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概率区间值直觉犹豫模糊Maclaurin对称平均算子及决策方法

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摘 要: 在概率区间值直觉犹豫模糊集(PIVIHFS)的基础上,引入Maclaurin对称平均算子和Archimedean范数,构建一种基于概率区间值直觉犹豫模糊Maclaurin对称平均(PIVIHFMSM)算子的多属性决策模型,用来刻画决策专家输入多个参数值的决策信息,决策者可根据风险偏好等主观意识选择合适的参数值进行决策,同时能保证决策信息的有效性和完整性,避免决策过程中的不确定性和决策信息缺失问题. 首先,回顾PIVIHFS的定义和排序方法以及Archimedean范数;其次,提出概率区间值直觉犹豫模糊Maclaurin对称平均(PIVIHFMSM)算子,研究其优良性质及常见形式;最后,提出一种基于PIVIHFWMSM算子的多属性决策方法,并进行比较分析,通过实例验证该方法的可行性和有效性. 拓展PIVIHFS理论和应用领域,为决策属性具有相关性和决策信息有可能缺失提供新思路.

关键词: 概率区间值直觉犹豫模糊集；Maclaurin对称平均算子；Archimedean范数；多属性决策

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Probabilistic interval-valued intuitionistic hesitant fuzzy Maclaurin symmetric mean operators and decision method

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Abstract: Based on the probabilistic interval-valued intuitionistic hesitant fuzzy set (PIVIHFS), this paper introduces the Maclaurin symmetric mean operator and the Archimedean norm, and constructs a multi-attribute decision model based on the PIVIHF Maclaurin symmetric mean (PIVIHFMSM) operator. The model is used to describe the decision information of the decision expert inputting multiple parameter values. The decision maker can select the appropriate parameter value according to subjective consciousness such as risk preference to make decision. The problems of uncertainty and lack of decision information in the decision-making process are avoided. Firstly, the definition and ordering method of the PIVIHFS and the Archimedean norm are reviewed. Then, the PIVIHFMSM operator is proposed, and its excellent properties and common forms are studied. Finally, the decision-making method based on the PIVZHFMSM operator is proposed, comparison analysis of which is conducted to prove its feasibility and effectiveness. This paper expands the PIVIHFS theory and application fields, and provides new ideas for the relevance of decision attributes and the possible lack of decision information.

Keywords: probabilistic interval-valued intuitionistic hesitant fuzzy set；Maclaurin symmetric mean operator；Archimedean norm；multi-attribute decision making

0 引 言

随着决策过程中不确定性的增加,精确值的决策方法已不适用实际的决策. 自文献[1]提出模糊集(FS)以来,许多学者在模糊决策上有了大量发现和研究. 文献[2]基于FS,提出了直觉模糊集(IFS),可以同时考虑隶属度、非隶属度两个方面,使不确定决策信

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息的描述更加科学有效;考虑到决策者在进行决策时往往会存在主观偏好和犹豫不决的态度,使决策结果存在一定偏差;文献[3]拓展FS提出了犹豫模糊集(HFS),用若干个实数刻画隶属度,使决策信息更接近真实的决策过程;精确值的隶属度与非隶属度不能满足决策需要,对此,文献[4]拓展IFS,提出了区间值直觉模糊集(IVIFS),利用[0,1]之间的区间值表达FS中的隶属度和非隶属度程度;文献[5]开发了一种基于IFS的相关系数和算法并拓展到IVIFS环境中;在不同的FS环境下,文献[6]使用区间值犹豫模糊集(IVHFS)表达不确定性群决策过程中的评估信息;文献[7]将HFS扩展到IVIHS环境中,提出了区间值直觉犹豫模糊集(IVIHFS)的概念;文献[8]使用三角模糊数表达隶属度与非隶属度,提出了三角直觉模糊数(TIFS);文献[9]利用TIFS,建立了三角模糊层次分析法和模糊模式识别,解决了农产品滞销风险等级的决策问题.

在实际决策中会有一定的信息缺失,IVHFS不能准确表达每个HFS的可能性大小.为此,文献[10]提出了概率犹豫模糊集(PHFS),可以保留比HFS更多的信息;文献[11]基于属性权重完全未知的情况提出了基于概率犹豫模糊熵的多属性决策方法;文献[12]在概率犹豫模糊环境下引入Archimedean范数,提出了新的概率犹豫模糊信息集成算子;文献[13]提出了概率区间值犹豫模糊集(PIVHFS)和相应的集成算子,研究了一种新的多属性群决策(MAGDM)模型;文献[14]定义了概率区间值直觉犹豫模糊集(PIVIHFS)作为模糊集的扩展数学表达式,建立了一些基于PIVIHFS的多属性决策方法;文献[15]定义了概率区间值犹豫Pythagorean模糊集(PIVHPFS),为企业选择合作伙伴提供了一个科学、快速、准确的应用算法.

以上研究大多假设算子集成对属性评价的输入参数都是独立的,但在现实决策中,有可能会根据决策者的风险偏好等主观意识有不同的决策结果,因此无法证明其结果的合理性.而Maclaurin对称平均(MSM)算子[16]可以反映多个输入参数之间的相互关系.对于相同的参数集合,MSM相对于参数单调递减,决策者可以根据风险偏好选择参数值.因此,MSM算子在解决属性独立的多属性决策问题时更加灵活有效[17].文献[18]开发了一些基于MSM的算子处理直觉模糊信息的决策问题;文献[19]将MSM引入到HFS环境中,提出了对应的集成算子.

在实际决策中还未有概率区间值直觉犹豫模糊集(PIVIHFS)与MSM算子集成的研究,为保证决策信息的有效性和完整性,本文基于MSM和Archimedean范数定义概率区间值直觉犹豫模糊Maclaurin对称平均(PIVIHFMSM)算子,并研究其具有的优良性质及常见形式;基于PIVIHFWMSM算子构建概率区间值直觉犹豫模糊MSM多属性决策模型,使决策者能够根据自身的决策偏好及风险态度等主观意识改变参数值,使决策结果更具有合理性,并考虑决策过程中属性的相关性以及决策信息的丢失问题.

