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任意切换下高能随机系统的神经网络预设控制

司文杰[1], 董训德[2†]

(1. 河南城建学院 电气与控制工程学院，河南 平顶山 467036；
2. 华南理工大学 自动化科学与工程学院，广州 510640)

摘 要: 针对一类不确定高能随机非线性系统,开展自适应神经网络backstepping控制研究,并保证在任意切换信号下的预设跟踪性能. 该高能系统假定系统动态和任意切换信号未知. 首先,利用预设性能控制,保证跟踪控制性能;其次,RBF神经网络用来克服未知系统动态,仅用到单一自适应更新参数,从而克服过参数问题;最后,基于公共的Lyapunov稳定性理论提出自适应神经网络控制策略,并减少了学习参数. 最终结果表明所设计的公共控制器能保证所有闭环信号半全局最终一致有界,并能在任意切换下保证预设的跟踪性能. 仿真结果进一步表明所提出方法的有效性.

关键词: 切换系统；自适应控制；RBF神经网络；高能随机非线性系统；公共Lyapunov函数；预设性能控制

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Adaptive neural prescribed performance control for uncertain high-power stochastic nonlinear systems under arbitrary switchings

SI Wen-jie[1], *DONG Xun-de*[2†]

(1. School of Electrical and Control Engineering，Henan University of Urban Construction，Pingdingshan 467036，China；2. School of Automation Science and Engineering，South China University of Technology，Guangzhou 510640，China)

Abstract: In this paper, the problem of adaptive neural backstepping control is investigated for uncertain high-power stochastic nonlinear systems with prescribed performance under arbitrary switchings. For the control of high-power nonlinear systems, it is assumed that unknown system dynamics and arbitrary switching signals are unknown. Firstly, by utilizing the prescribed performance control (PPC), the prescribed tracking control performance is ensured. Then, RBF neural networks are employed to deal with completely unknown system dynamics, and only one adaptive parameter is constructed to overcome the over-parameterization. Finally, based on the common Lyapunov stability method, the adaptive neural control method is proposed, which decreases the number of learning parameters. It is shown that the designed common controller can ensure that all the closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB), and the prescribed tracking control performance is guaranteed under arbitrary switchings. The simulation results show the effectiveness of the proposed scheme.

Keywords: switched systems；adaptive control；RBF neural networks；high-power stochastic nonlinear systems；common Lyapunov function；prescribed performance control

0 引 言

随机扰动是实际系统中真实存在的现象,切换信号也广泛存在于工程系统中,如网络系统、电路系统和运输系统. 切换系统中随机扰动的存在会进一步加大跟踪控制难度. 因此,本文试图去研究一类具有未知系统动态的高能切换系统,借助预设性能控制研究其跟踪控制性能,同时考虑随机扰动的影响.

因为高能项(high-power term)的存在使得一般的控制策略无法解决高能系统的跟踪控制问题. 近几年,高能系统得到了大量研究. 文献[1]针对高能系统研究了自适应控制方法; 文献[2]将时变反馈方法应用到高能系统的自适应稳定性问题. 然而,上面提到的控制设计无法解决系统动态未知的情况. 神经网络和模糊逻辑作为逼近器,为处理具有未知系统函数的控制器设计提供了一种有效途径. 文献[3]借助神经网络的逼近能力实现了系统状态未知的自适应

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†通讯作者. E-mail: audxd@scut.edu.cn.

神经网络输出反馈控制. 文献[4]设计了一种非严格反馈系统的神经网络控制器. 在文献[5],非严格反馈系统的控制器设计也考虑了随机扰动. 文献[6]同样针对随机非线性切换系统开展了模糊跟踪控制. 接着,将模糊/神经网络控制应用到实际对象上,如机器人的跟踪控制[7],机械臂的学习控制[8]. 文献[9-10]也针对高能系统的未知函数开展了跟踪控制研究. 然而上面提到的高能系统是在非切换信号下的控制器设计,并没有考虑跟踪性能.

实际的工程中通常需要控制策略满足某种性能指标,例如预定的超调和收敛速度. 针对该问题,预设性能控制(prescribed performance control, PPC)得到了大量研究,例如,基于扰动观测器的预设性能控制方法[11],事件激发的预设性能控制[12],具有输入饱和的预设性能跟踪控制策略[13]. 需要注意的是,随机扰动存在于真实的系统,据作者所知,现有文献中很少存在切换信号下针对高能系统的跟踪性能分析,随机扰动的存在也会加剧系统的跟踪控制难度.

基于上面分析,本文研究一类高能非线性系统在未知任意切换信号下的预设性能跟踪控制策略. 基于公共Lyapunov函数理论,得出的控制器可以保证闭环系统中所有信号有界. 本文的贡献总结如下:

1) 针对高能系统,提出一种新的控制方法, 相比于文献[1-2],本文考虑了未知的系统动态.

2) 随机扰动被研究,使设计的控制器更具有实用价值. 与文献[10]相比,任意切换信号是未知的,并考虑了跟踪性能.

3) 代替更新神经网络权值向量,本文直接更新神经网络权值范数,因此在backstepping运算中只用到了单一的更新参数.

