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基于动态事件触发的状态饱和复杂网络递推滤波

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摘 要: 以一类具有状态饱和与非线性耦合的复杂网络为研究对象,针对无线通信传输负荷大的问题,提出一种动态事件传输机制下的递推滤波算法,以达到保证滤波性能的同时减轻通信网络传输负担的目的. 首先,构造此类时变复杂网络的数学模型;然后,设计具有较低保守性的递推滤波器,计算滤波器的增益,并对所设计的递推滤波算法进行有界性分析;最后,给出一个仿真实例. 从仿真结果可以看出,动态事件触发机制的引入能够降低无线通信网络传输负担,达到节约能量的目的;同时,所提出的递推滤波算法能够保证此类复杂网络在动态事件触发机制下仍具有较好的滤波效果,验证了所提出算法的有效性.

关键词: 动态事件触发机制；复杂网络；状态饱和；非线性耦合；递推滤波；有界性

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Recursive filtering for state saturated complex networks under dynamic event-triggered mechanism

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Abstract: As for the problem of heavy transmission burden of wireless communication, a recursive filtering algorithm based on dynamic-event transmission mechanism is proposed for a class of complex networks with state saturations and nonlinear coupling strengths in this paper, so as to guarantee the filtering performance and reduce the transmission burden of wireless communication networks. Firstly, the mathematical model of this class of the complex network is constructed. Then, a recursive filter with less conservatism is designed, the gain of the recursive filter is calculated and the performance of the proposed recursive filtering algorithm is analyzed. Finally, a simulation example is implemented. The simulation results shows that the introduction of dynamic event-triggered mechanism can reduce the transmission burden of the wireless communication network and achieve the purpose of saving energy. The recursive filtering algorithm proposed can ensure that this class of complex networks possess good filtering effect under the dynamic event-triggered mechanism, which verifies the effectiveness of the proposed algorithm.

Keywords: dynamic event-triggered mechanism；complex networks；state saturations；nonlinear coupling strengths；recursive filtering；boundedness

0 引 言

复杂网络在近几十年里已成为各界学者热衷的研究对象[1-3]. 针对复杂网络内部的状态估计问题,已有的一些滤波/状态估计方法(例如 H_∞ 估计、集员滤波)可根据测量数据去估计动态系统的内部状态,一直是控制领域研究的热点[4-6]. 与独立节点的系统滤

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波/状态估计相比,复杂网络由于节点间复杂的耦合关系使得滤波/状态估计实现难度更大.

随着当前实际工程对实时性要求的提高,递推滤波方法由于其突出的在线计算能力,受到越来越多专家学者们的关注[7-10]. 在对复杂网络的研究中,通常假设网络内部节点间的耦合强度为常数或是线性的[11],但由于不可预期的干扰 (如噪声、数据传输拥堵等),这种假设在许多实际应用中是不符合现实情况的, 例如在跟踪多个相互作用的智能体时,它们的运动关系是由非线性耦合项描述的[12].

此外,当前大多数滤波器/估计器的设计都是基于时间触发机制的,这种数据传输方式会使得不必要的数据也被发送,从而导致通信网络带宽及能源的浪费. 为了节约资源,一些学者研究了基于事件触发通信机制的滤波/状态估计问题[13-14]. 基本思想是传感器只发送有用的信息至相应的滤波器/估计器中,而数据是否发送由制定的与阈值有关的事件发生器决定. 这类机制主要包括静态触发机制 (static event-triggered mechanism, SETM) 和动态触发机制 (dynamic event-triggered mechanism, DETM). SETM 情况下,阈值是一个固定的常数标量; DETM 情况下,阈值是一个动态调节的标量. DETM 与 SETM 相比,数据是否被发送能够根据实际情况动态地调节. 因此, DETM 最近获得了一些研究人员的关注[15-16].

在实际的复杂网络中,由于设备装置功率、电压、幅值等的限制,状态饱和现象是经常发生的,这将影响系统的性能,甚至导致系统不稳定. 此时常规的系统分析与综合方法将不再适用,因此研究具有状态饱和的复杂网络是非常必要的. 目前,针对这类系统的滤波问题已经获得了一些专家学者的关注[17-18].

本文的研究目标是以具有非线性耦合的状态饱和复杂网络为对象,提出一种在动态事件传输机制下具有较低保守性的递推滤波算法,在保证滤波性能的同时减轻无线通信网络传输负担. 本文的主要工作如下: 1) 在动态事件触发机制下,研究一类具有非线性耦合的状态饱和复杂网络的递推滤波问题; 2) 求取滤波误差协方差的上界,并将该上界最小化求取滤波器增益; 3) 证明滤波误差协方差的上界是有界的. 综合以上,本文的研究工作既具有理论意义又具有工程意义.

1 问题描述

考虑如下一类由 N 个节点组成的具有状态饱和与非线性耦合的复杂网络:

$$\begin{cases} x_{i,k+1} = S_i\Big(A_{i,k}x_{i,k} + g(x_{i,k}) + \\ \qquad \sum_{j=1}^{N}\gamma_{ij}h(x_{j,k})\Big) + B_{i,k}\omega_{i,k}, \\ y_{i,k} = C_{i,k}x_{i,k} + v_{i,k}. \end{cases} \tag{1}$$