1 基本知识

1.1 概率区间值直觉犹豫模糊集(PIVIHFSs)

定义1 设X为论域,X上的一个概率区间值直觉犹豫模糊集(PIVIHFSs)H定义[7]为

$$H=\{(x,([\underset{\sim}{u},\tilde{u}],[\underset{\sim}{v},\tilde{v}]),p)|x\in X\}. \tag{1}$$

PIVIHFSs是由一系列概率区间值直觉犹豫模糊元(PIVIHFEs) $h=\{(([\underset{\sim}{u},\tilde{u}],[\underset{\sim}{v},\tilde{v}]),p\}$组成,每个PIVIHFE是由一组区间值直觉模糊数(IVIFN)与概率(p)构成,p为其对应IVIFN的可能性大小.当h是一个无穷的PIVIHFE时,$p(x)$是一个连续的概率分布,$p(x)\in[0,1]$,$\int_{p\in\mathrm{PIVIHFE}}p(x)\mathrm{d}x\leqslant 1$;当$h$是一个有限的PIVIHFE时,可将其定义为$h_i=\{((([\underset{\sim}{u}_i^{(k)},\tilde{u}_i^{(k)}],[\underset{\sim}{v}_i^{(k)},\tilde{v}_i^{(k)}]),{p_i}^{(k)})|k=1,2,\ldots,l\}$.其中:$i=1,2,\ldots,n$;$l$为正整数,用来表示PIVIHFE中包含元素的个数,概率满足${p_i}^{(k)}\in[0,1]$且$\sum\limits_{k=1}^{l}{p_i}^{(k)}\leqslant 1$.其模糊区间$[\underset{\sim}{\pi}_i^{(k)},\tilde{\pi}_i^{(k)}]$可通过$\underset{\sim}{\pi}_i^{(k)}=1-\tilde{u}_i^{(k)}-\tilde{v}_i^{(k)}$,$\tilde{\pi}_i^{(k)}=1-\underset{\sim}{u}_i^{(k)}-\underset{\sim}{v}_i^{(k)}$计算.

设l_1和l_2分别为概率区间值直觉犹豫模糊元PIVIHFE$_1$和PIVIHFE$_2$中的元素个数.为了方便计算,规定$l_1=l_2$,假设PIVIHFE中元素的数量为l,为了比较不同PIVIHFE的大小,定义以下得分函数和精确函数.

定义2 设X为一个有限集,$h_i=\{((([\underset{\sim}{u}_i^{(k)},\tilde{u}_i^{(k)}],[\underset{\sim}{v}_i^{(k)},\tilde{v}_i^{(k)}]),{p_i}^{(k)})\}$是与$X$相关的有限PIVIHFE,其中$i=1,2,\cdots,n$,$k=1,2,\cdots,l$,得分函数定义[15]为

$$S(h_i)=\sum_{k=1}^{L(\mathrm{PIVIHFE})}p_i^{(k)}\times\frac{(\underset{\sim}{u}_i^{(k)}-\tilde{v}_i^{(k)})+(\tilde{u}_i^{(k)}-\underset{\sim}{v}_i^{(k)})}{2}, \tag{2}$$

精确函数定义为

$$E(h_i)=\sum_{k=1}^{L(\text{PIVIHFE})} p_i^{(k)}\times\frac{2-\underset{\sim}{\pi}_i^{(k)}-\tilde{\pi}_i^{(k)}}{2}. \quad (3)$$

其中: $p_i^{(k)}\in[0,1], 0\leqslant\sum_{k=1}^{L(\text{PIVIHFE})}p_i^{(k)}\leqslant 1$.

定义3　若 h_i 为两个PIVIHFEs, 则:

1) 如果 $S(h_1)>S(h_2)$, 则 $h_1\succ h_2$, 即 h_1 优于 h_2.

2) 如果 $S(h_1)<S(h_2)$, 则 $h_1\prec h_2$, 即 h_1 劣于 h_2.

3) 如果 $S(h_1)=S(h_2)$, 则有

① 当 $E(h_1)>E(h_2)$ 时, 有 $h_1\succ h_2$, 即 h_1 优于 h_2;

② 当 $E(h_1)<E(h_2)$ 时, 有 $h_1\prec h_2$, 即 h_1 劣于 h_2;

③ 当 $E(h_1)=E(h_2)$ 时, 有 $h_1\simeq h_2$, 说明 h_1 与 h_2 无异.

1.2　Archimedean范数

关于聚合算子的大多数现有信息都是基于代数乘法和加法运算, 本文代数乘法和加法运算均以Archimedean范数的形式出现[20].

严格的Archimedean T-范数可通过严格单调递减加性算子 $g:[0,1]\to[0,+\infty]$ 表示为 $T(x,y)=g^{-1}(g(x)+g(y))$, 其中 $g(1)=0$. 根据对偶原则可知, 严格的Archimedean S-范数可以表示为 $S(x,y)=f^{-1}(f(x)+f(y))$, 其中 $f(t)=g(1-t)$, 则 $f(t)$ 是单调递增函数且有 $f(0)=0$.

定义4　令 $h=\{(([\underset{\sim}{u}^{(k)},\tilde{u}^{(k)}],[\underset{\sim}{v}^{(k)},\tilde{v}^{(k)}]),p^{(k)})\}$, $h_i=\{(([\underset{\sim}{u}_i^{(k)},\tilde{u}_i^{(k)}],[\underset{\sim}{v}_i^{(k)},\tilde{v}_i^{(k)}]),p_i^{(k)})\}$ 为3个PIVIHFEs, 其中 $i=1,2$, $k=1,2,\ldots,l$, 定义运算[11]如下:

1) $h^c=\{(([1-\tilde{u}^{(k)},1-\underset{\sim}{u}^{(k)}],[1-\tilde{v}^{(k)},1-\underset{\sim}{v}^{(k)}]),$
$p^{(k)})|k=1,2,\ldots,l\}$;

2) $h_1\oplus h_2=\{(([f^{-1}(f(\underset{\sim}{u}_1^{(k)})+f(\underset{\sim}{u}_2^{(k)})),$
$f^{-1}(f(\tilde{u}_1^{(k)})+f(\tilde{u}_2^{(k)}))],$
$[g^{-1}(g(\underset{\sim}{v}_1^{(k)})+g(\underset{\sim}{v}_2^{(k)})),$
$g^{-1}(g(\tilde{v}_1^{(k)})+g(\tilde{v}_2^{(k)}))]),$
$\overline{p_1^{(k)}+p_2^{(k)}})|k=1,2,\ldots,l\}$;

