具有全状态约束和未建模动态的严格反馈系统有限时间自适应动态面控制

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具有全状态约束和未建模动态的严格反馈系统有限时间自适应动态面控制

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摘 要: 针对一类具有全状态约束、未建模动态和动态扰动的严格反馈非线性系统,通过构造非线性滤波器,并利用Young’s不等式,提出一种新的有限时间自适应动态面控制方法. 引入非线性映射处理全状态约束,将有约束系统变成无约束系统,利用径向基函数逼近未知光滑函数,利用辅助系统产生的动态信号处理未建模动态. 对于变换后的系统,利用改进的动态面控制和有限时间方法设计的控制器结构简单,移去现有有限时间控制中出现的“奇异性”问题,可加快系统的收敛速度. 理论分析表明,闭环系统中的所有信号在有限时间内有界,全状态不违背约束条件. 数值算例的仿真结果表明,所提出的自适应动态面控制方案是有效的.

关键词: 全状态约束；非线性映射；动态面控制；有限时间稳定；未建模动态

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Finite-time adaptive dynamic surface control for strict-feedback systems with full state constraints and unmodeled dynamics

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Abstract: By constructing nonlinear filters and using Young’s inequality, a new adaptive finite-time control method is proposed for a class of strict-feedback nonlinear systems with full state constraints, unmodeled dynamics and dynamic disturbances in this paper. The constrained system is transformed into an unconstrained system by introducing the nonlinear mapping. The radial basis function neural networks are utilized to approximate unknown nonlinear smooth functions. A dynamic signal produced by a auxiliary system is used to deal with unmodeled dynamics. Using the modified dynamic surface control technology and finite-time control method, a simple controller is developed. The singularity problem in the existing finite -time control is removed, and the converging speed of the system is accelerated. Theoretical analysis shows that all signals in the closed-loop system are bounded in finite time. Full state constraints are not triggered. Simulation results of numerical example show that the proposed approach is effective.

Keywords: full state constraints；nonlinear mapping；dynamic surface control；finite-time stability；unmodeled dynamics

0 引 言

随着现代工业过程日趋复杂,严重的不确定性和非线性常常影响控制系统的性能. 如何处理多种不确定性,消除他们对系统性能的影响是控制理论工作者亟待解决的理论问题. 自从Kanellakopoulos等[1]和Swaroop等[2]提出后推和动态面设计以来,两种设计方法被广泛运用到非线性系统的控制器设计中[3-6],相比于后推设计,动态面设计结构更简单,避免了后推设计中的过参数化问题,简化了控制器的设计.

未建模动态常存在于实际工业系统中. 系统在建模时产生的建模误差会对系统的稳定性产生负面影响. 文献[7]利用可测量的动态信号估计未建模动态,并给出未建模动态指数输入状态实用稳定的定义. 文献[8]结合动态面控制设计技术处理一类具有未建模动态和未知高频增益的输出反馈系统. 文献

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[9] 针对具有输出约束和未建模动态系统的自适应输出反馈系统,利用障碍李亚普诺夫函数并引入一阶辅助系统处理动态不确定项. 文献 [10] 针对一类具有动态不确定性的随机非线性系统,提出一种分散自适应模糊输出反馈的控制策略. 文献 [11] 针对具有未建模动态和状态时滞系统设计了稳定的自适应控制器. 文献 [12-14] 研究了一类具有未建模动态的纯反馈系统的控制问题. 文献 [7-14] 阐述了未建模动态对系统的影响,并通过引入一个动态信号处理未建模动态对系统的影响,但均未考虑到状态约束对整个系统的影响. 在实际系统的运行过程中,常常需要对系统进行一定的约束以保证系统的安全性和高效性,如果违背了约束条件,系统的性能和安全性则会受到严重影响. 因此,在控制器的设计过程中,对约束的考虑十分必要. 文献 [15-17] 针对具有状态约束的不同类型的非线性系统进行了研究,对本文的研究具有很大的启发作用.

近二十年来,有限时间控制受到了学者们的广泛关注,并取得了许多重要进展. 有限时间控制的关键特征是系统的状态在有限时间内达到平衡状态,此后并一直保持在平衡状态. 为了解决不连续控制器引起的颤振现象,文献 [18-19] 首次建立了关于有限时间稳定性的 Lyapunov 理论和齐次系统理论. 在此理论的基础上,研究了很多关于非线性系统的有限时间稳定性问题. 其中: 文献 [20-21] 均采用 Lyapunov-Krasovskii 泛函讨论了时变时滞系统的有限时间稳定性,但是未讨论关于涉及全状态约束的严格反馈系统. 文献 [22] 给出了非线性系统有限时间控制方法的研究发展. 文献 [23] 研究了一类不确定非线性系统的有限时间指令滤波自适应容错控制,利用补偿信号并结合动态面控制方法,解决了控制器的"奇异性"问题和控制器设计的"复杂性爆炸"问题,为解决有限时间控制问题带来了新的思路. 文献 [24] 研究了一类具有输出约束的 MIMO 非线性系统的定时控制. 文献 [25] 利用后推技术结合神经网络等证明了系统的稳定性,但后推技术造成了计算的复杂性且存在奇异性问题. 文献 [24-25] 均采用后推技术,每一步都需要对虚拟控制律进行反复求导,若系统阶数增大,则控制器的结构更加复杂,增加计算的复杂性. 文献 [26-27] 采用了动态面控制,避免了后推设计的"复杂性"问题. 文献 [26] 研究了一类 MIMO 非严格反馈系统的有限时间的自适应模糊动态面控制,但是仍存在"奇异性"问题. 文献 [27] 采用动态面的控制方法,并且提出了一种新的一阶滤波器的形式,但在虚拟控制器的设计中使用了符号函数,使得虚拟控制律不可导,可能导致算法失效. 文献 [28] 对控制增益及虚拟系数已知的一类严格反馈非线性系统,基于后推设计提出一种有限时间自适应控制策略.

本文研究了一类具有全状态约束、未建模动态和动态不确定性的严格反馈非线性系统有限时间控制问题,提出一种自适应控制策略. 主要贡献如下:

1) 对于具有未建模动态和全状态约束的严格反馈非线性系统,通过可逆非线性映射将有约束的非线性系统转化为无约束的纯反馈非线性系统,利用一阶辅助系统产生动态信号处理未建模动态,设计一阶非线性滤波器代替传统动态面控制设计中的 1 阶线性低通滤波器,并利用 Young's 不等式,提出一种新的有限时间自适应动面控制策略. 与文献 [27-28] 相比,考虑了未建模动态和全状态约束对控制系统性能的影响,无需假设虚拟控制增益函数已知及虚拟控制的导数有界性[27],对闭环系统给出了严格的稳定性证明.

2) 所设计的虚拟控制连续可导, 从而避免了文献 [25] 设计的虚拟控制导数可能发生奇异性以及神经网络逼近函数的不连续性情况,同时稳定性分析中无需假设逼近误差有界. 此外,所提出控制方案避免了文献 [27-28] 所设计的虚拟控制包含符号函数,保证了虚拟控制的可导性.

3) 为了实现有限时间控制,在参数自适应调节律中增加了额外 σ-修正项. 通过引入输入不确定项消除了未知控制增益函数给控制器的设计带来的困难,同时避免了采用积分型李亚普诺夫函数处理未知增益对控制器设计带来的复杂推导[16].

1　问题描述与预备知识

1.1　基本假设与预备知识

考虑如下一类具有全状态约束和未建模动态的严格反馈系统:

$$\begin{cases}\dot{\xi}=q(\xi,x,t);\\ \dot{x}_i=f_i(\bar{x}_i)+g_i(\bar{x}_i)x_{i+1}+d_i(\xi,x,t),\\ \qquad i=1,2,\dots,n-1;\\ \dot{x}_n=f_n(\bar{x}_n)+g_n(\bar{x}_n)u+d_n(\xi,x,t);\\ y=x_1.\end{cases}\tag{1}$$

其中: $\bar{x}_i=[x_1,\dots,x_i]^{\mathrm{T}}\in R^i(i=1,2,\dots,n)$ 为系统的状态向量,所有的状态满足约束条件 $x_i\in\Omega_{x_i}=\{x_i:-k_{b_{i1}}<x_i<k_{b_{i2}}\}$, $k_{b_{i1}}$、$k_{b_{i2}}$ 为正的设计常数; $\xi\in R^{n_0}$ 为未建模动态; $y\in R$ 为系统的输出; $u\in R$ 为系统的输入; $f_i(\bar{x}_i)$ 为未知的光滑非线性函数; $g_i(\bar{x}_i)$ 为未知控制增益; $d_i(\xi,x,t)$ 为未知不确定扰动

项.

