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基于周期动态事件触发的网络化切换系统的分析与控制

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摘 要： 针对网络化系统研究具有模态依赖平均驻留时间 (MDADT) 和异步的线性切换系统的周期动态事件触发 H_∞ 控制. 与已有切换系统事件触发方式不同,周期动态事件触发引入动态事件触发参数,且具有最小的事件触发间隔下界,在降低数据发送数量的同时,能够有效避免奇诺行为;利用 MDADT 方法和多 Lyapunov 函数技术,给出闭环系统具有指数稳定和 H_∞ 加权性能的充分条件;在此基础上,针对周期动态事件触发机制中可能存在的异步问题,提出相应的控制器设计准则. 最后,通过算例验证所得结果的优越性.

关键词： 切换系统；模态依赖平均驻留时间；多 Lyapunov 函数；异步切换；周期动态事件触发

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The anlysis and control for networked switched systems based on periodic event-trigger

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Abstract: This paper is concerned with periodic dynamic event-triggered H_∞ control for networked switched linear systems with both mode-dependent average dwell time (MDADT) and asynchronous switching. Compared with the existing event-triggered results of the switched systems, the periodic dynamic event-triggered scheme introduces dynamic event-triggered parameters and has the minimum lower bound of event-triggered interval, which can effectively reduce the number of data packages and avoid the Zeno behavior. By using the MDADT approach and the multiple Lyapunov functions technique, sufficient conditions are proposed so that the close-loop system is exponential stable with a weighted H_∞ performance. Furthermore, the corresponding criterion is proposed to design a set of controllers for the switched system with asynchronism. Finally, an example is provided to verify the superiority of the acquired results.

Keywords: switched systems；mode-dependent average dwell time；multiple Lyapunov functions；asynchronous switching；periodic dynamic event-triggered

0 引 言

切换系统是由一定数量的子系统和一个协调子系统间的切换信号组成的一类重要的混杂系统[1]. 在过去的20年中,切换系统因其在混沌发生器、交通控制、智能机器人等诸多领域的广泛应用而备受关注[2-3].

当前,人们对满足平均驻留时间 (ADT) 切换的切换系统进行了大量的研究[4-5],然而 ADT 切换信号的平均驻留时间 τ_α 是与系统模态无关的. 切换系统的最小容许 ADT 往往由两个模态相关参数限定. 显然,由于未考虑各子系统的特性,基于 ADT 的分析和综合结论是相对保守的. 因此,研究人员考虑不同子系统的影响,将 ADT 切换扩展到 MDADT 切换,并取得了一些有意义的成果[6].

近年来,伴随着网络技术的飞速发展,网络化控制系统 (NCSs) 具有成本低、灵活性强、安装维护简单等[7] 优点,受到了研究人员的广泛关注. 为了更为有效地利用通讯资源,事件触发的思想被提出并得到了广泛的关注和研究[8]. 与此同时,在切换系统的研究中,一个广泛采用的假设是,切换系统的子系统与相

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应的控制器是匹配的,即同步的[9]. 然而,在网络化的切换系统中,尤其在引入事件触发机制(ETC)后,控制器模态与运行的子系统模态之间不匹配是不可避免的. 为了便于系统稳定性分析和综合问题的设计,本文引入最小模态依赖驻留时间的假设,以保证在事件触发区间内最多出现一次切换. 此外,受文献[10]的启发,引入周期动态ETC,这种触发机制具有周期触发的特点,具有最小的事件触发间隔下界,从而有效避免了奇诺行为;同时,采用动态事件触发参数,与一般的事件触发机制相比较,进一步降低了数据发送的数量,节约了网络通讯资源.

本文考虑基于MDADT切换的网络化切换系统周期性事件触发控制,采用多Lyapunov函数方法,分析保证切换系统具有全局一致指数稳定和H_∞性能指标的充分条件;基于周期动态事件触发机制,设计模态依赖的控制器,使其在触发周期内存在异步时依然能够保证系统的要求. 最后,通过数值仿真算例验证本文方法的有效性.

1 问题形成

考虑下面连续时间线性切换系统:

$$\begin{cases}\dot{x}(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}u(t)+E_{\sigma(t)}\omega(t),\\ z(t)=L_{\sigma(t)}x(t).\end{cases}\tag{1}$$

其中: $x(t)\in\boldsymbol{R}^n$为状态向量; $u(t)\in\boldsymbol{R}^q$为控制输入; $z(t)\in\boldsymbol{R}^p$为测量输出; $\omega(t)\in\boldsymbol{R}^m$为属于$L_2[0,\infty)$的外部扰动; $A_{\sigma(t)},B_{\sigma(t)},E_{\sigma(t)}$和$L_{\sigma(t)}$为适当的矩阵函数; $\sigma(t):[0,\infty)\to S=1,2,\dots,M$为切换信号的分段常值函数. $\{(t_0,\sigma(t_0)),(t_1,\sigma(t_1)),\dots,(t_k,\sigma(t_k)),\dots,|\sigma(t_k)\in S,k\in\boldsymbol{N}\}$代表切换序列,其中$t_0$和$x(t_0)$分别定义为初始时刻和初始状态,且切换点满足$t_0<t_1<\dots<t_k<\dots$. 当$t\in[t_k,t_{k+1})$时,第$\sigma(t_k)$个子系统正在运行.

本文引入事件触发器,考虑如下的周期动态ETC:

$$\begin{aligned}b_{v+1}h=&b_vh+\min_{l}\{lh|\eta((b_v+l)h)+\\&\theta(\delta_ix^{\mathrm{T}}((b_v+l)h)\varPhi_ix((b_v+l)h)-\\&e_x^{\mathrm{T}}(t)\varPhi_ie_x(t))\leqslant 0,\ l\in\boldsymbol{N}\}.\end{aligned}\tag{2}$$

其中: $e_x(t)=x((b_v+l)h)-x(b_vh)$, $\theta>0$, $0<\delta_i<1$, $\varPhi_i>0$为事件触发参数, $(b_v+l)h$和b_vh分别为采样点和事件触发时刻; $\eta(t)$为内部动态变量,满足微分方程

$$\dot{\eta}(t)=-\lambda\eta(t)+\delta_ix^{\mathrm{T}}(t)\varPhi_ix(t)-e_x^{\mathrm{T}}(t)\varPhi_ie_x(t),\tag{3}$$

这里$\lambda>0$, $\eta(0)=\eta_0\geqslant 0$.

定义1[11] 对于任意$T>t>0$和任意切换信号$\sigma(t)$, $N_{\sigma p}(t,T)$定义为第p个子系统在时间区间(t,T)内的切换次数, $T_p(t,T)$定义为第p个子系统在时间区间(t,T)内总的运行时长,其中$p\in S$. 如果存在正数N_{0p}和$\tau_{\alpha p}$满足不等式

$$N_{\sigma p}(t,T)\leqslant N_{0p}+\frac{T_p(t,T)}{\tau_{\alpha p}},$$

则称$\tau_{\alpha p}>0$为模态依赖的平均驻留时间, N_{0p}为模态依赖的抖动界.

