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# 多电机驱动系统的一致性控制

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**摘　要:** 以单摆系统为例,研究一类多电机驱动系统的一致性控制问题. 针对一个由6个直流电动机驱动的单摆系统,提出一种基于比例积分观测器的一致性协议控制设计方法. 利用 $H_\infty$ 技术,所提观测器可以在估计系统状态的同时,得到未知输入和可测噪声的有效估计,在此基础上构建分布式一致性控制协议,并将求解观测器增益矩阵和一致性增益矩阵转化为求解线性矩阵不等式的问题. 最后,对某给定参数的多电动机驱动的单摆系统进行Matlab仿真,结果表明所提方法是正确且有效的.

**关键词:** 多电动机驱动；一致性控制；比例积分观测器；未知输入；可测噪声；线性矩阵不等式

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## Consensus control of multi motor drive systems

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**Abstract:** In this paper, the problem of consensus control for a class of multi motor drive systems is studied. For a single pendulum system driven by six direct-current motors, a consensus protocol control design method based on a proportional integral observer is proposed. Based on $H_\infty$ technology, the proposed observer can estimate the states of the systems and obtain the effective estimation of unknown inputs and measurable noises. On this basis, a distributed consensus control protocol is constructed. The solution of observer gain matrixes and consensus gain matrixes is transformed into solving linear matrix inequality problems. The Matlab simulation of a single pendulum system driven by multiple motors with given parameters shows that the proposed method is correct and effective.

**Keywords:** multi motor drive；consensus control；proportional integral observer；unknown input；measurable noise；linear matrix inequality

## 0　引　言

近年来,多智能体控制一直是学术界的研究热点,如集群控制、编队问题等[1-4]. 作为多智能体领域的基本问题,一致性控制也受到了许多学者的关注. 多智能体系统通过智能体之间的信息交互,按照一致性控制协议,使所有智能体达到一致的系统状态. 随着多智能体系统一致性研究的越发深入,在理论研究和实际应用中都取得了丰硕的研究成果[5-14].

作为一种常见的多智能体系统,多电动机驱动系统在工业领域得到了非常广泛的应用. 当多电动机驱动系统中存在共模电流时:文献[15]提出了一种无屏蔽和屏蔽电缆的高频模型;文献[16]提出了一种新型的偏差耦合控制结构,能够对电动机驱动系统的同步性能与跟踪性能进行解耦;文献[17]提出了一种新颖的速度和位置双闭环反馈控制方式,有效提升了多电动机驱动的多关节机器人的表现性能. 针对含有不确定性参数的多电动机驱动系统:文献[18]提出了一种基于最优保性能鲁棒Funnel控制方法;文献[19]提出了一种预测控制方法,有效解决了电动汽车多电机驱动系统的能量最优分配策略. 针对一类多电机卷绕系统的故障检测和隔离问题:文献[20]提出了一种基于滑模观测器的故障检测和隔离策略;文献[21]针对只有部分状态可测的多电动机驱动系统,基于未知输入观测器,实现了系统状态一致问题并对系统进行了故障检测.

以上研究大多未考虑系统中含有的未知输入或

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可测噪声,实际系统中未知干扰或噪声不可避免,为了得到更加实用的结果,研究中有必要给予考虑. 同时,多电机驱动系统的一致性控制研究广泛应用于工业自动化等领域[22-23]. 基于此,本文研究同时包含未知输入和可测噪声的多电机驱动系统,提出一种基于比例积分观测器的一致性控制协议设计方法. 首先,介绍图论的相关知识并构建由6个直流电动机驱动的单摆系统;然后,设计比例积分观测器并对系统的稳定性进行分析;最后,对所构建的单摆系统进行仿真分析,并给出本文的结论.

## 1 问题描述与模型建立

多智能体系统研究中,通常用$G=(\nu,\varepsilon,A_0)$表示一个无向图. 其中:$\nu=\{1,2,\dots,N\}$表示图中节点的集合,$\varepsilon\subseteq\nu\times\nu$表示边的集合,$A_0=[a_{ij}]$表示图$G$的非负邻接矩阵. $N_i=\{j:(i,j)\in\varepsilon\}$表示节点$i$的所有邻节点的集合. 令$D_{\rm in}(i)=\sum\limits_{j\in N_i}a_{ij}$表示节点$i$的度,则图$G$的拉普拉斯矩阵定义为$L_a=D_{\rm in}-A_0$,其中$D_{\rm in}={\rm diag}\{D_{\rm in}(1),D_{\rm in}(2),\dots,D_{\rm in}(N)\}$.

**引理1**[24] 无向图$G$的拉普拉斯矩阵$L_a$具有如下性质:

1) ${\rm rank}(L_a)=N-1$;

2) 矩阵$L_a$有且仅有一个特征值为0, 0特征值对应的特征向量的元素均为1,矩阵$L_a$的其余特征值均为非负实数.

根据引理1,存在一个正交矩阵$\varPi$,使得

$$L_a=\varPi\varLambda\varPi^{\rm T}.$$

其中:$\varLambda={\rm diag}\{0,\lambda_2,\dots,\lambda_N\}$,元素$0,\lambda_2,\dots,\lambda_N$表示矩阵$L_a$的$N$个特征值.

考虑如图1所示的拓扑结构的多电动机驱动单摆系统.

