状态时延和全状态约束下的多智能体系统自适应事件触发控制

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摘 要: 研究有向通信图下非线性多智能体系统的一致控制问题. 首先,通过引入性能函数,使输出误差满足预定性能; 其次, 采用障碍 Lyapunov 函数, 保证所有状态满足约束条件, 结合李雅普诺夫-克拉索夫斯基 (Lyapunov-Krasovskii, LK) 泛函和杨氏不等式消除状态时延的影响, 利用径向基函数神经网络 (radial basis function neural networks, RBF NNs) 逼近未知非线性函数; 再次, 设计自适应事件触发控制器, 实现所有智能体的输出一致性, 并基于 Lyapunov 稳定性理论证明闭环系统半全局有界稳定; 最后, 通过对比仿真验证所设计控制策略的有效性.

关键词: 多智能体系统；状态时延；事件触发控制；全状态约束

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Adaptive event-triggered control for multi-agent systems with state time-delays and full state constraints

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Abstract: This paper studies the consensus control problem for a class of nonlinear multi-agent systems under a directed communication graph. Firstly, the performance function is introduced to make the output error satisfy prescribed performance. Then, the full state constraints problem is solved by employing the Barrier Lyapunov function. The Lyapunov-Krasovskii functional and Young's inequality are used to eliminate the effects of state time-delays. The radial basis function neural networks are used to approximate the unknown nonlinear function. Furthermore, an adaptive event-triggered controller is proposed, such that the outputs of all agents can achieve the consensus. According to the Lyapunov stability theory, the closed-loop systems are semi-globally bounded stability. Finally, the validity of the designed control strategy is verified by simulation.

Keywords: multi-agent systems；state time-delays；event-triggered control；full state constraints

0 引 言

近年来,多智能体系统协同控制在工程领域应用广泛,引起了学术界的关注. 多智能体协同控制主要包括一致性问题、集群问题、编队问题等,与传统的单一智能体相比,多智能体系统由多个子系统构成,通过智能体之间的通信、协调合作完成许多非常复杂的实际问题,使得多智能体系统具有更强的鲁棒性和更好的可调控性,并且被应用在传感器网络、工业流程、无人机、移动机器人的协调控制等领域[1-7].

多智能体系统大量使用嵌入式微处理器,这意味着通信带宽和功率都是有限的. 事件触发策略可以减少智能体之间的通信频率,从而避免一些不必要的通信,只有在某个重要事件发生时控制信号才更新. 但是,事件触发机制可能出现 Zeno 现象,即触发事件会在有限时间内被无数次触发. 为了解决该问题,文献 [8] 研究了事件触发机制下的二阶多智能

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体系统的一致性问题,更多地节省了通讯和计算资源. 文献[9]进一步研究了多智能体系统的事件触发一致性问题,提出了一种分布式事件触发控制策略. 文献[10]针对多智能体系统的编队控制和跟踪问题,提出了分布式事件触发控制策略,减少了智能体之间的通信次数. 文献[11-12]针对多智能体系统在有向通信拓扑下的固定时间事件触发跟踪控制问题,提出了事件触发控制策略,降低控制器的更新频率.

然而,上述文献没有考虑状态时延对系统的影响,许多物理系统中普遍存在状态时延问题,它是系统不稳定的根源之一. 文献[13]既考虑了事件触发问题也考虑了状态时延问题,提出了新的事件触发控制策略. 为了解决状态时延对系统的影响,文献[14-16]针对具有状态时延的线性多智能体系统一致性问题,提出了一致性控制策略. 实际上,几乎所有物理系统本质上都是非线性的. 近年来,人们对具有状态时延的非线性多智能体系统的一致性问题进行了研究. 文献[17]研究了具有状态时延的多智能体系统的一致性问题,构造了LK函数以消除状态时延的影响. 文献[18]研究了一类具有噪声和状态时延的非线性多智能体系统,提出了一种鲁棒控制方法. 文献[19]研究了具有状态时延严反馈形式的非线性多智能体系统领导跟随一致性问题,利用LK泛函提出了自适应一致性控制策略. 文献[20]针对具有状态时延的多智能体系统领导跟随一致性问题,首次提出了固定有向拓扑条件下的多智能体系统输出反馈一致控制策略. 然而,上述文献均没有考虑状态约束的影响. 文献[21]研究了具有全状态约束和外界干扰的非线性多智能体系统的一致跟踪问题. 文献[22]考虑了具有全状态约束的多智能体系统,提出了事件触发自适应控制策略. 文献[23]考虑了全状态约束的影响,提出了自适应输出反馈控制策略,没有考虑状态时延的影响. 文献[24-25]针对具有全状态约束的非线性系统提出了自适应控制策略,没有考虑状态时延的影响.

文献[10-12]只考虑事件触发控制问题,文献[14-17]只考虑状态时延的影响,文献[21, 23]只考虑了状态受限问题,而同时考虑状态受限、状态时延、事件触发等因素下多智能体系统一致控制问题的成果较少. 本文同时考虑状态时延、状态受限以及外界干扰,在处理状态时延和状态受限时构造恰当的Lyapunov函数. 其由两部分构成:一是障碍Lyapunov函数,作用是保证所有状态满足约束要求;另一部分是LK泛函,作用是消除状态时延对系统的影响. 在处理外界干扰时,同时将外界干扰和LK泛函求导后的部分项定义为非线性函数,利用神经网络进行逼近. 考虑状态时延和状态受限问题,会给Lyapunov函数的设计带来一定的难度,同时设计有效处理外界干扰的神经网络也需要综合考虑状态时延和状态受限的影响. 在此基础上,为了节约控制资源和成本,需要利用有效的事件触发机制以达到减少控制器更新次数的目的. 因此,综合考虑状态受限、状态时延、外界干扰的影响,会加大多智能体一致性事件触发控制器的设计难度.

本文针对状态时延、全状态约束、外界干扰等问题,首次结合LK泛函和障碍Lyapunov函数,既可以消除状态时延的影响,又能够保证系统中所有状态满足约束要求. 进一步引入相对阈值事件触发机制,减少控制器更新次数,提出自适应事件触发控制策略. 所采用的相对阈值事件触发机制较文献[22]能够固定阈值事件触发机制产生的事件触发率更低,节约控制过程的资源和成本. 采用 Lyapunov 稳定性理论证明闭环系统半全局有界稳定,最后利用对比仿真验证所提出控制方法的有效性.