假设1 假设变事件触发区间的长度$h_v=b_{v+1}h-b_vh$的上界为H,并且满足

$$0<h_v\leqslant H\leqslant\tau_{dp}.$$

其中: τ_{dp}为第p个子系统的驻留时间, H为事件触发区间的最大时长.

定义2[12] 当$\omega(t)=0$时,如果系统$x(t)$的解对于任意$k>0$和$\epsilon>0$满足

$$\|x(t)\|\leqslant k\mathrm{e}^{-\epsilon(t-t_0)}\|x_{t_0}\|_{c1},\ \forall t>t_0,$$

其中$\|x_{t_0}\|_{c1}=\sup\limits_{-d_M\leqslant\theta\leqslant 0}\{x(t+\theta),\dot{x}(t+\theta)\}$,则切换信号$\sigma(t)$下闭环系统(5)的平衡点$x^*=0$被称为全局一致渐近稳定(GUES)的.

定义3[13] 对于给定常数$\alpha_p>0$和$\gamma>0$,下面的闭环系统(5)全局一致指数稳定且具有加权H_∞性能指标$\tilde{\gamma}$,如果在零初始条件下$\phi(\theta)=0,\theta\in[-d_M,0]$,则对于任意非零$\omega(t)\in L_2[0,\infty)$,下面不等式成立:

$$\int_{t_0}^{\infty}\mathrm{e}^{\sum\limits_{p=1}^{M}\{-\alpha_pT_p(t_0,t)\}}\|z(t)\|^2\mathrm{d}t\leqslant\tilde{\gamma}^2\int_{t_0}^{\infty}\|\omega(t)\|^2\mathrm{d}t.$$

引理1[14] 对于任意矩阵$M=\begin{bmatrix}R&G\\ *&R\end{bmatrix}\geqslant 0$, $R>0$,变量$0\leqslant d(t)\leqslant d$,向量函数$\dot{x}:[-d,0]\to\boldsymbol{R}^n$使得下面不等式成立:

$$-d\int_{t-d}^{t}\dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\leqslant\vartheta^{\mathrm{T}}(t)Z\vartheta(t),\tag{4}$$

其中

$$\vartheta(t)=[x^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-d(t))\quad x^{\mathrm{T}}(t-d)]^{\mathrm{T}},$$

$$Z=\begin{bmatrix}-R&R-G&G\\ *&[G-R]_s&R-G\\ *&*&-R\end{bmatrix}.$$

考虑状态反馈控制器

$$u(t)=K_{\hat{\sigma}(t)}x(t),$$

可以得到闭环系统如下:

$$\begin{aligned}&\dot{x}(t)=A_{\sigma(t)}x(t)+B_{\sigma(t)}K_{\hat{\sigma}(t)}x(t)+E_{\sigma(t)}\omega(t),\\&z(t)=L_{\sigma(t)}x(t).\end{aligned}\tag{5}$$

其中: $K_{\hat{\sigma}(t)}$是需要设计的状态反馈控制器增益, $\hat{\sigma}(t)\in S$是控制器的切换信号. 考虑到零阶保持器(ZOH)的作用,控制器的输入可以表示为

$$\bar{x}(t) = x(b_\nu h),\ t \in [b_\nu h + \tau_{b_\nu}, b_{\nu+1}h + \tau_{b_{\nu+1}}),$$

其中$\tau_{b_\nu} > 0$为传输时滞,且满足$\tau_m < \tau_{b_\nu} < \tau_M$. 为了便于讨论传输时滞对控制器模态切换的影响,信号的保持间隔$[b_\nu h + \tau_{b_\nu}, b_{\nu+1}h + \tau_{b_{\nu+1}})$分为如下的子区间:

$$\begin{cases} \Upsilon_0 = [b_\nu h + \tau_{b_\nu}, b_\nu h + h + \tau_{b_\nu}), \\ \Upsilon_1 = [b_\nu h + h + \tau_{b_\nu}, b_\nu h + 2h + \tau_{b_\nu}), \\ \quad \vdots \\ \Upsilon_{j_M^*} = [b_\nu h + j_M^* h + \tau_{b_\nu}, b_{\nu+1}h + \tau_{b_{\nu+1}}). \end{cases} \tag{6}$$

其中

$$j_M^* = \min\{j | b_\nu h + (j+1)h + \tau_{b_\nu} \geqslant b_{\nu+1}h + \tau_{b_{\nu+1}}\}.$$

分段时变时滞$d(t)$和误差向量$e_x(t)$定义为

$$d(t) = \begin{cases} t - b_\nu h,\ t \in \Upsilon_0; \\ t - b_\nu h - h,\ t \in \Upsilon_1; \\ \quad \vdots \\ t - b_\nu h - j_M^* h,\ t \in \Upsilon_{j_M^*}. \end{cases} \tag{7}$$

$$e_x(t) = \begin{cases} x(b_\nu h) - x(b_\nu h),\ t \in \Upsilon_0; \\ x(b_\nu h + h) - x(b_\nu h),\ t \in \Upsilon_1; \\ \quad \vdots \\ x(b_\nu h + j_M^* h) - x(b_\nu h),\ t \in \Upsilon_{j_M^*}. \end{cases} \tag{8}$$

那么,控制器的控制输入$\bar{x}(t)$可以重写为

$$\bar{x}(t) = x(b_\nu h) = x(t - d(t)) - e_x(t). \tag{9}$$

定义$d_m = \tau_m, d_M = \tau_M + h$,其中$d_m \leqslant d(t) \leqslant d_M$. 因此,闭环系统(5)可以重构为

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\hat{\sigma}(t)}x(t - d(t)) - \\ \qquad B_{\sigma(t)}K_{\hat{\sigma}(t)}e_x(t) + E_{\sigma(t)}\omega(t); \\ z(t) = L_{\sigma(t)}x(t); \\ x(\theta) = \phi(\theta),\ \theta \in [-d_M, 0]. \end{cases} \tag{10}$$

其中初始函数$\phi(\theta)$在区间$[-d_M, 0]$内连续可微.

2 控制器设计

首先研究切换系统H_∞性能,本节推导了保证闭环系统(5)指数稳定和H_∞性能的充分条件.