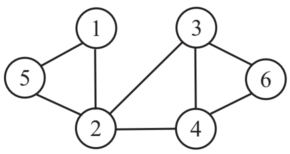


**图1 通信拓扑**

图1中,每个单摆系统的动力学方程如下[20]:

$$u=L_0\frac{{\rm d}i}{{\rm d}t}+Ri+k_E\dot{\theta},\tag{1}$$

$$ml^2\ddot{\theta}=-mgl\sin\theta+k_Ti.\tag{2}$$

其中:$u$表示电枢绕组端电压,$L_0$表示电枢回路自感系数,$i$表示电流,$R$表示回路电阻,$m$表示摆球质量,$g$表示重力加速度,$l$表示摆杆长度,$\theta$表示摆角,$k_T$表示转矩常数,$k_E$表示电动势常数. 考虑每个单摆系统均存在有界的未知输入,因控制大多由计算机完成,对原系统进行离散化可得

$$\begin{cases}x_{i1}(k+1)=T_sx_{i2}(k)+x_{i1}(k),\\ x_{i2}(k+1)=T_s\Big(-\dfrac{g}{l}\sin(x_{i1}(k))+\dfrac{k_T}{ml^2}x_{i3}(k)\Big)+\\ \qquad x_{i2}(k)+d_i(k),\\ x_{i3}(k+1)=T_s\Big(-\dfrac{R}{L_0}x_{i3}(k)-\dfrac{k_E}{L_0}x_{i2}(k)+\\ \qquad \dfrac{1}{L_0}u_i(k)\Big)+x_{i3}(k)+d_i(k).\end{cases}\tag{3}$$

其中:$x_{i1}(k)$表示单摆摆角;$x_{i2}(k)$表示单摆角速度;$x_{i3}(k)$表示电动机电流;$u_i(k)$表示控制输入;$d_i(k)$表示有界的未知输入信号,假设$d_i(k)=-0.7\sin(25k)$;$T_s$表示采样时间. 令$x_i(k)=(x_{i1}(k),x_{i2}(k),x_{i3}(k))^{\rm T}$,则式(3)可整理如下:

$$x_i(k+1)=\begin{bmatrix}1 & T_s & 0\\ 0 & 1 & T_s\dfrac{k_T}{ml^2}\\ 0 & -T_s\dfrac{k_E}{L_0} & 1-T_s\dfrac{R}{L_0}\end{bmatrix}x_i(k)+\begin{bmatrix}0\\0\\T_s\dfrac{1}{L_0}\end{bmatrix}u_i(k)+\begin{bmatrix}0\\1\\1\end{bmatrix}d_i(k)+\begin{bmatrix}0\\T_s\\0\end{bmatrix}\Big(-\frac{g}{l}\sin(x_{i1}(k))\Big),$$
$$i=1,2,\dots,6.\tag{4}$$

假设每个单摆只有角位移$x_{i1}(k)$和电流$x_{i3}(k)$可测,且输出通道含可测噪声,则输出可表示为

$$y_i(k)=\begin{bmatrix}1&0&0\\0&0&1\end{bmatrix}x_i(k)+\begin{bmatrix}1\\1\end{bmatrix}w_i(k).\tag{5}$$

其中:$y_i(k)$表示系统输出;$w_i(k)$表示可测噪声,并假设$w_i(k)=-0.5\sin(25k)$.

由式(4)和(5)可知,第$i$个子系统的模型如下所示:

$$\begin{cases}x_i(k+1)=Ax_i(k)+Bu_i(k)+Dd_i(k)+E\varphi_i(k),\\ y_i(k)=Cx_i(k)+Fw_i(k),\ i=1,2,\dots,6.\end{cases}\tag{6}$$

其中:$x_i(k)\in\boldsymbol{R}^3$, $u_i(k)\in\boldsymbol{R}^1$, $y_i(k)\in\boldsymbol{R}^2$, $d_i(k)\in\boldsymbol{R}^1$分别表示系统状态向量、系统输入向量、系统输出向量和未知输入向量;系数矩阵$A,B,C,D,E$表示已知的适维矩阵. 非线性函数$\varphi_i(k)=-\dfrac{g}{l}\sin(x_{i1}(k))\in\boldsymbol{R}^1$满足Lipschitz条件,即对于任意$x_i(k)$, $x_j(k)\in\boldsymbol{R}^3$,可求得Lipschitz常数$\gamma>0$,使得

$$\|\varphi_i(k)-\varphi_j(k)\| \leqslant \gamma\|x_i(k)-x_j(k)\|$$

成立.

## 2 状态估计与一致性控制

定义广义向量 $\chi_i(k) = [x_i(k)^{\mathrm{T}} \quad w_i(k)^{\mathrm{T}}]^{\mathrm{T}}, \vartheta = [I \ \ 0], \tilde{A} = [A \ \ 0], \tilde{C} = [C \ \ F]$,系统(6)可以写成

$$\begin{cases} \vartheta\chi_i(k+1) = \\ \tilde{A}\chi_i(k) + Bu_i(k) + Dd_i(k) + E\varphi_i(k), \\ y_i(k) = \tilde{C}\chi_i(k), \end{cases} \tag{7}$$

其中 $\varphi_i(k) = \varphi_i(\vartheta\chi_i(k), k)$. 显然

$$\operatorname{rank}\left(\begin{bmatrix}\vartheta \\ \tilde{C}\end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} I & 0 \\ C & F\end{bmatrix}\right) = n + s.$$

因此,存在列满秩矩阵 $M$ 和普通矩阵 $N$,使得

$$M\vartheta + N\tilde{C} = I.$$

其中: $M \in \boldsymbol{R}^{4\times3}, N \in \boldsymbol{R}^{4\times2}$. 则系统(7)可写为

$$\begin{cases} \chi_i(k+1) = M\tilde{A}\chi_i(k) + MBu_i(k) + \\ \qquad MDd_i(k) + ME\varphi_i(k) + \\ \qquad Ny_i(k+1), \\ y_i(k) = \tilde{C}\chi_i(k). \end{cases} \tag{8}$$

本文采用文献[25]中的比例积分观测器设计方法对每个子节点系统设计一个如下观测器:

$$\begin{cases} \hat{\chi}_i(k+1) = M\tilde{A}\hat{\chi}_i(k) + MBu_i(k) + MD\hat{d}_i(k) + \\ \qquad ME\hat{\varphi}_i(k) + Ny_i(k+1) + \\ \qquad L_1(y_i(k) - \tilde{C}\hat{\chi}_i(k)), \\ \hat{d}_i(k+1) = \hat{d}_i(k) + L_2(y_i(k) - \tilde{C}\hat{\chi}_i(k)). \end{cases} \tag{9}$$

其中: $\hat{\chi}_i(k) \in \boldsymbol{R}^4$ 表示系统状态向量 $\chi_i(k)$ 的估计, $\hat{\varphi}_i(k) \in \boldsymbol{R}^1$ 表示非线性函数 $\varphi_i(k)$ 的估计. 同时,观测器增益矩阵 $L_1 \in \boldsymbol{R}^{4\times2}$, $L_2 \in \boldsymbol{R}^{1\times2}$.

