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直觉模糊集的Choquet积分相关测度及其决策应用

刘卫锋†, 何 霞, 常 娟

(郑州航空工业管理学院 数学学院，郑州 450015)

摘 要: Qu 给出的直觉模糊集的Choquet 积分相关系数的计算公式与相关系数的性质相矛盾. 为此, 通过一个实例说明 Qu 定义的直觉模糊 Choquet 积分相关系数定义存在的问题, 并结合相关系数的性质证明, 分析问题出现的原因; 然后, 针对存在的问题, 以直觉模糊集的 Choquet 积分相关指标为基础, 给出新的直觉模糊集的 Choquet 积分信息能量的概念, 定义新的直觉模糊集的 Choquet 积分相关系数, 并讨论相关系数的性质; 最后, 利用新定义的直觉模糊集的 Choquet 积分相关测度, 推导出方案与正理想方案之间的 Choquet 积分相关系数计算公式, 据此提出一种直觉模糊多属性决策方法, 并通过实例分析以及方法对比, 说明所提出方法的可行性和有效性.

关键词: 直觉模糊集；相关系数；模糊测度；Choquet 积分；决策；应用

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Choquet integral correlation measures of intuitionistic fuzzy sets and its application in decision making

LIU Wei-feng, HE Xia, CHANG Juan

(School of Mathematics，Zhengzhou University of Aeronautics，Zhengzhou 450015，China)

Abstract: The Choquet integral correlation coefficient between intuitionistic fuzzy sets and the computing formula are obtained by Qu, but the results have contradiction with the nature of the Choquet integral correlation coefficient between intuitionistic fuzzy sets. It is found from the practical example that the range of the Choquet integral correlation coefficient between intuitionistic fuzzy sets is not appropriate, and through the study on the proof of its nature of the Choquet integral correlation coefficient, the cause of the problem existed is analyzed. Then based on the Choquet integral correlation of intuitionistic fuzzy sets, the new Choquet integral information energy of the intuitionistic fuzzy set and the Choquet integral correlation coefficient between intuitionistic fuzzy sets are defined, and their natures are also discussed. Finally, the Choquet correlation coefficient between the alternative and the positive ideal alterative is derived, and a method is developed to solve the multiple attribute decision making problem using the Choquet correlation coefficient, and an example is used to illustrate the feasibility and effectiveness of the proposed method.

Keywords: intuitionistic fuzzy set；correlation coefficient；fuzzy measure；Choquet integral；decision making；application

0 引 言

直觉模糊集[1] 作为模糊集[2] 的一种推广, 同时考虑了元素属于集合的隶属度和非隶属度, 故在描述客观世界的模糊性或不确定性现象时较模糊集更准确、更有力、更灵活. 目前, 关于直觉模糊集的研究已经取得了系列成果, 涉及直觉模糊集的理论基础[3-5]、直觉模糊决策理论[5-13]、直觉模糊积分[14]、直觉模糊逻辑[15] 等.

相关性作为概率统计中的一个重要概念, 在经济管理、工程科学等领域发挥着重要作用. 由于研究对象的不确定性、模糊性日益增强, 许多学者尝试将相关性推广到模糊集领域, 并取得了丰硕的研究成果, 极大地丰富了相关性理论. 如果考虑到研究对象的类型, 这些相关测度可以分为模糊集的相关测度[16-21]、直觉模糊集的相关测度[22-35]、犹豫模糊集的相关测度[36-39]、对偶犹豫模糊集的相关测度[40-45] 以及其他类型模糊集的相关测度[46-50] 等. 但是, 上述直觉模糊相关测度[22-34] 都是基于这样假设的, 即认

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责任编委: 李登峰.

†通讯作者. E-mail: lwf0519@163.com.

为属性之间是彼此独立的,也即认为属性的重要性是满足可加性的. 而事实上,在决策过程中,属性之间往往存在着相互作用或相关. 因此,为了更好地反映出实际决策状况并得到更加客观的决策结果,研究反映出属性相互作用或相关的相关测度就变得极其重要. 为此,Qu等[35]将Choquet积分应用到直觉模糊相关测度中,提出了直觉模糊集的Choquet积分信息能量、相关指标和相关系数等概念,推广了已有的加权直觉模糊信息能量、相关指标和相关系数. 但是,经过分析发现,Qu等定义的直觉模糊Choquet积分相关系数与其取值范围存在着矛盾,因此有必要重新研究并提出克服该矛盾的相关系数,从而达到修正Qu等定义的直觉模糊集的Choquet积分相关系数的目的.

结合上面分析,本文首先结合一个具体实例,说明Qu等定义的直觉模糊Choquet积分相关系数出现了大于1的情况,与其取值范围不相符;紧接着,结合该文中直觉模糊Choquet积分相关系数性质的证明,分析其大于1的症结所在;在此基础上,结合直觉模糊集Choquet积分相关指标的概念,定义由相关指标诱导的直觉模糊集的Choquet积分信息能量和直觉模糊集的Choquet积分相关系数,并研究其相关性质;最后,推导出方案与正理想方案之间的直觉模糊集Choquet积分相关系数的计算公式,并结合该计算公式提出一种直觉模糊多属性决策方法,并通过应用实例以及相关决策方法对比说明所提出方法的可行性和有效性.

1 相关概念

定义1[2] 设X为论域,称$A = \{\langle x,\mu_A(x),\nu_A(x)\rangle|x\in X\}$为$X$上的一个直觉模糊集. 其中:$\mu_A: X\to[0,1]$,$\nu_A: X\to[0,1]$是$X$上的模糊集;$\mu_A(x)$,$\nu_A(x)$分别表示$X$上元素$x$属于$A$的隶属度和非隶属度,且对于$\forall x\in X,\mu_A(x),\nu_A(x)\in[0,1]$,有$\mu_A(x)+\nu_A(x)\leqslant 1$.