1　问题描述与预备知识

1.1　问题描述

考虑由N个智能体组成的非线性多智能体系统,每个智能体的模型为

$$\begin{aligned}&\dot{x}_{i,m}=x_{i,m+1}+g_{i,m}(\bar{x}_{i,m}(t-\tau_{i,m}))+d_{i,m}(\bar{x}_{i,m}(t)),\\&\dot{x}_{i,n}=u_i+g_{i,n}(x_i(t-\tau_{i,n}))+d_{i,n}(x_i(t)),\\&y_i=x_{i,1},\ m=1,2,\ldots,n-1.\end{aligned}\tag{1}$$

其中: $i=1,2,\ldots,N$; $x_i=[x_{i,1},\ldots,x_{i,n}]^{\rm T}\in R^n$和$u_i\in R$分别为第$i$个智能体的状态向量和控制输入; $\bar{x}_{i,m}=[x_{i,1},\ldots,x_{i,m}]^{\rm T}\in R^m$; $g_{i,m}(\bar{x}_{i,m}(t-\tau_{i,m}))$为未知光滑的非线性函数; $\tau_{i,m}$为系统的未知状态时延; $\tau_{\max}$为时延上界; $d_{i,m}$为系统的外界干扰. 为了简化符号,省略每一项中的t,如$x_i(t-\tau_{i,n})$表示为$x_i(\tau_{i,n})$.

假设 1　外界干扰$d_{i,m}$满足$|d_{i,m}(\bar{x}_{i,m})|\leqslant|\rho_{i,m}(\bar{x}_{i,m})|$,其中$\rho_{i,m}(\cdot)$为未知函数.

假设 2　时延函数$g_{i,m}(\bar{x}_{i,m})$满足不等式$|g_{i,m}(\bar{x}_{i,m})|\leqslant\sum\limits_{h=1}^{m}\varpi_{i,m,h}(x_{i,h})$,其中$\varpi_{i,m,h}(x_{i,h})$为未知正函数.

注 1　文献[26]假设时延项满足$|h_i(\bar{x}_i(t))|\leqslant\sum\limits_{j=1}^{i}|e_j(t)|q_{i,j}(\bar{e}_j(t))$. 文献[27]针对单个系统作了类似假设2的假设. 文献[17]进一步将假设扩展到多智

能体系统,其中非线性函数$\varpi_{i,m,h}(\cdot)$假设已知. 本文中假设2并不要求$\varpi_{i,m,h}(\cdot)$已知,放松了文献[17]中的假设.

本文控制目标是针对多智能体系统(1),结合反步控制设计自适应事件触发控制策略,使系统达到半全局有界稳定. 一致性输出误差满足预定性能且所有状态满足约束条件. 所有智能体实现对期望轨迹的跟踪,且在事件触发机制下,有效减少控制器更新次数,无Zeno行为发生.

考虑状态满足如下约束:

$$|x_{i,m}| < \beta_{i,m},\ m = 1, 2, \ldots, n, \tag{2}$$

其中$\beta_{i,m}$为正常数.

输出误差$\tilde{z}_{i,1}$的预定性能描述为

$$-l_{\min}\mu(t) < \tilde{z}_{i,1}(t) < l_{\max}\mu(t),\ \forall t \geqslant 0. \tag{3}$$

其中: 性能函数$\mu(t)$有界且严格单调递减, 形式为$\mu(t) = (\mu_0 - \mu_\infty)\mathrm{e}^{-\upsilon t} + \mu_\infty$; $l_{\min}$和$l_{\max}$为可调参数; υ、μ_0和μ_∞为正实数, $\mu_0 = \mu(0)$, $\mu_0 > \mu_\infty$; 误差初值满足$-l_{\min}\mu(0) < \tilde{z}_{i,1}(0) < l_{\max}\mu(0)$.

1.2 预备知识

1.2.1 图 论

定义有向图$G = (V, E, A)$. 其中: $V = \{\nu_1, \nu_2, \ldots, \nu_N\}$和$E \subseteq V \times V$分别为节点集合和边集合; 节点$j$到$i$的边记为$(\nu_j, \nu_i) \in E$, 表示智能体$i$能够接收到智能体$j$的信息; $A = [a_{ij}]_{N\times N}$为邻接矩阵. 如果$(\nu_i, \nu_j) \in E$, 则$a_{ij} \geqslant 0$, 否则$a_{ij} = 0$. 集合$N_i = \{j \in V | (i, j) \in E, i \neq j\}$表示第$i$个智能体的邻居节点的集合. 度矩阵定义为$D = \mathrm{diag}(d_1, d_2, \ldots, d_N)$, 其中$d_i = \sum_{j\in N_i} a_{ij}$. 有向图$G$的拉普拉斯矩阵为$L = D - A$. 领导者记为$\nu_0$, 定义$B = \mathrm{diag}(b_1, b_2, \ldots, b_N)$, 当第$i$个节点可以接收到领导者的信息时$b_i = 1$, 否则$b_i = 0$.

假设3 [17] 若有向图G具有一个生成树, 即存在一条路径能从根节点到所有其他节点, 则矩阵$L + B$是非奇异的, 虚拟领导者的期望轨迹为y_0.

引理1 [19] 定义向量$z_1 = [z_{1,1}, z_{2,1}, \ldots, z_{N,1}]^{\mathrm{T}}$, $Y = [y_1, y_2, \ldots, y_N]^{\mathrm{T}}$, $Y_d = [y_0, y_0, \ldots, y_0]^{\mathrm{T}}$, Y_d是N维的, 有$\|Y - Y_d\| \leqslant \|z_1\|/\underline{\zeta}(L + B)$, 其中$\underline{\zeta}(L + B)$为矩阵$L + B$的最小奇异值.

引理2 [19] $\varOmega_{z_{i,m}} = \{z_{i,m} | |z_{i,m}| < 0.881\,4\delta_{i,m}\}$ $(i = 1, 2, \ldots, N, m = 1, 2, \ldots, n)$为所定义的紧集, 若$z_{i,m} \notin \varOmega_{z_{i,m}}$, 则$1 - 2\tanh^2(z_{i,m}/\delta_{i,m}) \leqslant 0$, 其中$\delta_{i,m} > 0$为常数.

引理3 [23] 对于任意正常数k_{bl}, 如果满足不等式$|z_{i,l}| < k_{bl}$, $z_{i,l} \in R$, 则有

$$\log \frac{k_{b,l}^2}{k_{b,l}^2 - z_{i,l}^2} < \frac{z_{i,l}^2}{k_{b,l}^2 - z_{i,l}^2}.$$

引理4 [20](Young's不等式) 对于$\forall (x, y) \in R^n$, 均有不等式

$$xy \leqslant \frac{q^M}{M}|x|^M + \frac{1}{Hq^H}|y|^H$$

成立. 其中$q > 0$, $M > 1$, $H > 1$, $(M-1)(H-1) = 1$.

引理5 [19] 对于任意$\varsigma > 0$和$\xi \in R$, 满足不等式$0 \leqslant |\xi| - \xi\tanh(\xi/\varsigma) \leqslant 0.278\,5\varsigma$.

1.2.2 神经网络

利用神经网络逼近未知非线性函数

$$f(Z) \approx W^{\mathrm{T}}S(Z).$$

其中: $Z \in \aleph \subset R^n$, $W \in R^{p\times m}$为权重矩阵; $S(Z) = [S_1(Z), S_2(Z), \ldots, S_p(Z)]^{\mathrm{T}} \Big/ \sum_{i=1}^{p} S_i(Z)$为激活函数; p为神经元数量. 选取高斯函数

$$S_i(Z) = \exp\Big(-\frac{(Z - \mu_i)^{\mathrm{T}}(Z - \mu_i)}{\ell_i^2}\Big),$$
$$i = 1, 2, \ldots, p,$$

其中$\mu_i = [\mu_{i1}, \ldots, \mu_{in}]^{\mathrm{T}}$和$\ell_i$分别为高斯函数的中心和宽度.