基于观测器(9)的估计结果,设计如下分布式一致性控制协议:

$$u_i(k) = K\sum_{j\in N_i} a_{ij}(\hat{\chi}_j(k) - \hat{\chi}_i(k)), \tag{10}$$

其中 $K \in \boldsymbol{R}^{1\times4}$ 表示一致性控制增益矩阵.

定义状态估计误差 $e_i(k) = \chi_i(k) - \hat{\chi}_i(k)$,未知输入估计误差 $\eta_i(k) = d_i(k) - \hat{d}_i(k)$,由式(8)和(9)可得

$$e_i(k+1) = \chi_i(k+1) - \hat{\chi}_i(k+1) =$$
$$(M\tilde{A} - L_1\tilde{C})e_i(k) + MD\eta_i(k) + ME\tilde{\varphi}_i(k), \tag{11}$$
$$\eta_i(k+1) = d_i(k+1) - \hat{d}_i(k+1) =$$
$$\eta_i(k) - L_2\tilde{C}e_i(k) + \Delta d_i(k). \tag{12}$$

其中: $\tilde{\varphi}_i(k) = \varphi_i(k) - \hat{\varphi}_i(k), \Delta d_i(k) = d_i(k+1) - d_i(k)$.

将式(11)和(12)写在一起,可得

$$\begin{bmatrix} e_i(k+1) \\ \eta_i(k+1)\end{bmatrix} =$$
$$\begin{bmatrix} M\tilde{A} - L_1\tilde{C} & MD \\ -L_2\tilde{C} & I\end{bmatrix}\begin{bmatrix} e_i(k) \\ \eta_i(k)\end{bmatrix} +$$
$$\begin{bmatrix} ME \\ 0\end{bmatrix}\tilde{\varphi}_i(k) + \begin{bmatrix} 0 \\ I\end{bmatrix}\Delta d_i(k) =$$
$$\left(\begin{bmatrix} M\tilde{A} & MD \\ 0 & I\end{bmatrix} - \begin{bmatrix} L_1 \\ L_2\end{bmatrix}[\tilde{C} \ \ 0]\right)\begin{bmatrix} e_i(k) \\ \eta_i(k)\end{bmatrix} +$$
$$\begin{bmatrix} ME \\ 0\end{bmatrix}\tilde{\varphi}_i(k) + \begin{bmatrix} 0 \\ I\end{bmatrix}\Delta d_i(k). \tag{13}$$

取观测器估计误差

$$\sigma_i(k) = \begin{bmatrix} e_i(k) \\ \eta_i(k)\end{bmatrix}, \ \bar{A} = \begin{bmatrix} M\tilde{A} & MD \\ 0 & I\end{bmatrix}, \ \bar{C} = [\tilde{C} \ \ 0],$$
$$\bar{E} = \begin{bmatrix} ME \\ 0\end{bmatrix}, \ H = \begin{bmatrix} 0 \\ I\end{bmatrix}, \ L = \begin{bmatrix} L_1 \\ L_2\end{bmatrix},$$

则式(13)可写成

$$\sigma_i(k+1) = (\bar{A} - L\bar{C})\sigma_i(k) + \bar{E}\tilde{\varphi}_i(k) + H\Delta d_i(k). \tag{14}$$

定义

$$\sigma(k) = (\sigma_1^{\mathrm{T}}(k), \sigma_2^{\mathrm{T}}(k), \ldots, \sigma_6^{\mathrm{T}}(k))^{\mathrm{T}},$$
$$\tilde{\varphi}(k) = (\tilde{\varphi}_1^{\mathrm{T}}(k), \tilde{\varphi}_2^{\mathrm{T}}(k), \ldots, \tilde{\varphi}_6^{\mathrm{T}}(k))^{\mathrm{T}},$$
$$\Delta d(k) = (\Delta d_1^{\mathrm{T}}(k), \Delta d_2^{\mathrm{T}}(k), \ldots, \Delta d_6^{\mathrm{T}}(k))^{\mathrm{T}},$$

进一步可得

$$\sigma(k+1) = (I_6 \otimes (\bar{A} - L\bar{C}))\sigma(k) + (I_6 \otimes \bar{E})\tilde{\varphi}(k) + (I_6 \otimes H)\Delta d(k). \tag{15}$$

定义系统状态和未知输入同步误差分别为

$$\delta_i(k) = \chi_i(k) - \frac{1}{6}\sum_{j=1}^{6}\chi_j(k) = \chi_i(k) - \bar{\chi}_i(k), \tag{16}$$

$$\varepsilon_i(k) = d_i(k) - \frac{1}{6}\sum_{j=1}^{6}d_j(k) = d_i(k) - \bar{d}(k). \tag{17}$$

根据式(8)和(16)有

$$\delta_i(k+1) = M\tilde{A}\delta_i(k) + MD\varepsilon_i(k) + ME\bar{\varphi}_i(k) + MBK\sum_{j\in N_i}a_{ij}(\hat{\chi}_j(k) - \hat{\chi}_i(k)) + \frac{1}{6}\sum_{i=1}^{6}\sum_{j\in N_i}MBKa_{ij}(\hat{\chi}_j(k) - \hat{\chi}_i(k)) + N\tilde{C}\Delta\delta_i(k) + N\tilde{C}\delta_i(k). \tag{18}$$