定义2[20] 设$X=\{x_1,x_2,\cdots,x_n\}$为论域,$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为$X$上的直觉模糊集,$A$的信息能量定义为

$$E_{\rm IF}(A)=\sum_{i=1}^n(\mu_A^2(x_i)+\nu_B^2(x_i)).$$

A与B的相关指标定义为

$$C_{\rm IF}(A,B)=\sum_{i=1}^n(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)).$$

定义3 设$X=\{x_1,x_2,\cdots,x_n\}$为论域,$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为$X$上的直觉模糊集,$A$的加权信息能量定义为

$$E_{\rm WIF}(A)=\sum_{i=1}^n w_i(\mu_A^2(x_i)+\nu_B^2(x_i)).$$

A与B的加权相关指标定义为

$$C_{\rm WIF}(A,B)=\sum_{i=1}^n w_i(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)).$$

其中:w_i表示$x_i(i=1,2,\cdots,n)$的重要性,且$w_i\geqslant 0,\sum_{i=1}^n w_i=1$.

定义3没有考虑到犹豫度,如果考虑到犹豫度,该定义就是Xu等[22]定义的加权相关指标.

虽然定义3中的信息能量和相关指标考虑了x_i的权重,但是却假设x_i之间是相互独立的,而事实上,x_i之间往往存在相互作用,为此Qu等[35]在模糊测度和Choquet积分等概念基础上,将Choquet积分引入到信息能量、相关指标以及相关系数的定义中,从而反映出x_i之间存在的相互作用.

定义4[51] 设2^X为有限集X的幂集,若映射$g: 2^X\to[0,1]$满足条件: 1) $g(\varPhi)=0,g(X)=1$;2) 若$A,B\in 2^X$且$A\subseteq B$,则$g(A)\leqslant g(B)$. 则称g是X上的模糊测度.

考虑到模糊测度计算复杂,Sugeno又提出了计算较为简单的λ-模糊测度.

定义5[51] 给定$\lambda\in(-1,\infty)$,2^X为有限集X的幂集,若映射$g: 2^X\to[0,1]$满足条件: 1) $g_\lambda(X)=0$;2) 若$A,B\in 2^X$且$A\bigcap B=\varPhi$,则$g_\lambda(A\bigcup B)=g_\lambda(A)+g_\lambda(B)+\lambda g_\lambda(A)g_\lambda(B)$. 则称$g_\lambda$是$X$上的$\lambda$-模糊测度.

定义6[52] 设f为定义在$X=\{x_1,x_2,\cdots,x_n\}$上的非负函数,g_λ是定义在X上的λ-模糊测度,则f关于g_λ的离散Choquet积分定义为

$$\int f{\rm d}\mu=\sum_{i=1}^n f(x_{\sigma(i)})[g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})].$$

其中:$(\sigma(1),\sigma(2),\cdots,\sigma(n))$是$(1,2,\cdots,n)$的一个置换,且满足$f(x_{\sigma(1)})\leqslant f(x_{\sigma(2)})\leqslant\cdots\leqslant f(x_{\sigma(n)})$;$F_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\cdots,x_{\sigma(n)}\},F_{\sigma(n+1)}=0$.

定义7[35] 设$X=\{x_1,x_2,\cdots,x_n\}$为论域,$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为$X$上的直觉模糊集,$g_\lambda$是定义在$X$上的$\lambda$-模糊测度,$A$的Choquet积分信息能量定义

为

$$E_{\mathrm{IFC}}(A)=\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))(\mu_A^2(x_{\sigma(i)})+\nu_A^2(x_{\sigma(i)})),$$

其中$\mu_A^2(x_{\sigma(1)})+\nu_B^2(x_{\sigma(1)})\leqslant\mu_A^2(x_{\sigma(2)})+\nu_B^2(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A^2(x_{\sigma(n)})+\nu_B^2(x_{\sigma(n)})$.

A与B的Choquet积分相关指标定义为

$$C_{\mathrm{IFC}}(A,B)=\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))\times(\mu_A(x_{\sigma(i)})\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\nu_B(x_{\sigma(i)})).$$

其中

$$\mu_A(x_{\sigma(1)})\mu_B(x_{\sigma(1)})+\nu_A(x_{\sigma(1)})\nu_B(x_{\sigma(1)})\leqslant\mu_A(x_{\sigma(2)})\mu_B(x_{\sigma(2)})+\nu_A(x_{\sigma(2)})\nu_B(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A(x_{\sigma(n)})\mu_B(x_{\sigma(n)})+\nu_A(x_{\sigma(n)})\nu_B(x_{\sigma(n)}),$$

$$F_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\cdots,x_{\sigma(n)}\},F_{\sigma(n+1)}=0,$$

这里$(\sigma(1),\sigma(2),\cdots,\sigma(n))$是$(1,2,\cdots,n)$的一个置换.