控制目标是设计一个控制器u使得输出y能够跟踪期望轨迹y_d,保证所有的状态满足$x_i \in \Omega_{x_i} = \{x_i : -k_{b_{i1}} < x_i < k_{b_{i2}}\}$,并通过调节设计参数使得闭环系统的所有信号在有限时间内有界,跟踪误差收敛到一个小的残差内.

定义1 [7] 对于系统$\dot{\xi} = q(\xi, x, t)$,如果存在K_∞类函数$\bar{\alpha}_1$、$\bar{\alpha}_2$和Lyapunov函数$V_0(\xi)$,使得

$$\bar{\alpha}_1(\|\xi\|) \leqslant V_0(\xi) \leqslant \bar{\alpha}_2(\|\xi\|), \tag{2}$$

以及存在两个已知常数$c > 0, d \geqslant 0$和一个K_∞类函数$\varLambda(\cdot)$,使得

$$\frac{\mathrm{d}V_0(\xi)}{\mathrm{d}\xi} q(\xi, x, t) \leqslant -cV_0(\xi) + \varLambda(|x_1|) + d, \tag{3}$$

则称未建模动态是指数输入状态实用稳定的(exponentially input-state-practically stable, exp-ISpS).

假设1 [4,16] 期望轨迹向量$x_d = [y_d, \dot{y}_d, \ddot{y}_d]^{\mathrm{T}} \in \Omega_d$是可量测的,其中$\Omega_d = \{x_d : y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leqslant B_0\}$,且$|y_d| \leqslant B_1 < \min\{k_{b_{11}}, k_{b_{12}}\}$,$B_0$、$B_1$为已知正常数.

假设2 [7] 对于未知扰动$d_i(\xi, x, t), i = 1, 2, \ldots, n$,存在未知非负连续函数$\varDelta_{i1}(\cdot)$、未知非负连续单调递增函数$\varDelta_{i2}(\cdot)$,使得

$$|d_i(\xi, x, t)| \leqslant \varDelta_{i1}(\|\bar{x}_i\|) + \varDelta_{i2}(\|\xi\|), \tag{4}$$

其中$\|\cdot\|$为欧氏范数.

假设3 [16] 未知控制增益$g_n(\bar{x}_n)$符号已知,且存在未知正常数$g_{i0} (i = 1, 2, \ldots, n)$,满足

$$0 < g_{i0} \leqslant |g_i(\bar{x}_i)|, \ \forall \bar{x}_i \in R^i. \tag{5}$$

不失一般性,假设$g_n(\bar{x}_n) > 0, \forall \bar{x}_n \in R^n$.

假设4 [7] 对于系统$\dot{\xi} = q(\xi, x, t)$,未建模动态ξ是指数输入状态实用稳定的.

引理1 [7] $V_0(\xi)$是系统$\dot{\xi} = q(\xi, x, t)$的一个指数输入状态实用稳定Lyapunov函数,对于任意常数$\bar{c}_f \in (0, c)$,任意初始时间$t_0 > 0$,任意初始状态$\xi_0 = \xi(t_0)$,$r_0 > 0$和任意K_∞类函数$\bar{\varLambda}|x_1|) \geqslant \varLambda(|x_1|)$,存在有限时间$T_0 = \max\{0, \ln[V_0(\xi_0)/r_0]/(c - \bar{c}_f)\} \geqslant 0$,非负函数$D(t_0, t)$,设计动态信号$r$如下:

$$\dot{r} = -\bar{c}_f r + \bar{\varLambda}(|x_1|) + d. \tag{6}$$

当$t \geqslant t_0 + T_0$时,$D(t_0, t) = 0$且$V_0(\xi) \leqslant r(t) + D(t_0, t)$. 不失一般性,$\bar{\varLambda}(|x_1|) = \varLambda(|x_1|)$.

引理2 [22,24] 考虑非线性系统$\dot{x} = f(x)$,如果存在一个光滑正定函数$V(x)$和常数$\alpha > 0, \beta > 0, q > 1, 0 < p < 1, 0 < C < \infty$,满足如下情况:

$$\dot{V}(x) \leqslant -\alpha V^p(x) - \beta V^q(x) + C, \tag{7}$$

则非线性系统$\dot{x} = f(x)$实用有限时间稳定.

引理3 [26] 对于任意$\eta_1, \eta_2, \ldots, \eta_N \in R, 0 < s \leqslant 1$,有

$$\Big[\sum_{i=1}^{N} |\eta_i|\Big]^s \leqslant \sum_{i=1}^{N} |\eta_i|^s \leqslant N^{1-s} \Big[\sum_{i=1}^{N} |\eta_i|\Big]^s. \tag{8}$$

引理4 [29](Young's不等式) 对于任意$x, y \in R$, $\varepsilon > 0, p > 1, q > 1$,且$1/p + 1/q = 1$,下述不等式成立:

$$xy \leqslant \frac{\varepsilon^p}{p} |x|^p + \frac{1}{q\varepsilon^q} |y|^q. \tag{9}$$

特例,当$p = 2, q = 2, \varepsilon = 1$时,有$xy \leqslant x^2/2 + y^2/2$;当$p = 2, q = 2, \varepsilon = 1/\sqrt{2}$时,有$xy \leqslant x^2/4 + y^2$.

1.2 神经网络

定义紧集$\varPi_{Z_i} = \{Z_i | \|Z_i\| \leqslant M_{Z_i}\} \subset R^{L_i}$,其中$M_{Z_i} > 0$为设计常数. 若利用径向基函数神经网络$\theta_i^{*\mathrm{T}} \phi_i(Z_i)$在紧集$\varPi_{Z_i}$上对未知连续函数$\varPhi_i(Z_i)$进行逼近,则有

$$\varPhi_i(Z_i) = \theta_i^{*\mathrm{T}} \phi_i(Z_i) + \varepsilon_i(Z_i). \tag{10}$$

其中: $Z_i = [\bar{s}_{i+1}^{\mathrm{T}}, z_i, y_i, r]^{\mathrm{T}} \in R^{L_i}$, $L_i = i + 4$, $i = 2, 3, \ldots, n-1$, $Z_1 = [\bar{s}_2^{\mathrm{T}}, z_2, \dot{y}_d, r]^{\mathrm{T}}$, $Z_n = [\bar{s}_n^{\mathrm{T}}, z_n, y_n, r]^{\mathrm{T}}$;基向量$\phi_i(Z_i) = [\varphi_{i,1}(Z_i), \ldots, \varphi_{i,l_i}(Z_i)]^{\mathrm{T}} \in R^{l_i}$,基函数$\varphi_{i,j}(Z_i)$选择为如下高斯函数:

$$\varphi_{i,j}(Z_i) = \mathrm{e}^{-\frac{(Z_i - \mu_{ij})^{\mathrm{T}} (Z_i - \mu_{ij})}{\phi_{ij}^2}}, \tag{11}$$

μ_{ij}和ϕ_{ij}分别为高斯函数的中心和宽度,l_i为神经网络的节点数,$i = 1, 2, \ldots, n, j = 1, 2, \ldots, l_i$,理想权向量$\theta^*$定义为

$$\theta_i^* = \arg \min_{\theta_i \in R^{l_i}} \big[\sup_{Z_i \in \Omega_{Z_i}} |\theta_i^{\mathrm{T}} \phi_i(Z_i) - \varPhi_i(Z_i)|\big]. \tag{12}$$

1.3 全状态约束

为了处理全状态约束,定义如下非线性映射:

$$s_i = \ln \frac{k_{b_{i1}} + x_i}{k_{b_{i2}} - x_i}, \ i = 1, 2, \ldots, n, \tag{13}$$

其中$k_{b_{i1}}, k_{b_{i2}} > 0$为两个已知的设计常数. 因此,式(13)的逆映射为

$$x_i = k_{b_{i2}} - \frac{k_{b_{i1}} + k_{b_{i2}}}{\mathrm{e}^{s_i} + 1}, \ i = 1, 2, \ldots, n. \tag{14}$$