定理1 给定正常数$\alpha_p, \beta_p, d_m, d_M, \mu_p > 1$和$0 < \delta_p < 1$,如果存在正定矩阵$P_p, Q_1^p, Q_2^p, R_1^p, R_2^p, \varPhi_p$和合适维数的$G_p$满足$\begin{bmatrix} R_2^p & G_p \\ * & R_2^p \end{bmatrix} \geqslant 0$,对于$\forall (p,q) \in S \times S$, $p \neq q$满足

$$\begin{bmatrix} \varPi_1 & \varPi_2 \\ * & \varPi_3 \end{bmatrix} < 0; \tag{11}$$

$$\begin{bmatrix} \tilde{\varPi}_1 & \tilde{\varPi}_2 \\ * & \tilde{\varPi}_3 \end{bmatrix} < 0; \tag{12}$$

$$P_p < \mu_p P_q; \tag{13}$$

$$Q_1^p < \mu_p \mathrm{e}^{-(\alpha_p+\beta_p)d_m} Q_1^q,\ Q_2^p < \mu_p \mathrm{e}^{-(\alpha_p+\beta_p)d_M} Q_2^q; \tag{14}$$

$$R_1^p < \mu_p \mathrm{e}^{-(\alpha_p+\beta_p)d_m} R_1^q,\ R_2^p < \mu_p \mathrm{e}^{-(\alpha_p+\beta_p)d_M} R_2^q. \tag{15}$$

其中

$$\varPi_1 = \begin{bmatrix} \varPi_{11} & \varPi_{12} & 0 & 0 & -P_pB_pK_p \\ * & \varPi_{22} & \varPi_{23} & \mathrm{e}^{-\alpha_p d_M}G_p & 0 \\ * & * & \varPi_{33} & \varPi_{34} & 0 \\ * & * & * & -\mathrm{e}^{-\alpha_p d_M}R_2^p & 0 \\ * & * & * & * & -\varPhi_p \end{bmatrix},$$

$$\varPi_2 = \begin{bmatrix} d_m A_p^{\mathrm{T}} & (d_M - d_m)A_p^{\mathrm{T}} \\ 0 & 0 \\ d_m K_p^{\mathrm{T}} B_p^{\mathrm{T}} & (d_M - d_m)K_p^{\mathrm{T}} B_p^{\mathrm{T}} \\ 0 & 0 \\ -d_m K_p^{\mathrm{T}} B_p^{\mathrm{T}} & -(d_M - d_m)K_p^{\mathrm{T}} B_p^{\mathrm{T}} \end{bmatrix},$$

$$\varPi_3 = \begin{bmatrix} -(R_1^p)^{-1} & 0 \\ * & -(R_2^p)^{-1} \end{bmatrix},$$

$$\tilde{\varPi}_1 = \begin{bmatrix} \tilde{\varPi}_{11} & \tilde{\varPi}_{12} & 0 & 0 & -P_qB_pK_q \\ * & \tilde{\varPi}_{22} & \tilde{\varPi}_{23} & \mathrm{e}^{\beta_p d_m}G_q & 0 \\ * & * & \tilde{\varPi}_{33} & \tilde{\varPi}_{34} & 0 \\ * & * & * & -\mathrm{e}^{\beta_p d_m}R_2^q & 0 \\ * & * & * & * & -\varPhi_p \end{bmatrix},$$

$$\tilde{\varPi}_2 = \begin{bmatrix} d_m A_p^{\mathrm{T}} & (d_M - d_m)A_p^{\mathrm{T}} \\ 0 & 0 \\ d_m K_q^{\mathrm{T}} B_p^{\mathrm{T}} & (d_M - d_m)K_q^{\mathrm{T}} B_p^{\mathrm{T}} \\ 0 & 0 \\ -d_m K_q^{\mathrm{T}} B_p^{\mathrm{T}} & -(d_M - d_m)K_q^{\mathrm{T}} B_p^{\mathrm{T}} \end{bmatrix},$$

$$\tilde{\varPi}_3 = \begin{bmatrix} -(R_1^q)^{-1} & 0 \\ * & -(R_2^q)^{-1} \end{bmatrix},$$

$$\varPi_{11} = A_p^{\mathrm{T}}P_p + P_pA_p + Q_1^p + \alpha_p P_p + \mathrm{e}^{-\alpha_p d_m}R_1^p,$$

$$\varPi_{12} = P_pB_pK_p - \mathrm{e}^{-\alpha_p d_m}R_1^p,$$

$$\varPi_{22} = -\mathrm{e}^{-\alpha_p d_m}Q_1^p + \mathrm{e}^{-\alpha_p d_m}Q_2^p + \mathrm{e}^{-\alpha_p d_m}R_1^p - \mathrm{e}^{-\alpha_p d_M}R_2^p,$$

$$\tilde{\varPi}_{34} = \mathrm{e}^{\beta_p d_m}(R_2^q - G_q),$$

$$\varPi_{23} = \mathrm{e}^{-\alpha_p d_M}(R_2^p - G_p),$$

$$\tilde{\varPi}_{12} = P_qB_pK_q - R_1^q,$$

$$\varPi_{33} = \delta_p\varPhi_p + \mathrm{e}^{-\alpha_p d_M}[G_p - R_2^p]_s,$$

$\Pi_{34} = \mathrm{e}^{-\alpha_p d_M}(R_2^p - G_p)$,

$\tilde{\Pi}_{23} = \mathrm{e}^{\beta_p d_m}(R_2^q - G_q)$,

$\tilde{\Pi}_{11} = A_p^{\mathrm{T}} P_q + P_q A_p + Q_1^q - \beta_p P_q + R_1^q$,

$\tilde{\Pi}_{22} = -\mathrm{e}^{\beta_p d_m} Q_1^q + \mathrm{e}^{\beta_p d_m} Q_2^q + R_1^q - \mathrm{e}^{\beta_p d_m} R_2^q$,

$\tilde{\Pi}_{33} = \delta_p \Phi_p + \mathrm{e}^{\beta_p d_m}[G_q - R_2^q]_s$.

那么,当$\omega(t) = 0$时,闭环系统(5)是全局一致渐近稳定的,切换信号满足

$$\tau_{\alpha p} \geqslant \frac{\alpha_p}{(\alpha_p + \beta_p)\tau_{dp} + \ln \mu_p}. \tag{16}$$

证明 首先,证明内部动态变量在任意时刻都保持非负. 通过结合式(2)和(3),对于任意$t \in \boldsymbol{R}^+$,下式成立:

$$\dot{\eta}(t) + \lambda\eta(t) =$$
$$\delta_{\sigma(t)} x^{\mathrm{T}}(t)\Phi_{\sigma(t)} x(t) - e_x^{\mathrm{T}}(t)\Phi_{\sigma(t)} e_x(t) \geqslant -\frac{\eta(t)}{\theta},$$

其中$\eta(0) = \eta_0 \geqslant 0$. 根据比较定理,可以推导得到

$$\eta(t) \geqslant \eta_0 \mathrm{e}^{-(\lambda + \frac{1}{\theta})t},\ \forall t \in \boldsymbol{R}^+,$$

这意味着$\eta(t) \geqslant 0$. 为了便于分析,假设$\alpha_p \leqslant \lambda$,可以得到

$$\dot{\eta}(t) \leqslant \dot{\eta}(t) + \alpha_p\eta(t) \leqslant$$
$$\dot{\eta}(t) + \lambda\eta(t) =$$
$$\delta_{\sigma(t)} x^{\mathrm{T}}(t)\Phi_{\sigma(t)} x(t) - e_x^{\mathrm{T}}(t)\Phi_{\sigma(t)} e_x(t). \tag{17}$$