定义8 [35] 设$X=\{x_1,x_2,\cdots,x_n\}$为论域,$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为$X$上的直觉模糊集,$g_\lambda$是定义在$X$上的$\lambda$-模糊测度,$A$与$B$的直觉模糊Choquet积分相关系数定义为

$$\rho_{\mathrm{IFC}}(A,B)=\frac{C_{\mathrm{IFC}}(A,B)}{\sqrt{E_{\mathrm{IFC}}(A)}\sqrt{E_{\mathrm{IFC}}(B)}}.$$

其中

$$C_{\mathrm{IFC}}(A,B)=\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)),$$

$$E_{\mathrm{IFC}}(A)=\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))(\mu_A^2(x_i)+\nu_A^2(x_i)),$$

$$E_{\mathrm{IFC}}(B)=\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))(\mu_B^2(x_i)+\nu_B^2(x_i)),$$

$$\mu_A^2(x_{\sigma(1)})+\nu_A^2(x_{\sigma(1)})\leqslant\mu_A^2(x_{\sigma(2)})+\nu_A^2(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A^2(x_{\sigma(n)})+\nu_A^2(x_{\sigma(n)}),$$

$$\mu_B^2(x_{\sigma(1)})+\nu_B^2(x_{\sigma(1)})\leqslant\mu_B^2(x_{\sigma(2)})+\nu_B^2(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_B^2(x_{\sigma(n)})+\nu_B^2(x_{\sigma(n)}),$$

$$\mu_A(x_{\sigma(1)})\mu_B(x_{\sigma(1)})+\nu_A(x_{\sigma(1)})\nu_B(x_{\sigma(1)})\leqslant\mu_A(x_{\sigma(2)})\mu_B(x_{\sigma(2)})+\nu_A(x_{\sigma(2)})\nu_B(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A(x_{\sigma(n)})\mu_B(x_{\sigma(n)})+\nu_A(x_{\sigma(n)})\nu_B(x_{\sigma(n)}),$$

$$F_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\cdots,x_{\sigma(n)}\},\ F_{\sigma(n+1)}=0,$$

这里$(\sigma(1),\sigma(2),\cdots,\sigma(n))$是$(1,2,\cdots,n)$的一个置换.

定理1 [35] 设$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为论域$X=\{x_1,x_2,\cdots,x_n\}$上的直觉模糊集,则有:

1) $\rho_{\mathrm{IFC}}(A,B)=\rho_{\mathrm{IFC}}(B,A)$;

2) $0\leqslant\rho_{\mathrm{IFC}}(A,B)\leqslant1$;

3) 若$A=B$,则$\rho_{\mathrm{IFC}}(A,B)=1$.

2 问题分析

2.1 计算实例

下面给出一个具体实例,说明按照定义8计算Choquet积分相关系数会使得$\rho_{\mathrm{IFC}}(A,B)\geqslant1$,从而出现与定理1相矛盾的结论.

例1 设$X=\{x_1,x_2,x_3\}$为论域,$A=\{\langle0.7,0.3\rangle,\langle0.8,0.1\rangle,\langle0.9,0.1\rangle\}$,$B=\{\langle0.6,0.2\rangle,\langle0,8,0.2\rangle,\langle0.8,0.1\rangle\}$是$X$上的直觉模糊集,$g_\lambda$是$X$上的$\lambda$-模糊测度. 其中:$g_\lambda(\varPhi)=0$,$g_\lambda(X)=1$,$g_\lambda(\{x_1\})=0.4$,$g_\lambda(\{x_2\})=0.5$,$g_\lambda(\{x_3\})=0.2$,$g_\lambda(\{x_1,x_2\})=0.95$,$g_\lambda(\{x_1,x_3\})=g_\lambda(\{x_2,x_3\})=0.8$.

表1 直觉模糊集A,B的Choquet积分相关系数

	x_1	x_2	x_3
A	$\langle0.7,0.3\rangle$	$\langle0.8,0.1\rangle$	$\langle0.9,0.1\rangle$
B	$\langle0.6,0.2\rangle$	$\langle0.8,0.2\rangle$	$\langle0.8,0.1\rangle$
$\mu_A\mu_B+\nu_A\nu_B$	0.48	0.66	0.73
$x_{\sigma(i)}$	$x_{\sigma(1)}$	$x_{\sigma(2)}$	$x_{\sigma(3)}$
$\mu_A^2+\nu_A^2$	0.58	0.65	0.82
$\mu_B^2+\nu_B^2$	0.40	0.68	0.65

按照定义8计算下式:

$$C_{\mathrm{IFC}}(A,B)=0.48\times(1-0.8)+0.66\times(0.8-0.2)+0.73\times(0.2-0)=0.638,$$

$$E_{\mathrm{IFC}}(A)=0.58\times(1-0.8)+0.65\times(0.8-0.2)+0.82\times(0.2-0)=0.67,$$

$$E_{\mathrm{IFC}}(B)=0.40\times(1-0.8)+0.65\times(0.8-0.2)+0.68\times(0.2-0)=0.606.$$

此时有

$$C_{\mathrm{IFC}}(A,B)^2=0.638^2=0.407\,044>0.406\,02=0.67\times0.606=E_{\mathrm{IFC}}(A)\times E_{\mathrm{IFC}}(B),$$

即有

$$\rho_{\mathrm{IFC}}(A,B)=\frac{C_{\mathrm{IFC}}(A,B)}{\sqrt{E_{\mathrm{IFC}}(A)}\sqrt{E_{\mathrm{IFC}}(B)}}>1.$$

显然,该结论与$0\leqslant\rho_{\mathrm{IFC}}(A,B)\leqslant 1$相矛盾.

2.2 理论分析

下面从理论上分析为什么例1按照定义8计算会出现$\rho_{\mathrm{IFC}}(A,B)>1$的现象.

首先回顾定理1中$\rho_{\mathrm{IFC}}(A,B)\leqslant 1$的证明过程,并在此基础上分析问题出现的原因.