3) $h_1\otimes h_2=\{(([g^{-1}(g(\underset{\sim}{u}_1^{(k)})+g(\underset{\sim}{u}_2^{(k)})),$
$g^{-1}(g(\tilde{u}_1^{(k)})+g(\tilde{u}_2^{(k)}))],$
$[f^{-1}(f(\underset{\sim}{v}_1^{(k)})+f(\underset{\sim}{v}_2^{(k)})),$
$f^{-1}(f(\tilde{v}_1^{(k)})+f(\tilde{v}_2^{(k)}))]),$
$\overline{p_1^{(k)}+p_2^{(k)}})|k=1,2,\cdots,l\}$;

4) $\lambda h=\{(([f^{-1}(\lambda f(\underset{\sim}{u}^{(k)})),f^{-1}(\lambda f(\tilde{u}^{(k)}))],$
$[g^{-1}(\lambda g(\underset{\sim}{v}^{(k)})),g^{-1}(\lambda g(\tilde{v}^{(k)}))]),$
$p^{(k)})|k=1,2,\cdots,l,\lambda>0\}$;

5) $h^\lambda=\{(([g^{-1}(\lambda g(\underset{\sim}{u}^{(k)})),g^{-1}(\lambda g(\tilde{u}^{(k)}))],$
$[f^{-1}(\lambda f(\underset{\sim}{v}^{(k)})),f^{-1}(\lambda f(\tilde{v}^{(k)}))]),$
$p^{(k)})|k=1,2,\cdots,l,\lambda>0\}$.

其中: 标准化概率 $\overline{p_1^{(k)}+p_2^{(k)}}=\dfrac{p_1^{(k)}+p_2^{(k)}}{\sum_{k=1}^{l}(p_1^{(k)}+p_2^{(k)})}$, $k=1,2,\ldots,l$, 因此 $\sum_{k=1}^{l}(\overline{p_1^{(k)}+p_2^{(k)}})=1$.

定理1　令 $h=\{(([\underset{\sim}{u}^{(k)},\tilde{u}^{(k)}],[\underset{\sim}{v}^{(k)},\tilde{v}^{(k)}]),p^{(k)})\}$, 且 $h_i=\{(([\underset{\sim}{u}_i^{(k)},\tilde{u}_i^{(k)}],[\underset{\sim}{v}_i^{(k)},\tilde{v}_i^{(k)}]),p_i^{(k)})\}$ 为3个PIVIHFEs, 其中 $i=1,2$, $k=1,2,\cdots,l$, 则有:

1) $h_1\oplus h_2=h_2\oplus h_1$;

2) $h_1\otimes h_2=h_2\otimes h_1$;

3) $\lambda(h_1\oplus h_2)=\lambda h_1\oplus\lambda h_2, \lambda>0$;

4) $\lambda_1 h\oplus\lambda_2 h=(\lambda_1+\lambda_2)h, \lambda_1,\lambda_2>0$;

5) $h^{\lambda_1}\otimes h^{\lambda_2}=h^{\lambda_1+\lambda_2}, \lambda_1,\lambda_2>0$;

6) $h_1{}^\lambda\otimes h_2{}^\lambda=(h_1\otimes h_2)^\lambda, \lambda>0$.

根据运算法则, 本文给出相应的概率区间值直觉犹豫模糊集成算子, 如: 令 $h_i(i=1,2,\ldots,n)$ 为一列PIVIHFE, 相关的权重向量 $w=(w_1,w_2,\ldots,w_n)^{\mathrm{T}}$, 满足 $w_i\geqslant 0$ 且 $\sum_{i=1}^{n}w_i=1$, 则有:

1) 概率区间值直觉犹豫模糊有序加权平均(PIVIHFOWA)算子

$$\text{PIVIHFOWA}(h_1,h_2,\ldots,h_n)=\overset{n}{\underset{i=1}{\oplus}}w_ih_i=$$
$$\Bigg\{\Bigg(\Bigg(\Bigg[1-\prod_{i=1}^{n}(1-\underset{\sim}{u}_i^{(k)})^{w_i},1-\prod_{i=1}^{n}(1-\tilde{u}_i^{(k)})^{w_i}\Bigg],$$
$$\Bigg[1-\prod_{i=1}^{n}(1-\underset{\sim}{v}_i^{(k)})^{w_i},1-\prod_{i=1}^{n}(1-\tilde{v}_i^{(k)})^{w_i}\Bigg]\Bigg),$$
$$\overline{\sum_{i=1}^{n}p_i^{(k)}}\Bigg)\Bigg|k=1,2,\ldots,l\Bigg\}. \quad (4)$$

2) 概率区间值直觉犹豫模糊有序加权几何(PIVIHFOWG)算子

$$\text{PIVIHFOWG}(h_1,h_2,\ldots,h_n)=\overset{n}{\underset{i=1}{\otimes}}h_i{}^{w_i}=$$
$$\Bigg\{\Bigg(\Bigg(\Bigg[\prod_{i=1}^{n}(\underset{\sim}{u}_i^{(k)})^{w_i},\prod_{i=1}^{n}(\tilde{u}_i^{(k)})^{w_i}\Bigg],$$

$$\Big[\prod_{i=1}^{n}(\underset{\sim}{v}_i^{(k)})^{w_i},\prod_{i=1}^{n}(\tilde{v}_i^{(k)})^{w_i}\Big]\Big),$$

$$\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\}.\tag{5}$$

1.3 Maclaurin对称平均算子

定义5 令$a_i(i=1,2,\ldots,n)$是一组非空实数,并且$r=1,2,\ldots,n$. 如果

$$\mathrm{MSM}^{(r)}(a_1,a_2,\ldots,a_n)=\left[\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}\prod\limits_{j=1}^{r}a_{i_j}}{C_n^r}\right]^{\frac{1}{r}},\tag{6}$$

则称$\mathrm{MSM}^{(r)}$为Maclaurin对称平均算子[16]. 其中$(i_1,i_2,\ldots,i_r)$遍历组合$(1,2,\ldots,n)$中所有的r元组, $C_n^r=\dfrac{n!}{r!(n-r)!}$为二项式系数.