引理6 [23] 对于任意$\varepsilon > 0$, 存在理想的神经网络$W^{*\mathrm{T}}S(Z)$, 使$f(Z) = W^{*\mathrm{T}}S(Z) + \varDelta(Z)$成立. 其中: $Z \in \aleph$, $\varDelta(Z)$为逼近误差且满足$|\varDelta(Z)| \leqslant \varepsilon$.

注2 同时考虑外界干扰、状态时延以及全状态约束的影响,提出分布式自适应事件触发控制策略,首次结合LK泛函和障碍Lyapunov函数,消除时延影响的同时保证了系统中所有状态均满足约束的要求. 进一步引入相对阈值事件触发机制,减少了控制器更新次数. 文献[22]没有考虑外界干扰和状态时延的影响. 另外,由仿真部分的对比结果可知,所采用的相对阈值事件触发机制比文献[22]的固定阈值事件触发机制产生的事件触发率更低,节约了控制过程的资源和成本.

2 自适应事件触发控制律设计

针对多智能体系统(1),基于反步控制设计自适应事件触发控制策略,设计过程包括n步,前$n-1$步设计虚拟控制输入,最后一步设计实际控制输入,引入事件触发机制,降低控制器更新频率. 在设计过程中,通过神经网络逼近未知非线性函数.

首先进行如下坐标变换:

$$\tilde{z}_{i,1} = \sum_{j\in N_i} a_{ij}(y_i - y_j) + b_i(y_i - y_0),$$
$$z_{i,m} = x_{i,m} - \alpha_{i,m-1},\ 2 \leqslant m \leqslant n, \tag{4}$$

其中$\alpha_{i,m-1}$为虚拟控制输入.

为了满足预定性能,进行如下等效变换:

$$\tilde{z}_{i,1}(t)=\mu(t)\varPsi_i(\omega_i(t)),\ t\geqslant 0.$$

其中: ω_i 为变换误差,且

$$\varPsi_i(\omega_i)=\frac{l_{\max}\mathrm{e}^{\omega_i}-l_{\min}\mathrm{e}^{-\omega_i}}{\mathrm{e}^{\omega_i}+\mathrm{e}^{-\omega_i}}.$$

由于函数 $\varPsi_i(\omega_i)$ 严格单调递增,且

$$\frac{\partial\varPsi_i}{\partial\omega_i}=\frac{2(l_{\max}+l_{\min})}{(\mathrm{e}^{\omega_i}+\mathrm{e}^{-\omega_i})^2}>0,$$

有

$$\omega_i(t)=\varPsi_i^{-1}\Big(\frac{\tilde{z}_{i,1}(t)}{\mu_i(t)}\Big)=\frac{1}{2}\ln\frac{\varPsi_i+l_{\min}}{l_{\max}-\varPsi_i}.$$

其导数为 $\dot{\omega}_i(t)=r_i(\dot{\tilde{z}}_{i,1}-\dot{\mu}\,\tilde{z}_{i,1}/\mu)$,其中

$$r_i=\frac{1}{2\mu}\Big[\frac{1}{\varPsi_i+l_{\min}}-\frac{1}{\varPsi_i-l_{\max}}\Big].$$

定义坐标变换

$$z_{i,1}(t)=\omega_i(t)-\frac{1}{2}\ln\frac{l_{\min}}{l_{\max}},$$

其导数为 $\dot{z}_{i,1}=r_i(\dot{\tilde{z}}_{i,1}-\dot{\mu}\,\tilde{z}_{i,1}/\mu)$.

为了简化符号,记 $\hbar_m=k_{bm}^2-z_{i,m}^2$. 非线性函数 $\psi_{i,m}(X_{i,m})$ 简化为 $\psi_{i,m}$, $m=1,2,\ldots,n$. $\hat{\theta}_{i,m}$ 是 $\theta_{i,m}$ 的估计,估计误差为 $\tilde{\theta}_{i,m}=\theta_{i,m}-\hat{\theta}_{i,m}$, $m=1,2,\ldots,n$. 接下来进行控制器的设计,具体步骤如下.

step 1: 设计虚拟控制输入 $\alpha_{i,1}$. 由式(4),对 $\tilde{z}_{i,1}$ 求导得

$$\dot{\tilde{z}}_{i,1}=\sum_{j\in N_i}a_{ij}(\dot{y}_i-\dot{y}_j)+b_i(\dot{y}_i-\dot{y}_0).\tag{5}$$

构造Lyapunov函数

$$V_{i,1}=\frac{1}{2}\log\frac{k_{b1}^2}{\hbar_1}+\frac{1}{2}\int_{t-\tau_{i,1}}^{t}\varpi_{i,1,1}^2(x_{i,1}(s))\mathrm{d}s+\frac{1}{2}\sum_{j\in N_i}\int_{t-\tau_{j,1}}^{t}\varpi_{j,1,1}^2(x_{j,1}(s))\mathrm{d}s+\frac{1}{2}\tilde{\theta}_{i,1}^2.\tag{6}$$

对 $V_{i,1}$ 求导,得

$$\begin{aligned}\dot{V}_{i,1}=&\\&\frac{z_{i,1}r_i}{\hbar_1}\Big((b_i+d_i)(z_{i,2}+\alpha_{i,1}+g_{i,1}(x_{i,1}(\tau_{i,1}))+\\&d_{i,1}(x_{i,1}))-\sum_{j\in N_i}a_{ij}(x_{j,2}+g_{j,1}(x_{j,1}(\tau_{j,1}))+\\&d_{j,1}(x_{j,1}))-b_i\dot{y}_0-\frac{\dot{\mu}\,\tilde{z}_{i,1}}{\mu}\Big)+\\&\frac{1}{2}[\varpi_{i,1,1}^2(x_{i,1}(t))-\varpi_{i,1,1}^2(x_{i,1}(\tau_{i,1}))]+\\&\frac{1}{2}\sum_{j\in N_j}[\varpi_{j,1,1}^2(x_{j,1}(t))-\varpi_{j,1,1}^2(x_{j,1}(\tau_{j,1}))]-\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}.\end{aligned}\tag{7}$$

由假设2和假设3,得

$$\begin{aligned}&\frac{z_{i,1}r_i}{\hbar_1}g_{i,1}\leqslant\frac{(b_i+d_i)z_{i,1}^2r_i^2}{2\hbar_1^2}+\frac{\varpi_{i,1,1}^2(x_{i,1}(\tau_{i,1}))}{2(b_i+d_i)},\\&\frac{z_{i,1}r_i}{\hbar_1}d_{i,1}\leqslant\frac{(b_i+d_i)c_{i,0,0}^2}{4}+\frac{z_{i,1}^2\rho_{i,1}^2r_i^2}{\hbar_1^2(b_i+d_i)c_{i,0,0}^2},\\&-\frac{z_{i,1}r_i}{\hbar_1}g_{j,1}\leqslant\frac{z_{i,1}^2r_i^2}{2\hbar_1^2}+\frac{\varpi_{j,1,1}^2(x_{j,1}(\tau_{j,1}))}{2},\\&-\frac{z_{i,1}r_i}{\hbar_1}d_{j,1}\leqslant\frac{c_{j,1,1}^2}{2}+\frac{z_{i,1}^2\rho_{j,1}^2r_i^2}{2\hbar_1^2c_{j,1,1}^2}.\end{aligned}\tag{8}$$