证明

$$\begin{aligned}
&C_{\mathrm{IFC}}(A,B)=\\
&\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))\times\\
&(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i))=\\
&\sqrt{g_\lambda(F_{\sigma(1)})-g_\lambda(F_{\sigma(2)})}\mu_A(x_1)\times\\
&\sqrt{g_\lambda(F_{\sigma(1)})-g_\lambda(F_{\sigma(2)})}\mu_B(x_1)+\\
&\sqrt{g_\lambda(F_{\sigma(2)})-g_\lambda(F_{\sigma(3)})}\mu_A(x_2)\times\\
&\sqrt{g_\lambda(F_{\sigma(2)})-g_\lambda(F_{\sigma(3)})}\mu_B(x_2)+\cdots+\\
&\sqrt{g_\lambda(F_{\sigma(n)})-g_\lambda(F_{\sigma(n+1)})}\mu_A(x_2)\times\\
&\sqrt{g_\lambda(F_{\sigma(n)})-g_\lambda(F_{\sigma(n+1)})}\mu_B(x_n)+\\
&\sqrt{g_\lambda(F_{\sigma(1)})-g_\lambda(F_{\sigma(2)})}\nu_A(x_1)\times\\
&\sqrt{g_\lambda(F_{\sigma(1)})-g_\lambda(F_{\sigma(2)})}\nu_B(x_1)+\\
&\sqrt{g_\lambda(F_{\sigma(2)})-g_\lambda(F_{\sigma(3)})}\nu_A(x_2)\times\\
&\sqrt{g_\lambda(F_{\sigma(2)})-g_\lambda(F_{\sigma(3)})}\nu_B(x_2)+\cdots+\\
&\sqrt{g_\lambda(F_{\sigma(n)})-g_\lambda(F_{\sigma(n+1)})}\nu_A(x_2)\times\\
&\sqrt{g_\lambda(F_{\sigma(n)})-g_\lambda(F_{\sigma(n+1)})}\nu_B(x_n).
\end{aligned}$$

于是,结合柯西-施瓦兹不等式有

$$\begin{aligned}
&C^2_{\mathrm{IFC}}(A,B)\leqslant\\
&\Big[\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))\times\\
&(\mu^2_A(x_{\sigma(i)})+\nu^2_A(x_{\sigma(i)}))\Big]\Big[\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-\\
&g_\lambda(F_{\sigma(i+1)}))(\mu^2_B(x_{\sigma(i)})+\nu^2_B(x_{\sigma(i)}))\Big].
\end{aligned}$$

两端开方后,同时除以右侧,即可得到$\rho_{\mathrm{IFC}}(A,B)\leqslant 1$. □

从上述证明可以发现,无论是$\mu_A(x_{\sigma(i)})\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\nu_B(x_{\sigma(i)})$还是$\mu^2_A(x_{\sigma(i)})+\nu^2_A(x_{\sigma(i)})$和$\mu^2_B(x_{\sigma(i)})+\nu^2_B(x_{\sigma(i)})$,三者的权重均为$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})$. 但必须注意的是,在整个证明过程中,$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})$及$\mu^2_A(x_{\sigma(i)})+\nu^2_A(x_{\sigma(i)})$和$\mu^2_B(x_{\sigma(i)})+\nu^2_B(x_{\sigma(i)})$中的$x_{\sigma(i)}$都是通过$\mu_A(x_{\sigma(i)})\times\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\nu_B(x_{\sigma(i)})$中的$x_{\sigma(i)}$确定的,即它们中的$x_{\sigma(i)}$对应于$\mu_A(x_{\sigma(i)})\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\times\nu_B(x_{\sigma(i)})$中的$x_{\sigma(i)}$,也即$\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\times\nu_B(x_i)(i=1,2,\cdots,n)$从小到大的排序决定了$x_{\sigma(i)}$,进而确定了权重$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})$,$\mu^2_A(x_{\sigma(i)})+\nu^2_A(x_{\sigma(i)})$和$\mu^2_B(x_{\sigma(i)})+\nu^2_B(x_{\sigma(i)})(i=1,2,\cdots,n)$. 由此可知定义8中要求条件$\mu_A(x_{\sigma(1)})\mu_B(x_{\sigma(1)})+\nu_A(x_{\sigma(1)})\nu_B(x_{\sigma(1)})\leqslant\mu_A(x_{\sigma(2)})\mu_B(x_{\sigma(2)})+\nu_A(x_{\sigma(2)})\times\nu_B(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A(x_{\sigma(n)})\mu_B(x_{\sigma(n)})+\nu_A(x_{\sigma(n)})\times\nu_B(x_{\sigma(n)})$成立是必须的,但是并没要求条件$\mu^2_A(x_{\sigma(1)})+\nu^2_A(x_{\sigma(1)})\leqslant\mu^2_A(x_{\sigma(2)})+\nu^2_A(x_{\sigma(2)})\leqslant\cdots\leqslant\mu^2_A(x_{\sigma(n)})+\nu^2_A(x_{\sigma(n)})$及$\mu^2_B(x_{\sigma(1)})+\nu^2_B(x_{\sigma(1)})\leqslant\mu^2_B(x_{\sigma(2)})+\nu^2_B(x_{\sigma(2)})\leqslant\cdots\leqslant\mu^2_B(x_{\sigma(n)})+\nu^2_B(x_{\sigma(n)})$成立. 因此,定义8中要求3组不等式同时成立,存在一定的问题,也是导致$\rho_{\mathrm{IFC}}(A,B)>1$的根源所在,故而定义8中条件$\mu^2_A(x_{\sigma(1)})+\nu^2_A(x_{\sigma(1)})\leqslant\mu^2_A(x_{\sigma(2)})+\nu^2_A(x_{\sigma(2)})\leqslant\cdots\leqslant\mu^2_A(x_{\sigma(n)})+\nu^2_A(x_{\sigma(n)})$及$\mu^2_B(x_{\sigma(1)})+\nu^2_B(x_{\sigma(1)})\leqslant\mu^2_B(x_{\sigma(2)})+\nu^2_B(x_{\sigma(2)})\leqslant\cdots\leqslant\mu^2_B(x_{\sigma(n)})+\nu^2_B(x_{\sigma(n)})$应该去掉,仅保留$\mu_A(x_{\sigma(1)})\mu_B(x_{\sigma(1)})+\nu_A(x_{\sigma(1)})\nu_B(x_{\sigma(1)})\leqslant\mu_A(x_{\sigma(2)})\times\mu_B(x_{\sigma(2)})+\nu_A(x_{\sigma(2)})\nu_B(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A(x_{\sigma(n)})\times\mu_B(x_{\sigma(n)})+\nu_A(x_{\sigma(n)})\nu_B(x_{\sigma(n)})$即可.