考虑下面的控制器依赖的Lyapunov函数:

$$V_{\hat{\sigma}(t)}(t) =$$
$$\alpha(t)V_{1,\hat{\sigma}(t)}(t) + (1 - \alpha(t))V_{2,\hat{\sigma}(t)}(t) + \eta(t). \tag{18}$$

其中

$$V_{\iota,\hat{\sigma}(t)}(t) =$$
$$x^{\mathrm{T}}(t)P_{\hat{\sigma}(t)} x(t) + \int_{t-d_m}^{t} \mathrm{e}^{\kappa_\iota(t-s)} x^{\mathrm{T}}(s) Q_1^{\hat{\sigma}(t)} x(s)\mathrm{d}s +$$
$$\int_{t-d_M}^{t-d_m} \mathrm{e}^{\kappa_\iota(t-s)} x^{\mathrm{T}}(s) Q_2^{\hat{\sigma}(t)} x(s)\mathrm{d}s +$$
$$d_m \int_{t-d_m}^{0}\int_{t+\theta}^{t} \mathrm{e}^{\kappa_\iota(t-s)} \dot{x}^{\mathrm{T}}(s) R_1^{\hat{\sigma}(t)} \dot{x}(s)\mathrm{d}s\mathrm{d}\theta +$$
$$(d_M - d_m)\int_{-d_M}^{-d_m}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s) R_2^{\hat{\sigma}(t)} \dot{x}(s)\mathrm{d}s\mathrm{d}\theta;$$

$\iota = 1, 2,\ \kappa_1 = -\alpha_p,\ \kappa_2 = \beta_p$;

$$\alpha(t) = \begin{cases} 1,\ \sigma(t) = \hat{\sigma}(t); \\ 0,\ \sigma(t) \neq \hat{\sigma}(t). \end{cases}$$

1) 当$\sigma(t) = \hat{\sigma}(t)$,子系统的模态和控制器模态同步. 在区间$t \in [b_\nu h + \tau_{b_\nu}, t_k)$内,求函数$V_{\hat{\sigma}(t)}(t) = V_{1,\hat{\sigma}(t)}(t) + \eta(t)$对$t$的微分可得

$$\dot{V}_{\hat{\sigma}(t)}(t) =$$
$$-\alpha_p V_{\hat{\sigma}(t)}(t) + \dot{\eta}(t) + \alpha_p\eta(t) +$$
$$x^{\mathrm{T}}(t)(\alpha_p P_{\sigma(t)} + A_{\sigma(t)}^{\mathrm{T}} P_{\sigma(t)} + P_{\sigma(t)} A_{\sigma(t)}) +$$
$$2x^{\mathrm{T}}(t)P_{\sigma(t)} B_{\sigma(t)} K_{\sigma(t)} x(t - d(t)) -$$
$$2x^{\mathrm{T}}(t)P_{\sigma(t)} B_{\sigma(t)} K_{\sigma(t)} e_x(t) -$$
$$\mathrm{e}^{-\alpha_p d_m} x^{\mathrm{T}}(t - d_m) Q_1^{\sigma(t)} x(t - d_m) +$$
$$\mathrm{e}^{-\alpha_p d_m} x^{\mathrm{T}}(t - d_m) Q_2^{\sigma(t)} x(t - d_m) +$$
$$d_m^2 \dot{x}^{\mathrm{T}}(t) R_1^{\sigma(t)} \dot{x}(t) + Q_1^{\sigma(t)} x(t) +$$
$$(d_M - d_m)^2 \dot{x}^{\mathrm{T}}(t) R_2^{\sigma(t)} \dot{x}(t) -$$
$$d_m \int_{t-d_m}^{t} \mathrm{e}^{-\alpha_p(t-s)} \dot{x}^{\mathrm{T}}(s) R_1^{\sigma(t)} \dot{x}(s)\mathrm{d}s -$$
$$(d_M - d_m)\int_{t-d_M}^{t-d_m} \mathrm{e}^{-\alpha_p(t-s)} \dot{x}^{\mathrm{T}}(s) R_2^{\sigma(t)} \dot{x}(s)\mathrm{d}s. \tag{19}$$

根据Jensen's不等式有

$$-d_m \int_{t-d_m}^{t} \mathrm{e}^{-\alpha_p(t-s)} \dot{x}^{\mathrm{T}}(s) R_1^{\sigma(t)} \dot{x}(s)\mathrm{d}s \leqslant$$
$$\mathrm{e}^{-\alpha_p d_m}[x(t) - x(t - d_m)]^{\mathrm{T}} R_1^{\sigma(t)}[x(t) - x(t - d_m)]. \tag{20}$$

根据引理1及关系$d_m < d(t) < d_M$有

$$-(d_M - d_m)\int_{t-d_M}^{t-d_m} \mathrm{e}^{-\alpha_p(t-s)} \dot{x}^{\mathrm{T}}(s) R_2^{\sigma(t)} \dot{x}(s)\mathrm{d}s \leqslant$$
$$\mathrm{e}^{-\alpha_p d_M} \xi^{\mathrm{T}}(t)\Pi_{\sigma(t)}\xi(t). \tag{21}$$

其中

$$\xi^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t - d_m)\ \ x^{\mathrm{T}}(t - d(t))\ \ x^{\mathrm{T}}(t - d_M)],$$

$$\Pi_{\sigma(t)} = \begin{bmatrix} -R_2^{\sigma(t)} & R_2^{\sigma(t)} - G_{\sigma(t)} & G_{\sigma(t)} \\ * & [G_{\sigma(t)} - R_2^{\sigma(t)}]_s & R_2^{\sigma(t)} - G_{\sigma(t)} \\ * & * & -R_2^{\sigma(t)} \end{bmatrix}.$$

当$t \in [b_\nu h + \tau_{b_\nu}, b_{\nu+1}h + \tau_{b_{\nu+1}})$时,周期动态事件触发条件(2)可以保证下面的不等式成立:

$$\delta_{\sigma(t)} x^{\mathrm{T}}(t - d(t))\Phi_{\sigma(t)} x(t - d(t)) -$$
$$e_x^{\mathrm{T}}(t)\Phi_{\sigma(t)} e_x(t) \geqslant 0. \tag{22}$$

由式(19) ~ (22)可知

$$\dot{V}_{\hat{\sigma}(t)}(t) \leqslant -\alpha_p V_{\hat{\sigma}(t)}(t) + \varsigma^{\mathrm{T}}(t)\Pi\varsigma(t), \tag{23}$$

其中

$$\Pi = \Pi_1 + \Lambda^{\mathrm{T}}[d_m^2 R_1^{\sigma(t)} + (d_M - d_m)^2 R_2^{\sigma(t)}]\Lambda,$$
$$\varsigma^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t - d_m), x^{\mathrm{T}}(t - d(t)),$$
$$x^{\mathrm{T}}(t - d_M), e_x^{\mathrm{T}}(t)],$$
$$\Lambda = [A_{\sigma(t)}\ \ 0\ \ B_{\sigma(t)} K_{\sigma(t)}\ \ 0\ \ -B_{\sigma(t)} K_{\sigma(t)}].$$