有

$$\dot{s}_i = \frac{\mathrm{e}^{s_i} + \mathrm{e}^{-s_i} + 2}{k_{b_{i1}} + k_{b_{i2}}} \dot{x}_i, \tag{15}$$

原系统(1)可改写为

$$\begin{cases} \dot{\xi} = q(t, \xi, x), \\ \dot{s}_i = F_i(\bar{s}_{i+1}) + s_{i+1} + D_i(t, \xi, \bar{s}_n), \\ \dot{s}_n = F_n(\bar{s}_n) + G_n(\bar{s}_n) u + D_n(t, \xi, \bar{s}_n). \end{cases} \tag{16}$$

其中

$\bar{s}_i = [s_1, s_2, \ldots, s_i]^{\mathrm{T}}, \ i = 1, 2, \ldots, n,$

$$K_i(s_i)=\frac{\mathrm{e}^{s_i}+\mathrm{e}^{-s_i}+2}{k_{b_{i1}}+k_{b_{i2}}}, \tag{17}$$

$$F_i(\bar{s}_{i+1})=K_i(s_i)[f_i(\bar{x}_i)+g_i(\bar{x}_i)x_{i+1}]-s_{i+1}, \tag{18}$$

$$F_n(\bar{s}_n)=K_n(s_n)f_n(\bar{x}_n),$$

$$G_n(\bar{s}_n)=K_n(s_n)g_n(\bar{x}_n), \tag{19}$$

$$D_i(t,\xi,\bar{s}_n)=K_i(s_i)d_i(\xi,x,t),\ i=1,2,\dots,n. \tag{20}$$

注1　根据式(20)和假设2,可知

$$|D_i(t,\xi,\bar{s}_n)|\leqslant K_i(s_i)[\varDelta_{i1}(\|\bar{x}_i\|)+\varDelta_{i2}(\|\xi\|)]. \tag{21}$$

2　控制器设计

为了设计自适应控制器,进行如下坐标变换:

$$\begin{cases}z_1=s_1-\hat{y}_d,\\ z_i=s_i-\omega_i,\ i=2,3,\dots,n.\end{cases} \tag{22}$$

其中: $\hat{y}_d=\ln\dfrac{k_{b_{11}}+y_d}{k_{b_{12}}-y_d}$, ω_i为以$\hbar_{i-1}$为输入的一阶滤波器的输出, $i=2,3,\dots,n$.

为了便于设计,定义一些符号如下:

$$\mathrm{sig}(\cdot)^{\vartheta}=|\cdot|^{\vartheta}\mathrm{sgn}(\cdot);$$

$$\lambda_i=\|\theta_i^*\|^2,\ \tilde{\lambda}_i=\hat{\lambda}_i-\lambda_i,\ 1\leqslant i\leqslant n;$$

$$\varOmega_{y_j}=\{y_j\in R|\ |y_j|<1\};$$

$$\varOmega_{y_j}^0=\{y_j\in R|\ |y_j|\geqslant 1\},\ 2\leqslant j\leqslant n;$$

$$\bar{z}_i=[z_1,z_2,\dots,z_i]^{\mathrm{T}},\ \bar{y}_j=[y_2,y_3,\dots,y_j]^{\mathrm{T}};$$

$$\bar{\hat{\lambda}}_i=[\hat{\lambda}_1,\hat{\lambda}_2,\dots,\hat{\lambda}_i]^{\mathrm{T}},\ 1\leqslant i\leqslant n,\ 2\leqslant j\leqslant n.$$

其中: $\hat{\lambda}_i$为λ_i在时间t上的估计, $y_j=\omega_j-\hbar_{j-1}$.

step 1: 定义第1个动态面$z_1=s_1-\hat{y}_d$, z_1关于时间t的导数为

$$\dot{z}_1=F_1(\bar{s}_2)+y_2+z_2+\hbar_1-\dot{\hat{y}}_d+D_1(t,\xi,\bar{s}_n). \tag{23}$$

取李雅普诺夫函数$V_1=z_1^2/2+\tilde{\lambda}_1^2/2$,对$V_1$关于$t$求导,得

$$\dot{V}_1=z_1[F_1(\bar{s}_2)+y_2+z_2+\hbar_1-\dot{\hat{y}}_d+D_1(t,\xi,\bar{s}_n)]+\tilde{\lambda}_1\dot{\hat{\lambda}}_1. \tag{24}$$

由定义1、假设2、引理3和引理4,可得

$$z_1D_1(t,\xi,\bar{s}_n)\leqslant z_1K_1(s_1)[\varDelta_{11}(|x_1|)+\varDelta_{12}(\|\xi\|)]\leqslant z_1^2K_1^2(s_1)[\varDelta_{11}(|x_1|)+\varDelta_{12}\circ\bar{\alpha}_1(r+D_0)]^2+\frac{1}{4}, \tag{25}$$

$$z_1z_2\leqslant\frac{1}{4}z_1^2+z_2^2, \tag{26}$$

$$z_1y_2\leqslant\frac{1}{4}z_1^2+y_2^2. \tag{27}$$

式(24)可化为

$$\dot{V}_1\leqslant z_1[F_1(\bar{s}_2)+\hbar_1-\dot{\hat{y}}_d+z_1K_1^2(s_1)[\varDelta_{11}(|x_1|)+\varDelta_{12}\circ\bar{\alpha}_1(r+D_0)]^2]+\tilde{\lambda}_1\dot{\hat{\lambda}}_1+\frac{1}{4}+\frac{1}{2}z_1^2+y_2^2+z_2^2. \tag{28}$$

令

$$\varPhi_1(Z_1)=F_1(\bar{s}_2)+\kappa_1|z_1|^{\frac{1}{2}}+\kappa_1z_1^3-\dot{\hat{y}}_d+z_1K_1^2(s_1)\times[\varDelta_{11}(|x_1|)+\varDelta_{12}\circ\bar{\alpha}_1(r+D_0)]^2-\kappa_1z_1,$$

其中$Z_1=[z_1,\bar{s}_2^{\mathrm{T}},\dot{\hat{y}}_d,r]^{\mathrm{T}}$, $\kappa_1>0$是一个设计常数. 根据式(28)得

$$\dot{V}_1\leqslant z_1[\theta_1^{*\mathrm{T}}\phi_1(Z_1)+\hbar_1-\kappa_1|z_1|^{\frac{1}{2}}-\kappa_1z_1^3+\kappa_1z_1]+z_1\varepsilon_1+\tilde{\lambda}_1\dot{\hat{\lambda}}_1+\frac{1}{4}+\frac{1}{2}z_1^2+y_2^2+z_2^2. \tag{29}$$

由Young's不等式得

$$z_1\theta_1^{*\mathrm{T}}\phi_1(Z_1)\leqslant\frac{\|\phi_1(Z_1)\|^2z_1^2\lambda_1}{2a_1^2}+\frac{a_1^2}{2}, \tag{30}$$

其中$a_1>0$为一个设计常数. 于是,式(29)可转化为

$$\dot{V}_1\leqslant z_1\Big[\frac{\|\phi_1(Z_1)\|^2z_1\lambda_1}{2a_1^2}+\hbar_1-\kappa_1|z_1|^{\frac{1}{2}}-\kappa_1{z_1}^3+\kappa_1z_1\Big]+\tilde{\lambda}_1\dot{\hat{\lambda}}_1+\frac{1}{4}+\frac{1}{2}z_1^2+y_2^2+z_2^2+\frac{a_1^2}{2}+z_1\varepsilon_1. \tag{31}$$

设计虚拟控制器

$$\hbar_1=-(\kappa_1+1)z_1-\frac{\|\phi_1(Z_1)\|^2z_1\hat{\lambda}_1}{2a_1^2}. \tag{32}$$

设计自适应律

$$\dot{\hat{\lambda}}_1=\frac{\|\phi_1(Z_1)\|^2z_1^2}{2a_1^2}-\sigma_{11}\hat{\lambda}_1-\sigma_{12}\hat{\lambda}_1^3, \tag{33}$$

其中$\sigma_{11}>0$, $\sigma_{12}>0$为设计常数. 将式(32)和(33)代入(31),得到

$$\dot{V}_1\leqslant-\kappa_1|z_1|^{\frac{3}{2}}-\kappa_1{z_1}^4+y_2^2+z_2^2-\sigma_{11}\tilde{\lambda}_1\hat{\lambda}_1-\sigma_{12}\tilde{\lambda}_1\hat{\lambda}_1^3+\frac{a_1^2}{2}+\frac{1}{4}-\frac{1}{2}z_1^2+z_1\varepsilon_1. \tag{34}$$