3 直觉模糊Choquet积分相关测度

根据上面分析定义新的直觉模糊Choquet积分信息能量和Choquet积分相关系数,并探讨它们的性质.

定义9 设$X=\{x_1,x_2,\cdots,x_n\}$为论域,$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为$X$上的直觉模糊集,$g_\lambda$是定义在$X$上的$\lambda$-模糊测度,$C_{\mathrm{IFC}}(A,B)$是定义7中$A$与$B$的Choquet积分相关指标,则直觉模糊集$A,B$的信息能量定义为

$$\begin{aligned}
E_{\mathrm{IIFC}}(A)=&\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))\times\\
&(\mu^2_A(x_{\sigma(i)})+\nu^2_A(x_{\sigma(i)})),\\
E_{\mathrm{IIFC}}(B)=&\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))\times\\
&(\mu^2_B(x_{\sigma(i)})+\nu^2_B(x_{\sigma(i)})),
\end{aligned}$$

并称$E_{\mathrm{IIFC}}(A)$,$E_{\mathrm{IIFC}}(B)$为由$C_{\mathrm{IFC}}(A,B)$诱导的信息能量. 其中

$\mu_A(x_{\sigma(1)})\mu_B(x_{\sigma(1)})+\nu_A(x_{\sigma(1)})\nu_B(x_{\sigma(1)})\leqslant$

$\mu_A(x_{\sigma(2)})\mu_B(x_{\sigma(2)})+\nu_A(x_{\sigma(2)})\nu_B(x_{\sigma(2)})\leqslant\cdots\leqslant$

$\mu_A(x_{\sigma(n)})\mu_B(x_{\sigma(n)})+\nu_A(x_{\sigma(n)})\nu_B(x_{\sigma(n)})$,

$F_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\cdots,x_{\sigma(n)}\},\ F_{\sigma(n+1)}=\varPhi$,

这里$(\sigma(1),\sigma(2),\cdots,\sigma(n))$是$(1,2,\cdots,n)$的一个置换,且$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})$及$\mu_A^2(x_{\sigma(i)})+\nu_A^2(x_{\sigma(i)})$和$\mu_B^2(x_{\sigma(i)})+\nu_B^2(x_{\sigma(i)})$中的$x_{\sigma(i)}$对应于$\mu_A(x_{\sigma(i)})\times\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\nu_B(x_{\sigma(i)})$中的$x_{\sigma(i)}$.

在直觉模糊Choquet积分信息能量$E_{\rm IIFC}(A)$, $E_{\rm IIFC}(B)$基础上,定义新的直觉模糊Choquet积分相关系数.

定义10 设$X=\{x_1,x_2,\cdots,x_n\}$为论域,$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为$X$上的直觉模糊集,$g_\lambda$是定义在$X$上的$\lambda$-模糊测度,$C_{\rm IFC}(A,B)$是$A$与$B$的Choquet积分相关指标,$E_{\rm IIFC}(A),E_{\rm IIFC}(B)$为由$C_{\rm IFC}(A,B)$诱导的直觉模糊Choquet积分信息能量,则$A,B$的Choquet积分相关系数定义为

$$\rho_{\rm IIFC}(A,B)=\frac{C_{\rm IFC}(A,B)}{\sqrt{E_{\rm IIFC}(A)}\sqrt{E_{\rm IIFC}(B)}}=\Big[\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))\times(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i))\Big]\Big/\Big\{\Big[\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))(\mu_A^2(x_{\sigma(i)})+\nu_A^2(x_{\sigma(i)}))\Big]^{\frac12}\Big[\sum_{i=1}^{n}(g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)}))(\mu_B^2(x_{\sigma(i)})+\nu_B^2(x_{\sigma(i)}))\Big]^{\frac12}\Big\}.$$

其中

$\mu_A(x_{\sigma(1)})\mu_B(x_{\sigma(1)})+\nu_A(x_{\sigma(1)})\nu_B(x_{\sigma(1)})\leqslant$

$\mu_A(x_{\sigma(2)})\mu_B(x_{\sigma(2)})+\nu_A(x_{\sigma(2)})\nu_B(x_{\sigma(2)})\leqslant\cdots\leqslant$

$\mu_A(x_{\sigma(n)})\mu_B(x_{\sigma(n)})+\nu_A(x_{\sigma(n)})\nu_B(x_{\sigma(n)})$,

$F_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\cdots,x_{\sigma(n)}\},\ F_{\sigma(n+1)}=\varPhi$,

这里$(\sigma(1),\sigma(2),\cdots,\sigma(n))$是$(1,2,\cdots,n)$的一个置换,且$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})$及$\mu_A^2(x_{\sigma(i)})+\nu_A^2(x_{\sigma(i)})$和$\mu_B^2(x_{\sigma(i)})+\nu_B^2(x_{\sigma(i)})$中的$x_{\sigma(i)}$对应于$\mu_A(x_{\sigma(i)})\times\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\nu_B(x_{\sigma(i)})$中的$x_{\sigma(i)}$.