1) 若$a\geqslant 0$且对于任意的i,都有$a_i=a$,则$\mathrm{MSM}^{(r)}(a,a,\ldots,a)=a$.

2) 若$a_i\leqslant b_i$,则$\mathrm{MSM}^{(r)}(a_1,a_2,\ldots,a_n)\leqslant\mathrm{MSM}^{(r)}(b_1,b_2,\ldots,b_n)$,其中$a_i$和$b_i$都是任意非负实数.

3) $\mathrm{MIN}(a_1,a_2,\ldots,a_n)\leqslant\mathrm{MSM}^{(r)}(a_1,a_2,\ldots,a_n)\leqslant\mathrm{MAX}(a_1,a_2,\ldots,a_n)$.

2 概率区间值直觉犹豫模糊Maclaurin对称平均(PIVIHFMSM)算子及其性质

定义6 令$h_i(i=1,2,\ldots,n)$是一组概率区间值直觉犹豫模糊元,且$r=1,2,\ldots,n$,若

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=\left[\frac{\bigoplus\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}\bigotimes\limits_{j=1}^{r}h_{i_j}}{C_n^r}\right]^{\frac{1}{r}},\tag{7}$$

则称$\mathrm{PIVIHFMSM}^{(r)}$为概率区间值直觉犹豫模糊麦克劳林对称平均算子,简记为$\mathrm{PIVIHFMSM}^{(r)}$算子. 其中$(i_1,i_2,\ldots,i_r)$遍历组合$(1,2,\ldots,n)$中所有的r元组, $C_n^r=\dfrac{n!}{r!(n-r)!}$为二项式系数.

定理2 假设$h_i(i=1,2,\ldots,n)$为一列PIVIHFE,则运用$\mathrm{PIVIHFMSM}^{(r)}$算子集成得到的结果仍为PIVIHFE,且有

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{r}g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}},$$

$$f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{r}g(\tilde{u}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}}\Big],$$

$$\Big[g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}g\Big(f^{-1}\Big(\sum\limits_{j=1}^{r}f(\underset{\sim}{v}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}},$$

$$g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}g\Big(f^{-1}\Big(\sum\limits_{j=1}^{r}f(\tilde{v}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}}\Big]\Big),$$

$$\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\}.\tag{8}$$

证明 1) 根据定义4和定义6可得

$$\bigotimes_{j=1}^{r}h_{i_j}=$$

$$\Big\{\Big(\Big(\Big[g^{-1}\Big(\sum_{j=1}^{r}g(\underset{\sim}{u}_{i_j}^{(k)})\Big),g^{-1}\Big(\sum_{j=1}^{r}g(\tilde{u}_{i_j}^{(k)})\Big)\Big],$$

$$\Big[f^{-1}\Big(\sum_{j=1}^{r}f(\underset{\sim}{v}_{i_j}^{(k)})\Big),f^{-1}\Big(\sum_{j=1}^{r}f(\tilde{v}_{i_j}^{(k)})\Big)\Big]\Big),$$

$$\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\},$$

$$\bigoplus_{1\leqslant i_1<\ldots<i_r\leqslant n}\bigotimes_{j=1}^{r}h_{i_j}=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\sum_{1\leqslant i_1<\ldots<i_r\leqslant n}f\Big(g^{-1}\Big(\sum_{j=1}^{r}g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)\Big),$$

$$f^{-1}\Big(\sum_{1\leqslant i_1<\ldots<i_r\leqslant n}f\Big(g^{-1}\Big(\sum_{j=1}^{r}g(\tilde{u}_{i_j}^{(k)})\Big)\Big)\Big)\Big],$$

$$\Big[g^{-1}\Big(\sum_{1\leqslant i_1<\ldots<i_r\leqslant n}g\Big(f^{-1}\Big(\sum_{j=1}^{r}f(\underset{\sim}{v}_{i_j}^{(k)})\Big)\Big)\Big),$$

$$g^{-1}\Big(\sum_{1\leqslant i_1<\ldots<i_r\leqslant n}g\Big(f^{-1}\Big(\sum_{j=1}^{r}f(\tilde{v}_{i_j}^{(k)})\Big)\Big)\Big)\Big]\Big),$$

$$\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\},$$

$$\Big(\frac{\bigoplus\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}\bigotimes\limits_{j=1}^{r}h_{i_j}}{C_n^r}\Big)^{\frac{1}{r}}=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{r}g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}},$$

$$f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{r}g(\tilde{u}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}}\Big],$$

$$\left[g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\underset{\sim}{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}},\right.$$

$$\left.g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\right]\Bigg),$$

$$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}\Bigg)\Bigg|k=1,2,\ldots,l\Bigg\},$$

所以式(8)成立.

2) 若要证PIVIHFMSM$^{(r)}$算子集成得到的结果仍为PIVIHFE,即证

$$f^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\geqslant$$

$$f^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\underset{\sim}{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\geqslant 0, \tag{9}$$

$$g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\geqslant$$

$$g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\underset{\sim}{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\geqslant 0, \tag{10}$$

$$f^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}+$$

$$g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\leqslant 1,$$

且$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}\in[0,1]$, $\sum_{k=1}^{l}\overline{\sum_{i=1}^{n} p_i{}^{(k)}}=1$.

根据$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}$的表达式可知$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}\in[0,1]$,显然$\sum_{k=1}^{l}\overline{\sum_{i=1}^{n} p_i{}^{(k)}}=1$成立. 因为对于任意$k=1,2,\ldots,l$, $i=1,2,\ldots,n$,有$0\leqslant\underset{\sim}{u}_{i_j}^{(k)}\leqslant\tilde{u}_{i_j}^{(k)}\leqslant 1$, $0\leqslant\underset{\sim}{v}_{i_j}^{(k)}\leqslant\tilde{v}_{i_j}^{(k)}\leqslant 1$, $g(t)$和$g^{-1}(t)$均为严格单调递减函数, $f(t)$和$f^{-1}(t)$均为严格单调递增函数,且$f(t)=g(1-t)$,故易证明式(9)和(10)成立.