由式(7)和(8),得

$$\begin{aligned}&\dot{V}_{i,1}\leqslant\\&\frac{z_{i,1}r_i}{\hbar_1}\Big[(b_i+d_i)\Big(z_{i,2}+\alpha_{i,1}+\frac{(b_i+d_i)z_{i,1}r_i}{2\hbar_1}+\\&\frac{\rho_{i,1}^2z_{i,1}r_i}{\hbar_1c_{i,0,0}^2(b_i+d_i)}\Big)-\sum_{j\in N_i}a_{ij}\Big(x_{j,2}+\frac{z_{i,1}r_i}{2\hbar_1}+\frac{z_{i,1}\rho_{j,1}^2r_i}{2\hbar_1c_{j,1,1}^2}\Big)-\\&b_i\dot{y}_0-\frac{\dot{\mu}\ \tilde{z}_{i,1}}{\mu}\Big]-\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}+\sum_{j\in N_i}\frac{c_{j,1,1}^2}{2}+\frac{c_{i,1,1}^2}{4}+\\&\frac{1}{2}\varpi_{i,1,1}^2(x_{i,1}(t))+\frac{1}{2}\sum_{j\in N_j}\varpi_{j,1,1}^2(x_{j,1}(t)).\end{aligned}$$

定义未知非线性函数 $\psi_{i,1}(X_{i,1})$,有

$$\begin{aligned}&\psi_{i,1}=\\&\frac{(b_i+d_i)z_{i,1}r_i}{2\hbar_1}+\frac{\rho_{i,1}^2z_{i,1}r_i}{\hbar_1c_{i,0,0}^2(b_i+d_i)}-\\&\frac{1}{(b_i+d_i)}\sum_{j\in N_i}a_{ij}\Big(\frac{z_{i,1}r_i}{2\hbar_1}+\frac{z_{i,1}\rho_{j,1}^2r_i}{2\hbar_1c_{j,1,1}^2}\Big)+\\&\frac{\hbar_1}{z_{i,1}r_i(b_i+d_i)}\tanh^2\Big(\frac{z_{i,1}}{\delta_{i,1}}\Big)\times\\&\Big[\varpi_{i,1,1}^2(x_{i,1}(t))+\sum_{j\in N_j}\varpi_{j,1,1}^2(x_{j,1}(t))\Big].\end{aligned}\tag{9}$$

其中: $c_{i,1,1}^2=(b_i+d_i)^2c_{i,0,0}^2$, $X_{i,1}=[x_{i,1},x_{j,1}]^{\mathrm{T}}$, $j\in N_i$. 由式(7)~(9),得

$$\begin{aligned}&\dot{V}_{i,1}\leqslant\\&\frac{z_{i,1}r_i}{\hbar_1}\Big[(b_i+d_i)(z_{i,2}+\alpha_{i,1}+\psi_{i,1})-\sum_{j\in N_i}a_{ij}x_{j,2}-\\&b_i\dot{y}_0-\frac{\dot{\mu}\,\tilde{z}_{i,1}}{\mu}\Big]-\tilde{\theta}_{i,1}\dot{\hat{\theta}}_{i,1}+\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,1}}{\delta_{i,1}}\Big)\Big)\times\\&\Big[\varpi_{i,1,1}^2(x_{i,1}(t))+\sum_{j\in N_j}\varpi_{j,1,1}^2(x_{j,1}(t))\Big]+\\&\frac{c_{i,1,1}^2}{4}+\sum_{j\in N_i}\frac{c_{j,1,1}^2}{2}.\end{aligned}\tag{10}$$

利用RBFNNs逼近未知非线性函数 $\psi_{i,1}$,根据引理6可得

$$\psi_{i,1}(X_{i,1})=W_{i,1}^{\mathrm{T}}S_{i,1}(X_{i,1})+\varDelta_{i,1}(X_{i,1}).$$

其中: 逼近误差 $\varDelta_{i,1}$ 满足 $|\varDelta_{i,1}(X_{i,1})|\leqslant\varepsilon_{i,1}$; $S_{i,1}^{\mathrm{T}}S_{i,1}\leqslant p_{i,1}$, $p_{i,1}$ 为神经元数量. 有

$$\frac{z_{i,1}r_i}{\hbar_1}\psi_{i,1}\leqslant$$

$$\frac{z_{i,1}^2\theta_{i,1}r_i^2}{\hbar_1^2c_{i,0,0}^2(b_i+d_i)}+\frac{(b_i+d_i)c_{i,0,0}^2}{4}+\frac{z_{i,1}^2r_i^2}{2\hbar_1^2}+\frac{\varepsilon_{i,1}^2}{2},\tag{11}$$

其中$\theta_{i,1}=p_{i,1}\|W_{i,1}\|_2^2$. 设计虚拟控制输入$\alpha_{i,1}$和自适应律分别为

$$\alpha_{i,1}=\frac{1}{b_i+d_i}\Big[-\frac{k_{i,1}z_{i,1}}{r_i}-\frac{(b_i+d_i)z_{i,1}r_i}{2\hbar_1}-\frac{z_{i,1}r_i\hat{\theta}_{i,1}}{c_{i,0,0}^2\hbar_1}+\sum_{j\in N_i}a_{i,j}x_{j,2}+b_i\dot{y}_0+\frac{\dot{\mu}\,\tilde{z}_{i,1}}{\mu}\Big],$$

$$\dot{\hat{\theta}}_{i,1}=\frac{z_{i,1}^2r_i^2}{\hbar_1^2c_{i,0,0}^2}-\sigma_{i,1}\hat{\theta}_{i,1}.\tag{12}$$

将式(11)和(12)代入(10),可得

$$\dot{V}_{i,1}\leqslant-\frac{k_{i,1}z_{i,1}^2}{\hbar_1}+\frac{(b_i+d_i)z_{i,1}z_{i,2}}{\hbar_1}+\sigma_{i,1}\tilde{\theta}_{i,1}\hat{\theta}_{i,1}+\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,1}}{\delta_{i,1}}\Big)\Big)\times\Big[\varpi_{i,1,1}^2(x_{i,1}(t))+\sum_{j\in N_j}\varpi_{j,1,1}^2(x_{j,1}(t))\Big]+c_{i,1},\tag{13}$$

其中$c_{i,1}=\dfrac{c_{i,1,1}^2}{2}+\sum\limits_{j\in N_i}\dfrac{c_{j,1,1}^2}{2}+\dfrac{\varepsilon_{i,1}^2}{2}$.