显然,当$\mu_A^2(x_i)+\nu_A^2(x_i)$和$\mu_B^2(x_i)+\nu_B^2(x_i)$的值从小到大排序情况恰好与$\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)$的值从小到大排序一致时,定义10中的直觉模糊Choquet积分相关系数就是Qu等定义的直觉模糊Choquet积分相关系数.

当$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})=g_\lambda(\{x_{\sigma(i)}\})$,并且$\mu_A^2(x_i)+\nu_A^2(x_i)$和$\mu_B^2(x_i)+\nu_B^2(x_i)$的值从小到大排序情况恰好与$\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)$的值从小到大排序一致时,定义10中的直觉模糊Choquet积分相关系数就退化为直觉模糊加权相关系数.

例2 (例1续) 若按照定义10计算$\rho_{\rm IIFC}(A,B)$,即求$E_{\rm IIFC}(A),E_{\rm IIFC}(B)$时,$\mu_A^2(x_i)+\nu_A^2(x_i)$和$\mu_B^2(x_i)+\nu_B^2(x_i)$不按照从小到大排序,而是按照$\mu_A(x_{\sigma(i)})\mu_B(x_{\sigma(i)})+\nu_A(x_{\sigma(i)})\nu_B(x_{\sigma(i)})$中的$x_{\sigma(i)}$来排序,则有

$$E_{\rm IIFC}(A)=0.58\times(1-0.8)+0.65\times(0.8-0.2)+0.82\times(0.2-0)=0.67,$$

$$E_{\rm IIFC}(B)=0.40\times(1-0.8)+0.68\times(0.8-0.2)+0.65\times(0.2-0)=0.618.$$

此时有

$$C_{\rm IFC}^2(A,B)=0.638^2=0.407\,044\leqslant 0.414\,06=0.67\times0.618=E_{\rm IIFC}(A)\times E_{\rm IIFC}(B),$$

故而有

$$\rho_{\rm IIFC}(A,B)=\frac{C_{\rm IFC}(A,B)}{\sqrt{E_{\rm IIFC}(A)}\sqrt{E_{\rm IIFC}(B)}}=0.991\,5\leqslant1.$$

定理2 设$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$,$B=\{\langle x_i,\mu_B(x_i),\nu_B(x_i)\rangle|x_i\in X\}$为论域$X=\{x_1,x_2,\cdots,x_n\}$上的直觉模糊集,则有:

1) $\rho_{\rm IIFC}(A,B)=\rho_{\rm IIFC}(B,A)$;

2) $0\leqslant\rho_{\rm IIFC}(A,B)\leqslant1$;

3) 若$A=B$,则$\rho_{\rm IIFC}(A,B)=1$.

4 决策应用

设$A=\{A_1,A_2,\cdots,A_m\}$为方案集,$C=\{c_1,c_2,\cdots,c_n\}$为属性集,$w=(w_1,w_2,\cdots,w_n)^{\rm T}$为属性权重向量,其中$w_i\geqslant0$且$\sum_{i=1}^{n}w_i=1$. 决策者给出的方案$A_i$在属性$c_j$下的属性值为直觉模糊值$\alpha_{ij}$. 其中:$\alpha_{ij}=\langle\mu_{\alpha_{ij}},\nu_{\alpha_{ij}}\rangle,i=1,2,\cdots,m,j=1,2,\cdots,n$. 于是得到直觉模糊决策矩阵$M=(\alpha_{ij})_{mn}$. 若此时属性之间不存在相互作用或相互交叉,则可以使用Ye[34]提出的直觉模糊相关系数求出每个方案与正理想方案$A^*=\{\langle x_i,1,0\rangle|x_i\in X\}$的相关系数,进而由相关系数的大小对方案进行排序择优. 但是,实际决策过程中,属性之间往往存在相互作用或相互交叉的情况,此时考虑使用文中定义的直觉模糊

Choquet积分相关系数求出每个方案与正理想方案的相关系数,并根据相关系数的大小实现方案的排序择优.为此,下面考虑直觉模糊集与正理想方案之间的Choquet积分相关系数.

定理3 设$A=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle|x_i\in X\}$以及正理想方案$A^*=\{\langle x_i,1,0\rangle|x_i\in X\}$均为论域$X=\{x_1,x_2,\cdots,x_n\}$上的直觉模糊集,$g_\lambda$是定义在$X$上的$\lambda$-模糊测度,则$A$与$A^*$的Choquet相关系数为

$$\rho_{\rm IIFC}(A,A^*)=\frac{\sum\limits_{i=1}^{n}[g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})]\mu_A(x_{\sigma(i)})}{\sqrt{\sum\limits_{i=1}^{n}[g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})][\mu_A^2(x_{\sigma(i)})+\nu_A^2(x_{\sigma(i)})]}}.$$

其中

$$\mu_A(x_{\sigma(1)})\leqslant\mu_A(x_{\sigma(2)})\leqslant\cdots\leqslant\mu_A(x_{\sigma(n)}),$$

$$F_{\sigma(i)}=\{x_{\sigma(i)},x_{\sigma(i+1)},\cdots,x_{\sigma(n)}\},\ F_{\sigma(n+1)}=\varPhi,$$

这里$g_\lambda(F_{\sigma(i)})-g_\lambda(F_{\sigma(i+1)})$和$\mu_A^2(x_{\sigma(i)})+\nu_A^2(x_{\sigma(i)})$中的$x_{\sigma(i)}$对应于$\mu_A(x_{\sigma(i)})$中的$x_{\sigma(i)}$.

定理3可由定义10直接证明.

显然,A与A^*的Choquet积分相关系数$\rho_{\rm IIFC}(A,A^*)$具有下面的性质.