又由$\tilde{u}_{i_j}^{(k)}+\tilde{v}_{i_j}^{(k)}\leqslant 1$,即$\tilde{u}_{i_j}^{(k)}\leqslant 1-\tilde{v}_{i_j}^{(k)}$可知,对于任意$k=1,2,\ldots,l$, $i=1,2,\ldots,n$,存在$f(\tilde{v}_{i_j}^{(k)})=g(1-\tilde{v}_{i_j}^{(k)})\leqslant g(\tilde{u}_{i_j}^{(k)})$,则

$$f^{-1}\left(\sum_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\leqslant f^{-1}\left(\sum_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)=$$

$$1-g^{-1}\left(\sum_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right),$$

$$g\left(f^{-1}\left(\sum_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)\geqslant g\left(1-g^{-1}\left(\sum_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)=$$

$$f\left(g^{-1}\left(\sum_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right),$$

故

$$\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\geqslant$$

$$\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}},$$

$$g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\leqslant$$

$$g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}=$$

$$1-f^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}},$$

即

$$g^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\left(f^{-1}\left(\sum\limits_{j=1}^{r} f(\tilde{v}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}+$$

$$f^{-1}\left(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\left(g^{-1}\left(\sum\limits_{j=1}^{r} g(\tilde{u}_{i_j}^{(k)})\right)\right)}{C_n^r}\right)^{\frac{1}{r}}\leqslant 1.$$

所以PIVIHFMSM$^{(r)}$算子集成得到的结果仍为PIVIHFE. 综上所述,定理2成立. □

基于上述PIVIHFMSM$^{(r)}$算子提出以下算子的性质.

性质1 (幂等性) 假设$h_i(i=1,2,\ldots,n)$为一列PIVIHFE,如果对于任意$i=1,2,\ldots,n$都有$h_i=h=\{(([\underset{\sim}{u}^{(k)},\tilde{u}^{(k)}],[\underset{\sim}{v}^{(k)},\tilde{v}^{(k)}]),p^{(k)})|k=1,2,\ldots,l\}$,则

$$\text{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=h. \tag{11}$$

性质2 (置换不变性) 假设$h_i(i=1,2,\ldots,n)$为一列PIVIHFE,如果是$h_i=(h_1,h_2,\ldots,h_n)$的一个任

意置换,则

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=\mathrm{PIVIHFMSM}^{(r)}(\tilde{h}_1,\tilde{h}_2,\ldots,\tilde{h}_n). \tag{12}$$

性质3 (单调性) 假设h_i和h'_i为两个PIVIHFE, 对于任意$i=1,2,\ldots,n$,如果有$\underset{\sim}{u}_i^{(k)}\geqslant \underset{\sim}{u'}_i^{(k)},\tilde{u}_i^{(k)}\geqslant \tilde{u}'^{(k)}_i,\underset{\sim}{v}_i^{(k)}\geqslant \underset{\sim}{v'}_i^{(k)},\tilde{v}_i^{(k)}\geqslant \tilde{v}'^{(k)}_i,p_i^{(k)}\geqslant p'^{(k)}_i$,则

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)\geqslant \mathrm{PIVIHFMSM}^{(r)}(h'_1,h'_2,\ldots,h'_n). \tag{13}$$

性质4 (有界性) 假设h_i为一系列PIVIHFE,对于任意$i=1,2,\ldots,n$,令

$$h^-=-\min_i h_i=\{(((\min_i \underset{\sim}{u}_i^{(k)},\min_i \tilde{u}_i^{(k)}],[\max_i \underset{\sim}{v}_i^{(k)},\max_i \tilde{v}_i^{(k)}]),\min_i p_i^{(k)})|k=1,2,\ldots,l\},$$

$$h^+=\max_i h_i=\{(((\max_i \underset{\sim}{u}_i^{(k)},\max_i \tilde{u}_i^{(k)}],[\min_i \underset{\sim}{v}_i^{(k)},\min_i \tilde{v}_i^{(k)}]),\max_i p_i^{(k)})|k=1,2,\ldots,l\},$$

则

$$h^-\leqslant \mathrm{PIVIHFMSM}^{(r)}\leqslant h^+. \tag{14}$$

限于篇幅,性质1～性质4证明过程略.

接下来介绍$\mathrm{PIVIHFMSM}^{(r)}$在属性值r取不同值时的几种特殊情况:

1) 当$r=1$时,有

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{1}g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)}{C_n^1}\Big)^{\frac{1}{1}},f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{1}g(\tilde{u}_{i_j}^{(k)})\Big)\Big)}{C_n^1}\Big)^{\frac{1}{1}}\Big],$$

$$\Big[g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1\leqslant n}g\Big(f^{-1}\Big(\sum\limits_{j=1}^{1}f(\underset{\sim}{v}_{i_j}^{(k)})\Big)\Big)}{C_n^1}\Big)^{\frac{1}{1}},g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1\leqslant n}g\Big(f^{-1}\Big(\sum\limits_{j=1}^{1}f(\tilde{v}_{i_j}^{(k)})\Big)\Big)}{C_n^1}\Big)^{\frac{1}{1}}\Big]\Big),\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\}=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1\leqslant i\leqslant n}f(\underset{\sim}{u}_i^{(k)})}{n}\Big),f^{-1}\Big(\frac{\sum\limits_{1\leqslant i\leqslant n}f(\tilde{u}_i^{(k)})}{n}\Big)\Big],\Big[g^{-1}\Big(\frac{\sum\limits_{1\leqslant i\leqslant n}g(\underset{\sim}{v}_i^{(k)})}{n}\Big),g^{-1}\Big(\frac{\sum\limits_{1\leqslant i\leqslant n}g(\tilde{v}_i^{(k)})}{n}\Big)\Big]\Big),\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\};$$