sgep 2: 设计虚拟控制输入$\alpha_{i,2}$. 构造Lyapunov函数

$$V_{i,2}=V_{i,1}+\frac{1}{2}\log\frac{k_{b2}^2}{\hbar_2}+\frac{1}{2}\tilde{\theta}_{i,2}^2+\frac{1}{2}\sum_{k=1}^{2}\sum_{h=1}^{k}\int_{t-\tau_{i,k}}^{t}\varpi_{i,k,h}^2(x_{i,h}(s))\mathrm{d}s+\frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{2}\sum_{h=1}^{k}\int_{t-\tau_{j,k}}^{t}\varpi_{j,k,h}^2(x_{j,h}(s))\mathrm{d}s.\tag{14}$$

对$V_{i,2}$求导,得

$$\dot{V}_{i,2}=\dot{V}_{i,1}+\frac{z_{i,2}}{\hbar_2}(z_{i,3}+\alpha_{i,2}+g_{i,2}(\bar{x}_{i,2}(\tau_{i,2}))-\dot{\alpha}_{i,1}+d_{i,2}(\bar{x}_{i,2}))-\tilde{\theta}_{i,2}\dot{\hat{\theta}}_{i,2}+\frac{1}{2}\sum_{k=1}^{2}\sum_{h=1}^{k}[\varpi_{i,k,h}^2(x_{i,h}(t))-\varpi_{i,k,h}^2(x_{i,h}(\tau_{i,k}))]+\frac{1}{2}\sum_{j\in N_j}\sum_{k=1}^{2}\sum_{h=1}^{k}[\varpi_{j,k,h}^2(x_{j,h}(t))-\varpi_{j,k,h}^2(x_{j,h}(\tau_{j,k}))].\tag{15}$$

由假设2和假设3,得

$$\frac{z_{i,2}}{\hbar_2}g_{i,2}\leqslant\frac{z_{i,2}^2}{2\hbar_2^2}+\sum_{h=1}^{2}\frac{\varpi_{i,2,h}^2(x_{i,h}(\tau_{i,2}))}{2},$$

$$\frac{z_{i,2}}{\hbar_2}d_{i,2}\leqslant\frac{c_{i,2,2}^2}{4}+\frac{z_{i,2}^2\rho_{i,2}^2(\bar{x}_{i,2})}{\hbar_2^2c_{i,2,2}^2}.\tag{16}$$

对虚拟控制律$\alpha_{i,1}$求导,得

$$\dot{\alpha}_{i,1}=\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\dot{x}_{i,1}+\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\dot{x}_{j,k}+\frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1}+\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial y_0^{(k)}}y_0^{(k+1)}+\frac{\partial\alpha_{i,1}}{\partial r_i}\dot{r}_i+\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial\mu^{(k)}}\mu^{(k+1)}=R_1+\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}(g_{i,1}(x_{i,1}(\tau_{i,1}))+d_{i,1}(x_{i,1}))+\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}(g_{j,k}(\bar{x}_{j,k}(\tau_{j,k}))+d_{j,k}(\bar{x}_{j,k})).\tag{17}$$

其中

$$R_1=\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}x_{i,2}+\sum_{k=1}^{2}\sum_{j\in N_i}\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}x_{j,k+1}+\frac{\partial\alpha_{i,1}}{\partial\hat{\theta}_{i,1}}\dot{\hat{\theta}}_{i,1}+\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial y_0^{(k)}}y_0^{(k+1)}+\frac{\partial\alpha_{i,1}}{\partial r_i}\dot{r}_i+\sum_{k=0}^{1}\frac{\partial\alpha_{i,1}}{\partial\mu^{(k)}}\mu^{(k+1)}.$$

定义未知非线性函数$\psi_{i,2}(X_{i,2})$,有

$$\psi_{i,2}=-R_1+\frac{z_{i,2}}{2\hbar_2}+\frac{\rho_{i,2}^2z_{i,2}}{\hbar_2c_{i,2,2}^2}+\frac{z_{i,2}}{2\hbar_2}\Big(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\Big)^2+\frac{z_{i,2}\rho_{i,1}^2}{2\hbar_2c_{i,2,1}^2}\Big(\frac{\partial\alpha_{i,1}}{\partial x_{i,1}}\Big)^2+\sum_{k=1}^{2}\sum_{j\in N_i}\Big(\frac{z_{i,2}}{2\hbar_2}\Big(\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\Big)^2+\frac{z_{i,2}\rho_{j,k}^2}{2\hbar_2c_{i,2,k}^2}\Big(\frac{\partial\alpha_{i,1}}{\partial x_{j,k}}\Big)^2\Big)+\frac{\hbar_2}{z_{i,2}}\tanh^2\Big(\frac{z_{i,2}}{\delta_{i,2}}\Big)\times\sum_{k=1}^{2}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t))+\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big],\tag{18}$$

其中$X_{i,2}=[\bar{x}_{i,2},\bar{x}_{j,3}]^{\mathrm{T}},j\in N_i$. 由式(16)～(18),得

$$\dot{V}_{i,2}\leqslant\dot{V}_{i,1}+\frac{z_{i,2}}{\hbar_2}(z_{i,3}+\alpha_{i,2}+\psi_{i,2})+\frac{c_{i,2,2}^2}{4}+\frac{c_{i,2,1}^2}{2}-\tilde{\theta}_{i,2}\dot{\hat{\theta}}_{i,2}+\sum_{k=1}^{2}\sum_{j\in N_i}\frac{c_{j,2,k}^2}{2}+\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,2}}{\delta_{i,2}}\Big)\Big)\times\sum_{k=1}^{2}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t))+\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big].\tag{19}$$

利用RBFNNs逼近未知非线性函数$\psi_{i,2}$,根据引理6可得

$$\psi_{i,2}(X_{i,2})=W_{i,2}^{\mathrm{T}}S_{i,2}(X_{i,2})+\varDelta_{i,2}(X_{i,2}).$$

其中:逼近误差$\varDelta_{i,2}$满足$|\varDelta_{i,2}(X_{i,2})|\leqslant\varepsilon_{i,2}$; $S_{i,2}^{\mathrm{T}}S_{i,2}\leqslant p_{i,2}$, $p_{i,2}$为神经元数量. 由引理4得

$$\frac{z_{i,2}}{\hbar_2}\psi_{i,2} \leqslant \frac{z_{i,2}^2\theta_{i,2}}{\hbar_2^2 c_{i,2,2}^2} + \frac{c_{i,2,2,}^2}{4} + \frac{z_{i,2}^2}{2\hbar_2^2} + \frac{\varepsilon_{i,2}^2}{2}, \tag{20}$$

其中$\theta_{i,2} = p_{i,2}\|W_{i,2}\|_2^2$. 设计虚拟控制律$\alpha_{i,2}$和自适应律分别为

$$\alpha_{i,2} = -k_{i,2}z_{i,2} - \frac{z_{i,2}}{2\hbar_2} - \frac{z_{i,2}\hat{\theta}_{i,2}}{c_{i,2,2}^2\hbar_2} - \frac{(b_i+d_i)z_{i,1}\hbar_1}{2},$$

$$\dot{\hat{\theta}}_{i,2} = \frac{z_{i,2}^2}{\hbar_2^2 c_{i,2,2}^2} - \sigma_{i,2}\hat{\theta}_{i,2}. \tag{21}$$