其中: $x_{i,k} \in \boldsymbol{R}^n$ 及 $y_{i,k} \in \boldsymbol{R}^m (i = 1, 2, \ldots, N)$ 分别为第 i 个节点的状态向量和测量输出; $g(\cdot)$ 和 $h(\cdot)$ 为连续可微的非线性函数; $\gamma_{ij} \geqslant 0$ 为节点间的耦合强度,有 $a_i = \sum_{j=1}^{N}\gamma_{ij}$; $\omega_{i,k} \in \boldsymbol{R}^{r_1}$ 及 $v_{i,k} \in \boldsymbol{R}^{r_2}$ 为具有协方差矩阵 $R_{iw,k}$ 和 $R_{iv,k}$ 的零均值高斯白噪声,分别代表过程噪声和测量噪声,设 $\omega_{i,k}$ 及 $v_{i,k}$ 对任意 i 和 k 都是互不相关的; $A_{i,k}$、$B_{i,k}$ 和 $C_{i,k}$ 为适维已知矩阵; $S(\cdot) : \boldsymbol{R}^n \mapsto \boldsymbol{R}^n$ 为定义的饱和函数,有

$$S_i(\sigma) = [S_{i1}(\sigma_{i1}), S_{i2}(\sigma_{i2}), \ldots, S_{in}(\sigma_{in})]^{\mathrm{T}}, \tag{2}$$

这里

$$\begin{aligned} &\sigma = [\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{in}]^{\mathrm{T}}; \\ &S_{il}(\sigma_{il}) = \mathrm{sgn}(\sigma_{il})\min\{\sigma_{il,\max}, |\sigma_{il}|\}, \\ &\qquad l = 1, 2, \ldots, n, \end{aligned} \tag{3}$$

$\sigma_{il,\max}$ 为 $\sigma_{i,\max}$ 的第 l 个分量, $\sigma_{i,\max}$ 为第 i 个节点的饱和水平向量, $\mathrm{sgn}(\cdot)$ 为符号函数.

本文考虑了 DETM 下的复杂网络滤波问题,与 SETM 不同的是, DETM 下当前传感器的测量数据是否被发送是由一个动态条件决定的. 定义第 i 个传感器节点的触发时刻为 $t_m^i (m = 0, 1, \ldots)$, t_m^i 由下面的动态条件[19] 确定:

$$t_{m+1}^i \triangleq \min_{k\in\boldsymbol{N}}\Big\{k > t_m^i | \zeta_{i,k}^{\mathrm{T}}\zeta_{i,k} - \lambda_i - \frac{1}{\varepsilon_i}\alpha_{i,k} > 0\Big\}. \tag{4}$$

其中: $\alpha_{i,k}$ 为动态辅助变量,有

$$\alpha_{i,k+1} = \mu_i\alpha_{i,k} + \lambda_i - \zeta_{i,k}^{\mathrm{T}}\zeta_{i,k},\ \alpha_{i,0} \geqslant 0; \tag{5}$$

$\lambda_i > 0$ 为给定的阈值; μ_i 和 ε_i 为给定的标量参数,满足 $0 < \mu_i < 1, \mu_i \geqslant \dfrac{1}{\varepsilon_i}$;

$$\zeta_{i,k} \triangleq y_{i,t_m^i} - y_{i,k} \tag{6}$$

为第 i 个传感器节点在最近触发时刻 t_m^i 和当前采样时刻 k 之间的测量值之差.

基于上述,构造如下结构的复杂网络 (1) 的第 i 个传感器节点的递推滤波器:

$$\begin{cases} \hat{x}_{i,k+1|k} = \\ S_i\Big(A_{i,k}\hat{x}_{i,k|k} + g(\hat{x}_{i,k|k}) + \sum_{j=1}^{N}\gamma_{ij}h(\hat{x}_{j,k|k})\Big), \\ \hat{x}_{i,k+1|k+1} = \hat{x}_{i,k+1|k} + K_{i,k+1}(C_{i,k+1}x_{i,k+1} - \\ \qquad C_{i,k+1}\hat{x}_{i,k+1|k} + \zeta_{i,k+1} + v_{i,k+1}). \end{cases} \tag{7}$$

其中: $\hat{x}_{i,k+1|k}$ 和 $\hat{x}_{i,k+1|k+1}$ 分别代表第 i 个传感器节点的状态 $x_{i,k}$ 对 $k+1$ 时刻状态的一步预测和估计, $K_{i,k+1}$ 为待确定的第 i 个传感器节点在 $k+1$ 时刻的增益矩阵. 为分析方便, 这里定义第 i 个节点的一步预测误差、滤波误差及相应的协方差分别为

$$\tilde{x}_{i,k+1|k}=x_{i,k+1}-\hat{x}_{i,k+1|k},$$
$$\tilde{x}_{i,k+1|k+1}=x_{i,k+1}-\hat{x}_{i,k+1|k+1},$$
$$\mathrm{Cov}_{i,k+1|k}=\boldsymbol{E}\{\tilde{x}_{i,k+1|k}\tilde{x}^{\mathrm{T}}_{i,k+1|k}\},$$
$$\mathrm{Cov}_{i,k+1|k+1}=\boldsymbol{E}\{\tilde{x}_{i,k+1|k+1}\tilde{x}^{\mathrm{T}}_{i,k+1|k+1}\}.$$

注 1 在DETM下, 只有满足条件(4)时, k 时刻的测量数据才会被第 i 个传感器节点发送至相应的滤波器. 从式(4)可以看出, 当 $\alpha_{i,k}=0$ 时, DETM条件退化为SETM条件, 因此DETM包含了SETM.

2 滤波器设计

首先给出后续分析用到的若干引理.

引理 1 [17] 对于任意 $x_1,x_2\in\boldsymbol{R}$, 存在一个实数 $\varrho_d\in[0,1]$, 使得下式成立:

$$S_d(x_1)-S_d(x_2)=\varrho_d(x_1-x_2),\ d=1,2,\dots,n, \tag{8}$$

其中 $S_d(\cdot)$ 为式(2)和(3)中定义的饱和函数.