定理4 1) $\rho_{\rm IIFC}(A,A^*)=\rho_{\rm IIFC}(A^*,A)$; 2) $0\leqslant\rho_{\rm IIFC}(A,A^*)\leqslant 1$; 3) 若$A=A^*$,则$\rho_{\rm IIFC}(A,A^*)=1$.

下面结合直觉模糊集与正理想方案之间的Choquet积分相关系数,提出一种直觉模糊集的多属性决策方法,步骤如下:

Step 1: 根据实际情况建立直觉模糊决策矩阵.

Step 2: 确定每个属性以及任意属性组合的模糊测度.

Step 3: 计算方案A_i与正理想方案A^*的直觉模糊Choquet积分相关系数$\rho_{\rm IIFC}(A_i,A^*)$.

Step 4: 根据相关系数$\rho_{\rm IIFC}(A_i,A^*)$的大小实现方案排序择优.

例3[35] 某汽车制造商的高科技制造中心打算选一个合适的绿色原料供应商,用以购买一个新产品的关键部件. 经过初步筛选后,有5家供应商入选,其评价准则为: c_1为绿色供应链组织; c_2为绿色供应链设计; c_3为绿色供应链投入; c_4为绿色供应链技术; c_5为绿色供应链教育. 为了避免彼此相互影响,要求决策者匿名根据上面5个属性对每个方案进行评价,并给出直觉模糊决策矩阵$M=(a_{ij})_{55}$, $a_{ij}=\langle\mu_{a_{ij}},\nu_{a_{ij}}\rangle(i,j=1,2,3,5)$为直觉模糊数. 试根据直觉模糊决策矩阵为该汽车制造商选择合适的供应商.

Step 1: 根据实际情况,由专家建立直觉模糊决策矩阵,见表2.

表2 直觉模糊决策矩阵

	c_1	c_2	c_3	c_4	c_5
A_1	$\langle 0.1,0.3\rangle$	$\langle 0.4,0.6\rangle$	$\langle 0.2,0.5\rangle$	$\langle 0.4,0.5\rangle$	$\langle 0.4,0.4\rangle$
A_2	$\langle 0.8,0.1\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0.5,0.4\rangle$	$\langle 0.8,0.1\rangle$
A_3	$\langle 0.3,0.6\rangle$	$\langle 0.3,0.5\rangle$	$\langle 0.2,0.5\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.5,0.4\rangle$
A_4	$\langle 0.7,0.2\rangle$	$\langle 0.6,0.3\rangle$	$\langle 0.1,0.6\rangle$	$\langle 0.3,0.7\rangle$	$\langle 0.8,0.2\rangle$
A_5	$\langle 0.5,0.4\rangle$	$\langle 0.2,0.4\rangle$	$\langle 0.4,0.4\rangle$	$\langle 0.2,0.7\rangle$	$\langle 0.5,0.4\rangle$

Step 2: 由文献[35]可知每个属性和任意属性组合的模糊测度,见表3.

表3 属性及属性组合的模糊测度

属性(组合)	模糊测度	属性(组合)	模糊测度	属性(组合)	模糊测度
$\varPhi$	0	$\{c_2,C_4\}$	0.74	$\{c_2,c_3,C_4\}$	0.85
$\{c_1\}$	0.45	$\{c_2,C_5\}$	0.74	$\{c_2,c_3,C_5\}$	0.89
$\{c_2\}$	0.55	$\{c_3,C_4\}$	0.71	$\{c_2,c_4,C_5\}$	0.86
$\{c_3\}$	0.50	$\{c_3,C_5\}$	0.71	$\{c_3,c_4,C_5\}$	0.84
$\{c_4\}$	0.40	$\{c_4,C_5\}$	0.65	$\{c_1,c_2,c_3,C_4\}$	0.97
$\{c_5\}$	0.40	$\{c_1,c_2,C_3\}$	0.91	$\{c_1,c_2,c_3,C_5\}$	0.97
$\{c_1,C_2\}$	0.77	$\{c_1,c_2,C_4\}$	0.88	$\{c_1,c_2,c_4,C_5\}$	0.92
$\{c_1,C_3\}$	0.74	$\{c_1,c_2,C_5\}$	0.88	$\{c_1,c_3,c_4,C_5\}$	0.94
$\{c_1,C_4\}$	0.68	$\{c_1,c_3,C_4\}$	0.86	$\{c_2,c_3,c_4,C_5\}$	0.91
$\{c_1,C_5\}$	0.68	$\{c_1,c_3,C_5\}$	0.86	$\{c_1,c_2,c_3,c_4,C_5\}$	1.00
$\{c_2,C_3\}$	0.68	$\{c_1,c_4,C_5\}$	0.83		

Step 3: 首先,计算方案A_1与正理想方案A^*的Choquet积分相关系数$\rho_{\rm IIFC}(A_1,A^*)$,即

$$C_{\rm IIFC}(A_1,A^*)=$$
$$0.1\times(1-0.96)+0.2\times(0.96-0.86)+$$
$$0.4\times(0.86-0.65)+0.4\times(0.65-0.4)+$$
$$0.4\times(0.4-0)=0.368,$$

$$E_{\rm IIFC}(A_1)=$$
$$0.1\times(1-0.96)+0.29\times(0.96-0.86)+$$
$$0.52\times(0.86-0.65)+0.41\times(0.65-0.4)+$$
$$0.32\times(0.4-0)=0.3727,$$