由式(20)和(21),得

$$\begin{aligned}&\dot{V}_{i,2} \leqslant \\ &-\sum_{m=1}^{2}\Big(\frac{k_{i,m}z_{i,m}^2}{\hbar_m} - \sigma_{i,m}\tilde{\theta}_{i,m}\hat{\theta}_{i,m}\Big) + \frac{z_{i,2}z_{i,3}}{\hbar_2} + \\ &\sum_{m=1}^{2}\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times \\ &\sum_{k=1}^{m}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t)) + \sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big] + c_{i,2},\end{aligned} \tag{22}$$

其中$c_{i,2} = \sum_{k=1}^{2}\sum_{h=1}^{k}\Big(\frac{c_{i,k,h}^2}{2} + \sum_{j\in N_i}\frac{c_{j,k,h}^2}{2}\Big) + \sum_{k=1}^{2}\frac{\varepsilon_{i,k}^2}{2}$.

step q ($q = 3,4,\dots,n-1$): 设计虚拟控制输入$\alpha_{i,q}$和自适应律分别为

$$\alpha_{i,q} = -k_{i,q}z_{i,q} - \frac{z_{i,q}}{2\hbar_q} - \frac{z_{i,q}\hat{\theta}_{i,q}}{c_{i,q,q}^2\hbar_n} - \frac{z_{i,q-1}\hbar_q}{\hbar_{q-1}},$$

$$\dot{\hat{\theta}}_{i,q} = \frac{z_{i,q}^2}{\hbar_q^2 c_{i,q,q}^2} - \sigma_{i,q}\hat{\theta}_{i,q}. \tag{23}$$

由式(23)可得

$$\begin{aligned}&\dot{V}_{i,q} \leqslant \\ &-\sum_{m=1}^{q}\Big(\frac{k_{i,m}z_{i,m}^2}{\hbar_m} - \sigma_{i,m}\tilde{\theta}_{i,m}\hat{\theta}_{i,m}\Big) + \frac{z_{i,q}z_{i,q+1}}{\hbar_q} + \\ &\sum_{m=1}^{q}\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times\sum_{k=1}^{m}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t)) + \\ &\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big] + c_{i,q},\end{aligned} \tag{24}$$

其中$c_{i,q} = \sum_{k=1}^{q}\sum_{h=1}^{k}\Big(\frac{c_{i,k,h}^2}{2} + \sum_{j\in N_i}\frac{c_{j,k,h}^2}{2}\Big) + \sum_{k=1}^{q}\frac{\varepsilon_{i,k}^2}{2}$.

step n: 构造Lyapunov函数

$$\begin{aligned}V_{i,n} =\ & V_{i,n-1} + \frac{1}{2}\log\frac{k_{bn}^2}{\hbar_n} + \frac{1}{2}\tilde{\theta}_{i,n}^2 + \\ &\frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{k}\int_{t-\tau_{i,k}}^{t}\varpi_{i,k,h}^2(x_{i,h}(s))\mathrm{d}s + \\ &\frac{1}{2}\sum_{j\in N_i}\sum_{k=1}^{n}\sum_{h=1}^{k}\int_{t-\tau_{j,k}}^{t}\varpi_{j,k,h}^2(x_{j,h}(s))\mathrm{d}s.\end{aligned} \tag{25}$$

对$V_{i,n}$求导,得

$$\begin{aligned}&\dot{V}_{i,n} = \\ &\dot{V}_{i,n-1} + \frac{z_{i,n}}{\hbar_n}(u_i + g_{i,n}(x_{i,n}(\tau_{i,n})) + d_{i,n}(x_{i,n}) - \\ &\dot{\alpha}_{i,n-1}) - \tilde{\theta}_{i,n}\dot{\hat{\theta}}_{i,n} + \frac{1}{2}\sum_{k=1}^{n}\sum_{h=1}^{k}[\varpi_{i,k,h}^2(x_{i,h}(t)) - \\ &\varpi_{i,k,h}^2(x_{i,h}(\tau_{i,k}))] + \frac{1}{2}\sum_{j\in N_j}\sum_{k=1}^{n}\sum_{h=1}^{k}[\varpi_{j,k,h}^2(x_{j,h}(t)) - \\ &\varpi_{j,k,h}^2(x_{j,h}(\tau_{j,k}))].\end{aligned} \tag{26}$$

对虚拟控制输入$\alpha_{i,n-1}$求导,得

$$\begin{aligned}&\dot{\alpha}_{i,n-1} = \\ &\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial x_{i,k}}(g_{i,k}(\bar{x}_{i,k}(\tau_{i,k})) + d_{i,k}(\bar{x}_{i,k})) + \\ &\sum_{k=1}^{n}\sum_{j\in N_i}\frac{\partial\alpha_{i,n-1}}{\partial x_{j,k}}(g_{j,k}(\bar{x}_{j,k}(\tau_{j,k})) + d_{j,k}(\bar{x}_{j,k})) + R_{n-1}.\end{aligned}$$

其中

$$\begin{aligned}&R_{n-1} = \\ &\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial x_{i,k}}x_{i,k+1} + \sum_{k=1}^{n}\sum_{j\in N_i}\frac{\partial\alpha_{i,n-1}}{\partial x_{j,k}}x_{j,k+1} + \\ &\sum_{k=1}^{n-1}\frac{\partial\alpha_{i,n-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k} + \sum_{k=0}^{n-1}\frac{\partial\alpha_{i,1}}{\partial y_0^{(k)}}y_0^{(k+1)}.\end{aligned}$$

定义未知非线性函数$\psi_{i,n}$,有

$$\begin{aligned}&\psi_{i,n} = \\ &-R_{n-1} + \sum_{k=1}^{n-1}\Big[\frac{\rho_{i,k}^2 z_{i,n}}{2\hbar_n c_{i,n,k}^2} + \frac{z_{i,n}}{2\hbar_n}\Big]\Big(\frac{\partial\alpha_{i,n-1}}{\partial x_{j,k}}\Big)^2 + \\ &\sum_{k=1}^{n}\sum_{j\in N_i}\Big[\frac{z_{i,n}}{2\hbar_n} + \frac{z_{i,n}\rho_{j,k}^2}{2\hbar_n c_{j,n,k}^2}\Big]\Big(\frac{\partial\alpha_{i,n-1}}{\partial x_{j,k}}\Big)^2 + \frac{z_{i,n}}{2\hbar_n} + \\ &\frac{\rho_{i,n}^2 z_{i,n}}{\hbar_n c_{i,n,n}^2} + \frac{\hbar_n}{z_{i,n}}\tanh^2\Big(\frac{z_{i,n}}{\delta_{i,n}\hbar_n}\Big)\times \\ &\sum_{k=1}^{n}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t)) + \sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big].\end{aligned} \tag{27}$$