引理 2 [20] 对于任意两个向量 $a,b\in\boldsymbol{R}^n$, 有下面的不等式成立:

$$ab^{\mathrm{T}}+ba^{\mathrm{T}}\leqslant\epsilon aa^{\mathrm{T}}+\epsilon^{-1}bb^{\mathrm{T}}, \tag{9}$$

其中 $\epsilon>0$ 为常数.

引理 3 [21] 已知适维矩阵 A、B、C 和 D, 其中 $CC^{\mathrm{T}}<I$. 令 P 为正定对称矩阵, ρ 为任意大于0的常数, 满足 $\rho^{-1}I-DPD^{\mathrm{T}}>0$, 则下面的不等式成立:

$$\begin{aligned}&(A+BCD)P(A+BCD)^{\mathrm{T}}\leqslant\\&A(P^{-1}-\rho D^{\mathrm{T}}D)^{-1}A^{\mathrm{T}}+\rho^{-1}BB^{\mathrm{T}}.\end{aligned} \tag{10}$$

引理 4 [22] 对于 $0\leqslant k\leqslant n$, 假定 $X=X^{\mathrm{T}}>0, \mathcal{M}_k(X)=\mathcal{M}^{\mathrm{T}}_k(X)\in\boldsymbol{R}^{L\times L}, \mathcal{N}_k(X)=\mathcal{N}^{\mathrm{T}}_k(X)\in\boldsymbol{R}^{L\times L}$, 则存在 $\mathcal{Q}=\mathcal{Q}^{\mathrm{T}}>X$ 使得

$$\begin{cases}\mathcal{M}_k(X)\geqslant\mathcal{M}_k(\mathcal{Q}),\\ \mathcal{N}_k(X)\geqslant\mathcal{M}_k(X)\end{cases} \tag{11}$$

成立, 那么下面差分方程的解 $\mathcal{T}_k$ 及 $\mathcal{U}_k$ 满足 $\mathcal{U}_k\geqslant\mathcal{T}_k$:

$$\begin{cases}\mathcal{T}_k=\mathcal{M}_k(\mathcal{T}_{k-1}),\\ \mathcal{U}_k=\mathcal{N}_k(\mathcal{U}_{k-1}),\\ \mathcal{T}_0=\mathcal{U}_0>0.\end{cases} \tag{12}$$

引理 5 动态辅助变量 $\alpha_{i,k}$ 存在一个上界 $\bar{\alpha}_{i,k}$, 即 $\alpha_{i,k}\leqslant\bar{\alpha}_{i,k}$, 其中

$$\bar{\alpha}_{i,k}\triangleq\mu_i^k\alpha_{i,0}+\lambda_i\frac{1-\mu_i^k}{1-\mu_i}. \tag{13}$$

证明 当 $k=0$ 时, 有 $\bar{\alpha}_{i,0}=\alpha_{i,0}$.

由式(5)可知, $\alpha_{i,k}\leqslant\mu_i\alpha_{i,k-1}+\lambda_i$. 通过代入计算, 容易推得

$$\alpha_{i,k}\leqslant\mu_i^k\alpha_{i,0}+\lambda_i\frac{1-\mu_i^k}{1-\mu_i}. \tag{14}$$

证明成立. □

下面给出主要结论.

定理 1 一步预测误差协方差 $\mathrm{Cov}_{i,k+1|k}$ 及滤波误差协方差 $\mathrm{Cov}_{i,k+1|k+1}$ 满足下面两个递推方程:

$$\begin{aligned}&\mathrm{Cov}_{i,k+1|k}=\\&B_{i,k}R_{iw,k}B^{\mathrm{T}}_{i,k}+\boldsymbol{E}\{\varPsi_{i,k}\mathcal{A}_{i,k}\mathrm{Cov}_{i,k|k}\mathcal{A}^{\mathrm{T}}_{i,k}\varPsi^{\mathrm{T}}_{i,k}\}+\\&\sum_{j=1}^{N}\gamma_{ij}\boldsymbol{E}\{\varPsi_{i,k}(\mathcal{A}_{i,k}\tilde{x}_{i,k|k}\tilde{x}^{\mathrm{T}}_{j,k|k}\mathcal{H}^{\mathrm{T}}_{j,k}+\\&\mathcal{H}_{j,k}\tilde{x}_{j,k|k}\tilde{x}^{\mathrm{T}}_{i,k|k}\mathcal{A}^{\mathrm{T}}_{i,k})\varPsi^{\mathrm{T}}_{i,k}\}+\\&\sum_{j=1}^{N}\sum_{z=1}^{N}\gamma_{ij}\gamma_{iz}\boldsymbol{E}\{\varPsi_{i,k}\mathcal{H}_{j,k}\tilde{x}_{j,k|k}\tilde{x}^{\mathrm{T}}_{z,k|k}\mathcal{H}^{\mathrm{T}}_{z,k}\varPsi^{\mathrm{T}}_{i,k}\},\end{aligned} \tag{15}$$

$$\begin{aligned}&\mathrm{Cov}_{i,k+1|k+1}=\\&(I-K_{i,k+1}C_{i,k+1})\mathrm{Cov}_{i,k+1|k}(I-K_{i,k+1}C_{i,k+1})^{\mathrm{T}}+\\&K_{i,k+1}\boldsymbol{E}\{\zeta_{i,k+1}\zeta^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}-\\&(I-K_{i,k+1}C_{i,k+1})\boldsymbol{E}\{\tilde{x}_{i,k+1|k}\zeta^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}-\\&K_{i,k+1}\boldsymbol{E}\{\zeta_{i,k+1}\tilde{x}^{\mathrm{T}}_{i,k+1|k}\}(I-K_{i,k+1}C_{i,k+1})^{\mathrm{T}}+\\&K_{i,k+1}\boldsymbol{E}\{\zeta_{i,k+1}v^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}+\\&K_{i,k+1}\boldsymbol{E}\{v_{i,k+1}\zeta^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}+\\&K_{i,k+1}R_{iv,k+1}K^{\mathrm{T}}_{i,k+1}.\end{aligned} \tag{16}$$