其中: $\bar{\varphi}_i(k)=\varphi_i(k)-\frac{1}{6}\sum_{j=1}^{6}\varphi_j(k)$, $\Delta\delta_i(k)=\delta_i(k+1)-\delta_i(k)$. 由无向拓扑图的对称性$a_{ij}=a_{ji}$可知

$$\frac{1}{6}\sum_{i=1}^{6}\sum_{j\in N_i}MBKa_{ij}(\hat{\chi}_j(k)-\hat{\chi}_i(k))=0, \tag{19}$$

并注意到

$$\begin{aligned}&\sum_{j\in N_i}a_{ij}(\hat{\chi}_j(k)-\hat{\chi}_i(k))=\\&\sum_{j\in N_i}a_{ij}((\chi_j(k)-\chi_i(k))+\\&(\hat{\chi}_j(k)-\chi_j(k))-(\hat{\chi}_i(k)-\chi_i(k)))=\\&\sum_{j\in N_i}a_{ij}(\chi_j(k)-\chi_i(k)+e_j-e_i).\end{aligned} \tag{20}$$

将式(19)和(20)代入(18)并假设$\Delta\delta_i(k)=0$可得

$$\begin{aligned}&\delta_i(k+1)=\\&(M\tilde{A}+N\tilde{C})\delta_i(k)+MD\varepsilon_i(k)+\\&MBK\sum_{j\in N_i}a_{ij}(\delta_j(k)-\delta_i(k))+\\&MBK\sum_{j\in N_i}a_{ij}(e_j(k)-e_i(k))+ME\bar{\varphi}_i(k).\end{aligned} \tag{21}$$

定义$\delta(k)=(\delta_1^{\mathrm T}(k),\delta_2^{\mathrm T}(k),\dots,\delta_6^{\mathrm T}(k))^{\mathrm T}$, $\bar{\varphi}(k)=(\bar{\varphi}_1^{\mathrm T}(k),\bar{\varphi}_2^{\mathrm T}(k),\dots,\bar{\varphi}_6^{\mathrm T}(k))^{\mathrm T}$, $\varepsilon(k)=(\varepsilon_1^{\mathrm T}(k),\varepsilon_2^{\mathrm T}(k),\dots,\varepsilon_6^{\mathrm T}(k))$. 同时, 令$e(k)=(I_6\otimes G)\sigma(k)$, 其中$G=[I\ \ 0]$, 则有

$$\begin{aligned}&\delta(k+1)=\\&(I_6\otimes(M\tilde{A}+N\tilde{C}))\delta(k)+\\&(I_6\otimes MD)\varepsilon(k)+(I_6\otimes ME)\bar{\varphi}(k)-\\&(L_a\otimes MBK)\delta(k)-(L_a\otimes MBKG)e(k).\end{aligned} \tag{22}$$

**定理1**　考虑由6个非线性智能体(6)构成的互联系统, 对于一个给定的$H_\infty$性能指标$\tau$, 拉普拉斯矩阵特征值$\lambda_i(i=1,2,\dots,6)$和大于0的标量常数$\gamma$、$\kappa$, 如果存在适当维数的对称正定矩阵$P_1$、$P_2$, 以及适维矩阵$Q_1$、$Q_2$使得线性矩阵不等式

$$\begin{bmatrix}\Omega_{11} & * & * & * & * & * & * & *\\ \Omega_{21} & \Omega_{22} & * & * & * & * & * & *\\ \Omega_{31} & \Omega_{32} & \Omega_{33} & * & * & * & * & *\\ \Omega_{41} & 0 & 0 & \Omega_{44} & * & * & * & *\\ \Omega_{51} & \Omega_{52} & \Omega_{53} & 0 & \Omega_{55} & * & * & *\\ \Omega_{61} & 0 & 0 & \Omega_{64} & 0 & \Omega_{66} & * & *\\ \Omega_{71} & \Omega_{72} & 0 & 0 & 0 & 0 & -P_2 & *\\ \Omega_{81} & 0 & 0 & 0 & 0 & 0 & 0 & -P_1\end{bmatrix}<0 \tag{23}$$

有解. 其中

$$\begin{aligned}&\Omega_{11}=-P_1+(\kappa\gamma^2+1)I,\\&\Omega_{21}=-\lambda_i(M\tilde{A}+N\tilde{C})^{\mathrm T}Q_2G,\\&\Omega_{31}=-\lambda_i(MD)^{\mathrm T}Q_2G,\ \Omega_{41}=H^{\mathrm T}(P_1\bar{A}-Q_1\bar{C}),\\&\Omega_{51}=-\lambda_i(ME)^{\mathrm T}Q_2G,\ \Omega_{61}=\bar{E}^{\mathrm T}(P_1\bar{A}-Q_1\bar{C}),\\&\Omega_{71}=\lambda_iQ_2G,\ \Omega_{81}=P_1\bar{A}-Q_1\bar{C},\\&\Omega_{22}=(M\tilde{A}+N\tilde{C})^{\mathrm T}P_2(M\tilde{A}+N\tilde{C})-P_2-\lambda_i(M\tilde{A}+\\&\qquad N\tilde{C})^{\mathrm T}Q_2-\lambda_iQ_2^{\mathrm T}(M\tilde{A}+N\tilde{C})+\kappa\gamma^2I,\\&\Omega_{32}=(MD)^{\mathrm T}P_2(M\tilde{A}+N\tilde{C})-\lambda_i(MD)^{\mathrm T}Q_2,\\&\Omega_{52}=(ME)^{\mathrm T}P_2(M\tilde{A}+N\tilde{C})-\lambda_i(ME)^{\mathrm T}Q_2,\\&\Omega_{72}=\lambda_iQ_2,\ \Omega_{33}=(MD)^{\mathrm T}P_2MD,\\&\Omega_{53}=(ME)^{\mathrm T}P_2MD,\ \Omega_{44}=H^{\mathrm T}P_1H-\tau^2I,\\&\Omega_{64}=\bar{E}^{\mathrm T}P_1H,\ \Omega_{55}=(ME)^{\mathrm T}P_2ME-\kappa I,\\&\Omega_{66}=\bar{E}^{\mathrm T}P_1\bar{E}-\kappa I.\end{aligned}$$

则误差系统(15)满足$H_\infty$性能指标$\|\sigma(k)\|\leqslant\tau\|\Delta d(k)\|$. 此时观测器增益矩阵$L=P_1^{-1}Q_1$, 一致性控制增益矩阵$K=((MB)^{\mathrm T}MB)^{-1}(MB)^{\mathrm T}P_2^{-1}Q_2$.