根据Shur补引理,可以推导出$\Pi < 0$和式(11)中的不等式是等价的,即

$$\dot{V}_{\sigma(t)}(t) + \alpha_p V_{\sigma(t)}(t) \leqslant 0. \tag{24}$$

2) 当$\sigma(t) \neq \hat{\sigma}(t)$时,子系统模态和控制器的模态是异步的. 对于Lyapunov函数选择正常数β_p,然后按照1)的证明过程,可以由式(12)推导出下面的不等式:

$$\dot V_{\sigma(t)}(t)-\beta_p V_{\sigma(t)}(t)\leqslant 0. \tag{25}$$

考虑网络延迟引起的传输时滞,定义触发点$\hat t_k=b_\nu h+\tau_{b_\nu}$. 当$t\in[\hat t_k,t_k)$时,第$\sigma(t_k)$个子系统和第$\sigma(t_k^-)$时的控制器正在运行,即$\sigma(t)\neq\hat\sigma(t)$. 当$t\in[t_k,\hat t_{k+1})$时,第$\sigma(t_k)$个子系统和与之匹配的控制器正在运行,即$\sigma(t)=\hat\sigma(t)$. 为了使符号更加简洁,令$T_\downarrow(t_0,t)$和$T_\uparrow(t_0,t)$分别代表$(t_0,t)$时间内子系统和控制器匹配和不匹配的总时长,$T_{p\downarrow}(t_0,t)$和$T_{p\uparrow}(t_0,t)$分别代表$(t_0,t)$时间内第$p$个子系统相匹配和不匹配的控制器控制的总时长.

基于以上的讨论和不等式条件(13)～(15)、(24)和(25),对于$\forall t\in[t_k,t_{k+1})$,可以得到

$$\begin{aligned}&V_{\hat\sigma(t)}(t)\leqslant\\&\mu_{\sigma(t_k)}\mathrm{e}^{\beta_{\sigma}(t_k)T_{p\uparrow}(t_k,t)-\alpha_\sigma(t_k)T_{p\downarrow}(t_k,t)}V_{\hat\sigma(t_k)}(\hat t_k)\leqslant\\&\cdots\leqslant\\&\exp\Big\{\sum_{p=1}^{M}[(\ln\mu_p+(\alpha_p+\beta_p)\tau_{dp})N_{0p}]\Big\}\times\\&\exp\Big\{\sum_{p=1}^{M}\Big[\Big(\frac{\ln\mu_p+(\alpha_p+\beta_p)\tau_{dp}}{\tau_{\alpha p}}-\\&\alpha_p\Big)T_p(t_0,t)\Big]\Big\}V_{\hat\sigma(t_0)}(\hat t_0).\end{aligned}\tag{26}$$

定义

$$\begin{aligned}&\lambda_m=\min\{\lambda_{\min}(P_{\hat\sigma(t)})\},\\&\lambda_M=\\&\max\{\lambda_{\max}(P_{\hat\sigma(t)})\}+d_m\max\{\lambda_{\max}(Q_1^{\hat\sigma(t)})\}+\\&(d_M-d_m)\max\{\lambda_{\max}(Q_2^{\hat\sigma(t)})\}+\\&\frac12 d_m^3\max\{\lambda_{\max}(R_1^{\hat\sigma(t)})\}+\\&\frac{d_M+d_m}{2}(d_M-d_m)^2\max\{\lambda_{\max}(R_2^{\hat\sigma(t)})\}.\end{aligned}$$

令

$$\kappa=\sqrt{\frac{\lambda_M}{\lambda_m}}\exp\Big\{\frac12\sum_{p=1}^{M}[(\ln\mu_p+(\alpha_p+\beta_p)\tau_{dp})N_{0p}]\Big\},$$

$$\epsilon=-\frac12\max\Big\{\sum_{p=1}^{M}\Big(\frac{\ln\mu_p+(\alpha_p+\beta_p)\tau_{dp}}{\tau_{\alpha p}}-\alpha_p\Big)\Big\},$$

结合不等式(16)和(26),可以得到

$$\|x(t)\|\leqslant\kappa\mathrm{e}^{-\epsilon(t-t_0)}\|x_{t_0}\|_{c1}. \tag{27}$$

根据定义2,可以得出结论:当$\omega(t)=0$时,闭环系统(5)在满足MDADT(16)的切换信号下是全局一致指数稳定的. □

考虑闭环系统(5)的加权H_∞性能,有如下定理.

定理2 对于闭环切换系统(6),给定正常数$\alpha_p,\beta_p,d_m,d_M,\mu_p>1$和$0<\delta_p<1$,如果存在正定矩阵$P_p,Q_1^p,Q_2^p,R_1^p,R_2^p,\varPhi_p$和合适维数的$G_p$满足$\begin{bmatrix}R_2^p&G_p\\ *&R_2^p\end{bmatrix}\geqslant 0$,则对于$\forall(p,q)\in S\times S,\ p\neq q$满足

$$\begin{bmatrix}\varOmega_1&\varOmega_2\\ *&\varPi_3\end{bmatrix}<0, \tag{28}$$

$$\begin{bmatrix}\tilde\varOmega_1&\tilde\varOmega_2\\ *&\tilde\varPi_3\end{bmatrix}<0. \tag{29}$$

其中

$$\varOmega_1=\begin{bmatrix}\varPi_1&\varOmega_{12}\\ *&\varOmega_{22}\end{bmatrix},\ \varOmega_{12}^{\mathrm T}=\begin{bmatrix}E_p^{\mathrm T}P_p^{\mathrm T}&0&0&0&0\\ L_p&0&0&0&0\end{bmatrix},$$

$$\varOmega_2=\begin{bmatrix}d_mA_p^{\mathrm T}&(d_M-d_m)A_p^{\mathrm T}\\0&0\\d_mK_p^{\mathrm T}B_p^{\mathrm T}&(d_M-d_m)K_p^{\mathrm T}B_p^{\mathrm T}\\0&0\\-d_mK_p^{\mathrm T}B_p^{\mathrm T}&-(d_M-d_m)K_p^{\mathrm T}B_p^{\mathrm T}\\d_mE_p^{\mathrm T}&(d_M-d_m)E_p^{\mathrm T}\\0&0\end{bmatrix},$$

$$\varOmega_{22}=\tilde\varOmega_{22}=\begin{bmatrix}-\gamma^2I&0\\0&-I\end{bmatrix},$$

$$\tilde\varOmega_1=\begin{bmatrix}\tilde\varPi_1&\tilde\varOmega_{12}\\ *&\tilde\varOmega_{22}\end{bmatrix},\ \tilde\varOmega_{12}^{\mathrm T}=\begin{bmatrix}E_p^{\mathrm T}P_q^{\mathrm T}&0&0&0&0\\ L_p&0&0&0&0\end{bmatrix},$$

$$\tilde\varOmega_2=\begin{bmatrix}d_mA_p^{\mathrm T}&(d_M-d_m)A_p^{\mathrm T}\\0&0\\d_mK_q^{\mathrm T}B_p^{\mathrm T}&(d_M-d_m)K_q^{\mathrm T}B_p^{\mathrm T}\\0&0\\-d_mK_q^{\mathrm T}B_p^{\mathrm T}&-(d_M-d_m)K_q^{\mathrm T}B_p^{\mathrm T}\\d_mE_p^{\mathrm T}&(d_M-d_m)E_p^{\mathrm T}\\0&0\end{bmatrix}.$$