其中

$$\varPsi_{i,k}\triangleq\mathrm{diag}\{\eta^{(1)}_{i,k},\eta^{(2)}_{i,k},\dots,\eta^{(n)}_{i,k}\},$$
$$\eta^{(\nu)}_{i,k}\in[0,1],\ \nu=1,2,\dots,n;$$
$$\mathcal{A}_{i,k}\triangleq A_{i,k}+(G_{i,k}+W_{i,k}M_{i,k}),$$
$$G_{i,k}=\frac{\partial g(x_{i,k})}{\partial x_{i,k}}\Big|_{x_{i,k}=\hat{x}_{i,k|k}};$$
$$\mathcal{H}_{j,k}\triangleq H_{j,k}+J_{j,k}L_{j,k},$$
$$H_{j,k}=\frac{\partial h(x_{j,k})}{\partial x_{j,k}}\Big|_{x_{j,k}=\hat{x}_{j,k|k}}.$$

$W_{i,k}$ 和 $H_{j,k}$ 为依赖于问题的尺度矩阵, $M_{i,k}$ 和 $L_{j,k}$ 分别为满足 $M_{i,k}M^{\mathrm{T}}_{i,k}\leqslant I$ 和 $L_{j,k}L^{\mathrm{T}}_{j,k}\leqslant I$ 的未知线性化误差矩阵.

证明 根据方程(1)及引理1有

$$\begin{aligned}&\tilde{x}_{i,k+1|k}=\\&B_{i,k}\omega_{i,k}+\varPsi_{i,k}\Big(A_{i,k}\tilde{x}_{i,k|k}+g(x_{i,k})-g(\hat{x}_{i,k|k})+\\&\sum_{j=1}^{N}\gamma_{ij}[h(x_{j,k})-h(\hat{x}_{j,k|k})]\Big).\end{aligned} \tag{17}$$

将非线性函数$g(\cdot)$和$h(\cdot)$分别在$x_{i,k}$和$x_{j,k}$处进行泰勒展开,有

$$g(x_{i,k})=g(\hat{x}_{i,k|k})+G_{i,k}\tilde{x}_{i,k|k}+o_g(|\tilde{x}_{i,k|k}|), \quad (18)$$

$$h(x_{j,k})=h(\hat{x}_{j,k|k})+H_{j,k}\tilde{x}_{j,k|k}+o_h(|\tilde{x}_{j,k|k}|). \quad (19)$$

其中:$o_g(|\tilde{x}_{i,k|k}|)$和$o_h(|\tilde{x}_{j,k|k}|)$分别为$g(x_{i,k})$和$h(x_{i,k})$的泰勒展开高阶项,$G_{i,k}=\dfrac{\partial g(x_{i,k})}{\partial x_{i,k}}\Big|_{x_{i,k}=\hat{x}_{i,k|k}}$和$H_{j,k}=\dfrac{\partial g(x_{j,k})}{\partial x_{j,k}}\Big|_{x_{j,k}=\hat{x}_{j,k|k}}$为雅克比矩阵. 参考文献[23]中对高阶项的处理办法,可得

$$o_g(|\tilde{x}_{i,k|k}|)=W_{i,k}M_{i,k}\tilde{x}_{i,k|k}, \quad (20)$$

$$o_h(|\tilde{x}_{j,k|k}|)=J_{j,k}L_{j,k}\tilde{x}_{j,k|k}. \quad (21)$$

将式(18)～(21)代入到(17)中,可求得

$$\tilde{x}_{i,k+1|k}=\Psi_{i,k}\mathcal{A}_{i,k}\tilde{x}_{i,k|k}+\Psi_{i,k}\sum_{j=1}^{N}\gamma_{ij}\mathcal{H}_{j,k}\tilde{x}_{j,k|k}+B_{i,k}\omega_{i,k}. \quad (22)$$

根据$\mathrm{Cov}_{i,k+1|k}=\boldsymbol{E}\{\tilde{x}_{i,k+1|k}\tilde{x}^{\mathrm{T}}_{i,k+1|k}\}$,可得式(15).

再根据式(1)和(7)可求得滤波误差$\tilde{x}_{i,k+1|k+1}$为

$$\tilde{x}_{i,k+1|k+1}=(I-K_{i,k+1}C_{i,k+1})\tilde{x}_{i,k+1|k}-K_{i,k+1}\zeta_{i,k+1}-K_{i,k+1}v_{i,k+1}, \quad (23)$$

则可求得式(16). □

注2 从式(15)和(16)可以看到,有不确定项包含在$\mathrm{Cov}_{i,k+1|k}$与$\mathrm{Cov}_{i,k+1|k+1}$中,显然这种情况下若要求得确定的滤波误差协方差阵,进而计算滤波器增益是不可能的. 因此在后面的处理中,采用一种可替代的解决办法,即求取滤波误差协方差阵的上界,然后利用该上界求取设计的滤波器增益.