**证明**　选取Lyapunov函数

$$V(k)=\sigma^{\mathrm T}(k)(I_6\otimes P_1)\sigma(k)+\delta^{\mathrm T}(k)(I_6\otimes P_2)\delta(k).$$

根据式(15)和(22)可得

$$\begin{aligned}&\Delta V(k)=\\&\sigma^{\mathrm T}(k+1)(I_6\otimes P_1)\sigma(k+1)+\delta^{\mathrm T}(k+1)\cdot\\&(I_6\otimes P_2)\delta(k+1)-\sigma^{\mathrm T}(k)(I_6\otimes P_1)\sigma(k)-\\&\delta^{\mathrm T}(k)(I_6\otimes P_2)\delta(k)=\\&\sigma^{\mathrm T}(k)(I_6\otimes((\bar{A}-L\bar{C})^{\mathrm T}P_1(\bar{A}-L\bar{C})-P_1))\sigma(k)+\\&\sigma^{\mathrm T}(k)(I_6\otimes((\bar{A}-L\bar{C})^{\mathrm T}P_1H))\Delta d(k)+\sigma^{\mathrm T}(k)\cdot\\&(I_6\otimes((\bar{A}-L\bar{C})^{\mathrm T}P_1\bar{E}))\bar{\varphi}(k)+\Delta d^{\mathrm T}(k)(I_6\otimes\\&(H^{\mathrm T}P_1(\bar{A}-L\bar{C})))\sigma(k)+\Delta d^{\mathrm T}(k)(I_6\otimes(H^{\mathrm T}P_1H))\cdot\\&\Delta d(k)+\Delta d^{\mathrm T}(k)(I_6\otimes(H^{\mathrm T}P_1\bar{E}))\bar{\varphi}(k)+\bar{\varphi}^{\mathrm T}(k)\cdot\\&(I_6\otimes(\bar{E}^{\mathrm T}P_1(\bar{A}-L\bar{C})))\sigma(k)+\bar{\varphi}^{\mathrm T}(k)(I_6\otimes\\&(\bar{E}^{\mathrm T}P_1H))\Delta d(k)+\bar{\varphi}^{\mathrm T}(k)(I_6\otimes(\bar{E}^{\mathrm T}P_1\bar{E}))\bar{\varphi}(k)+\\&\delta^{\mathrm T}(k)(I_6\otimes((M\tilde{A}+N\tilde{C})^{\mathrm T}P_2(M\tilde{A}+N\tilde{C})-P_2))\cdot\\&\delta(k)+\delta^{\mathrm T}(k)(I_6\otimes((M\tilde{A}+N\tilde{C})^{\mathrm T}P_2ME))\bar{\varphi}(k)+\\&\delta^{\mathrm T}(k)(I_6\otimes((M\tilde{A}+N\tilde{C})^{\mathrm T}P_2MD))\varepsilon(k)-\delta^{\mathrm T}(k)\cdot\\&(L_a\otimes((M\tilde{A}+N\tilde{C})^{\mathrm T}P_2MBK))\delta(k)-\delta^{\mathrm T}(k)(L_a\otimes\\&((M\tilde{A}+N\tilde{C})^{\mathrm T}P_2MBKG))\sigma(k)+\bar{\varphi}^{\mathrm T}(k)(I_6\otimes\\&((ME)^{\mathrm T}P_2(M\tilde{A}+N\tilde{C})))\delta(k)+\bar{\varphi}^{\mathrm T}(k)(I_6\otimes\\&((ME)^{\mathrm T}P_2ME))\bar{\varphi}(k)+\bar{\varphi}^{\mathrm T}(k)(I_6\otimes((ME)^{\mathrm T}P_2M\cdot\\&D))\varepsilon(k)-\bar{\varphi}^{\mathrm T}(k)(L_a\otimes((ME)^{\mathrm T}P_2MBK))\delta(k)-\end{aligned}$$

$\bar{\varphi}^{\mathrm{T}}(k)(L_a\otimes((ME)^{\mathrm{T}}P_2MBKG))\sigma(k)+\varepsilon^{\mathrm{T}}(k)(I_6\otimes (MD)^{\mathrm{T}}P_2(M\tilde{A}+N\tilde{C}))\delta(k)+\varepsilon^{\mathrm{T}}(k)(I_6\otimes(MD)^{\mathrm{T}}\cdot P_2ME)\bar{\varphi}(k)+\varepsilon^{\mathrm{T}}(k)(I_6\otimes(MD)^{\mathrm{T}}P_2MD)\varepsilon(k)- \varepsilon^{\mathrm{T}}(k)(L_a\otimes(MD)^{\mathrm{T}}P_2MBK)\delta(k)-\varepsilon^{\mathrm{T}}(k)(L_a\otimes (MD)^{\mathrm{T}}P_2MBKG)\sigma(k)-\delta^{\mathrm{T}}(k)(L_a\otimes(MBK)^{\mathrm{T}}\cdot P_2(M\tilde{A}+N\tilde{C}))\delta(k)-\delta^{\mathrm{T}}(k)(L_a\otimes(MBK)^{\mathrm{T}}P_2\cdot ME)\bar{\varphi}(k)-\delta^{\mathrm{T}}(k)(L_a\otimes(MBK)^{\mathrm{T}}P_2MD)\varepsilon(k)+ \delta^{\mathrm{T}}(k)(L_a^2\otimes(MBK)^{\mathrm{T}}P_2MBK)\delta(k)+\delta^{\mathrm{T}}(k)(L_a^2\otimes (MBK)^{\mathrm{T}}P_2MBKG)\sigma(k)-\sigma^{\mathrm{T}}(k)(L_a\otimes(MBK\cdot G)^{\mathrm{T}}P_2(M\tilde{A}+N\tilde{C}))\delta(k)-\sigma^{\mathrm{T}}(k)(L_a\otimes(MBKG)^{\mathrm{T}}\cdot P_2ME)\bar{\varphi}(k)-\sigma^{\mathrm{T}}(k)(L_a\otimes(MBKG)^{\mathrm{T}}P_2MD)\varepsilon(k)+ \sigma^{\mathrm{T}}(k)(L_a^2\otimes(MBKG)^{\mathrm{T}}P_2MBK)\delta(k)+\sigma^{\mathrm{T}}(k)\cdot (L_a^2\otimes(MBKG)^{\mathrm{T}}P_2MBKG)\sigma(k). \quad (24)$