1 问题描述和预备知识

1.1 预备知识

引理1[14] 对于随机系统$\mathrm{d}x = f(x,t)\mathrm{d}t + h(x,t)\mathrm{d}\omega, \forall x \in \boldsymbol{R}^n$有$\omega$是定义在完备概率空间$(\Omega, F, P)$上的$r$维相互独立的标准维纳过程向量,给定任意的$V(x,t) \in C^{2,1}$,定义如下的微分算子$L$:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2}\mathrm{Tr}\Big\{h^{\mathrm{T}}\frac{\partial V}{\partial x}h\Big\}, \tag{1}$$

其中$\mathrm{Tr}(A)$代表矩阵A的秩.

1.2 问题描述

考虑任意切换下的高能随机系统

$$\begin{aligned}
&\mathrm{d}x_i = (x_{i+1}^{p_i} + f_{i,\sigma}(\bar{x}_i))\mathrm{d}t + \psi_{i,\sigma}^{\mathrm{T}}(\bar{x}_i)\mathrm{d}\omega,\\
&\mathrm{d}x_n = (u^{p_n} + f_{n,\sigma}(\bar{x}_n))\mathrm{d}t + \psi_{n,\sigma}^{\mathrm{T}}(\bar{x}_n)\mathrm{d}\omega,\\
&y = x_1, 1 \leqslant i \leqslant n-1.
\end{aligned} \tag{2}$$

其中: $\bar{x}_i = [x_1, x_2, \ldots, x_i]^{\mathrm{T}} \in \boldsymbol{R}^i$、$u \in R$和$y \in R$分别是系统状态、系统输入和系统输出; $p_i \in R^* \triangleq \{\lambda \in R : \lambda \geqslant 1, \lambda$ 是奇数$\}$; 函数$f_i(\cdot) : R^i \to R$和$\psi_i(\cdot) : R^i \to R^r$是未知的; ω定义见引理1; $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$是切换信号.

注1 当高能项$p_i > 1$时,系统(2)被表达为高能(high-power)系统,实际上系统(2)可以表达更广泛的系统结构. 与高能系统[10, 15]相比,系统(2)考虑了随机扰动,随机扰动的存在使得控制器设计更具有实际应用价值. 文献[6]同样开展了在任意切换信号下的预设跟踪性能控制,但是高能项的存在使得该设计策略无法完成高能非线性系统的跟踪控制任务. 在文献[16]中系统函数是已知的,实际上针对未知系统动态的高能系统去设计控制器本身是一个复杂的问题.

本文控制的目标是设计一个自适应神经网络控制器,保证闭环系统内在任意切换的条件下所有信号有界,并使系统输出y能跟踪参考轨迹y_d,最终实现控制系统具有预定的跟踪性能.

假设1 参考轨迹$y_d(t)$及其k阶导数都是连续和有界的.

1.3 预设性能函数

跟踪误差$\nu_1 = y - y_d$,有如下的性能边界:

$$-\underline{\delta}\rho(t) < \nu_1(t) < \bar{\delta}\rho(t),\ \forall t \geqslant 0. \tag{3}$$

其中: $\underline{\delta}$和$\bar{\delta}$是正的常数; $\rho(t) = (\rho_0 - \rho_\infty)\mathrm{e}^{-\kappa t} + \rho_\infty$, κ、ρ_0和ρ_∞是正的设计参数.

为了描述不受限形式(3),给出如下的状态变换:

$$\nu_1(t) = \rho(t)R(\zeta_1). \tag{4}$$

其中: $R(\zeta_1) = \dfrac{\bar{\delta}\mathrm{e}^{\zeta_1} - \underline{\delta}\mathrm{e}^{-\zeta_1}}{\mathrm{e}^{\zeta_1} + \mathrm{e}^{-\zeta_1}}$, ζ_1是转换误差.

受限的跟踪误差条件(3)可以被转换为同等不受限的形式(4). 转化误差ζ_1可以表达为

$$\zeta_1(t) = R^{-1}\Big(\frac{\nu_1}{\rho}\Big) = \frac{1}{2}\ln\Bigg(\frac{\dfrac{\nu_1}{\rho} + \underline{\delta}}{\bar{\delta} - \dfrac{\nu_1}{\rho}}\Bigg), \tag{5}$$

$$\dot{\zeta}_1(t) = \gamma\Big(\dot{\nu}_1 - \frac{\dot{\rho}}{\rho}\nu_1\Big).$$

其中

$$\gamma = \frac{1}{2\rho}\left(\frac{1}{\dfrac{\nu_1}{\rho} + \underline{\delta}} - \frac{1}{\dfrac{\nu_1}{\rho} - \bar{\delta}}\right) > 0,$$

$$\dot{\rho} = -\kappa(\rho_0 - \rho_\infty)\mathrm{e}^{-\kappa t}.$$

2 自适应神经网络控制和稳定性分析

首先,定义$p = \max\{p_i, i = 1, 2, \ldots, n\}$, p_i是正的奇整数;然后,借助如下的坐标变换:

$$z_1 = \zeta_1,\ z_i = x_i - \alpha_{i-1},\ i = 2, 3, \ldots, n, \tag{6}$$

给出下面的定义,其中α_{i-1}是虚拟控制信号.

