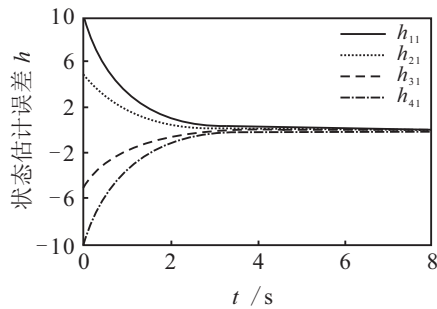


图6 状态估计误差轨迹 $h_{i1}(t)(i = 1, 2, 3, 4)$

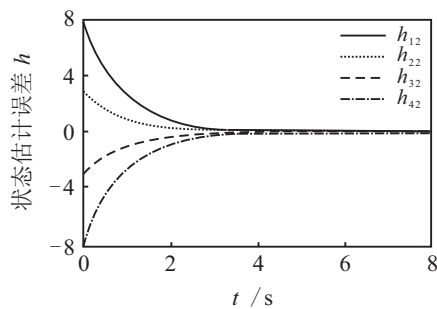


图7 状态估计误差轨迹 $h_{i2}(t)(i = 1, 2, 3, 4)$

4 结论

本文考虑了在通讯时滞情形下线性多智能体系统的一致性,将研究神经网络的控制思想应用于多智能体系统,提出了一种时滞线性多智能体系统一致性控制算法,并且运用图论、Lyapunov-Krasovskii函数分析了系统稳定性和求解线性矩阵不等式的方法得到时滞线性系统达到一致性的充分条件. 该算法最大的特点是利用观测器解决了系统状态量不可测的问题,观测器针对每个智能体估计出其状态信息,从而利用观测器的相对输出信息设计控制协议,能够使得多智能体系统状态达到一致. 本文研究的拓扑结构是无向图,根据已有经验并对比其他文献可知,当前工作可以推广到加权有向网络,下一步的工作内容是对该情况做进一步研究.

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