存在一个非负连续函数$\delta_1(\bar{z}_2,\hat{\lambda}_1,y_d,\dot{y}_d,y_2,r)$,使得

$$|\varepsilon_1(Z_1)|\leqslant\delta_1(\bar{z}_2,\hat{\lambda}_1,y_d,\dot{y}_d,y_2,r).$$

且有

$$z_1\varepsilon_1\leqslant\frac{1}{2}z_1^2+\frac{1}{2}\delta_1^2, \tag{35}$$

$$-\sigma_{12}\tilde{\lambda}_1\hat{\lambda}_1^3\leqslant-\sigma_{12}\Big(\frac{1}{2}\tilde{\lambda}_1^2\Big)^2+\frac{61\sigma_{12}}{3}\lambda_1^4, \tag{36}$$

$$-\sigma_{11}\tilde{\lambda}_1\hat{\lambda}_1 \leqslant -\gamma_1(\tilde{\lambda}_1^2)^{\frac{3}{4}} + \frac{\gamma_1^4}{\sigma_{11}^3} + \frac{\sigma_{11}}{2}\lambda_1^2, \quad (37)$$

其中$\gamma_1 > 0$为一个设计常数. 将式(35)～(37)代入(34),得到

$$\dot{V}_1 \leqslant -\kappa_1|z_1|^{\frac{3}{2}} - \kappa_1 z_1^4 + y_2^2 + z_2^2 - \gamma_1(\tilde{\lambda}_1^2)^{\frac{3}{4}} + \frac{\sigma_{11}}{2}\lambda_1^2 + \frac{\gamma_1^4}{\sigma_{11}^3} - \sigma_{12}\Big(\frac{1}{2}\tilde{\lambda}_1^2\Big)^2 + \frac{61\sigma_{12}}{3}\lambda_1^4 + \frac{a_1^2}{2} + \frac{1}{2}\delta_1^2 + \frac{1}{4}. \quad (38)$$

对于$i=2$,设计一阶滤波器

$$\tau_i\dot{\omega}_i = \mathrm{sig}(\hbar_{i-1}-\omega_i)^{\frac{1}{2}} + \mathrm{sig}(\hbar_{i-1}-\omega_i)^3,$$

$$-\dot{\omega}_i = \frac{1}{\tau_i}|y_i|^{\frac{1}{2}}\mathrm{sgn}(y_i) + \frac{1}{\tau_i}|y_i|^3\mathrm{sgn}(y_i) \leqslant \frac{1}{\tau_i^2}|y_i| + \frac{1}{\tau_i^2}y_i^6 + \frac{1}{2}, \quad (39)$$

其中$\tau_i > 0$为一个设计常数.

step i $(2 \leqslant i \leqslant n-1)$: 定义第$i$个动态面为$z_i = s_i - \omega_i$. z_i 关于时间 t 的导数为

$$\dot{z}_i = F_i(\bar{s}_{i+1}) + y_{i+1} + z_{i+1} + \hbar_i - \dot{\omega}_i + D_i(t,\xi,\bar{s}_n). \quad (40)$$

取李雅普诺夫函数

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\lambda}_i^2 + \frac{1}{2}y_i^2.$$

将V_i对t求导,得到

$$\dot{V}_i = \dot{V}_{i-1} + z_i[F_i(\bar{s}_{i+1}) + y_{i+1} + z_{i+1} + \hbar_i - \dot{\omega}_i + D_i(t,\xi,\bar{s}_n)] + \tilde{\lambda}_i\dot{\tilde{\lambda}}_i + y_i\dot{y}_i. \quad (41)$$

将式(39)代入(41),得到

$$\dot{V}_i \leqslant \dot{V}_{i-1} + z_i\Big[F_i(\bar{s}_{i+1}) + y_{i+1} + z_{i+1} + \hbar_i + \frac{1}{\tau_i^2}|y_i| + \frac{1}{\tau_i^2}y_i^6 + \frac{1}{2} + D_i(t,\xi,\bar{s}_n)\Big] + \tilde{\lambda}_i\dot{\tilde{\lambda}}_i + y_i\dot{y}_i. \quad (42)$$

由定义1、假设2、引理1和引理4,可得

$$z_iD_i(t,\xi,\bar{s}_n) \leqslant z_iK_i(s_i)[\varDelta_{i1}(\|\bar{x}_i\|) + \varDelta_{i2}(\|\xi\|)] \leqslant z_i^2K_i^2(s_i)[\varDelta_{i1}(\|\bar{x}_i\|) + \varDelta_{i2}\circ\bar{\alpha}_1(r+D_0)]^2 + \frac{1}{4}, \quad (43)$$

$$z_iz_{i+1} \leqslant \frac{1}{4}z_i^2 + z_{i+1}^2, \quad (44)$$

$$z_iy_{i+1} \leqslant \frac{1}{4}z_i^2 + y_{i+1}^2. \quad (45)$$

式(42)可转化为

$$\dot{V}_i \leqslant \sum_{j=1}^{i-1}\Big[-\kappa_j|z_j|^{\frac{3}{2}} - \kappa_jz_j^4 - \gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}} - \sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big] + \sum_{j=1}^{i-1}\Big[\frac{1}{2}\delta_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{\gamma_j^4}{\sigma_{j1}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4\Big] + \sum_{j=2}^{i-1}\Big[-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^{\frac{3}{4}} - \Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^2 + \frac{1}{4}\eta_j^2\Big] + z_i\Big(F_i(\bar{s}_{i+1}) + \hbar_i + \frac{1}{\tau_i^2}|y_i| + \frac{1}{\tau_i^2}y_i^6 + \frac{1}{2} + z_iK_i^2(s_i)[\varDelta_{i1}(\|\bar{x}_i\|) + \varDelta_{i2}\circ\bar{\alpha}_1(r+D_0)]^2\Big) + \frac{3}{2}z_i^2 + y_i^2 + \frac{i}{4} + \tilde{\lambda}_i\dot{\tilde{\lambda}}_i + y_i\dot{y}_i + z_{i+1}^2 + y_{i+1}^2, \quad (46)$$

其中$\kappa_j, \sigma_{j1}, \sigma_{j2}, \gamma_j > 0$为设计常数. 令

$$\varPhi_i(Z_i) = F_i(\bar{s}_{i+1}) + \frac{1}{\tau_i^2}|y_i| + \frac{1}{\tau_i^2}y_i^6 + \frac{1}{2} + \kappa_i|z_i|^{\frac{1}{2}} + \kappa_iz_i^3 + z_iK_i^2(s_i)[\varDelta_{i1}(\|\bar{x}_i\|) + \varDelta_{i2}\circ\bar{\alpha}_1(r+D_0)]^2 - \kappa_iz_i,$$

其中: $Z_i = [\bar{s}_{i+1}^{\mathrm{T}}, z_i, y_i, r]^{\mathrm{T}}, \kappa_i > 0$为一个设计常数. 且有

$$\dot{V}_i \leqslant -\sum_{j=1}^{i-1}\Big(\kappa_j|z_j|^{\frac{3}{2}} + \kappa_jz_j^4 + \gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}} + \sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big) + \sum_{j=1}^{i-1}\Big(\frac{1}{2}\delta_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{\gamma_j^4}{\sigma_{j1}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4\Big) + \sum_{j=2}^{i-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^{\frac{3}{4}} - \Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^2 + \frac{1}{4}\eta_j^2\Big) + \frac{3}{2}z_i^2 + y_i^2 + \frac{i}{4} + z_i(\theta_i^{*\mathrm{T}}\phi_i(Z_i) + \varepsilon_i + \hbar_i - k_i|z_i|^{\frac{1}{2}} - k_i{z_i}^3 + \kappa_iz_i) + \tilde{\lambda}_i\dot{\tilde{\lambda}}_i + y_i\dot{y}_i + z_{i+1}^2 + y_{i+1}^2. \quad (47)$$

由引理4得

$$z_i\theta_i^{*\mathrm{T}}\phi_i(Z_i) \leqslant \frac{\|\phi_i(Z_i)\|^2z_i^2\lambda_i}{2a_i^2} + \frac{a_i^2}{2}, \quad (48)$$