step 1: 考虑式(2),可以得到

$$\begin{aligned}&dz_1=\\&\gamma\Big(x_{i,2}^{p_1}+f_{1,\sigma(t)}(x_1)-\dot{y}_d-\frac{\dot{\rho}}{\rho}\nu_1\Big)dt+\\&\gamma(g_{1,\sigma(t)}^k(x_1))^{T}d\omega.\end{aligned}\tag{7}$$

选取Lyapunov函数$V_1=\dfrac{z_1^{p-p_1+4}}{p-p_1+4}+\dfrac{\tilde{\theta}_1^2}{2r_1}$,其中$r_1$是正的设计参数.

借助V_1函数的Itô微分并考虑式(7),可以获得

$$\begin{aligned}&LV_1=\\&z_1^{p-p_1+3}\gamma\Big(x_2^{p_1}+f_{1,\sigma(t)}(x_1)-\dot{y}_d-\frac{\dot{\rho}}{\rho}\nu_1\Big)+\\&\frac{p-p_1+3}{2}(\gamma g_{1,\sigma(t)})^{T}(\gamma g_{1,\sigma(t)})z_1^{p-p_1+2}+\frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{r_1}.\end{aligned}\tag{8}$$

依据Young不等式,可以得到

$$\begin{aligned}&\frac{p-p_1+3}{2}(\gamma g_{1,\sigma(t)})^{T}(\gamma g_{1,\sigma(t)})z_1^{p-p_1+2}\leqslant\\&\frac{p-p_1+2}{2}\xi_{1,\sigma(t)}^{\frac{p-p_1+3}{p-p_1+2}}z_1^{p-p_1+3}\|\gamma g_{1,\sigma(t)}\|^{\frac{2(p-p_1+3)}{p-p_1+2}}+\\&\frac{1}{2}\xi_{1,\sigma(t)}^{-(p-p_1+3)},\end{aligned}\tag{9}$$

其中$\xi_{1,\sigma(t)}$是正的设计参数.

考虑式(9),式(8)可写为

$$\begin{aligned}&LV_1\leqslant\\&z_1^{p-p_1+3}\gamma x_2^{p_1}+z_1^{p-p_1+3}\Big(\gamma f_{1,\sigma(t)}(x_1)+\\&\frac{p-p_1+2}{2}\xi_{1,\sigma(t)}^{\frac{p-p_1+3}{p-p_1+2}}\|\gamma g_{1,\sigma(t)}\|^{\frac{2(p-p_1+3)}{p-p_1+2}}-\\&\gamma\dot{y}_d-\gamma\frac{\dot{\rho}}{\rho}\nu_1\Big)+\frac{1}{2}\xi_{1,\sigma(t)}^{-(p-p_1+3)}+\frac{\tilde{\theta}_1\dot{\hat{\theta}}_1}{r_1}.\end{aligned}\tag{10}$$

未知非线性函数$F_{1,\sigma(t)}(Z_1)$被定义为

$$\begin{aligned}&F_{1,\sigma(t)}(Z_1)=\\&\gamma f_{1,\sigma(t)}(x_1)-\gamma\dot{y}_d-\gamma\frac{\dot{\rho}}{\rho}\nu_1+\\&\frac{p-p_1+2}{2}\xi_{1,\sigma(t)}^{\frac{p-p_1+3}{p-p_1+2}}\|\gamma g_{1,\sigma(t)}\|^{\frac{2(p-p_1+3)}{p-p_1+2}},\end{aligned}\tag{11}$$

其中$Z_1=[x_1,y_d,\dot{y}_d]^{T}$.

RBF NN被用来逼近

$$F_{1,\sigma(t)}(Z_1)=W_{1,\sigma(t)}^{*T}S_{1,\sigma(t)}(Z_1)+\epsilon_{1,\sigma(t)}(Z_1),\tag{12}$$

其中逼近误差$|\epsilon_{1,\sigma(t)}(Z_1)|\leqslant\epsilon_{1,\sigma(t)}^*$[17]. NN输入与第$k$个子系统相同,因此$S_{1,\sigma(t)}(Z_1):k\in M$是一样的,可重写为$S_1(Z_1)$.

基于Young不等式,得到

$$\begin{aligned}&z_1^{p-p_1+3}F_{1,\sigma(t)}\leqslant\\&\eta_{1,\sigma(t)}^{\frac{p+3}{p-p_1+3}}z_1^{p+3}\theta_1^*(S_1^{T}S_1)^{\frac{p+3}{2(p-p_1+3)}}+\\&\eta_{1,\sigma(t)}^{-\frac{p+3}{p_1}}+z_1^{p+3}+\epsilon_{1,\sigma(t)}^{*\frac{p+3}{p_1}}.\end{aligned}\tag{13}$$

其中:$\theta_1^*=\|W_{1,\max}^*\|^{\frac{p+3}{p-p_1+3}}$, $W_{1,\max}^*=\max\{W_{1,\sigma(t)}^*:\sigma(t)\in M\}$; $\eta_{1,\sigma(t)}$是正的设计参数.