$$\rho_{\text{IIFC}}(A_1, A^*) = \frac{0.368}{\sqrt{0.3727}} = 0.6028.$$

其次,可以求出

$$\rho_{\text{IIFC}}(A_2, A^*) = \frac{0.562}{\sqrt{0.577}} = 0.7399,$$

$$\rho_{\text{IIFC}}(A_3, A^*) = \frac{0.422}{\sqrt{0.4106}} = 0.6586,$$

$$\rho_{\text{IIFC}}(A_4, A^*) = \frac{0.656}{\sqrt{0.5632}} = 0.8741,$$

$$\rho_{\text{IIFC}}(A_2, A^*) = \frac{0.44}{\sqrt{0.3908}} = 0.7038.$$

Step 4: 由Choquet积分相关系数$\rho_{\text{IIFC}}(A_4, A^*) \geqslant \rho_{\text{IIFC}}(A_2, A^*) \geqslant \rho_{\text{IIFC}}(A_5, A^*) \geqslant \rho_{\text{IIFC}}(A_3, A^*) \geqslant \rho_{\text{IIFC}}(A_1, A^*)$可知,方案排序为

$$A_4 \succ A_2 \succ A_5 \succ A_3 \succ A_1,$$

即方案A_4是最优的.

下面分别使用文献[53-55]提出的直觉模糊Choquet积分多属性决策方法求出每个方案的综合值,并与文中Choquet积分相关测度方法进行比较.

首先,文献[53]的核心思想是求出直觉模糊决策矩阵的得分矩阵并将其规范化,对每个方案的规范化得分值进行Choquet积分集成.

由表2可以计算出得分矩阵和规范化得分矩阵分别为

$$S = \begin{bmatrix} -0.2 & -0.2 & -0.3 & -0.1 & 0 \\ 0.7 & -0.4 & -0.3 & 0.1 & 0.7 \\ -0.3 & -0.2 & -0.3 & 0 & 0.1 \\ 0.5 & 0.3 & -0.5 & -0.4 & 0.6 \\ 0.1 & -0.2 & 0 & -0.5 & 0.1 \end{bmatrix},$$

$$R = \begin{bmatrix} 0.1 & 0.2857 & 0.4 & 0.6667 & 0 \\ 1 & 0 & 0.4 & 1 & 1 \\ 0 & 0.2857 & 0.4 & 0.8333 & 0.1429 \\ 0.8 & 1 & 0 & 0.1667 & 0.8571 \\ 0.4 & 0.2857 & 1 & 0 & 0.1429 \end{bmatrix}.$$

于是,A_1的Choquet积分综合得分值为

$$\begin{aligned} z(A_1) = {} & 0 \times (1-0.97) + 0.1 \times (0.97-0.85) + \\ & 0.2857 \times (0.85-0.71) + \\ & 0.4 \times (0.71-0.4) + \\ & 0.6667 \times (0.4-0) = 0.4427, \end{aligned}$$

类似可求得

$$z(A_2) = 0.8750,\ z(A_3) = 0.5130,$$

$$z(A_4) = 0.8315,\ z(A_5) = 0.6531.$$

由方案的Choquet积分综合得分值$z(A_2) \geqslant z(A_4) \geqslant z(A_5) \geqslant z(A_3) \geqslant z(A_1)$得到方案排序为

$$A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_1.$$

其次,使用文献[54-55]的直觉模糊Choquet积分集成算子对每个方案的属性值进行集成,即

$$\begin{aligned} & z(A_1) = \\ & (1-0.92)\langle 0.2, 0.5\rangle \oplus (0.92-0.86)\langle 0.1, 0.3\rangle \oplus \\ & (0.86-0.65)\langle 0.4, 0.6\rangle \oplus (0.65-0.40)\langle 0.4, 0.5\rangle \oplus \\ & (0.40-0)\langle 0.4, 0.4\rangle = \langle 0.3709, 0.4608\rangle. \end{aligned}$$

类似地,可以得到

$$z(A_2) = \langle 0.7138, 0.1670\rangle,$$

$$z(A_3) = \langle 0.4315,\ 0.4623\rangle,$$

$$z(A_4) = \langle 0.6948, 0.2490\rangle,$$

$$z(A_5) = \langle 0.4482, 0.4068\rangle.$$

求出每个方案的得分函数为

$$s(A_1) = -0.0899,\ s(A_2) = 0.5468,$$

$$s(A_3) = -0.0308,\ s(A_4) = 0.4458,$$

$$s(A_5) = 0.0414.$$

由$s(A_2) \geqslant s(A_4) \geqslant s(A_5) \geqslant s(A_3) \geqslant s(A_1)$可知,方案排序为

$$A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_1.$$

由上面计算结果可知,使用文献[53-55]提出的直觉模糊Choquet积分方法得到的方案次序均为$A_2 \succ A_4 \succ A_5 \succ A_3 \succ A_1$,而使用本文方法得到的方案排序为$A_4 \succ A_2 \succ A_5 \succ A_3 \succ A_1$. 显然,除了最优方案不同之外,由3种决策方法得到的方案排序基本一致.

5　结　论

本文通过实例说明了现有直觉模糊Choquet积分相关系数存在的问题,结合其性质证明分析了问题出现的原因. 在此基础上,提出了新的直觉模糊集的Choquet积分信息能量,并定义了新的直觉模糊集的Choquet积分相关系数,同时讨论了其性质. 推导出每个方案到正理想方案的Choquet积分相关系数的计算公式,提出了基于该计算公式的直觉模糊多属性决策方法,并通过实例和相关决策方法的对比说明了所提出方法的可行性.

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作者简介

刘卫锋 (1976−), 男, 副教授, 从事模糊数学、决策理论等研究, E-mail: lwf0519@163.com;

何霞 (1976−), 女, 副教授, 从事决策理论的研究, E-mail: hexia0723@163.com;

常娟 (1979−), 女, 讲师, 从事决策理论的研究, E-mail: changjuan@zua.edu.cn.

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