其中$a_i > 0$为一个设计常数. 将式(48)代入(47),得到

$$\dot{V}_i \leqslant \sum_{j=1}^{i-1}\Big[-\kappa_j|z_j|^{\frac{3}{2}} - \kappa_jz_j^4 - \gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}} - \sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big] + \sum_{j=1}^{i-1}\Big(\frac{1}{2}\delta_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{\gamma_j^4}{\sigma_{j1}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4\Big) + \sum_{j=2}^{i-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^{\frac{3}{4}} - \Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^2 + \frac{1}{4}\eta_j^2\Big) + \frac{i}{4} + \frac{3}{2}z_i^2 + y_i^2 + y_{i+1}^2 + \frac{a_i^2}{2} +$$

$$z_i\Big[\frac{\|\phi_i(Z_i)\|^2 z_i\lambda_i}{2a_i^2}+\hbar_i-\kappa_i|z_i|^{\frac{1}{2}}-\kappa_i {z_i}^3+\kappa_i z_i\Big]+$$
$$\tilde{\lambda}_i\dot{\hat{\lambda}}_i+y_i\dot{y}_i+z_{i+1}^2+z_i\varepsilon_i. \tag{49}$$

设计虚拟控制律

$$\hbar_i=-(\kappa_i+2)z_i-\frac{\|\phi_i(Z_i)\|^2 z_i\hat{\lambda}_i}{2a_i^2}. \tag{50}$$

设计自适应律

$$\dot{\hat{\lambda}}_i=\frac{\|\phi_i(Z_i)\|^2 z_i^2}{2a_i^2}-\sigma_{i1}\hat{\lambda}_i-\sigma_{i2}\hat{\lambda}_i^3, \tag{51}$$

其中$\sigma_{i1}>0,\sigma_{i2}>0$为两个设计常数. 将式(50)和(51)代入(49),得到

$$\dot{V}_i\leqslant$$
$$\sum_{j=1}^{i}\Big[-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_j z_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big]+$$
$$\sum_{j=1}^{i}\Big(\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big)+\frac{i}{4}+$$
$$\sum_{j=2}^{i-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^{\frac{3}{4}}-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^2+\frac{1}{4}\eta_j^2\Big)+$$
$$y_i^2+y_i\dot{y}_i+z_{i+1}^2+y_{i+1}^2-\frac{1}{2}z_i^2+z_i\varepsilon_i+\sum_{j=1}^{i-1}\frac{1}{2}\delta_j^2. \tag{52}$$

存在一个非负连续函数 $\delta_i(\bar{z}_{i+1},\bar{y}_{i+1},\bar{\hat{\lambda}}_i,y_d,\dot{y}_d,r)$,使得$|\varepsilon_i(Z_i)|\leqslant\delta_i(\bar{z}_{i+1},\bar{y}_{i+1},\bar{\hat{\lambda}}_i,y_d,\dot{y}_d,r)$. 且有

$$z_i\varepsilon_i\leqslant\frac{1}{2}z_i^2+\frac{1}{2}\delta_i^2. \tag{53}$$

将式(53)代入(52),得到

$$\dot{V}_i\leqslant$$
$$\sum_{j=1}^{i}\Big(-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_j z_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big)+$$
$$\sum_{j=1}^{i}\Big(\frac{1}{2}\delta_j^2+\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big)+\frac{i}{4}+$$
$$\sum_{j=2}^{i-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^{\frac{3}{4}}-\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^2+\frac{1}{4}\eta_j^2\Big)+$$
$$y_i^2+y_i\dot{y}_i+z_{i+1}^2+y_{i+1}^2. \tag{54}$$

引入一阶滤波器

$$\dot{\omega}_i=\frac{1}{\tau_i}\mathrm{sig}(\omega_i-\hbar_{i-1})^{\frac{1}{2}}+\frac{1}{\tau_i}\mathrm{sig}(\omega_i-\hbar_{i-1})^3=$$
$$-\frac{1}{\tau_i}\mathrm{sig}(y_i)^{\frac{1}{2}}-\frac{1}{\tau_i}\mathrm{sig}(y_i)^3. \tag{55}$$

其中: $\hbar_{i-1}$和ω_i分别为一阶滤波器的输入和输出, τ_i为正的设计常数. 存在一个非负连续函数$\eta_i(\bar{s}_{i+1},\bar{z}_i,\bar{y}_i,\bar{\hat{\lambda}}_i,y_d,\dot{y}_d,\ddot{y}_d,r)$,使得

$$|-\dot{\hbar}_{i-1}|\leqslant\eta_i(\bar{s}_{i+1},\bar{z}_i,\bar{y}_i,\bar{\hat{\lambda}}_i,y_d,\dot{y}_d,\ddot{y}_d,r), \tag{56}$$

$$y_i\dot{y}_i=y_i(\dot{\omega}_i-\dot{\hbar}_{i-1})=$$
$$-\frac{1}{\tau_i}y_i^{\frac{3}{2}}-\frac{1}{\tau_i}y_i^4-y_i\dot{\hbar}_{i-1}\leqslant$$
$$-\frac{1}{\tau_i}|y_i|^{\frac{3}{2}}-\frac{1}{\tau_i}y_i^4+y_i^2+\frac{1}{4}\eta_i^2. \tag{57}$$

将式(57)代入(54),得到

$$\dot{V}_i\leqslant$$
$$\sum_{j=1}^{i}\Big(-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_j z_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big)+$$
$$\sum_{j=1}^{i}\Big(\frac{1}{2}\delta_j^2+\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big)+$$
$$\frac{i}{4}+\sum_{j=2}^{i-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}}-\Big(\frac{1}{\tau_j}-2\Big)y_j^4+\frac{1}{4}\eta_j^2\Big)+$$
$$z_{i+1}^2+y_{i+1}^2+2y_i^2-\frac{1}{\tau_i}|y_i|^{\frac{3}{2}}-\frac{1}{\tau_j}y_i^4+\frac{1}{4}\eta_i^2. \tag{58}$$

1) $y_i\in\Omega_{y_i}^0, 2y_i^2\leqslant 2(y_i^2)^2+2(y_i^2)^{\frac{3}{4}}$,由式(58)得

$$\dot{V}_i\leqslant$$
$$\sum_{j=1}^{i}\Big[-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_j z_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big]+$$
$$\sum_{j=1}^{i}\Big[\frac{1}{2}\delta_j^2+\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big]+$$
$$\frac{i}{4}+z_{i+1}^2+y_{i+1}^2+$$
$$\sum_{j=2}^{i}\Big(-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}}-\Big(\frac{1}{\tau_j}-2\Big)y_j^4+\frac{1}{4}\eta_j^2\Big). \tag{59}$$

2) $y_i\in\Omega_{y_i}, 2y_i^2\leqslant 2(y_i^2)^{\frac{3}{4}}+2(y_i^2)^2$,由式(58)得

$$\dot{V}_i\leqslant$$
$$\sum_{j=1}^{i}\Big[-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_j {z_j}^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big]+$$
$$\sum_{j=1}^{i}\Big(\frac{1}{2}\delta_j^2+\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}{\lambda_j}^4\Big)+$$
$$\frac{i}{4}+z_{i+1}^2+y_{i+1}^2+$$
$$\sum_{j=2}^{i}\Big(-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}}-\Big(\frac{1}{\tau_j}-2\Big)y_j^4+\frac{1}{4}\eta_j^2\Big). \tag{60}$$

综合上述两种情况,可知

$$\dot{V}_i\leqslant$$
$$\sum_{j=1}^{i}\Big[-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_j z_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big]+$$
$$\sum_{j=1}^{i}\Big(\frac{1}{2}\delta_j^2+\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big)+$$
$$\frac{i}{4}+z_{i+1}^2+y_{i+1}^2+$$

$$\sum_{j=2}^{i}\Big(-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}}-\Big(\frac{1}{\tau_j}-2\Big)y_j^4+\frac{1}{4}\eta_j^2\Big). \quad (61)$$

设计一阶滤波器

$$\tau_n\dot{\omega}_n=\mathrm{sig}(\hbar_{n-1}-\omega_n)^{\frac{1}{2}}+\mathrm{sig}(\hbar_{n-1}-\omega_n)^3,$$
$$-\dot{\omega}_n=\frac{1}{\tau_n}|y_n|^{\frac{1}{2}}\mathrm{sgn}(y_n)+\frac{1}{\tau_n}|y_n|^3\mathrm{sgn}(y_n)\leqslant \frac{1}{\tau_n^2}|y_n|+\frac{1}{\tau_n^2}y_n^6+\frac{1}{2}, \quad (62)$$