虚拟信号选择为

$$\begin{aligned}&\alpha_1=\\&-z_1\Big\{\frac{1}{\gamma}(k_1+\bar{\eta}_1^{\frac{p+3}{p-p_1+3}}\hat{\theta}_1(S_1^{T}S_1)^{\frac{p+3}{2(p-p_1+3)}}+1)\Big\}^{\frac{1}{p_1}}.\end{aligned}\tag{14}$$

其中:$\hat{\theta}_1$是θ_1^*的估计值,有$\tilde{\theta}_1=\hat{\theta}_1-\theta_1^*$; $\bar{\eta}_1=\max\{\eta_{1,\sigma(t)}:\sigma(t)\in M\}$. 令$\alpha_1=-z_1\varrho_1$.

设计控制率为

$$\dot{\hat{\theta}}_1=r_1\bar{\eta}_1^{\frac{p+3}{p-p_1+3}}(S_1^{T}S_1)^{\frac{p+3}{2(p-p_1+3)}}z_1^{p+3}-\beta_1\hat{\theta}_1,\tag{15}$$

其中β_1是设计参数.

考虑式(14)和(15),有

$$-\frac{\beta_1}{r_1}\tilde{\theta}_1\hat{\theta}_1\leqslant-\frac{\beta_1}{2r_1}\tilde{\theta}_1^2+\frac{\beta_1}{2r_1}\theta_1^{*2},$$

可得

$$\begin{aligned}&LV_1\leqslant\\&-k_1z_{i,1}^{p+3}+z_1^{p-p_1+3}\gamma(x_2^{p_1}-\alpha_1^{p_1})-\frac{\beta_1}{2r_1}\tilde{\theta}_1^2+\\&\frac{\beta_1}{2r_1}\theta_1^{*2}+\underline{\eta}_1^{-\frac{p+3}{p_1}}+\bar{\epsilon}_1^{*\frac{p+3}{p_1}}+\frac{1}{2}\underline{\xi}_1^{-(p-p_1+3)}.\end{aligned}\tag{16}$$

其中

$$\begin{aligned}&\underline{\eta}_1=\min\{\eta_{1,\sigma(t)}:\sigma(t)\in M\},\\&\bar{\epsilon}_1^*=\max\{\epsilon_{1,\sigma(t)}^*:\sigma(t)\in M\},\\&\underline{\xi}_1=\min\{\xi_{1,\sigma(t)}:\sigma(t)\in M\}.\end{aligned}$$

因为$z_2=x_2-\alpha_1$,基于Young不等式,可以获得

$$z_1^{p-p_1+3}\gamma(x_2^{p_1}-\alpha_1^{p_1})\leqslant z_1^{p+3}+z_2^{p-p_2+3}\bar{\varrho}_1,\tag{17}$$

其中

$$\begin{aligned}&\bar{\varrho}_1=\\&z_2^{p_2}((2^{p_1-2}\gamma p_1)^{(p+3/p_1)}+(2^{p_1-2}\gamma p_1\varrho_1^{p_1-1})^{p+3}+\\&(p_1\gamma\varrho_1^{p_1-1})^{p+3}).\end{aligned}$$

考虑式(17),式(16)能重新整理为

$$\begin{aligned}&LV_1\leqslant\\&-(k_1-1)z_1^{p+3}+z_2^{p-p_2+3}\bar{\varrho}_1-\frac{\beta_1}{2r_1}\tilde{\theta}_1^2+\\&\frac{\beta_1}{2r_1}\theta_1^{*2}+\underline{\eta}_1^{-\frac{p+3}{p_1}}+\bar{\epsilon}_1^{*\frac{p+3}{p_1}}+\frac{1}{2}\underline{\xi}_1^{-(p-p_1+3)}.\end{aligned}\tag{18}$$

step $i(i=2,3,\dots,n-1)$: 相同的设计过程应用在step i中. 定义$z_i=x_i-\alpha_{i-1}$,并得出其微分形式

$$\begin{aligned}&dz_i=\\&(x_{i+1}^{p_i}+f_{i,\sigma(t)}(\bar{x}_i)+h_{i,\sigma(t)}(y)-l\alpha_{i-1})dt+\end{aligned}$$

$$\Big(g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big)^{\rm T}{\rm d}\omega, \tag{19}$$

其中

$$l\alpha_{i-1}=\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}(x_{c+1}^{p_c}+f_{c,\sigma(t)}(\bar{x}_i))+\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_c}\dot{\hat{\theta}}_c+\sum_{c=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(c)}}y_d^{(c+1)}+\frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}g_{p,\sigma(t)}^{\rm T}g_{q,\sigma(t)}.$$

选择Lyapunov函数

$$V_i=\frac{1}{p-p_i+4}z_i^{p-p_i+4}+\frac{1}{2r_i}\tilde{\theta}_i^2,$$

其中r_i是正的设计参数.