定理2 考虑式(15)和(16)中给出的一步预测误差协方差$\mathrm{Cov}_{i,k+1|k}$及滤波误差协方差$\mathrm{Cov}_{i,k+1|k+1}$,令ϵ_1、ρ_1、ρ_2及ρ_3为正标量. 假定下面的递推方程

$$\begin{aligned}&\Pi_{i,k+1|k}=\\&(1+a_i\epsilon_1)[\bar{A}_{i,k}(\Pi^{-1}_{i,k|k}-\rho_1I)^{-1}\bar{A}^{\mathrm{T}}_{i,k}+\\&\rho_1^{-1}W_{i,k}W^{\mathrm{T}}_{i,k}]+(\epsilon_1^{-1}+a_i)\times\\&\sum_{j=1}^{N}\gamma_{ij}[H_{j,k}(\Pi^{-1}_{j,k|k}-\rho_2I)^{-1}H^{\mathrm{T}}_{j,k}+\\&\rho_2^{-1}J_{i,k}J^{\mathrm{T}}_{i,k}]+B_{i,k}R_{iw,k}B^{\mathrm{T}}_{i,k}\end{aligned} \quad (24)$$

与

$$\begin{aligned}&\Pi_{i,k+1|k+1}=\\&(1+\rho_3)(I-K_{i,k+1}C_{i,k+1})\Pi_{i,k+1|k}\times\\&(I-K_{i,k+1}C_{i,k+1})^{\mathrm{T}}+\\&K_{i,k+1}\Big[(1+\rho_3^{-1})\Big(\lambda_i+\frac{1}{\varepsilon_i}\bar{\alpha}_{i,k+1}\Big)I+\\&(2\delta_{k+1,t^i_{m+1}}-1)R_{iv,k+1}\Big]K^{\mathrm{T}}_{i,k+1}\end{aligned} \quad (25)$$

有正定解$\Pi_{i,k+1|k}$及$\Pi_{i,k+1|k+1}$,其中$\epsilon_1^{-1}I-\Pi_{i,k|k}>0$,初始条件为$\mathrm{Cov}_{0|0}\leqslant\Pi_{0|0}$,则矩阵$\Pi_{i,k+1|k}$为$\mathrm{Cov}_{i,k+1|k}$的一个上界,$\Pi_{i,k+1|k+1}$为$\mathrm{Cov}_{i,k+1|k+1}$的一个上界,即

$$\mathrm{Cov}_{i,k+1|k}\leqslant\Pi_{i,k+1|k}, \quad (26)$$

$$\mathrm{Cov}_{i,k+1|k+1}\leqslant\Pi_{i,k+1|k+1}, \quad (27)$$

且滤波器增益为

$$K_{i,k+1}=(1+\rho_3)\Pi_{i,k+1|k}C^{\mathrm{T}}_{i,k+1}\Omega^{-1}_{i,k+1}. \quad (28)$$

其中

$$\begin{aligned}&\bar{A}_{i,k}=A_{i,k}+G_{i,k},\\&\Omega_{i,k+1}=(1+\rho_3)C_{i,k+1}\Pi_{i,k+1|k}C^{\mathrm{T}}_{i,k+1}+\\&\quad(1+\rho_3^{-1})\Big(\lambda_i+\frac{1}{\varepsilon_i}\bar{\alpha}_{i,k+1}\Big)I+\\&\quad(2\delta_{k+1,t^i_{m+1}}-1)R_{iv,k+1}.\end{aligned} \quad (29)$$

$\delta_{k+1,t^i_{m+1}}$为克罗内克函数,即

$$\delta_{k+1,t^i_{m+1}}=\begin{cases}1,\ k+1=t^i_{m+1};\\0,\ k+1\neq t^i_{m+1}.\end{cases} \quad (30)$$

证明 采用数学归纳法证明. 考虑初始条件$\mathrm{Cov}_{0|0}\leqslant\Pi_{0|0}$,并假定$\mathrm{Cov}_{i,k|k}\leqslant\Pi_{i,k|k}$.

利用引理2,从式(15)可推得

$$\begin{aligned}&\mathrm{Cov}_{i,k+1|k}\leqslant\\&(1+a_i\epsilon_1)(\bar{A}_{i,k}+W_{i,k}M_{i,k})\mathrm{Cov}_{i,k|k}\times\\&(\bar{A}_{i,k}+W_{i,k}M_{i,k})^{\mathrm{T}}+(\epsilon_1^{-1}+a_i)\times\\&\sum_{j=1}^{N}\gamma_{ij}(H_{j,k}+J_{i,k}L_{i,k})\mathrm{Cov}_{j,k|k}\times\\&(H_{j,k}+J_{i,k}L_{i,k})^{\mathrm{T}}+B_{i,k}R_{iw,k}B^{\mathrm{T}}_{i,k}.\end{aligned} \quad (31)$$

再根据式(16)、引理2及引理3即可推得

$$\begin{aligned}&\mathrm{Cov}_{i,k+1|k+1}\leqslant\\&(1+\rho_3)(I-K_{i,k+1}C_{i,k+1})\mathrm{Cov}_{i,k+1|k}\times\\&(I-K_{i,k+1}C_{i,k+1})^{\mathrm{T}}+(1+\rho_3^{-1})\times\\&K_{i,k+1}\boldsymbol{E}\{\zeta_{i,k+1}\zeta^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}+\\&K_{i,k+1}\boldsymbol{E}\{\zeta_{i,k+1}v^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}+\\&K_{i,k+1}\boldsymbol{E}\{v_{i,k+1}\zeta^{\mathrm{T}}_{i,k+1}\}K^{\mathrm{T}}_{i,k+1}+\\&K_{i,k+1}R_{iv,k+1}K^{\mathrm{T}}_{i,k+1}.\end{aligned} \quad (32)$$