2) 当$r=2$时,有

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<i_2\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{2}g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)}{C_n^2}\Big)^{\frac{1}{2}},f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<i_2\leqslant n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{2}g(\tilde{u}_{i_j}^{(k)})\Big)\Big)}{C_n^2}\Big)^{\frac{1}{2}}\Big],$$

$$\Big[g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<i_2\leqslant n}g\Big(f^{-1}\Big(\sum\limits_{j=1}^{2}f(\underset{\sim}{v}_{i_j}^{(k)})\Big)\Big)}{C_n^2}\Big)^{\frac{1}{2}},g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<i_2\leqslant n}g\Big(f^{-1}\Big(\sum\limits_{j=1}^{2}f(\tilde{v}_{i_j}^{(k)})\Big)\Big)}{C_n^2}\Big)^{\frac{1}{2}}\Big]\Big),\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\}=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{2\sum\limits_{1\leqslant i<j\leqslant n}f(\underset{\sim}{u}_i^{(k)}\underset{\sim}{u}_j^{(k)})}{n(n-1)}\Big)^{\frac{1}{2}},f^{-1}\Big(\frac{2\sum\limits_{1\leqslant i<j\leqslant n}f(\tilde{u}_i^{(k)}\tilde{u}_j^{(k)})}{n(n-1)}\Big)^{\frac{1}{2}}\Big],$$

$$\Big[g^{-1}\Big(\frac{2\sum\limits_{1\leqslant i<j\leqslant n}g(\underset{\sim}{v}_i^{(k)}\underset{\sim}{v}_j^{(k)})}{n(n-1)}\Big)^{\frac{1}{2}},g^{-1}\Big(\frac{2\sum\limits_{1\leqslant i<j\leqslant n}g(\tilde{v}_i^{(k)}\tilde{v}_j^{(k)})}{n(n-1)}\Big)^{\frac{1}{2}}\Big]\Big),\overline{\sum_{i=1}^{n}p_i^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\};$$

3) 当$r=n$时,有

$$\mathrm{PIVIHFMSM}^{(r)}(h_1,h_2,\ldots,h_n)=$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1=i_1<\ldots<i_n=n}f\Big(g^{-1}\Big(\sum\limits_{j=1}^{n}g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)}{C_n^n}\Big)^{\frac{1}{n}},$$

$$f^{-1}\Big(\frac{\sum\limits_{1=i_1<\ldots<i_n=n} f\Big(g^{-1}\Big(\sum\limits_{j=1}^{n} g(\tilde{u}_{i_j}^{(k)})\Big)\Big)}{C_n^n}\Big)^{\frac{1}{n}}\Big],$$

$$\Big[g^{-1}\Big(\frac{\sum\limits_{1=i_1<\ldots<i_n=n} g\Big(f^{-1}\Big(\sum\limits_{j=1}^{n} f(\underset{\sim}{v}_{i_j}^{(k)})\Big)\Big)}{C_n^n}\Big)^{\frac{1}{n}},$$

$$g^{-1}\Big(\frac{\sum\limits_{1=i_1<\ldots<i_n=n} g\Big(f^{-1}\Big(\sum\limits_{j=1}^{n} f(\tilde{v}_{i_j}^{(k)})\Big)\Big)}{C_n^n}\Big)^{\frac{1}{n}}\Big]\Big),$$

$$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\} =$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\sum_{1=i_1<\ldots<i_n=n} f\Big(\prod_{j=1}^{n} \underset{\sim}{u}_j^{(k)}\Big)\Big)^{\frac{1}{n}},$$

$$f^{-1}\Big(\sum_{1=i_1<\ldots<i_n=n} f\Big(\prod_{j=1}^{n} \tilde{u}_j^{(k)}\Big)\Big)^{\frac{1}{n}}\Big],$$

$$\Big[g^{-1}\Big(\sum_{1=i_1<\ldots<i_n=n} g\Big(\prod_{j=1}^{n} \underset{\sim}{v}_j^{(k)}\Big)\Big)^{\frac{1}{n}},$$

$$g^{-1}\Big(\sum_{1=i_1<\ldots<i_n=n} g\Big(\prod_{j=1}^{n} \tilde{v}_j^{(k)}\Big)\Big)^{\frac{1}{n}}\Big]\Big),$$

$$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\}.$$

3 基于PIVIHFWMSM算子的多属性决策模型及其应用

3.1 PIVIHFWMSM算子

定理3 假设$h_i = \{(((\underset{\sim}{u}_i^{(k)}, \tilde{u}_i^{(k)}], [\underset{\sim}{v}_i^{(k)}, \tilde{v}_i^{(k)}]), p_i{}^{(k)})\}$是一系列PIVIHFE, 其中$i = 1,2,\ldots,n$, 相关的权重向量为$w=(w_1,w_2,\ldots,w_n)^{\mathrm{T}}$, 满足$w_i \geqslant 0$且$\sum\limits_{i=1}^{n} w_i = 1$, 若

$$\mathrm{PIVIHFWMSM}^{(r)}(h_1,h_2,\ldots,h_n) =$$

$$\left[\frac{\bigoplus\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} \bigotimes\limits_{j=1}^{r} h_{i_j}{}^{w_{i_j}}}{C_n^r}\right]^{\frac{1}{r}} =$$

$$\Big\{\Big(\Big(\Big[f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\Big(g^{-1}\Big(\sum\limits_{j=1}^{r} w_{i_j} g(\underset{\sim}{u}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}},$$

$$f^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} f\Big(g^{-1}\Big(\sum\limits_{j=1}^{r} w_{i_j} g(\tilde{u}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}}\Big],$$

$$\Big[g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\Big(f^{-1}\Big(\sum\limits_{j=1}^{r} w_{i_j} f(\underset{\sim}{v}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}},$$

$$g^{-1}\Big(\frac{\sum\limits_{1\leqslant i_1<\ldots<i_r\leqslant n} g\Big(f^{-1}\Big(\sum\limits_{j=1}^{r} w_{i_j} f(\tilde{v}_{i_j}^{(k)})\Big)\Big)}{C_n^r}\Big)^{\frac{1}{r}}\Big]\Big),$$

$$\overline{\sum_{i=1}^{n} p_i{}^{(k)}}\Big)\Big|k=1,2,\ldots,l\Big\}, \tag{15}$$

则称PIVIHFWMSM$^{(r)}$为概率区间值直觉犹豫模糊加权麦克劳林对称平均算子. 其中$(i_1,i_2,\ldots,i_r)$遍历组合$(1,2,\ldots,n)$中所有的r元组, $C_n^r = \dfrac{n!}{r!(n-r)!}$为二项式系数. 证明过程与定理2类似, 在此不加赘述.