由Lipschitz条件假设可知, $\|\bar{\varphi}(k)\|\leqslant\gamma\|\delta(k)\|$, $\|\tilde{\varphi}(k)\|\leqslant\gamma\|e(k)\|$, 又易知 $\|e(k)\|\leqslant\|\sigma(k)\|$, 所以 $\|\tilde{\varphi}(k)\|\leqslant\gamma\|\sigma(k)\|$, 同时定义

$$\varrho(k)=(\varrho_1^{\mathrm{T}}(k),\varrho_2^{\mathrm{T}}(k),\ldots,\varrho_6^{\mathrm{T}}(k))^{\mathrm{T}}=(\Pi^{\mathrm{T}}\otimes I_5)\sigma(k),$$
$$\zeta(k)=(\zeta_1^{\mathrm{T}}(k),\zeta_2^{\mathrm{T}}(k),\ldots,\zeta_6^{\mathrm{T}}(k))^{\mathrm{T}}=(\Pi^{\mathrm{T}}\otimes I_4)\delta(k),$$
$$\xi(k)=(\xi_1^{\mathrm{T}}(k),\xi_2^{\mathrm{T}}(k),\ldots,\xi_6^{\mathrm{T}}(k))^{\mathrm{T}}=(\Pi^{\mathrm{T}}\otimes I_1)\varepsilon(k),$$
$$\psi(k)=(\psi_1^{\mathrm{T}}(k),\psi_2^{\mathrm{T}}(k),\ldots,\psi_6^{\mathrm{T}}(k))^{\mathrm{T}}=(\Pi^{\mathrm{T}}\otimes I_1)\bar{\varphi}(k),$$
$$\omega(k)=(\omega_1^{\mathrm{T}}(k),\omega_2^{\mathrm{T}}(k),\ldots,\omega_6^{\mathrm{T}}(k))^{\mathrm{T}}=(\Pi^{\mathrm{T}}\otimes I_1)\tilde{\varphi}(k),$$
$$\varpi(k)=(\varpi_1^{\mathrm{T}}(k),\varpi_2^{\mathrm{T}}(k),\ldots,\varpi_6^{\mathrm{T}}(k))^{\mathrm{T}}=(\Pi^{\mathrm{T}}\otimes I_1)\Delta d(k).$$

将式(24)中的变量 $\sigma(k)$、$\delta(k)$、$\varepsilon(k)$、$\bar{\varphi}(k)$、$\tilde{\varphi}(k)$、$\Delta d(k)$ 分别转换成变量 $\varrho(k)$、$\zeta(k)$、$\xi(k)$、$\psi(k)$、$\omega(k)$、$\varpi(k)$, 可得