从式(32)可以看到其中存在不确定项,因此不可能获得$\mathrm{Cov}_{i,k+1|k+1}$的准确值,这里考虑采用求取

$\text{Cov}_{i,k+1|k+1}$上界的替代办法. 为求解此上界,回顾前述与式(6)相关的定义,进而可知,当$k+1=t^i_{m+1}$时,有$\zeta_{i,k+1}=y_{i,t^i_{m+1}}-y_{i,k+1}=0$,则$\boldsymbol{E}\{\zeta_{i,k+1}v^{\mathrm{T}}_{i,k+1}\}=0$; 当$k+1\neq t^i_{m+1}$时,有$\zeta_{i,k+1}\neq 0$,可求得$\boldsymbol{E}\{\zeta_{i,k+1}v^{\mathrm{T}}_{i,k+1}\}=-R_{iv,k+1}$. 因此可知

$$\boldsymbol{E}\{\zeta_{i,k+1}v^{\mathrm{T}}_{j,k+1}\}=-(1-\delta_{k+1,t^i_{m+1}})R_{iv,k+1}. \tag{33}$$

考虑DETM条件(4)及引理5,可知对于任意$k\in\boldsymbol{N}$有

$$\zeta_{i,k+1}\zeta^{\mathrm{T}}_{i,k+1}\leqslant\Big(\lambda_i+\frac{1}{\varepsilon_i}\bar{\alpha}_{i,k+1}\Big)I. \tag{34}$$

结合式(33)及(34),式(32)可写为

$$\begin{aligned}&\text{Cov}_{i,k+1|k+1}\leqslant\\&(1+\rho_3)(I-K_{i,k+1}C_{i,k+1})\text{Cov}_{i,k+1|k}\times\\&(I-K_{i,k+1}C_{i,k+1})^{\mathrm{T}}+\\&K_{i,k+1}\Big[(1+\rho_3^{-1})\Big(\lambda_i+\frac{1}{\varepsilon_i}\bar{\alpha}_{i,k+1}\Big)I+\\&(2\delta_{k+1,t^i_{m+1}}-1)R_{iv,k+1}\Big]K^{\mathrm{T}}_{i,k+1}.\end{aligned} \tag{35}$$

回顾前述的假定条件$\text{Cov}_{i,k|k}\leqslant\varPi_{i,k|k}$,并利用引理4可推得式(24)与(25).

接下来计算这个上界$\varPi_{i,k+1|k+1}$的迹对$K_{i,k+1}$的导数,并令其为零,即令$\dfrac{\partial\text{tr}(\varPi_{i,k+1|k+1})}{\partial K_{i,k+1}}=0$,则可求得如式(28)所示的滤波器增益. □

注3　本文针对式(32)中$\boldsymbol{E}\{\zeta_{i,k+1}v^{\mathrm{T}}_{i,k+1}\}$包含不确定项的问题,并未采取常规的放大办法,而是技巧性地利用克罗内克函数采用式(33)所示的处理方式. 与放大的方法相比,本文的处理方法保守性较低.

注4　从定理2可以看出,滤波误差协方差上界矩阵$\varPi_{i,k+1|k+1}$的维数是$n\times n$,与复杂网络的规模N无关. 显然,与矩阵增广方法相比,本文给出的滤波算法降低了设计的保守性,具有更高的计算效率,提高了工程实施的灵活性.

3　有界性分析

下面分析滤波误差协方差阵的有界性. 首先给出如下假设.

假设1　对于任意i和k,存在正标量$\underline{r}_{iw}$、$\bar{r}_{iw}$、$\underline{b}_i$和$\bar{b}_i$使得

$$\underline{r}_{iw}\leqslant\|R_{iw,k+1}\|\leqslant\bar{r}_{iw},\ \underline{b}_i\leqslant\|B_{i,k}\|\leqslant\bar{b}_i.$$

定理3　考虑具有N个节点的时变复杂网络(1),基于定理2与假设1,则滤波误差协方差的上界$\varPi_{i,k+1|k+1}$是有界的,即

$$\varPi_{i,k+1|k+1}\leqslant(1+\rho_3)(\pi_{i,k}+\bar{b}_i^2\bar{r}_{iw})I. \tag{36}$$

其中

$$\begin{aligned}&\pi_{i,k}=\\&(1+a_i\epsilon_1)[\text{tr}(\bar{A}_{i,k}(\varPi^{-1}_{i,k|k}-\rho_1 I)^{-1}\bar{A}^{\mathrm{T}}_{i,k})+\\&\rho_1^{-1}\text{tr}(W_{i,k}W^{\mathrm{T}}_{i,k})]+(\epsilon_1^{-1}+a_i)\times\\&\sum_{j=1}^{N}\gamma_{ij}[\text{tr}(H_{j,k}(\varPi^{-1}_{j,k|k}-\rho_2 I)^{-1}H^{\mathrm{T}}_{j,k})+\\&\rho_2^{-1}\text{tr}(J_{i,k}J^{\mathrm{T}}_{i,k})].\end{aligned} \tag{37}$$

证明　根据定理2中的式(24),有

$$\varPi_{i,k+1|k}\leqslant\pi_{i,k}I+B_{i,k}R_{iw,k}B^{\mathrm{T}}_{i,k}, \tag{38}$$