3.2 算法模型

设方案集$X = \{x_1,x_2,\ldots,x_m\}$, $C = \{C_1,C_2,\ldots,C_n\}$为属性集, 属性权重向量$w = (w_1,w_2,\ldots,w_n)^{\mathrm{T}}$, 满足$w_j \geqslant 0$且$\sum\limits_{j=1}^{n} w_j = 1$. 为了从上述方案中选择一个综合性能最高的方案, 现邀请相关领域专家运用概率区间值直觉犹豫模糊集($i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$)对方案进行综合评估, 进而构建一个概率区间值直觉犹豫模糊决策矩阵$H = (h_{ij})_{m\times n}$. 具体步骤如下.

step 1: 根据各备选方案的决策信息, 构建概率区间值直觉犹豫模糊决策矩阵$H = (h_{ij})_{m\times n}$.

step 2: 决策信息标准化. 如果所有的属性均为效益型, 则不需对决策矩阵$H = (h_{ij})_{m\times n}$进行标准化处理; 否则, 运用如下方法对原始矩阵进行标准化: 当C_j为效益型时$\tilde{h}_{ij} = h_{ij}$, 当C_j为成本型时$\tilde{h}_{ij} = h_{ij}^c$.

step 3: 利用PIVIHFOWA算子、PIVIHFOWG算子、PIVIHFWMSM$^{(r)}$算子计算各备选方案x_i的综合属性信息$\tilde{h}_i$.

step 4: 计算各备选方案综合属性信息$\tilde{h}_i$得分函数值$S(\tilde{h}_i)$和精确函数值$E(\tilde{h}_i)$.

step 5: 对各备选方案进行优劣排序, 选出综合性能最高的方案.

3.3 算例分析

风险投资非常注重风险项目及风险企业的成长性、可操作性和可靠性等, 即注重远期利益的实现. 现考虑选择从如下几个方面对4个风险投资企业x_i($i = 1,2,3,4$)进行风险项目及风险企业评价, 分别为C_1风险性、C_2保证性、C_3成长性、C_4可操作性, 这些评价环节的有机组合构成了风险投资的评价指标体系. 公司邀请一位专家顾问对以上4个属性进行评估, 其中评价指标权重为$w = (0.22, 0.20, 0.28, 0.30)^{\mathrm{T}}$. 该算例数据改编自文献[21].

表 1 专家给出概率区间值直觉犹豫模糊决策矩阵 $H=(h_{ij})_{m\times n}$

x	C_1	C_2	C_3	C_4
x_1	$\{([0.3,0.5],[0.2,0.4],0.5),([0.4,0.6],[0.1,0.3],0.5)\}$	$\{([0.2,0.4],[0.3,0.5],0.4),([0.1,0.3],[0.4,0.6],0.6)\}$	$\{([0.3,0.4],[0.4,0.6],0.3),([0.2,0.4],[0.1,0.3],0.7)\}$	$\{([0.1,0.3],[0.2,0.4],0.5),([0.2,0.3],[0.4,0.6],0.5)\}$
x_2	$\{([0.2,0.4],[0.3,0.5],0.6),([0.1,0.3],[0.2,0.4],0.4)\}$	$\{([0.3,0.5],[0.4,0.5],0.5),([0.2,0.4],[0.1,0.3],0.5)\}$	$\{([0.2,0.3],[0.2,0.4],0.3),([0.2,0.4],[0.3,0.5],0.7)\}$	$\{([0.3,0.5],[0.2,0.4],0.5),([0.1,0.3],[0.4,0.6],0.5)\}$
x_3	$\{([0.3,0.5],[0.2,0.4],0.4),([0.1,0.3],[0.2,0.4],0.6)\}$	$\{([0.4,0.6],[0.1,0.3],0.6),([0.2,0.4],[0.3,0.5],0.4)\}$	$\{([0.5,0.6],[0.3,0.4],0.3),([0.2,0.4],[0.3,0.5],0.7)\}$	$\{([0.2,0.4],[0.3,0.5],0.5),([0.1,0.3],[0.4,0.6],0.5)\}$
x_4	$\{([0.3,0.5],[0.2,0.4],0.7),([0.4,0.6],[0.2,0.3],0.3)\}$	$\{([0.4,0.6],[0.2,0.4],0.6),([0.3,0.5],[0.2,0.3],0.4)\}$	$\{([0.1,0.3],[0.4,0.6],0.5),([0.3,0.5],[0.2,0.4],0.5)\}$	$\{([0.2,0.4],[0.3,0.5],0.4),([0.4,0.6],[0.3,0.4],0.6)\}$

step 1: 根据决策信息,构建概率区间值直觉犹豫模糊决策矩阵 $H=(h_{ij})_{m\times n}$,如表1所示.

step 2: 由于4种属性指标 $C_j(j=1,2,3,4)$ 均为效益型指标,专家决策信息矩阵无需进行标准化.

step 3: 根据决策信息矩阵,利用PIVIHFOWA算子、PIVIHFOWG算子和PIVIHFWMSM$^{(r)}$算子计算各备选方案的综合属性 $\tilde{h}_i(i=1,2,\ldots,m)$. 为方便计算,假设 $g(t)=-\ln(t)$,则由PIVIHFOWA算子得到的综合属性值为

$$x_1=\{([0.2248,0.3963],[0.2814,0.4836],0.425),([0.2312,0.4072],[0.2652,0.4708],0.575)\};$$
$$x_2=\{([0.2517,0.4281],[0.2666,0.4442],0.475),([0.1495,0.3499],[0.2762,0.4794],0.525)\};$$
$$x_3=\{([0.3570,0.5255],[0.2420,0.4142],0.450),([0.1495,0.3499],[0.3117,0.5132],0.550)\};$$
$$x_4=\{([0.2420,0.4450],[0.2909,0.4929],0.550),([0.3539,0.5548],[0.2314,0.3599],0.450)\}.$$

由PIVIHFOWG算子得到的综合属性值为

$$x_1=\{([0.1990,0.3854],[0.2634,0.4685],0.425),([0.2028,0.3787],[0.2000,0.4243],0.575)\};$$
$$x_2=\{([0.2449,0.4126],[0.2512,0.4393],0.475),([0.1395,0.3444],[0.2401,0.4540],0.525)\};$$
$$x_3=\{([0.3246,0.5104],[0.2203,0.4038],0.450),([0.1395,0.3444],[0.2991,0.5028],0.550)\};$$
$$x_4=\{([0.2069,0.4204],[0.2742,0.4791],0.550),([0.3484,0.5497],[0.2259,0.3545],0.450)\}.$$