对于非零$\omega(t)\in L_2[0,\infty)$,事件触发闭环系统(5)是全局一致指数稳定的,且具有给定的H_∞扰动衰减性能指标$\tilde\gamma$,切换信号$\sigma(t)$满足(16). 其中:$\tilde\gamma=\sqrt{\rho}\gamma,\rho=\exp\Big\{\sum_{p=1}^{M}[(\ln\mu_p+(\alpha_p+\beta_p)\tau_{dp})N_{0p}]\Big\}$.

证明 式(11)和(12)可以由式(28)和(29)推导得到. 根据定理1,可以保证当$\omega(t)=0$时,系统(5)是指数稳定的. 构造式(18)所示的Lyapunov方程,用定理1相同的方法,可以得到

$$\begin{cases}\dot V_{\hat\sigma(t)}(t)+\alpha_{\sigma(t)}V_{\hat\sigma(t)}(t)+\varGamma(t)\leqslant 0,\ t\in[\hat t_k,t_k);\\ \dot V_{\hat\sigma(t)}(t)-\beta_{\sigma(t)}V_{\hat\sigma(t)}(t)+\varGamma(t)\leqslant 0,\ t\in[t_k,\hat t_{k+1}).\end{cases}\tag{30}$$

其中$\varGamma(t)=z^{\mathrm T}(t)z(t)-\gamma^2\omega^{\mathrm T}(t)\omega(t)$. 对于$\forall t\in[\hat t_k,\hat t_{k+1})$,对式(30)两边进行积分,由式(13)～(15)可

得

$$V_{\hat\sigma(t)}(t)\leqslant$$
$$\mu_{\sigma(t_k)}\mathrm{e}^{\beta_{\sigma(t_k)}T_{p\uparrow}(t_k,t)-\alpha_{\sigma(t_k)}T_{p\downarrow}(t_k,t)}V_{\hat\sigma(t_k)}(\hat t_k)-$$
$$\int_{t_k}^{t}\mathrm{e}^{\beta_{\sigma(t_k)}T_{p\uparrow}(s,t)-\alpha_{\sigma(t_k)}T_{p\downarrow}(s,t)}\varGamma(s)\mathrm{d}s\leqslant$$
$$\cdots\leqslant$$
$$\exp\Big\{\sum_{p=1}^{M}[\beta_pT_{p\uparrow}(t_0,t)-\alpha_pT_{p\downarrow}(t_0,t)]\Big\}\times$$
$$\prod_{p=1}^{M}(\mu_p)^{N_{\sigma p}(t_0,t)}V_{\hat\sigma(t_0)}(\hat t_0)-$$
$$\int_{t_0}^{t}\exp\Big\{\sum_{p=1}^{M}[\beta_pT_{p\uparrow}(s,t)-\alpha_pT_{p\downarrow}(s,t)]\Big\}\times$$
$$\prod_{p=1}^{M}(\mu_p)^{N_{\sigma p}(s,t)}\varGamma(s)\mathrm{d}s. \tag{31}$$

在零初始条件下,可以得到

$$\int_{t_0}^{t}\exp\Big\{\sum_{p=1}^{M}[-\alpha_pT_p(s,t)+(\alpha_p+\beta_p)T_{p\uparrow}(s,t)+$$
$$N_{\sigma p}(s,t)\ln\mu_p]\Big\}\varGamma(s)\mathrm{d}s\leqslant 0.$$

因为$\varGamma(t)=z^{\mathrm T}(t)z(t)-\gamma^2\omega^{\mathrm T}(t)\omega(t)$,有

$$\int_{\hat t_0}^{t}\exp\Big\{\sum_{p=1}^{M}[-\alpha_pT_p(s,t)-(\alpha_p+\beta_p)T_{p\uparrow}(\hat t_0,s)-$$
$$N_{\sigma p}(\hat t_0,s)\ln\mu_p]\Big\}z^{\mathrm T}(s)z(s)\mathrm{d}s\leqslant$$
$$\gamma^2\int_{\hat t_0}^{t}\exp\Big\{\sum_{p=1}^{M}[-\alpha_pT_p(s,t)-(\alpha_p+\beta_p)T_{p\uparrow}(\hat t_0,s)-$$
$$N_{\sigma p}(\hat t_0,s)\ln\mu_p]\Big\}\omega^{\mathrm T}(s)\omega(s)\mathrm{d}s.$$

由定义1和式(16),有

$$\sum_{p=1}^{M}[-\alpha_pT_p(s,t)-((\alpha_p+$$
$$\beta_p)\tau_{dp}+\ln\mu_p)N_{\sigma p}(t_0,s)]\geqslant$$
$$\sum_{p=1}^{M}[-\alpha_pT_p(t_0,t)-((\alpha_p+\beta_p)\tau_{dp}+\ln\mu_p)N_{0p}].$$

由$-(\alpha_p+\beta_p)T_{p\uparrow}(t_0,s)-N_{\sigma p}(t_0,s)\ln\mu_p\leqslant 0$,可以得出结论

$$\int_{t_0}^{t}\exp\Big\{\sum_{p=1}^{M}[-\alpha_pT_p(t_0,s)]\Big\}z^{\mathrm T}(s)z(s)\mathrm{d}s\leqslant$$
$$\tilde\gamma^2\int_{t_0}^{t}\omega^{\mathrm T}(s)\omega(s)\mathrm{d}s. \tag{32}$$

对式(32)两边由$t=\hat t_0$到∞进行积分,可知

$$\int_{t_0}^{\infty}\exp\Big\{\sum_{p=1}^{M}[-\alpha_pT_p(t_0,s)]\Big\}z^{\mathrm T}(s)z(s)\mathrm{d}s\leqslant$$
$$\tilde\gamma^2\int_{t_0}^{\infty}\omega^{\mathrm T}(s)\omega(s)\mathrm{d}s. \tag{33}$$

即闭环系统(5)实现了加权H_∞性能指标. □

接下来进行控制器和触发机制的联合设计.