其中$\pi_{i,k}$即式(37)所示. 将式(28)代入(25)中,可得

$$\begin{aligned}&\varPi_{i,k+1|k+1}=\\&(1+\rho_3)\varPi_{i,k+1|k}-(1+\rho_3)^2\varPi_{i,k+1|k}C^{\mathrm{T}}_{i,k+1}\times\\&\varOmega^{-1}_{i,k+1}C_{i,k+1}\varPi^{\mathrm{T}}_{i,k+1|k}.\end{aligned} \tag{39}$$

式(39)中$\varPi_{i,k+1|k}$、$C^{\mathrm{T}}_{i,k+1}$、$\varOmega^{-1}_{i,k+1}$、$C_{i,k+1}$和$\varPi^{\mathrm{T}}_{i,k+1|k}$显然是正半定的,因此可得

$$\varPi_{i,k+1|k+1}\leqslant(1+\rho_3)\varPi_{i,k+1|k}. \tag{40}$$

结合式(38)～(40)及假设1即证得定理3. □

4　仿真算例

考虑式(1)描述的具有如下4个节点的复杂网络:

$$A_{1,k}=\begin{bmatrix}1.5\sin(0.01k) & 0.45\\ 0.2 & -0.5\sin(0.2k+2)\end{bmatrix},$$

$$A_{2,k}=\begin{bmatrix}2.35 & \cos(0.1k)\\ 0.1 & -0.3\end{bmatrix},$$

$$A_{3,k}=\begin{bmatrix}-0.25\sin(0.1k) & -0.5\\ 0.2 & 1.4\sin(0.1k)\end{bmatrix},$$

$$A_{4,k}=\begin{bmatrix}0.5\cos(0.1k) & 0.5\\ 1.3 & -0.6\end{bmatrix}.$$

测量噪声$v_{i,k}$与过程噪声$\omega_{i,k}(i=1,2,3,4)$皆为零均值白噪声,其协方差依次分别为

$$R_{1v,k}=\text{diag}\{0.2,0.1\},\ R_{2v,k}=\text{diag}\{0.2,0.3\},$$

$$R_{3v,k}=\text{diag}\{0.1,0.1\},\ R_{4v,k}=\text{diag}\{0.2,0.3\};$$

$$R_{1\omega,k}=0.2,\ R_{2\omega,k}=0.1,$$

$$R_{3\omega,k}=0.4,\ R_{4\omega,k}=0.3.$$

非线性函数$g(x_{i,k})$为

$$g(x_{i,k})=\begin{bmatrix}x^{(1)}_{i,k}+\sin(x^{(1)}_{i,k}x^{(2)}_{i,k})\\ 0.5x^{(2)}_{i,k}+\sin(x^{(1)}_{i,k}x^{(2)}_{i,k})\end{bmatrix}.$$

不失一般性,设置复杂网络的非线性耦合函数$h(x_{i,k})=g(x_{i,k})$. 复杂网络节点间的耦合强度为$\gamma_{ij}=0.5(i=1,2,3,4;j=1,2,3,4)$. 系统的参数矩阵$B_{i,k}$及$C_{i,k}$分别为

$$B_{1,k}=[0.05,0.02]^{\mathrm{T}},\ B_{2,k}=[-0.03,0.02]^{\mathrm{T}},$$

$$B_{3,k}=[0.02,0.06]^{\mathrm{T}},\ B_{4,k}=[0.04,-0.01]^{\mathrm{T}};$$

$$C_{1,k}=\begin{bmatrix}0.6 & 0.25\\0.3 & 0.2\end{bmatrix},\ C_{2,k}=\begin{bmatrix}0.5 & 0.6\\0.5 & 0.2\end{bmatrix},$$

$$C_{3,k}=\begin{bmatrix}0.8 & -1.2\\0.6 & 0.8\end{bmatrix},\ C_{4,k}=\begin{bmatrix}0.85 & 0.95\\0.5 & 0.3\end{bmatrix}.$$

饱和函数

$$S_{1,\max}=[5,26]^{\mathrm{T}},\ S_{2,\max}=[18,3]^{\mathrm{T}},$$

$$S_{3,\max}=[25,8]^{\mathrm{T}},\ S_{4,\max}=[9,15]^{\mathrm{T}}.$$

设置参数

$$\rho_1=1,\ \rho_2=20,\ \rho_3=2,$$

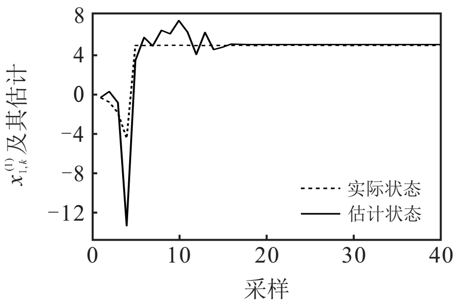
$$\lambda_i=0.01,\ \mu_i=\frac{1}{2},\ \varepsilon_i=3,\ i=1,2,3,4.$$