$\Delta V(k)\leqslant \sum_{i=1}^{6}(\varrho^{\mathrm{T}}(k)((\bar{A}-L\bar{C})^{\mathrm{T}}P_1(\bar{A}-L\bar{C})-P_1)\varrho(k)+ \varrho^{\mathrm{T}}(k)(\bar{A}-L\bar{C})^{\mathrm{T}}P_1H\varpi(k)+\varrho^{\mathrm{T}}(k)(\bar{A}-L\bar{C})^{\mathrm{T}}\cdot P_1\bar{E}\omega(k)+\varpi^{\mathrm{T}}(k)H^{\mathrm{T}}P_1(\bar{A}-L\bar{C})\varrho(k)+ \varpi^{\mathrm{T}}(k)H^{\mathrm{T}}P_1H\varpi(k)+\varpi^{\mathrm{T}}(k)H^{\mathrm{T}}P_1\bar{E}\omega(k)+ \omega^{\mathrm{T}}(k)\bar{E}^{\mathrm{T}}P_1(\bar{A}-L\bar{C})\varrho(k)+\omega^{\mathrm{T}}(k)\bar{E}^{\mathrm{T}}P_1H\varpi(k)+ \omega^{\mathrm{T}}(k)\bar{E}^{\mathrm{T}}P_1\bar{E}\omega+\zeta^{\mathrm{T}}(k)((M\tilde{A}+N\tilde{C})^{\mathrm{T}}P_2(M\tilde{A}+ N\tilde{C})-P_2)\zeta(k)+\zeta^{\mathrm{T}}(k)(M\tilde{A}+N\tilde{C})^{\mathrm{T}}P_2ME\psi(k)+ \zeta^{\mathrm{T}}(k)(M\tilde{A}+N\tilde{C})^{\mathrm{T}}P_2MD\xi(k)-\zeta^{\mathrm{T}}(k)\lambda_i(M\tilde{A}+ N\tilde{C})^{\mathrm{T}}P_2MBK\zeta(k)-\zeta^{\mathrm{T}}(k)\lambda_i(M\tilde{A}+N\tilde{C})^{\mathrm{T}}P_2M\cdot BKG\varrho(k)+\psi^{\mathrm{T}}(k)(ME)^{\mathrm{T}}P_2(M\tilde{A}+N\tilde{C})\zeta(k)+ \psi^{\mathrm{T}}(k)(ME)^{\mathrm{T}}P_2ME\psi(k)+\psi^{\mathrm{T}}(k)(ME)^{\mathrm{T}}P_2MD\cdot \xi(k)-\psi^{\mathrm{T}}(k)\lambda_i(ME)^{\mathrm{T}}P_2MBK\zeta(k)-\psi^{\mathrm{T}}(k)\lambda_i\cdot (ME)^{\mathrm{T}}P_2MBKG\varrho(k)+\xi^{\mathrm{T}}(k)(MD)^{\mathrm{T}}P_2(M\tilde{A}+ N\tilde{C})\zeta(k)+\xi^{\mathrm{T}}(k)(MD)^{\mathrm{T}}P_2ME\psi(k)+\xi^{\mathrm{T}}(k)\cdot (MD)^{\mathrm{T}}P_2MD\varepsilon(k)-\xi^{\mathrm{T}}(k)\lambda_i(MD)^{\mathrm{T}}P_2MBK\zeta(k)- \xi^{\mathrm{T}}(k)\lambda_i(MD)^{\mathrm{T}}P_2MBKG\varrho(k)-\zeta^{\mathrm{T}}(k)\lambda_i(MBK)^{\mathrm{T}}\cdot P_2(M\tilde{A}+N\tilde{C})\zeta(k)-\zeta^{\mathrm{T}}(k)\lambda_i(MBK)^{\mathrm{T}}P_2ME\cdot \psi(k)-\zeta^{\mathrm{T}}(k)\lambda_i(MBK)^{\mathrm{T}}P_2MD\xi(k)+\zeta^{\mathrm{T}}(k)\lambda_i^2\cdot (MBK)^{\mathrm{T}}P_2MBK\zeta(k)+\zeta^{\mathrm{T}}(k)\lambda_i^2(MBK)^{\mathrm{T}}P_2\cdot MBKG\varrho(k)-\varrho^{\mathrm{T}}(k)\lambda_i(MBKG)^{\mathrm{T}}P_2(M\tilde{A}+ N\tilde{C})\zeta(k)-\varrho^{\mathrm{T}}(k)\lambda_i(MBKG)^{\mathrm{T}}P_2ME\psi(k)- \varrho^{\mathrm{T}}(k)\lambda_i(MBKG)^{\mathrm{T}}P_2MD\xi(k)+\varrho^{\mathrm{T}}(k)\lambda_i^2\cdot (MBKG)^{\mathrm{T}}P_2MBK\zeta(k)+\varrho^{\mathrm{T}}(k)\lambda_i^2(MBKG)^{\mathrm{T}}\cdot P_2MBKG\varrho(k)+\kappa(\gamma^2\varrho^{\mathrm{T}}(k)\varrho(k)-\omega^{\mathrm{T}}(k)\omega(k))+ \kappa(\gamma^2\zeta^{\mathrm{T}}(k)\zeta(k)-\psi^{\mathrm{T}}(k)\psi(k)). \quad (25)$

定义如下性能指标[26]:

$$J=\sum_{k=1}^{\infty}(\sigma^{\mathrm{T}}(k)\sigma(k)-\tau^2\Delta d^{\mathrm{T}}(k)\Delta d(k)). \quad (26)$$

将式(26)中的变量 $\sigma(k)$、$\Delta d(k)$ 替换为 $\varrho(k)$、$\varpi(k)$, 在零初始条件下可得

$$J\leqslant\sum_{k=1}^{\infty}(\Delta V(k)+\sigma^{\mathrm{T}}(k)\sigma(k)-\tau^2\Delta d^{\mathrm{T}}(k)\Delta d(k)). \quad (27)$$

定义 $\iota=\begin{bmatrix}\varrho^{\mathrm{T}}(k) & \zeta^{\mathrm{T}}(k) & \xi^{\mathrm{T}}(k) & \psi^{\mathrm{T}}(k) & \omega^{\mathrm{T}}(k) & \varpi^{\mathrm{T}}(k)\end{bmatrix}^{\mathrm{T}}$ 有

$$\Delta V(k)+\varrho^{\mathrm{T}}(k)\varrho(k)-\tau^2\varpi^{\mathrm{T}}(k)\varpi(k)\leqslant\iota^{\mathrm{T}}\Phi\iota.$$

其中

$$\Phi=\begin{bmatrix}\Phi_{11} & * & * & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * & * & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & * & * & * \\ \Phi_{41} & 0 & 0 & \Phi_{44} & * & * \\ \Phi_{51} & \Phi_{52} & \Phi_{53} & 0 & \Phi_{55} & * \\ \Phi_{61} & 0 & 0 & \Phi_{64} & 0 & \Phi_{66}\end{bmatrix}, \quad (28)$$

$$\Phi_{11}=(\bar{A}-L\bar{C})^{\mathrm{T}}P_1(\bar{A}-L\bar{C})-P_1+\lambda_i^2G^{\mathrm{T}}Q_2P_2^{-1}\cdot Q_2G+(\kappa\gamma^2+1)I,$$
$$\Phi_{21}=-\lambda_i(M\tilde{A}+N\tilde{C})^{\mathrm{T}}Q_2G+\lambda_i^2Q_2^{\mathrm{T}}P_2^{-1}Q_2G,$$
$$\Phi_{31}=-\lambda_i(MD)^{\mathrm{T}}Q_2G,\ \Phi_{41}=H^{\mathrm{T}}(P_1\bar{A}-Q_1\bar{C}),$$
$$\Phi_{51}=-\lambda_i(ME)^{\mathrm{T}}Q_2G,\ \Phi_{61}=\bar{E}^{\mathrm{T}}(P_1\bar{A}-Q_1\bar{C}),$$
$$\Phi_{22}=(M\tilde{A}+N\tilde{C})^{\mathrm{T}}P_2(M\tilde{A}+N\tilde{C})-P_2- \lambda_i(M\tilde{A}+N\tilde{C})^{\mathrm{T}}Q_2-\lambda_iQ_2^{\mathrm{T}}(M\tilde{A}+N\tilde{C})+ \kappa\gamma^2I+\lambda_i^2Q_2P_2^{-1}Q_2,$$
$$\Phi_{32}=(MD)^{\mathrm{T}}P_2(M\tilde{A}+N\tilde{C})-\lambda_i(MD)^{\mathrm{T}}Q_2,$$