给出V_i的微分算子L为

$$LV_i=z_i^{p-p_i+3}(x_{i+1}^{p_i}+f_{i,\sigma(t)}(\bar{x}_i)-l\alpha_{i-1})+\frac{1}{r_i}\tilde{\theta}_i\dot{\hat{\theta}}_i+\frac{p-p_i+3}{2}z_i^{p-p_i+2}\Big(g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big)^{\rm T}\Big(g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big). \tag{20}$$

基于Young不等式,有如下不等式成立:

$$\frac{p-p_i+3}{2}\Big(g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big)^{\rm T}\Big(g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big)z_i^{p-p_i+2}\leqslant\frac{p-p_i+2}{2}\xi_{i,\sigma(t)}^{\frac{p-p_i+3}{p-p_i+2}}z_i^{p-p_i+3}\Big\|g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big\|^{\frac{2(p-p_i+3)}{p-p_i+2}}+\frac{1}{2}\xi_{i,\sigma(t)}^{-(p-p_i+3)}, \tag{21}$$

其中$\xi_{i,\sigma(t)}$是正的设计参数.

定义未知函数

$$F_{i,\sigma(t)}(Z_i)=f_{i,\sigma(t)}-l\alpha_{i-1}+\frac{p-p_i+2}{2}\xi_{i,\sigma(t)}^{\frac{p-p_i+3}{p-p_i+2}}\times\Big\|g_{i,\sigma(t)}-\sum_{c=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_c}g_{c,\sigma(t)}\Big\|^{\frac{2(p-p_i+3)}{p-p_i+2}}+\bar{\varrho}_{i-1}, \tag{22}$$

其中$Z_i=[x_1,\ldots,x_i,y_d,\ldots,y_d^{(i)},\hat{\theta}_1,\ldots,\hat{\theta}_{i-1}]^{\rm T}$.

RBF NN $W_{i,\sigma(t)}^{*\rm T}S_{i,\sigma(t)}$被用来辨识$F_{i,\sigma(t)}(Z_i)=W_{i,\sigma(t)}^{*\rm T}S_{i,\sigma(t)}(Z_i)+\epsilon_{i,\sigma(t)}(Z_i)$, 其中逼近误差$|\epsilon_{i,\sigma(t)}(Z_i)|\leqslant\epsilon_{i,\sigma(t)}^*$. 由于子系统的径向基函数$S_{i,\sigma(t)}(Z_i):\sigma(t)\in M$相同,可简写为$S_i(Z_i)$.

相似于式(13),基于Young不等式,得出

$$z_i^{p-p_i+3}F_{i,\sigma(t)}\leqslant\eta_{i,\sigma(t)}^{\frac{p+3}{p-p_i+3}}z_i^{p+3}\theta_i^*(S_i^{\rm T}S_i)^{\frac{p+3}{2(p-p_i+3)}}+\eta_{i,\sigma(t)}^{-\frac{p+3}{p_i}}+z_i^{p+3}+\epsilon_{i,\sigma(t)}^{*\frac{p+3}{p_i}}. \tag{23}$$

其中

$$\theta_i^*=\|W_{i,\max}^*\|^{\frac{p+3}{p-p_i+3}},$$

$$W_{i,\max}^*=\max\{W_{i,\sigma(t)}^*:\sigma(t)\in M\}.$$

构造虚拟控制信号和自适应控制率

$$\alpha_i=-z_i\{k_i+\bar{\eta}_i^{\frac{p+3}{p-p_i+3}}\hat{\theta}_i(S_i^{\rm T}S_i)^{\frac{p+3}{2(p-p_i+3)}}+1\}^{\frac{1}{p_i}}. \tag{24}$$

其中:k_i是正的设计参数,$\bar{\eta}_i=\max\{\eta_{i,\sigma(t)}:\sigma(t)\in M\}$,$\hat{\theta}_i$是$\theta_i^*$的估计值且有$\tilde{\theta}_i=\hat{\theta}_i-\theta_i^*$. 令$\alpha_i=-z_i\varrho_i$,

$$\dot{\hat{\theta}}_i=r_i\bar{\eta}_i^{\frac{p+3}{p-p_i+3}}(S_i^{\rm T}S_i)^{\frac{p+3}{2(p-p_i+3)}}z_i^{p+3}-\beta_i\hat{\theta}_i, \tag{25}$$

β_i是正的设计参数.

因为$z_i=x_i-\alpha_i$,相似于式(17),有

$$z_i^{p-p_i+3}(x_{i+1}^{p_i}-\alpha_i^{p_i})\leqslant z_i^{p+3}+z_{i+1}^{p-p_{i+1}+3}\bar{\varrho}_i,$$

其中

$$\bar{\varrho}_i=z_{i+1}^{p_{i+1}}((2^{p_i-2}p_i)^{(p+3/p_i)}+(2^{p_i-2}p_i\varrho_i^{p_i-1})^{p+3}+(p_i\varrho_i^{p_i-1})^{p+3}).$$