其中τ_n为一个正的设计常数.

step n: 定义第n个动态面为$z_n=s_n-\omega_n$. z_n 关于时间t的导数为

$$\dot{z}_n=F_n(\bar{s}_n)+(G_n(\bar{s}_n)-1)u+u-\dot{\omega}_n+D_n(t,\xi,\bar{s}_n). \quad (63)$$

令

$$\varDelta(u)=(G_n(\bar{s}_n)-1)u. \quad (64)$$

取李雅普诺夫函数

$$V_n=V_{n-1}+\frac{1}{2}z_n^2+\frac{1}{2}\tilde{\lambda}_n^2+\frac{1}{2}y_n^2,$$

对V_n关于t求导,得

$$\dot{V}_n=\dot{V}_{n-1}+z_n[F_n(\bar{s}_n)+\varDelta(u)+u-\dot{\omega}_n+D_n(t,\xi,\bar{s}_n)]+\tilde{\lambda}_n\dot{\hat{\lambda}}_n+y_n\dot{y}_n. \quad (65)$$

将式(62)代入(65),得到

$$\dot{V}_n\leqslant \dot{V}_{n-1}+z_n\Big[F_n(\bar{s}_n)+\varDelta(u)+u+\frac{1}{\tau_n^2}|y_n|+\frac{1}{\tau_n^2}y_n^6+\frac{1}{2}+D_n(t,\xi,\bar{s}_n)\Big]+\tilde{\lambda}_n\dot{\hat{\lambda}}_n+y_n\dot{y}_n. \quad (66)$$

由定义1、假设2、引理1和引理4,可得

$$z_nD_n(t,\xi,\bar{s}_n)\leqslant z_nK_n(s_n)[\varDelta_{n1}(\|\bar{x}_n\|)+\varDelta_{n2}(\|\xi\|)]\leqslant z_n^2K_n^2(s_n)[\varDelta_{n1}(\|\bar{x}_n\|)+\varDelta_{n2}\circ\bar{\alpha}_1(r+D_0)]^2+\frac{1}{4}, \quad (67)$$

$$z_n\varDelta(u)\leqslant\frac{1}{2}z_n^2+\frac{1}{2}\|\varDelta(u)\|^2. \quad (68)$$

式(66)可转化为

$$\dot{V}_n\leqslant \dot{V}_{n-1}+z_n\Big[F_n(\bar{s}_n)+u+\frac{1}{\tau_n^2}|y_n|+\frac{1}{\tau_n^2}y_n^6+z_nK_n^2(s_n)[\varDelta_{n1}(\|\bar{x}_n\|)+\varDelta_{n2}\circ\bar{\alpha}_1(r+D_0)]^2\Big]+\tilde{\lambda}_n\dot{\hat{\lambda}}_n+y_n\dot{y}_n+\frac{1}{2}z_n^2+\frac{1}{2}\|\varDelta(u)\|^2+\frac{3}{4}. \quad (69)$$

令

$$\varPhi_n(Z_n)=F_n(\bar{s}_n)+\frac{1}{\tau_n^2}|y_n|+\frac{1}{\tau_n^2}y_n^6+\frac{1}{2}+\kappa_n|z_n|^{\frac{1}{2}}+\kappa_n{z_n}^3+z_nK_n^2(s_n)[\varDelta_{n1}(\|\bar{x}_n\|)+\varDelta_{n2}\circ\bar{\alpha}_1(r+D_0)]^2-\kappa_nz_n.$$

其中: $Z_n=[\bar{s}_n^{\mathrm{T}},z_n,y_n,r]^{\mathrm{T}}$, $\kappa_n>0$为一个设计常数. 因此有

$$\dot{V}_n\leqslant \sum_{j=1}^{n-1}\Big[-\kappa_j|z_j|^{\frac{3}{2}}-k_jz_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big]+\sum_{j=1}^{n-1}\Big(\frac{1}{2}\delta_j^2+\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big)+z_n^2+y_n^2+\sum_{j=2}^{n-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}}-\Big(\frac{1}{\tau_j}-2\Big)y_j^4+\frac{1}{4}\eta_j^2\Big)+z_n(\theta_n^{*\mathrm{T}}\phi_n(Z_n)+\varepsilon_n+u-k_n|z_n|^{\frac{1}{2}}-\kappa_n{z_n}^3+\kappa_nz_n)+\tilde{\lambda}_n\dot{\hat{\lambda}}_n+y_n\dot{y}_n+\frac{n}{4}+\frac{1}{2}z_n^2+\frac{1}{2}\|\varDelta(u)\|^2. \quad (70)$$

由Young's不等式,得

$$z_n\theta_n^{*\mathrm{T}}\phi_n(Z_n)\leqslant\frac{\|\phi_n(Z_n)\|^2z_n^2\lambda_n}{2a_n^2}+\frac{a_n^2}{2}, \quad (71)$$

其中$a_n>0$为一个设计常数.

设计控制律

$$u=-(\kappa_n+2)z_n-\frac{\|\phi_n(Z_n)\|^2z_n\hat{\lambda}_n}{2a_n^2}, \quad (72)$$

设计自适应律

$$\dot{\hat{\lambda}}_n=\frac{\|\phi_n(Z_n)\|^2z_n^2}{2a_n^2}-\sigma_{n1}\hat{\lambda}_n-\sigma_{n2}\hat{\lambda}_n^3, \quad (73)$$

其中$\sigma_{n1}>0,\sigma_{n2}>0$为两个设计常数. 将式(71)、(72)和(73)代入(70),得到

$$\dot{V}_n\leqslant \sum_{j=1}^{n}\Big(-\kappa_j|z_j|^{\frac{3}{2}}-\kappa_jz_j^4-\gamma_j(\tilde{\lambda}_j^2)^{\frac{3}{4}}-\sigma_{j2}\Big(\frac{1}{2}\tilde{\lambda}_j^2\Big)^2\Big)+\sum_{j=1}^{n}\Big(\frac{1}{2}a_j^2+\frac{\sigma_{j1}}{2}\lambda_j^2+\frac{\gamma_j^4}{\sigma_{j1}^3}+\frac{61\sigma_{j2}}{3}\lambda_j^4\Big)+y_n^2+\sum_{j=2}^{n-1}\Big(-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}}-\Big(\frac{1}{\tau_j}-2\Big)y_j^4+\frac{1}{4}\eta_j^2\Big)+y_n\dot{y}_n+\frac{n}{4}+\sum_{j=1}^{n-1}\frac{1}{2}\delta_j^2-\frac{1}{2}z_n^2+z_n\varepsilon_n+\frac{1}{2}\|\varDelta(u)\|^2. \quad (74)$$

存在一个非负连续函数$\delta_n(\bar{z}_n,\bar{y}_n,\bar{\hat{\lambda}}_n,y_d,\dot{y}_d,r)$,使得

$$|\varepsilon_n(Z_n)|\leqslant\delta_n(\bar{z}_n,\bar{y}_n,\bar{\hat{\lambda}}_n,y_d,\dot{y}_d,r).$$

且有

$$z_n\varepsilon_n \leqslant \frac{1}{2}z_n^2 + \frac{1}{2}\delta_n^2. \tag{75}$$

将式(75)代入(74),得到

$$\begin{aligned}&\dot V_n \leqslant\\&\sum_{j=1}^{n}\Big[-\kappa_j|z_j|^{\frac{3}{2}} - \kappa_j z_j^4 - \gamma_j(\tilde\lambda_j^2)^{\frac{3}{4}} - \sigma_{j2}\Big(\frac{1}{2}\tilde\lambda_j^2\Big)^2\Big]+\\&\sum_{j=1}^{n}\Big(\frac{1}{2}\delta_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{\gamma_j^4}{\sigma_{j1}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4 + \frac{j}{4}\Big)+\\&y_n^2 - \sum_{j=2}^{n-1}\Big[\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}} + \Big(\frac{1}{\tau_j}-2\Big)y_j^4 - \frac{1}{4}\eta_j^2\Big]+\\&y_n\dot y_n + \frac{1}{2}\|\varDelta(u)\|^2.\end{aligned} \tag{76}$$