由式(26)和(27),得

$$\begin{aligned}&\dot{V}_{i,n} \leqslant \\ &\dot{V}_{i,n-1} + \frac{z_{i,n}}{\hbar_n}(u_i + \psi_{i,n}) + \frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,n}}{\delta_{i,n}}\Big)\Big)\times \\ &\sum_{k=1}^{n}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t)) + \sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big] - \\ &\tilde{\theta}_{i,n}\dot{\hat{\theta}}_{i,n} + \frac{c_{i,n,n}^2}{4} + \sum_{k=1}^{n-1}\frac{c_{i,n,k}^2}{2} + \sum_{k=1}^{n}\sum_{j\in N_i}\frac{c_{j,n,k}^2}{2}.\end{aligned} \tag{28}$$

为了节约智能体有限的计算资源和能源的消耗,

受文献[9,28]的启发,本文采用如下事件触发机制:

$$u_i(t)=\omega_i(t_{i,k}),\ \forall t\in[t_{i,k},t_{i,k+1}),\tag{29}$$

$$t_{i,k+1}=\inf\{t\in R||e_i(t)|\geqslant\eta_i|u_i(t)|+m_i\}.\tag{30}$$

其中: $\inf\{\cdot\}$ 为下确界;测量误差 $e_i(t)=\omega_i(t)-u_i(t)$; ς_i、$0<\eta_i<1$、$m_i>0$、$\bar{m}_i>m_i/(1-\eta_i)$ 为需要设计的参数. 第 i 个智能体第 k 个触发时刻为 $t_{i,k}$, $k\in z^+$, 事件触发控制策略表示从触发时刻 $t_{i,k}$ 到下一触发时刻 $t_{i,k+1}$ 之间, $u_i(t)$ 均保持 ω_i 在 $t_{i,k}$ 时刻的控制输入, 直到下一触发时刻 $t_{i,k+1}$ 才会更新控制输入. 设计自适应控制律和自适应律分别为

$$\omega_i(t)=-(1+\eta_i)\Big(\alpha_{i,n}\tanh\Big(\frac{z_{i,n}\alpha_{i,n}}{\hbar_n\varsigma_i}\Big)+\bar{m}_1\tanh\Big(\frac{z_{i,n}\bar{m}_1}{\hbar_n\varsigma_i}\Big)\Big),\tag{31}$$

$$\alpha_{i,n}=-k_{i,n}z_{i,n}-\frac{z_{i,n}}{2\hbar_n}-\frac{z_{i,n}\hat{\theta}_{i,n}}{c_{i,n,n}^2\hbar_n}-\frac{z_{i,n-1}\hbar_n}{\hbar_{n-1}},\tag{32}$$

$$\dot{\hat{\theta}}_{i,n}=\frac{z_{i,n}^2}{\hbar_n^2c_{i,n,n}^2}-\sigma_{i,n}\hat{\theta}_{i,n}.\tag{33}$$

由式(30)可得

$$\omega_i(t)=(1+\lambda_1(t)\eta_i)u_i(t)+\lambda_2(t)m_i.\tag{34}$$

其中: $t\in[t_{i,k},t_{i,k+1}]$, $\lambda_1(t)$ 和 $\lambda_2(t)$ 满足 $|\lambda_1(t)|\leqslant 1$ 和 $|\lambda_2(t)|\leqslant 1$. 由式(34)可得

$$u_i(t)=\frac{\omega_i(t)}{1+\lambda_1(t)\eta_i}-\frac{\lambda_2(t)m_i}{1+\lambda_1(t)\eta_i}.\tag{35}$$

因为 $\forall\xi\in R,\varepsilon>0,-\xi\tanh(\xi/\varsigma)\leqslant 0$, 由式(31)可知 $z_{i,n}\omega_i(t)\leqslant 0$, 又 $|\lambda_1(t)|\leqslant 1,|\lambda_2(t)|\leqslant 1$, 所以有

$$z_{i,n}\omega_i(t)/(1+\lambda_1(t)\eta_i)\hbar_n\leqslant z_{i,n}\omega_i(t)/(1+\eta_i)\hbar_n,$$

$$|\lambda_2(t)m_i/(1+\lambda_1(t))\eta_i|\leqslant m_i/(1-\eta_i),$$

由式(35)得

$$\dot{V}_{i,n}\leqslant \dot{V}_{i,n-1}-\Big|\frac{z_{i,n}\alpha_{i,n}}{\hbar_n}\Big|-\Big|\frac{z_{i,n}\bar{m}_i}{\hbar_n}\Big|-\tilde{\theta}_{i,n}\dot{\hat{\theta}}_{i,n}+\frac{z_{i,n}^2}{2\hbar_n^2}+\Big|\frac{z_{i,n}m_i}{(1-\eta_i)\hbar_n}\Big|+\frac{z_{i,n}^2\theta_{i,n}}{\hbar_n^2c_{i,n,n}^2}+\sum_{k=1}^{n}\frac{c_{i,n,k}^2}{2}+\frac{\varepsilon_{i,n}^2}{2}+0.557\varsigma_i+\sum_{k=1}^{n}\sum_{j\in N_i}\frac{c_{j,n,k}^2}{2}+\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,n}}{\delta_{i,n}}\Big)\Big)\times\sum_{k=1}^{n}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t))+\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big].\tag{36}$$

由式(36)得

$$\dot{V}_{i,n}\leqslant -\sum_{m=1}^{n}\Big(\frac{k_{i,m}z_{i,m}^2}{\hbar_m}-\sigma_{i,m}\tilde{\theta}_{i,m}\hat{\theta}_{i,m}\Big)+C_i+\sum_{m=1}^{n}\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times\sum_{k=1}^{m}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t))+\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big],\tag{37}$$

$$C_i=\sum_{k=1}^{n}\sum_{h=1}^{k}\Big(\frac{c_{i,k,h}^2}{2}+\sum_{j\in N_i}\frac{c_{j,k,h}^2}{2}\Big)+\sum_{k=1}^{n}\frac{\varepsilon_{i,k}^2}{2}+0.557\varsigma_i.\tag{38}$$

注3 相对阈值事件触发控制策略是设计与控制器 u_i 幅值相关的可变阈值. 当控制信号 u_i 幅值维持较大时,较大的阈值可以避免频繁触发;当系统稳定, u_i 幅值维持较小时,较小的阈值可以获得更好的控制性能.

注4 与传统的相对阈值事件触发机制相比,式(30)的常数项 m_i 可以保证事件触发机制的触发间隔 t^* 存在下界,因为控制信号 $u_i(t)$ 可能收敛到零. 由式(30)可知,当 $u_i(t)$ 为0时, m_i 的大小决定了触发间隔 t^* 的下界.

注5 文献[19]针对具有状态时延的多智能体系统,利用LK泛函消除状态时延的影响,提出了自适应一致性控制策略. 本文在文献[19]的基础上,引入性能函数,使输出误差满足预定性能,利用障碍Lyapunov函数,保证状态满足约束条件. 此外,为了减少控制器更新频率,提出了自适应事件触发控制策略,且有效避免了Zeno行为.