定理3 对于线性切换系统(5),令$\alpha_p,\beta_p,d_m,d_M,\mu_p>1$和$0<\delta_p<1$为给定正常数,如果存在正定矩阵$X_p,Y_p,\tilde Q_1^p,\tilde Q_2^p,\tilde R_1^p,\tilde R_2^p,\tilde\varPhi_p$和合适维数的$\tilde G_p$满足$\begin{bmatrix}\tilde R_2^p & \tilde G_p\\ * & \tilde R_2^p\end{bmatrix}\geqslant 0$,则对于$\forall(p,q)\in S\times S,\ p\neq q$满足

$$\begin{bmatrix}\varXi_1 & \varXi_2\\ * & \varXi_3\end{bmatrix}<0, \tag{34}$$
$$\begin{bmatrix}\tilde\varXi_1 & \tilde\varXi_2\\ * & \tilde\varXi_3\end{bmatrix}<0, \tag{35}$$
$$\begin{bmatrix}-\mu_pX_q & X_q\\ * & -X_p\end{bmatrix}<0, \tag{36}$$
$$\begin{bmatrix}-\mu_p\mathrm{e}^{-(\alpha_p+\beta_p)d_m}\tilde Q_1^q & X_q\\ * & \tilde Q_1^p-2X_p\end{bmatrix}<0, \tag{37}$$
$$\begin{bmatrix}-\mu_p\mathrm{e}^{-(\alpha_p+\beta_p)d_M}\tilde Q_2^q & X_q\\ * & \tilde Q_2^p-2X_p\end{bmatrix}<0, \tag{38}$$
$$\begin{bmatrix}-\mu_p\mathrm{e}^{-(\alpha_p+\beta_p)d_m}\tilde R_1^q & X_q\\ * & \tilde R_1^p-2X_p\end{bmatrix}<0, \tag{39}$$
$$\begin{bmatrix}-\mu_p\mathrm{e}^{-(\alpha_p+\beta_p)d_M}\tilde R_2^q & X_q\\ * & \tilde R_2^p-2X_p\end{bmatrix}<0, \tag{40}$$
$$\tau_{\alpha p}\geqslant\frac{\alpha_p}{(\alpha_p+\beta_p)\tau_{dp}+\ln\mu_p}. \tag{41}$$

其中

$$\varXi_1=\begin{bmatrix}\varXi_{11} & \varXi_{12}\\ * & \varXi_{22}\end{bmatrix},\ \varXi_{12}^{\mathrm T}=\begin{bmatrix}E_p^{\mathrm T} & 0 & 0 & 0 & 0\\ L_pX_p^{\mathrm T} & 0 & 0 & 0 & 0\end{bmatrix},$$
$$\tilde\varXi_1=\begin{bmatrix}\tilde\varXi_{11} & \tilde\varXi_{12}\\ * & \tilde\varXi_{22}\end{bmatrix},\ \tilde\varXi_{12}^{\mathrm T}=\begin{bmatrix}E_p^{\mathrm T} & 0 & 0 & 0 & 0\\ L_pX_q^{\mathrm T} & 0 & 0 & 0 & 0\end{bmatrix},$$
$$\varXi_{11}=\begin{bmatrix}\varXi_{1111} & \varXi_{1112} & 0 & 0 & -B_pY_p\\ * & \varXi_{1122} & \varXi_{1123} & \mathrm{e}^{-\alpha_pd_M}\tilde G_p & 0\\ * & * & \varXi_{1133} & \varXi_{1134} & 0\\ * & * & * & -\mathrm{e}^{-\alpha_pd_M}\tilde R_2^p & 0\\ * & * & * & * & -\tilde\varPhi_p\end{bmatrix},$$
$$\tilde\varXi_{11}=\begin{bmatrix}\tilde\varXi_{1111} & \tilde\varXi_{1112} & 0 & 0 & -B_pY_q\\ * & \tilde\varXi_{1122} & \tilde\varXi_{1123} & \mathrm{e}^{\beta_pd_m}\tilde G_q & 0\\ * & * & \tilde\varXi_{1133} & \tilde\varXi_{1134} & 0\\ * & * & * & -\mathrm{e}^{\beta_pd_m}\tilde R_2^q & 0\\ * & * & * & * & -\tilde\varPhi_p\end{bmatrix},$$
$$\varXi_{22}=\tilde\varXi_{22}=\begin{bmatrix}-\gamma^2I & 0\\ 0 & -I\end{bmatrix},$$

$$\Xi_2=\begin{bmatrix} d_mX_pA_p^{\mathrm{T}} & (d_M-d_m)X_pA_p^{\mathrm{T}}\\ 0 & 0\\ d_mY_p^{\mathrm{T}}B_p^{\mathrm{T}} & (d_M-d_m)Y_p^{\mathrm{T}}B_p^{\mathrm{T}}\\ 0 & 0\\ -d_mY_p^{\mathrm{T}}B_p^{\mathrm{T}} & -(d_M-d_m)Y_p^{\mathrm{T}}B_p^{\mathrm{T}}\\ d_mE_p^{\mathrm{T}} & (d_M-d_m)E_p^{\mathrm{T}}\\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Xi}_2=\begin{bmatrix} d_mX_qA_p^{\mathrm{T}} & (d_M-d_m)X_qA_p^{\mathrm{T}}\\ 0 & 0\\ d_mY_q^{\mathrm{T}}B_p^{\mathrm{T}} & (d_M-d_m)Y_q^{\mathrm{T}}B_p^{\mathrm{T}}\\ 0 & 0\\ -d_mY_q^{\mathrm{T}}B_p^{\mathrm{T}} & -(d_M-d_m)Y_q^{\mathrm{T}}B_p^{\mathrm{T}}\\ d_mE_p^{\mathrm{T}} & (d_M-d_m)E_p^{\mathrm{T}}\\ 0 & 0 \end{bmatrix},$$

$$\Xi_3=\begin{bmatrix}\tilde{R}_1^p-2X_p & 0\\ * & \tilde{R}_1^p-2X_p\end{bmatrix},$$

$$\tilde{\Xi}_3=\begin{bmatrix}\tilde{R}_1^q-2X_q & 0\\ * & \tilde{R}_1^q-2X_q\end{bmatrix},$$

$$\Xi_{1111}=X_pA_p^{\mathrm{T}}+A_pX_p+\tilde{Q}_1^p+\alpha_pX_p+\mathrm{e}^{-\alpha_pd_m}\tilde{R}_1^p,$$

$$\Xi_{1112}=B_pY_p-\mathrm{e}^{-\alpha_pd_m}\tilde{R}_1^p,$$

$$\tilde{\Xi}_{1112}=B_pY_q-\tilde{R}_1^q,$$

$$\Xi_{1122}=-\mathrm{e}^{-\alpha_pd_m}\tilde{Q}_1^p+\mathrm{e}^{-\alpha_pd_m}\tilde{Q}_2^p+\mathrm{e}^{-\alpha_pd_m}\tilde{R}_1^p-\mathrm{e}^{-\alpha_pd_M}\tilde{R}_2^p,$$