初始值设为

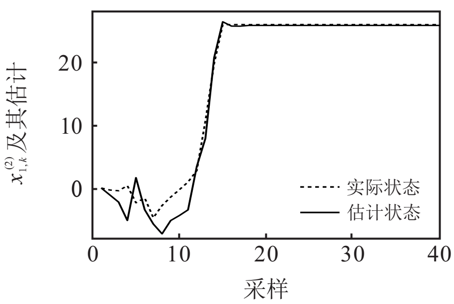
$$x_{1,0}=[-0.3,0.1]^{\mathrm{T}},\ x_{2,0}=[-0.7,0.2]^{\mathrm{T}},$$

$$x_{3,0}=[0.5,-0.2]^{\mathrm{T}},\ x_{4,0}=[0.3,-0.25]^{\mathrm{T}}.$$

实际状态值$x_{i,k}^{(j)}$及其估计$\hat{x}_{i,k}^{(j)}$ $(i=1,2,3,4;j=1,2)$分别绘制,如图1～图4所示. 由图可见,所设计的滤波器对于式(1)描述的复杂网络在动态事件触发机制下仍具有较好的滤波效果. 表1给出了DETM与SETM下,当$\lambda_i(i=1,2,3,4)$分别取0.007、0.01及0.012时各节点的事件触发率对比情况. 从表1可以观察到,与SETM相比,DETM下的事件触发率更低. 综合以上仿真结果可以看出,本文设计的滤波算法实现了在进一步降低无线网络传输负担、节约能量的同时,保证滤波性能的目的,验证了设计方法的有效性.

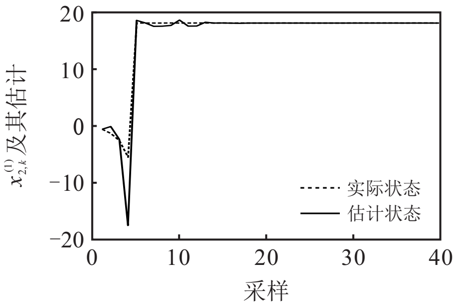


(a) 状态$x_{1,k}^{(1)}$及其估计

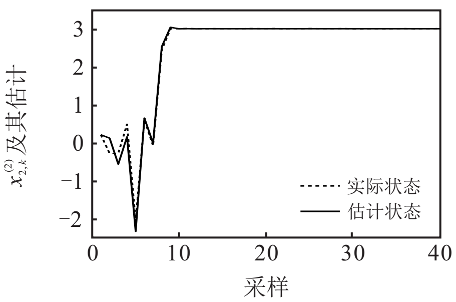


(b) 状态$x_{1,k}^{(2)}$及其估计

图1 实际状态$x_{1,k}$及其估计

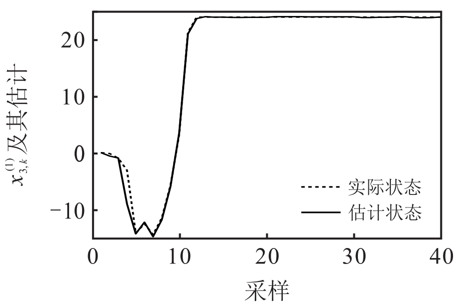


(a) 状态$x_{2,k}^{(1)}$及其估计

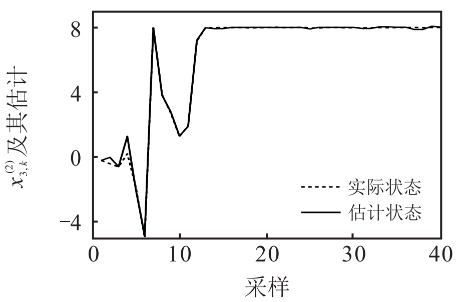


(b) 状态$x_{2,k}^{(2)}$及其估计

图2 实际状态$x_{2,k}$及其估计



(a) 状态$x_{3,k}^{(1)}$及其估计

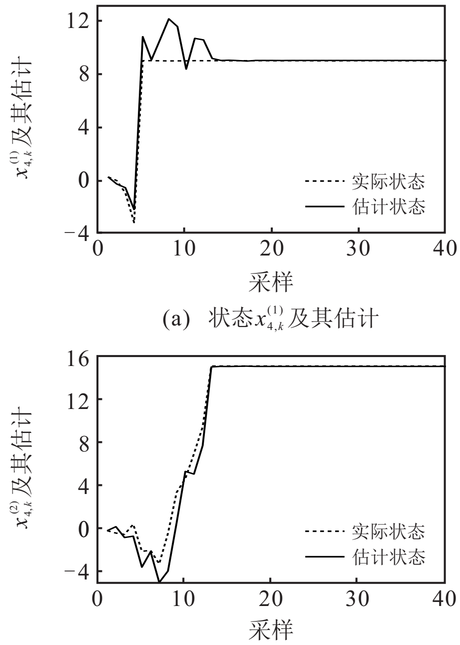


(b) 状态$x_{3,k}^{(2)}$及其估计

图3 实际状态$x_{3,k}$及其估计

表1 两种触发机制下各节点触发率 %

触发方式	节点1	节点2	节点3	节点4	平均触发率
DETM ($\lambda_i=0.007/0.01/0.012$)	65/32.5/32.5	17.5/17.5/17.5	65/55/47.5	25/25/25	37.5
SETM ($\lambda_i=0.007/0.01/0.012$)	70/67.5/60	17.5/17.5/17.5	80/60/57.5	25/25/25	45.625



(a) 状态$x_{4,k}^{(1)}$及其估计

(b) 状态$x_{4,k}^{(2)}$及其估计

图 4　实际状态$x_{4,k}$及其估计

5 结　论

本文以带有非线性耦合的状态饱和复杂网络为对象,在动态事件触发机制下,研究了递推滤波问题. 针对该类复杂网络,提出了一种新的且保守性较低的滤波器设计方案,并分析了滤波误差协方差的有界性. 最后通过一个仿真实例验证了提出的滤波器设计方案在DETM下是有效的. 未来,将研究不同传输协议下的递推滤波问题、时变复杂网络的同步问题和离散复杂网络的控制问题,并开展滤波器设计对系统性能影响的相关分析.

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