设计一阶滤波器

$$\dot\omega_n = -\frac{1}{\tau_n}|y_n|^{\frac{1}{2}}\mathrm{sgn}(y_n) - \frac{1}{\tau_n}|y_n|^3\mathrm{sgn}(y_n). \tag{77}$$

其中: $y_n=\omega_n-\hbar_{n-1}$, τ_n 为正的设计常数. 存在一个非负连续函数 $\eta_n(\bar s_n,\bar z_n,\bar y_n,\bar{\hat\lambda}_n,y_d,\dot y_d,\ddot y_d,r)$,使得

$$|-\dot\hbar_{n-1}| \leqslant \eta_n(\bar s_n,\bar z_n,\bar y_n,\bar{\hat\lambda}_n,y_d,\dot y_d,\ddot y_d,r), \tag{78}$$

$$y_n\dot y_n \leqslant -\frac{1}{\tau_n}|y_n|^{\frac{3}{2}} - \frac{1}{\tau_n}y_n^4 + y_n^2 + \frac{1}{4}\eta_n^2. \tag{79}$$

将式(79)代入(76),得到

$$\begin{aligned}&\dot V_n \leqslant\\&-\sum_{j=1}^{n}\Big[\kappa_j|z_j|^{\frac{3}{2}} + \kappa_j z_j^4 + \gamma_j(\tilde\lambda_j^2)^{\frac{3}{4}} + \sigma_{j2}\Big(\frac{1}{2}\tilde\lambda_j^2\Big)^2\Big]+\\&\sum_{j=1}^{n}\Big[\frac{1}{2}\delta_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{\gamma_j^4}{\sigma_{j1}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4\Big]+\\&2y_n^2 + \frac{1}{2}\|\Delta(u)\|^2-\\&\sum_{j=2}^{n-1}\Big[\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}} + \Big(\frac{1}{\tau_j}-2\Big)y_j^4 - \frac{1}{4}\eta_j^2\Big]-\\&\frac{1}{\tau_n}|y_n|^{\frac{3}{2}} - \frac{1}{\tau_n}y_n^4 + \frac{1}{4}\eta_n^2 + \frac{n}{4}.\end{aligned} \tag{80}$$

类似于式(59)和(60)的讨论,得到 $2y_n^2 \leqslant 2(y_n^2)^{\frac{3}{4}} + 2(y_n^2)^2$. 进一步根据式(80),得

$$\begin{aligned}&\dot V_n \leqslant\\&-\sum_{j=1}^{n}\Big[\kappa_j|z_j|^{\frac{3}{2}} + \kappa_j z_j^4 + \gamma_j(\tilde\lambda_j^2)^{\frac{3}{4}} + \sigma_{j2}\Big(\frac{1}{2}\tilde\lambda_j^2\Big)^2\Big]+\\&\sum_{j=1}^{n}\Big[\frac{1}{4}\delta_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{\gamma_j^4}{\sigma_{j1}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4 + \frac{j}{4}\Big]+\\&\sum_{j=2}^{n}\Big[-\Big(\frac{1}{\tau_j}-2\Big)|y_j|^{\frac{3}{2}} - \Big(\frac{1}{\tau_j}-2\Big)y_j^4 + \frac{1}{4}\eta_j^2\Big]+\\&\|\Delta(u)\|^2/2.\end{aligned} \tag{81}$$

3　稳定性分析

定义如下紧集:

$$\varOmega = \{[\bar z_n^{\mathrm T},\bar y_n^{\mathrm T},\bar{\tilde\lambda}_n^{\mathrm T},r]^{\mathrm T}: V_n \leqslant P\}, \tag{82}$$

其中 $P>0$ 为任意正常数. 非负连续函数 $\eta_i(\cdot)$ 在紧集 $\varOmega\times\varOmega_d$ 上存在最大值 M_i,非负连续函数 $\delta_i(\cdot)$ 在紧集 $\varOmega\times\varOmega_d$ 上存在最大值 N_i.

定理 1　考虑由系统(1)、控制律(72)、虚拟控制律(32)、(50)和自适应律(33)、(51)、(73)组成的闭环系统,若假设1～假设4成立,则对于任意初始条件 $V_n(0)\leqslant P$,选取适当的正常数 σ_{i2}、τ_i、k_i、γ_i,整个闭环系统中的所有信号有界,且系统状态满足约束条件 $x_i\in\varOmega_{x_i}$. 设计常数 σ_{i2}、τ_i、k_i 和 γ_i 满足

$$\begin{cases}\kappa_i>0,\ \gamma_i>0,\ \sigma_{i2}>0,\ i=1,2,\dots,n;\\ \dfrac{1}{\tau_j}>2,\ j=2,3,\dots,n;\\ \alpha = 2^{\frac{3}{4}}\min\Big\{\kappa_i, \dfrac{1}{\tau_j}-2, \gamma_i\Big\};\\ \beta = \min\Big\{\dfrac{4\kappa_i}{3n}, \dfrac{\sigma_{i2}}{3n}, \dfrac{4}{3(n-1)}\Big(\dfrac{1}{\tau_j}-2\Big)\Big\}.\end{cases} \tag{83}$$

证明　若 $V_n\leqslant P$,则 $\bar z_n$、$\bar{\tilde\lambda}_n$、$\bar y_n$, r 有界. 因为 $s_1=z_1+\hat y_d$, $s_i=z_i+y_i+\hbar_{i-1}$,所以 $\hbar_i$、s_i 有界,进一步可知 $x_i\in\varOmega_{x_i}$. 因为 $\bar s_n$、z_n、$\hat\lambda_n$、y_n、r 有界,由式(72)可知控制律 u 有界. 根据式(64),假设3和控制律 u 有界,可以得到 $\varDelta(u)$ 有界,则存在一个正的常数 c 满足 $\|\varDelta(u)\|\leqslant c$.

定义如下Lyapunov函数:

$$V_n = \sum_{j=1}^{n}\frac{1}{2}z_j^2 + \sum_{j=1}^{n}\frac{1}{2}\tilde\lambda_j^2 + \sum_{j=2}^{n}\frac{1}{2}y_j^2. \tag{84}$$

V_n 关于时间 t 的导数为

$$\begin{aligned}&\dot V_n \leqslant\\&-\sum_{j=1}^{n}\kappa_j(z_j^2)^{\frac{3}{4}} - \sum_{j=2}^{n}\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^{\frac{3}{4}}-\\&\sum_{j=1}^{n}\gamma_j(\tilde\lambda_j^2)^{\frac{3}{4}} - \sum_{j=1}^{n}\kappa_j(z_j^2)^2-\\&\sum_{j=2}^{n}\Big(\frac{1}{\tau_j}-2\Big)(y_j^2)^2 - \sum_{j=1}^{n}\sigma_{j2}\Big(\frac{1}{2}\tilde\lambda_j^2\Big)^2 + \bar C,\end{aligned} \tag{85}$$

其中

$$\begin{aligned}&\bar C =\\&\sum_{j=1}^{n}\Big[\frac{1}{2}N_j^2 + \frac{1}{2}a_j^2 + \frac{\sigma_{j1}}{2}\lambda_j^2 + \frac{2}{{\sigma_{j1}}^3} + \frac{61\sigma_{j2}}{3}\lambda_j^4\Big]+\\&\sum_{j=2}^{n}\frac{1}{4}M_j^2 + \frac{n}{4} + \frac{1}{2}c^2.\end{aligned}$$

于是式(85)可简化为

$$\dot V_n \leqslant -\alpha V_n^{\frac{3}{4}} - \beta V_n^2 + \bar C. \tag{86}$$

其中

$$\alpha = 2^{\frac{3}{4}}\min\left\{\kappa_1,\ldots,\kappa_n,\frac{1}{\tau_2}-2,\ldots,\frac{1}{\tau_n}-2,\gamma_1,\ldots,\gamma_n\right\},$$

$$\beta=\min\left\{\frac{4\kappa_1}{3n},\ldots,\frac{4\kappa_n}{3n},\frac{\sigma_{12}}{3n},\ldots,\frac{\sigma_{n2}}{3n},\frac{4}{3(n-1)}\left(\frac{1}{\tau_2}-2\right),\ldots,\frac{4}{3(n-1)}\left(\frac{1}{\tau_n}-2\right)\right\}.$$

进而有

$$\dot V_n\leqslant-\alpha V_n^{\frac{3}{4}}-\beta V_n^2+\bar C\leqslant-\alpha V_n^{\frac{3}{4}}+\bar C. \tag{87}$$

$\forall 0<\nu\leqslant 1$,将式(87)改写为

$$\dot V_n\leqslant-\alpha\nu V_n^{\frac{3}{4}}-(1-\nu)\alpha V_n^{\frac{3}{4}}+\bar C. \tag{88}$$

考虑两个集合

$$\bar\varOmega_x=\left\{x(t)|V_n^{\frac{3}{4}}\leqslant\frac{\bar C}{(1-\nu)\alpha}\right\},$$

$$\varOmega_x=\left\{x(t)|V_n^{\frac{3}{4}}>\frac{\bar C}{(1-\nu)\alpha}\right\},$$

讨论如下两种情况.