$$\tilde{\Xi}_{1134}=\mathrm{e}^{\beta_pd_m}(\tilde{R}_2^q-\tilde{G}_q),$$

$$\Xi_{1123}=\mathrm{e}^{-\alpha_pd_M}(\tilde{R}_2^p-\tilde{G}_p),$$

$$\tilde{\Xi}_{1123}=\mathrm{e}^{\beta_pd_m}(\tilde{R}_2^q-\tilde{G}_q),$$

$$\Xi_{1133}=\delta_p\tilde{\Phi}_p+\mathrm{e}^{-\alpha_pd_M}[\tilde{G}_p-\tilde{R}_2^p]_s,$$

$$\Xi_{1134}=\mathrm{e}^{-\alpha_pd_M}(\tilde{R}_2^p-\tilde{G}_p),$$

$$\tilde{\Xi}_{1111}=X_qA_p^{\mathrm{T}}+A_pX_q+\tilde{Q}_1^q-\beta_pX_q+\tilde{R}_1^q,$$

$$\tilde{\Xi}_{1122}=-\mathrm{e}^{\beta_pd_m}\tilde{Q}_1^q+\mathrm{e}^{\beta_pd_m}\tilde{Q}_2^q+\tilde{R}_1^q-\mathrm{e}^{\beta_pd_m}\tilde{R}_2^q,$$

$$\tilde{\Xi}_{1133}=\delta_p\tilde{\Phi}_p+\mathrm{e}^{\beta_pd_m}[\tilde{G}_q-\tilde{R}_2^q]_s.$$

模态依赖的控制器参数可以由以下等式计算:

$$K_p=Y_pX_p.$$

证明　定义$X_p=P_p^{-1},Y_p=K_pX_p,\tilde{Q}_1^p=X_pQ_1^pX_p,\tilde{Q}_2^p=X_pQ_2^pX_p,\tilde{R}_1^p=X_pR_1^pX_p,\tilde{R}_2^p=X_pR_2^pX_p,\tilde{\Phi}_p=X_p\Phi_pX_p$. 对不等式(28)左右同乘$\mathrm{diag}\{X_p,X_p,X_p,X_p,X_p,I,I,I,I\}$,利用Shur补引理,可以得到不等式(34). 同理,可以看出(35)和(29)是等价的. 对不等式(13)～(15)左右同乘X_q,利用Shur补引理,得到(36)～(40)与(13)～(15)是等价的. 因此,从定理2可以知道,具有异步性的事件触发闭环系统是全局一致指数稳定的,且具有指定的H_∞干扰抑制性能$\tilde{\gamma}$. □

3　仿真算例

考虑脉宽调制驱动的升压-降压转换电路,参照文献[15]中的具体过程,利用归一化等方法,其可以建模为如下的切换系统:

$$\begin{bmatrix}\dfrac{\mathrm{d}V_o}{\mathrm{d}t}\\ \dfrac{\mathrm{d}i_L}{\mathrm{d}t}\end{bmatrix}=\begin{bmatrix}\dfrac{1}{RC} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}V_o\\ i_L\end{bmatrix}+\begin{bmatrix}\dfrac{i_L}{C}\\ -\dfrac{V_o}{L}\end{bmatrix}\bar{u}+\begin{bmatrix}0\\ \dfrac{V_{\mathrm{in}}}{L}\end{bmatrix},$$

其中$\bar{u}=1-u$. 在不同模式下,升压-降压转换电路的系统矩阵可以表示为

$$u=0:A_1=\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix},\ B_1=\begin{bmatrix}0\\ 1\end{bmatrix};\tag{42}$$

$$u=1:A_2=\begin{bmatrix}-1 & 0\\ 0 & 0\end{bmatrix},\ B_2=\begin{bmatrix}0\\ 1\end{bmatrix}.\tag{43}$$

定义状态变量$x(t)=[V_o,i_L]^{\mathrm{T}}$,控制输入$u(t)=V_{\mathrm{in}}$. 假设其他的矩阵参数为

$$E_1=[0.1;\ -0.1],\ E_2=[0.2;\ 0.1];$$
$$L_1=[-0.2\ \ 0.1],\ L_2=[0.1\ \ 0.4].$$

其余系统参数分别为$\alpha_1=1.2,\alpha_2=1.3,\beta_1=0.3,\beta_2=0.2,\mu_1=1.6,\mu_2=1.9,\theta=800,\lambda=1.5,\delta_1=0.5,\delta_2=0.3,\tau_{d1}=0.8,\tau_{d2}=0.8,H=0.8$. 外部扰动设为$\omega(t)=0.3/(1+t)$. 根据定理3,可得$\tau_{\alpha1}^*=1.3917,\tau_{\alpha2}^*=0.8399$. 选取$\tau_{\alpha1}=1.5,\tau_{\alpha2}=0.9$,设置初始状态为$[0.1,\ -0.1]$,根据定理3求得的控制器和事件驱动条件,可得系统状态的轨迹如图1所示,切换信号和控制器切换如图2所示,闭环切换系统对应的事件触发释放间隔如图3所示. 由图1和图2可以看出,基于事件触发的状态反馈控制器在异步的情况下也能保证切换系统的指数稳定.

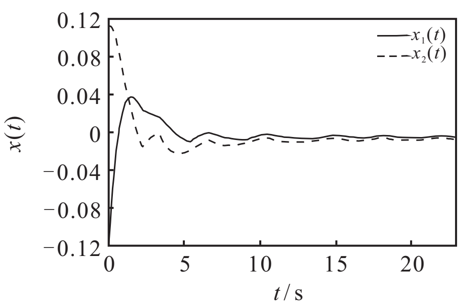

图1　系统状态的状态轨迹

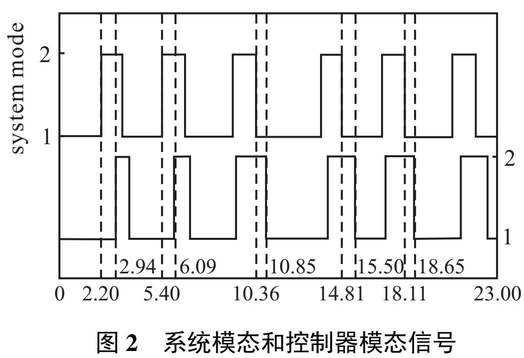

图2　系统模态和控制器模态信号

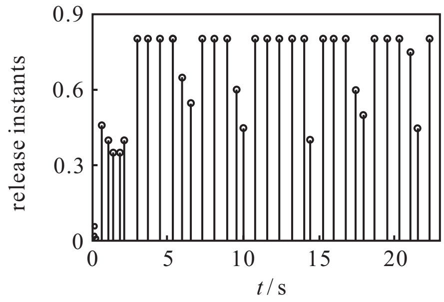

图 3 基于周期动态ETC机制(式(2))的事件触发过程

4 结 论

基于周期动态事件触发机制, 本文研究了MDADT下切换系统的H_∞控制问题. 通过构造控制器依赖的Lyapunov函数, 给出了闭环切换系统全局一致指数稳定的充分条件, 设计了周期动态事件触发参数和模态依赖控制器. 通过升压-降压变换器仿真实验, 验证了所得结果的优越性.

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