考虑到$-\dfrac{\beta_i}{r_i}\tilde{\theta}_i\hat{\theta}_i\leqslant-\dfrac{\beta_i}{2r_i}\tilde{\theta}_i^2+\dfrac{\beta_i}{2r_i}\theta_i^{*2}$,可得

$$LV_i\leqslant-(k_i-1)z_i^{p+3}+z_{i+1}^{p-p_{i+1}+3}\bar{\varrho}_i+\underline{\eta}_i^{-\frac{p+3}{p_i}}+\bar{\epsilon}_i^{*\frac{p+3}{p_i}}-z_i^{p-p_i+3}\bar{\varrho}_{i-1}-\frac{\sigma_i}{2r_i}\tilde{\theta}_i^2+\frac{\sigma_i}{2r_i}\theta_i^{*2}+\frac{1}{2}\underline{\xi}_i^{-(p-p_i+3)}. \tag{26}$$

其中

$$\underline{\eta}_i=\min\{\eta_{i,\sigma(t)}:\sigma(t)\in M\},$$

$$\bar{\epsilon}_i^*=\max\{\epsilon_{i,\sigma(t)}^*:\sigma(t)\in M\},$$

$$\underline{\xi}_i=\min\{\xi_{i,\sigma(t)}:\sigma(t)\in M\}.$$

step n: $z_n=x_n-\alpha_{n-1}$,求其微分

$${\rm d}z_n=(u_i^{p_n}+f_{n,\sigma(t)}(x_i)-l\alpha_{n-1}){\rm d}t+\Big(g_{n,\sigma(t)}-\sum_{c=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_c}g_{c,\sigma(t)}\Big)^{\rm T}{\rm d}\omega. \tag{27}$$

选择Lyapunov函数

$$V_n=\frac{1}{p-p_n+4}z_n^{p-p_n+4}+\frac{1}{2r_n}\tilde{\theta}_n^2,$$

其中r_n是设计参数.

求出V_n 的微分算子 L,参考式(8)和(20),第n步

的设计过程与第i步相同,限于篇幅,省略推导过程.

未知非线性函数$F_n(Z_n)$定义为

$$\begin{aligned}&F_{n,\sigma(t)}(Z_n)=\\&-l\alpha_{n-1}+\frac{p-p_{n,\sigma(t)}+2}{2}\xi_{n,\sigma(t)}^{\frac{p-p_n+3}{p-p_n+2}}\Big\|g_{n,\sigma(t)}+\\&+\bar{\varrho}_{n-1}-\sum_{c=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_c}g_{c,\sigma(t)}\Big\|^{\frac{2(p-p_n+3)}{p-p_n+2}},\end{aligned}\tag{28}$$

其中

$$Z_n=[x_1,\dots,x_{n_i},y_d,\dots,y_d^{(n)},\hat{\theta}_1,\dots,\hat{\theta}_{n-1}]^{\mathrm{T}}.$$

用神经网络逼近$F_{n,\sigma(t)}$,基于Young不等式,得

$$\begin{aligned}&z_n^{p-p_n+3}F_{n,\sigma(t)}\leqslant\\&\eta_{n,\sigma(t)}^{-\frac{p+3}{p_n}}+z_n^{p+3}+\epsilon_{n,\sigma(t)}^{*\frac{p+3}{p_n}}+\\&\eta_{n,\sigma(t)}^{\frac{p+3}{p-p_n+3}}z_n^{p+3}\theta_n^*(S_{n,\sigma(t)}^{\mathrm{T}}S_{n,\sigma(t)})^{\frac{p+3}{2(p-p_n+3)}}.\end{aligned}\tag{29}$$

其中

$$\theta_n^*=\|W_{n,\max}^*\|^{\frac{p+3}{p-p_n+3}},$$
$$W_{n,\max}^*=\max\{W_{n,\sigma(t)}^*:\sigma(t)\in M\}.$$

$|\epsilon_{n,\sigma(t)}(Z_n)|\leqslant\epsilon_{n,\sigma(t)}^*$是逼近误差.

构造控制器

$$u=-z_n\{k_n+\bar{\eta}_n^{\frac{p+3}{p-p_n+3}}\hat{\theta}_n(S_n^{\mathrm{T}}S_n)^{\frac{p+3}{2(p-p_n+3)}}+1\}^{\frac{1}{p_n}}.\tag{30}$$

其中: k_n是正的设计参数; $\bar{\eta}_n=\max\{\eta_{n,\sigma(t)}:\sigma(t)\in M\}$; $\hat{\theta}_n$是θ_n^*的估计值,并有$\tilde{\theta}_n=\hat{\theta}_n-\theta_n^*$.

$$\dot{\hat{\theta}}_n=r_n\bar{\eta}_n^{\frac{p+3}{p-p_n+3}}(S_n^{\mathrm{T}}S_n)^{\frac{p+3}{2(p-p_n+3)}}z_n^{p+3}-\beta_n\hat{\theta}_n,\tag{31}$$

β_n是正的设计参数.