由PIVIHFWMSM$_1(r=1)$算子得到的综合属性值为

$$x_1=\{([0.0497,0.0963],[0.0658,0.1171],0.425),([0.0507,0.0947],[0.0500,0.1061],0.575)\};$$
$$x_2=\{([0.0612,0.1032],[0.0628,0.1098],0.475),([0.0349,0.0861],[0.0600,0.1135],0.525)\};$$
$$x_3=\{([0.0812,0.1276],[0.0551,0.1009],0.450),([0.0349,0.0861],[0.0748,0.1257],0.550)\};$$
$$x_4=\{([0.0517,0.1051],[0.0686,0.1198],0.550),([0.0871,0.1374],[0.0565,0.0886],0.450)\}.$$

由PIVIHFWMSM$_2(r=2)$算子得到的综合属性值为

$$x_1=\{([0.0148,0.0399],[0.0225,0.0535],0.425),([0.0152,0.0388],[0.0149,0.0461],0.575)\};$$
$$x_2=\{([0.0202,0.0442],[0.0210,0.0485],0.475),([0.0087,0.0337],[0.0196,0.0510],0.525)\};$$
$$x_3=\{([0.0308,0.0646],[0.0172,0.0428],0.450),([0.0087,0.0337],[0.0273,0.0594],0.550)\};$$
$$x_4=\{([0.0311,0.0454],[0.0239,0.0553],0.550),([0.0343,0.0679],[0.0179,0.0352],0.450)\}.$$

由PIVIHFWMSM$_3(r=3)$算子得到的综合属性值为

$$x_1=\{([0.0765,0.1245],[0.0948,0.1440],0.425),([0.0740,0.1208],[0.0804,0.1364],0.575)\};$$
$$x_2=\{([0.0903,0.1323],[0.0936,0.1379],0.475),([0.0604,0.1156],[0.0828,0.1363],0.525)\};$$
$$x_3=\{([0.1111,0.1543],[0.0782,0.1260],0.450),([0.0604,0.1156],[0.1032,0.1509],0.550)\};$$
$$x_4=\{([0.0823,0.1356],[0.0948,0.1440],0.550),([0.1142,0.1602],[0.0833,0.1156],0.450)\}.$$

由PIVIHFWMSM$_4(r=4)$算子得到的综合属性值为

$$x_1=\{([0.6679,0.7879],[0.7164,0.8273],0.425),$$

$([0.6711, 0.7845], [0.6687, 0.8071], 0.575)\};$

$x_2 = \{([0.7035, 0.8015], [0.7079, 0.8141], 0.475),$

$([0.6111, 0.7661], [0.7000, 0.8208], 0.525)\};$

$x_3 = \{([0.7548, 0.8452], [0.6851, 0.7971], 0.450),$

$([0.6111, 0.7661], [0.7395, 0.8421], 0.550)\};$

$x_4 = \{([0.6744, 0.8052], [0.7237, 0.8320], 0.550),$

$([0.7683, 0.8611], [0.6894, 0.7716], 0.450)\}.$

step 4: 计算上述各算子集成后的综合属性值的得分函数,并进行优劣排序,如表2所示.

step 5: 根据得分函数值的大小进行风险性能排序,得分函数值越大说明其综合条件越优,选择最佳企业.

表2 各算子计算得到的得分函数和排序结果

算子	x_1	x_2	x_3	x_4	排序结果
PIVIHFOWA	−0.0488	−0.1281	−0.1628	0.1587	$x_4 \succ x_1 \succ x_2 \succ x_3$
PIVIHFOWG	−0.0214	−0.1051	−0.1590	0.1589	$x_4 \succ x_1 \succ x_2 \succ x_3$
PIVIHFWMSM_1	−0.0054	−0.0530	−0.0398	0.0397	$x_4 \succ x_1 \succ x_3 \succ x_2$
PIVIHFWMSM_2	−0.0035	−0.0141	−0.0222	0.0246	$x_4 \succ x_1 \succ x_2 \succ x_3$
PIVIHFWMSM_3	−0.0110	−0.0216	−0.0391	0.0378	$x_4 \succ x_1 \succ x_2 \succ x_3$
PIVIHFWMSM_4	−0.0101	−0.0718	−0.1022	0.0842	$x_4 \succ x_1 \succ x_2 \succ x_3$

由Matlab可视化了最佳风险投资企业选择,由图1分析可知,基于PIVIHFOWA算子、PIVIHFOWG算子和$\text{PIVIHFMSM}^{(r)}$算子决策方法选择的最佳企业均为x_4,说明本文提出的决策方法具有合理性和有效性. 本文提出的算子具有以下优点:

1) 由于参数r取值的变化,属性值的排序结果也会出现细微的变化,表明本文构建的算子更具灵活性;

2) 决策者根据自身的决策偏好及风险态度改变参数,满足多种决策需求,更具一般性,而PIVIHFOWA算子和PIVIHFOWG算子不具有可变参数;

3) 利用本文提出的决策方法处理属性值具有相关性的多属性决策问题会更加科学并尽可能地避免了决策过程中的信息缺失.

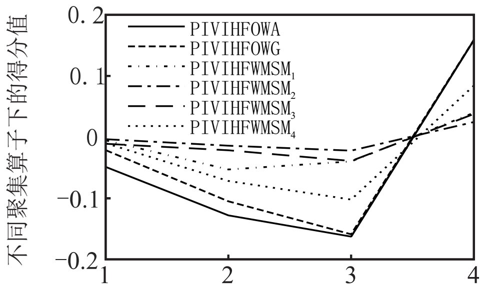

图1 不同聚集算子下的得分值

4 结论

本文在PIVIHFS环境下引入Maclaurin对称平均算子,并基于Archimedean T-范数和Archimedean S-范数提出了概率区间值直觉犹豫模糊Maclaurin对称平均算子($\text{PIVIHFMSM}^{(r)}$);其次,根据参数r的变化对算子$\text{PIVIHFMSM}^{(r)}$的几种特殊情况进行了讨论;最后,将该模型应用于风险投资企业进行风险项目及风险企业评价,并且将本文提出的$\text{PIVIHFMSM}^{(r)}$算子与其他算子相比较,结果说明该决策方法具有灵活性和有效性,决策结果能够满足决策者的多种需求. 后续将进一步研究决策过程中参数的选择以及基于毕达哥拉斯模糊集和语言术语集等其他模糊数下Maclaurin 对称平均算子的多属性决策问题.

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