$$\Phi_{52} = (ME)^{\mathrm{T}}P_2(M\tilde{A} + N\tilde{C}) - \lambda_i(ME)^{\mathrm{T}}Q_2,$$
$$\Phi_{33} = (MD)^{\mathrm{T}}P_2MD,\ \Phi_{53} = (ME)^{\mathrm{T}}P_2MD,$$
$$\Phi_{44} = H^{\mathrm{T}}P_1H - \tau^2 I,\ \Phi_{64} = \bar{E}^{\mathrm{T}}P_1H,$$
$$\Phi_{55} = (ME)^{\mathrm{T}}P_2ME - \kappa I,$$
$$\Phi_{66} = \bar{E}^{\mathrm{T}}P_1\bar{E} - \kappa I.$$

依据Schur补引理,$\Phi < 0$等价于式(23). 因此,如果式(23)成立,则可以得到性能指标$J < 0$,即误差系统(15)满足$H_\infty$性能指标$\|\sigma(k)\| \leqslant \tau\|\Delta d(k)\|$. □

## 3 仿真研究

为了验证本文所提方法的正确性和有效性,下面对具体的例子进行仿真验证. 某电动机驱动单摆系统的参数见表1[20].

**表1 多电动机驱动单摆系统参数**

| 参数名称 | 参数数值 |
|---|---|
| $L$ / H | 1 |
| $R$ / Ω | 1 |
| $m$ / g | 0.425 |
| $l$ / m | 20 |
| $g$ / (m/s$^2$) | 10 |
| $k_T$ / (N·m /Arms) | 0.17 |
| $k_E$ / (Vrms /(rad /s)) | 0.1 |

代入系统(4)可得

$$A = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ 0 & -0.1T & 1-T \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

$$D = [0\ \ 1\ \ 1]^{\mathrm{T}},\ E = [0\ \ T\ \ 0]^{\mathrm{T}}.$$

可以求得矩阵$M$, $N$分别为

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix},\ N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

取$\kappa = 1, \tau = 10, \gamma = 0.5$,代入线性矩阵不等式(23),可求得增益矩阵$L$和$K$分别为

$$L = \begin{bmatrix} 0.4990 & -0.0013 \\ -0.9898 & 2.0020 \\ -0.9996 & 2.0034 \\ -0.4990 & 0.0013 \\ 1.6656 & -3.3555 \end{bmatrix},$$

$$K = [153.5246\ \ -11.7750\ \ 336.5984\ \ 153.5247].$$

仿真结果如图2和图3所示,图2和图3中实线表示真实状态,点线表示状态估计. 可以看出,所提方法估计出的系统状态、未知输入和可测噪声都取得了满意的效果,并且系统在一致性控制方面的效果也较为满意.

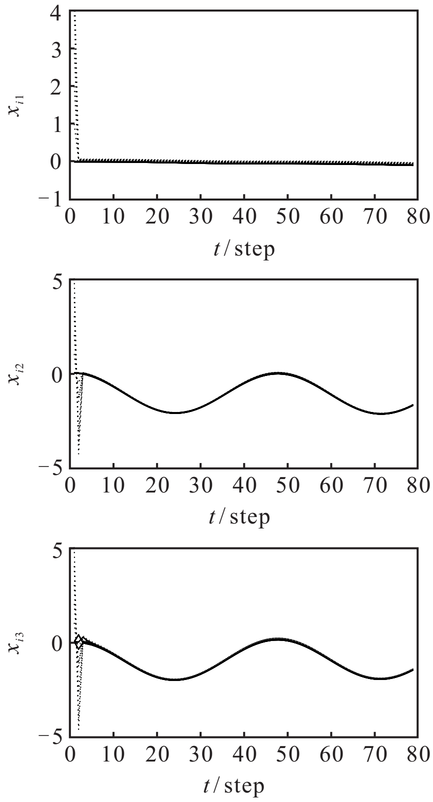


**图2 系统状态及其估计**

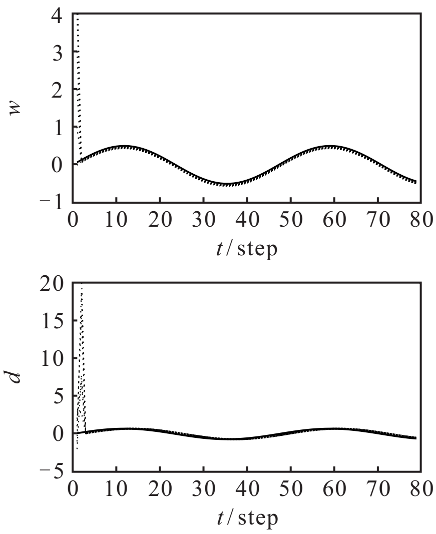


**图3 可测噪声和未知输入及其估计**

## 4 结 论

针对一类由6个直流电动机驱动的单摆系统,在同时含有未知输入和可测噪声的条件下,提出了一种基于比例积分观测器的一致性协议控制方法,利用$H_\infty$技术,所提观测器可以在估计系统状态的同时给出未知输入和可测噪声的有效估计. 仿真结果表明了所提方法的可行性和正确性.

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