考虑到$-\dfrac{\beta_n}{r_n}\tilde{\theta}_n\hat{\theta}_n\leqslant-\dfrac{\beta_n}{2r_n}\tilde{\theta}_n^2+\dfrac{\beta_n}{2r_n}\theta_n^{*2}$,以及式(28)～(30),有如下不等式成立:

$$\begin{aligned}&LV_n\leqslant\\&-k_nz_n^{p+3}+\underline{\eta}_n^{-\frac{p+3}{p_n}}+\bar{\epsilon}_n^{*\frac{p+3}{p_n}}-z_n^{p-p_n+3}\bar{\varrho}_{n-1}-\\&\frac{\beta_n}{2r_n}\tilde{\theta}_n^2+\frac{\beta_n}{2r_n}\theta_n^{*2}+\frac{1}{2}\underline{\xi}_n^{-(p-p_n+3)}.\end{aligned}\tag{32}$$

其中

$$\underline{\eta}_n=\min\{\eta_{n,\sigma(t)}:\sigma(t)\in M\},$$
$$\bar{\epsilon}_n^*=\max\{\epsilon_{n,\sigma(t)}^*:\sigma(t)\in M\},$$
$$\underline{\xi}_n=\min\{\xi_{n,\sigma(t)}:\sigma(t)\in M\}.$$

考虑整体的Lyapunov函数$V=\sum_{i=1}^{n}V_i$,有式(18),(26)和(32),选择$k_i-1>0,k_n>0$和$\beta_i/2r_i>0$,得

$$\begin{aligned}&LV\leqslant\\&-\sum_{i=1}^{n-1}(k_i-1)z_i^{p+3}-k_nz_n^{p+3}-\sum_{i=1}^{n}\frac{\beta_i}{2r_i}\tilde{\theta}_i^2+\\&\sum_{i=1}^{n}\Big(\underline{\eta}_i^{-\frac{p+3}{p_i}}+\bar{\epsilon}_i^{*\frac{p+3}{p_i}}+\frac{\beta_i}{2r_i}\theta_i^{*2}+\frac{1}{2}\underline{\xi}_i^{-(p-p_i+3)}\Big)\leqslant\\&-\alpha_0V+\Psi_0.\end{aligned}\tag{33}$$

其中

$$\begin{aligned}&\alpha_0=\min_{1\leqslant i\leqslant n}\{(p-p_i+4)(k_i-1)\Psi_1^{\frac{p_i-1}{p+3}},\\&\quad(p-p_n+4)k_n\Psi_1^{\frac{p_n-1}{p+3}},\sigma_i\},\\&\Psi_0=\Psi_1\Big(\Big(\sum_{i=1}^{n-1}(k_i-1)+k_n\Big)+1\Big),\\&\Psi_1=\sum_{i=1}^{n}\Big(\underline{\eta}_i^{-\frac{p+3}{p_i}}+\bar{\epsilon}_i^{*\frac{p+3}{p_i}}+\frac{\beta_i}{2r_i}\theta_i^{*2}+\frac{1}{2}\underline{\xi}_i^{-(p-p_i+3)}\Big).\end{aligned}$$

基于文献[18],如下的不等式成立:

$$\frac{\mathrm{d}E[V]}{\mathrm{d}t}\leqslant-\alpha_0E[V]+\Psi_0.\tag{34}$$

对式(34)进行积分处理,得

$$0\leqslant E[V]\leqslant\Big(V(0)-\frac{\Psi_0}{\alpha_0}\Big)\mathrm{e}^{-\alpha_0t}+\frac{\Psi_0}{\alpha_0}.\tag{35}$$

因此可以得出闭环系统中所有信号都是SGUUB.

设定$D_0=\Psi_0/\alpha_0$,可以得出

$$\lim_{t\to\infty}E[|z_1|]\leqslant(D_0(p-p_1+4))^{\frac{1}{p-p_1+4}}.\tag{36}$$

下面给出本文的主要结论.

定理1 考虑切换系统(2)满足假设1,对于任意切换信号$\sigma(t)$,有控制器(30)以及自适应参数(15),(25)和(31).针对有界的初始条件,设计的控制策略可以保证闭环系统下所有信号有界,并且跟踪误差ν_1在时间$t\geqslant0$和在预定的界限内收敛到零值小的领域内.

3 仿真研究

为验证所提出方法的有效性,考虑如下的高阶随机非线性切换系统:

$$\Sigma_1:\begin{cases}\mathrm{d}x_1=(x_2^{p_1}+1.5x_1^2\cos x_1)\mathrm{d}t+\\\qquad 0.15\sin x_1x_1\mathrm{d}\omega,\\\mathrm{d}x_2=\Big(u^{p_2}+\cos x_2\dfrac{x_1^2}{1+x_1^2+x_2^2}\Big)\mathrm{d}t+\\\qquad 0.45\sin x_1\cos x_2\mathrm{d}\omega,\\y=x_1.\end{cases}\tag{37}$$

$$\Sigma_2:\begin{cases}\mathrm{d}x_1=\Big(x_2^{p_1}+\dfrac{1-\cos x_1}{1+x_1^2}\Big)\mathrm{d}t+\\\qquad 0.3\sin x_1^2\mathrm{d}\omega,\\\mathrm{d}x_2=(u^{p_2}+x_1\mathrm{e}^{-0.5x_2^2})\mathrm{d}t+\\\qquad 0.3x_1\cos(x_1+x_2)\mathrm{d}\omega,\\y=x_1.\end{cases}\tag{38}$$

其中: $p_1 = 3$ 和 $p_2 = 5$. 控制目的是设计自适应控制器使得所有闭环信号有界, 并且保证 y 能跟踪参考信号 $y_d = 0.8\sin t + 1.2\cos(1.5t)$.