情况1 当$x(t)\in\varOmega_x$时,将式(88)改写为

$$\dot V_n\leqslant-\alpha\nu V_n^{\frac{3}{4}}. \tag{89}$$

对式(89)在$[0,T]$上积分,有

$$\int_0^T\frac{\dot V_n}{V_n^{\frac{3}{4}}}\mathrm{d}t\leqslant\int_0^T-\alpha\nu\mathrm{d}t. \tag{90}$$

由式(90)得到

$$T\leqslant T_{\max}=\frac{4}{\alpha\nu}\left[V_n^{\frac{1}{4}}(0)-\left(\frac{\bar C}{(1-\nu)\alpha}\right)^{\frac{1}{3}}\right],$$

这里$V_n(0)$是$V_n(x)$的初值. 所以 $\forall t\geqslant T_{\max},x(t)\in\bar\varOmega_x$.

情况2 当$x(t)\in\bar\varOmega_x$时,由情况1可知,$x(t)$的轨迹不会超出集合$\bar\varOmega_x$.

综上两种情况可得,V_n在有限时间内有界. 进一步,由式(84)可知

$$|z_1|\leqslant\sqrt{2\left(\frac{\bar C}{(1-\nu)\alpha}\right)^{\frac{4}{3}}}. \tag{91}$$

定理1得证. □

4 仿真算例

为了验证所提出控制方案的有效性,对下述数值算例进行仿真和比较.

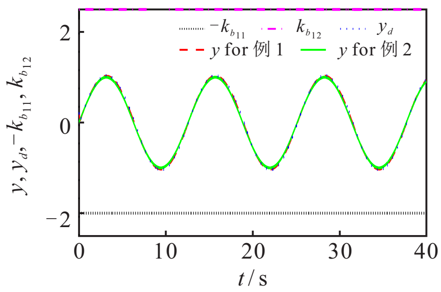
考虑一个具有全状态约束和未建模动态的严格反馈系统

$$\begin{cases}\dot\xi=-\xi+0.5x_1^2\sin(x_1t),\\ \dot x_1=x_1\mathrm{e}^{-0.5x_1}+(1+x_1^2)x_2+0.2\xi x_1\sin(x_2t),\\ \dot x_2=x_1x_2^2+(3-\cos(x_1x_2))u+0.1\xi\cos(0.5x_2t),\\ y=x_1.\end{cases}$$

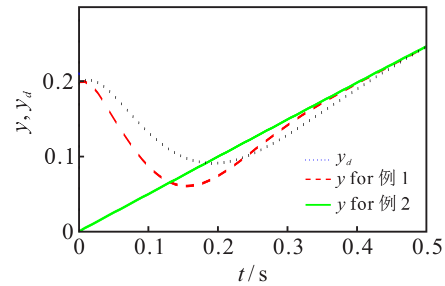
选取期望信号$y_d=0.5\sin(0.5t)$. 设计动态信号$\dot r=-r+1.5x_1^4+0.8$. 选取初值:$x_1(0)=0.2,x_2(0)=0.1,\hat\lambda_1(0)=2,\hat\lambda_2(0)=0.5,\omega_2(0)=0.1,r(0)=0.1,\xi(0)=0.1$. 选取设计常数: $k_{b_{11}}=2,k_{b_{12}}=2,k_{b_{21}}=2,k_{b_{22}}=2.5,l_1=l_2=9$.

例1 本文设计参数选取如下:$\kappa_1=16,\kappa_2=12,a_1=a_2=1,\sigma_{11}=0.01,\sigma_{12}=4,\sigma_{21}=0.01,\sigma_{22}=4,\tau_2=0.01,\gamma_1=\gamma_2=50$. 仿真结果如图1～图6所示.

例2 文献[16]设计参数选取如下:$\kappa_1=10,\kappa_2=15,a_1=a_2=1,\sigma_1=\sigma_2=0.01,\gamma_1=\gamma_2=50,\tau_2=0.01$. 仿真结果如图1～图6所示.



(a) 情况1



(b) 情况2

图 1 输出y和期望信号y_d

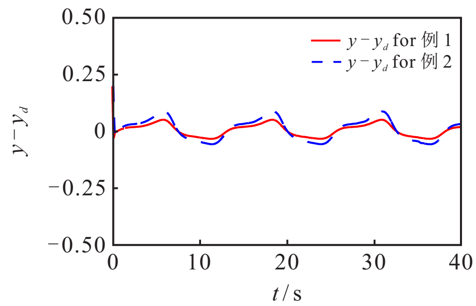


图 2 跟踪误差 $y-y_d$

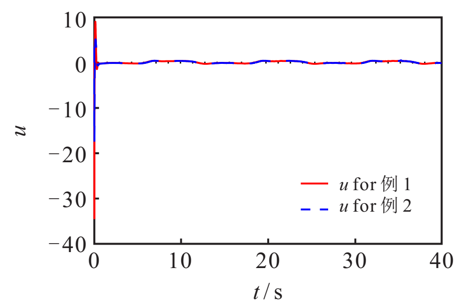


图 3 控制信号u

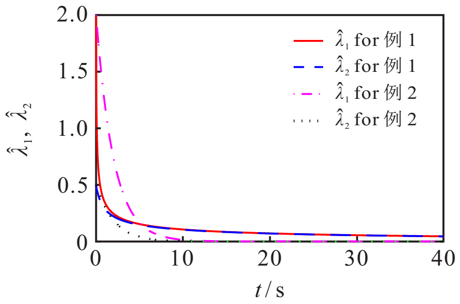


图 4　自适应调节参数 $\hat{\lambda}_1$、$\hat{\lambda}_2$

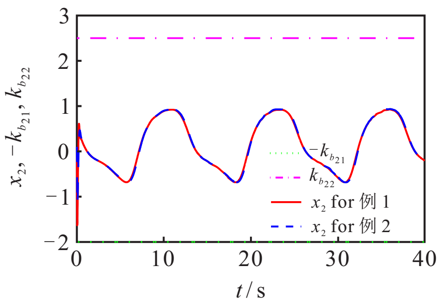


图 5　状态 x_2 和约束 $-k_{b_{21}}$、$k_{b_{22}}$

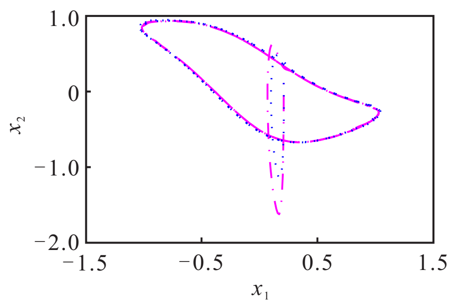


图 6　x_1、x_2 的相平面轨迹

由图1、图2、图5和图6可知,输出信号 y 可以很好地跟踪期望轨迹,且所有的状态满足约束条件. 例1的跟踪性能优于例2,并且具有较快的收敛速度. 图3和图4表明控制信号 u 和自适应调节参数 $\hat{\lambda}_1$、$\hat{\lambda}_2$ 有界.

5　结　论

本文针对一类具有全状态约束、未建模动态和动态不确定性的严格反馈非线性系统,提出了一种新的自适应动态面控制方案. 通过引入一一映射将有约束的非线性系统转化为无约束的非线性系统,结合改进的动态面控制技术、Young's不等式和神经网络,设计出一种结构简单的控制器,既可以避免后推设计中的"复杂性爆炸"问题,又可以解决现有有限时间控制中的奇异性问题. 与已有的线性滤波器相比,该滤波器对滤波器参数的要求降低,系统收敛时间缩短. 理论分析表明,闭环系统中的所有信号在有限时间内有界,且系统的状态满足约束条件. 仿真结果验证了所提出控制方案的有效性.

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