初始状态 $[x_1(0), x_2(0)]^{\mathrm{T}} = [0.48, 0.11]^{\mathrm{T}}$, $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$. 预设性能参数为

$$\rho_0 = 1.4,\ \rho_\infty = 0.1,\ \kappa = 2.8,\ \bar{\delta} = \underline{\delta} = 1.$$

在仿真中, 设计参数选择为

$$k_1 = 38, k_2 = 15,\ r_1 = r_2 = 1,\ \bar{\eta}_1 = \bar{\eta}_2 = 8,$$

$$\beta_1 = 0.85,\ \beta_2 = 0.3.$$

基于设计的控制器, RBF 神经网络被用来逼近未知的非线性函数.

仿真结果如图1 ～ 图5所示. 图2给出了在预设性能控制 (PPC) 下跟踪误差 (加粗实线) 和没有预设性能控制下的跟踪误差 (点虚线), 说明跟踪误差被约束在预定的界限内, 切换信号 $\sigma(t)$ 展示在图5中. 由图1 ～ 图5可以看出闭环信号都是有界的.

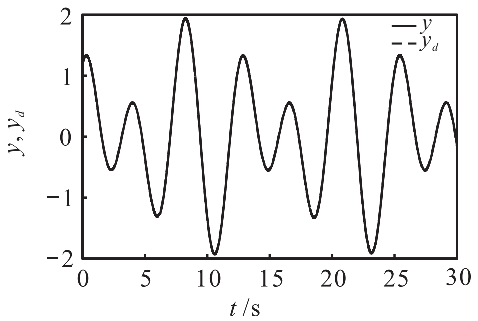

图 1　系统的跟踪性能

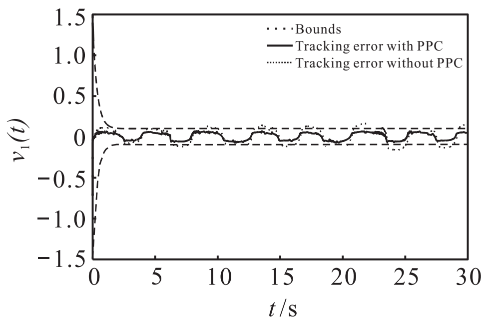

图 2　跟踪误差

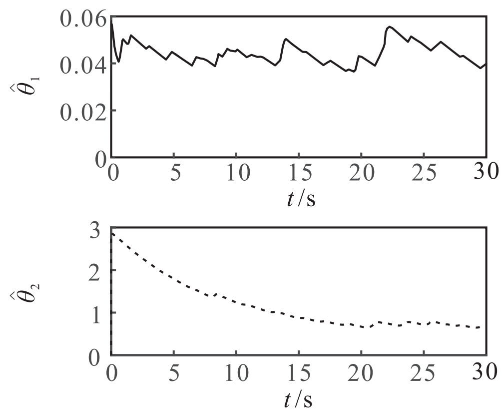

图 3　自适应参数 $\hat{\theta}_1$ 和 $\hat{\theta}_2$

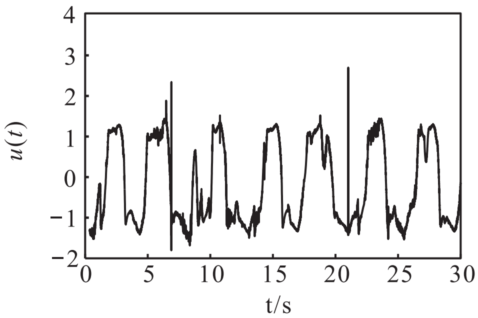

图 4　控制输入信号 $u(t)$

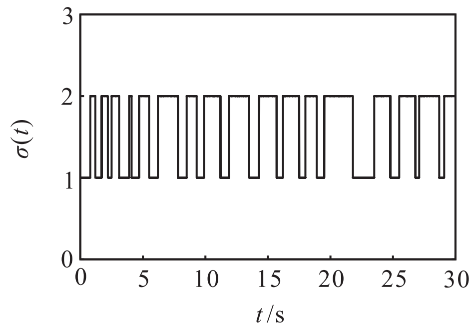

图 5　切换信号 $\sigma(t)$

4　结　论

本文研究了在任意切换信号下的不确定高能随机非线性系统的跟踪控制问题, 并保证预设跟踪性能. 最终展示提出的控制器能保证闭环系统中所有信号有界, 与跟踪误差在任意切换信号下保持在预定的界限范围内.

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作者简介

司文杰 (1985−), 男, 副教授, 博士, 从事自适应神经网络控制等研究, E-mail: siwenjie2008@163.com;

董训德 (1985−), 男, 副教授, 博士, 从事确定学习及动态模式识别等研究, E-mail: audxd@scut.edu.cn.

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