3 稳定性分析

基于Lyapunov稳定性理论对闭环系统的稳定性进行分析,总结为如下定理.

定理1 基于假设1∼假设3,考虑系统(1)在控制器(31)和事件触发机制(29)、(30)的作用下,系统中所有信号都是半全局有界的,所有智能体的输出实现一致,一致性输出误差满足预定性能且所有状态满足约束条件. 事件触发间隔 $[t_{i,k},t_{i,k+1}]$ 存在一个下界 t^*, $t^*>0$, 不会发生Zeno行为.

证明 选取Lyapunov函数 $V=\sum_{i=1}^{N}V_{i,n}$. 由式(37)可得

$$\dot{V}\leqslant -\sum_{i=1}^{N}\sum_{m=1}^{n}\Big(\frac{k_{i,m}z_{i,m}^2}{\hbar_m}-\sigma_{i,m}\tilde{\theta}_{i,m}\hat{\theta}_{i,m}\Big)+\sum_{i=1}^{N}C_i+\sum_{i=1}^{N}\sum_{m=1}^{n}\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times\sum_{k=1}^{m}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t))+\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big],\tag{39}$$

其中$\sigma_{i,m}\tilde{\theta}_{i,m}\hat{\theta}_{i,m} \leqslant -\frac{\sigma_{i,m}\tilde{\theta}_{i,m}^2}{2} + \frac{\sigma_{i,m}\theta_{i,m}^2}{2}$. 由式(38)得

$$\dot{V} \leqslant -\sum_{i=1}^{N}\sum_{m=1}^{n}\Big(\frac{k_{i,m}z_{i,m}^2}{\hbar_m} + \frac{\sigma_{i,m}}{2}\tilde{\theta}_{i,m}^2\Big) + C + \sum_{i=1}^{N}\sum_{m=1}^{n}\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times \sum_{k=1}^{m}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t)) + \sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big]. \tag{40}$$

则式(39)可写为

$$\dot{V} \leqslant -aV(t) + \pi. \tag{41}$$

其中

$$C = \sum_{i=1}^{N} C_i . a = \min\{k_{i,m}, \sigma_{i,m}\},$$

$$\pi = C + \sum_{i=1}^{N}\sum_{m=1}^{n}\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times \sum_{k=1}^{m}\sum_{h=1}^{k}\Big[\varpi_{i,k,h}^2(x_{i,h}(t)) + \sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big].$$

对不等式(40)左右两边同时乘以e^{at}且积分,得

$$\int_0^t \mathrm{e}^{at}\dot{V}\mathrm{d}t \leqslant \int_0^t -a\mathrm{e}^{at}V(t)\mathrm{d}t + \int_0^t \mathrm{e}^{at}\pi\mathrm{d}t.$$

进一步得

$$\int_0^t (\mathrm{e}^{at}V)'\mathrm{d}t = \int_0^t \mathrm{e}^{at}\pi\mathrm{d}t.$$

由式(40)得

$$\frac{1}{2}z_{i,1}^2 \leqslant V(t) \leqslant \Big(V(0)-\frac{\pi}{a}\Big)\mathrm{e}^{-at} + \frac{\pi}{a}. \tag{42}$$

由引理1得

$$\lim_{t\to\infty}\|Y - Y_d\| \leqslant \frac{\sqrt{2\pi/a}}{\underline{\varsigma}(L+B)}. \tag{43}$$

由式(42)可知,通过选择合适的参数可以使输出误差收敛于以原点为中心的有界邻域内. 结合式(6)和(41),得

$$|z_{i,m}| < k_{bm}\sqrt{1-\mathrm{e}^{-2(V(0)-\pi/a)\mathrm{e}^{-at}-2\pi/a}} \leqslant k_{bm}. \tag{44}$$

令

$$\bar{\aleph} = \{X_{i,l}(t) | |z_{i,l}| < k_{bl}\sqrt{1-\mathrm{e}^{-2(V(0)-\pi/a)\mathrm{e}^{-at}-2\pi/a}}\}.$$

由式(43)可知,误差约束在区间$(-k_{bm}, k_{bm})$内. 通过以上分析,系统在自适应事件触发控制器作用下达到半全局有界稳定,误差满足预先设定的条件. □

注6　利用神经网络逼近非线性函数,神经网络是导致半全局稳定的主要原因,根据引理6, Z满足$Z\subset\aleph$. 文献[29]利用神经网络逼近紧集$\bar{\Omega}$上的任意连续函数,与最终收敛的集合Ω满足关系$\Omega\subset\bar{\Omega}$,系统状态的初值在紧集$\bar{\Omega}$内. 紧集$\aleph$和误差信号收敛集$\bar{\aleph}$满足$\bar{\aleph}\subset\aleph$. 由神经网络控制器(31)、(32)和(38)可知,系统状态必须向衰减Lyapunov函数的方向移动,因此状态一直保持在紧集$\bar{\aleph}$内并最终收敛到紧集$\aleph$.

注7　基于障碍Lyapunov函数理论,通常将状态约束问题转化为误差约束问题进行分析. 由于y_0和$\tilde{z}_{i,1}$有界,根据式(2)可知, $y_0 < Y_0, \tilde{z}_{i,1} < k_{b1}, x_{i,1}$有界且满足$|y_i| = |x_{i,1}| < \beta_{i1}$. 根据式(43)可知,误差$z_{i,2}$有界,即$|z_{i,2}| < k_{b2}, k_{b2}$是正数. 假设正常数$\zeta_{i,2}$满足$|\alpha_{i,1}| < \zeta_{i,1}$,可得$|x_{i,2}| = |z_{i,2}| + |\alpha_{i,1}| < k_{b2} + \zeta_{i,1} < \beta_{i,2}$. 同理, $|x_{i,m}| < \beta_{i,m} (m = 3, 4, \ldots, n)$也有界. 基于上述分析,设计的控制策略可以保证所有状态满足预先设定的约束要求.

下面分析所采用的事件触发机制(29)和(30)无Zeno行为发生. Zeno行为是指在有限时间内发生无数次事件触发,如果事件被触发无数次,则事件触发机制无效,不能节约资源,系统也难以稳定. 因为

$$\frac{\mathrm{d}}{\mathrm{d}t}|e_i(t)| = \frac{\mathrm{d}}{\mathrm{d}t}(e_i \times e_i)^{\frac{1}{2}} = \mathrm{sign}(e_i)\dot{e}_i \leqslant |w_i|$$

已经说明了系统中的所有状态都是有界的,所以必然存在正数ξ,使得$|\dot{w}_i| \leqslant \xi$. 由$e_i(t_k) = 0, \lim_{t\to t_{k+1}}|e_i(t)| = m_i$可知,事件触发时间间隔的下界$t^*$满足$t^* \geqslant m_i/\xi$. 因此所采用的事件触发机制(29)和(30)避免了Zeno行为.

以下分析多智能体系统的一致性误差,可分为3种情形.

情况1　若$z_{i,m} \in \Omega_{z_{i,m}}, i = 1, 2, \ldots, N, m = 1, 2, \ldots, n$,则$|z_{i,m}| < 0.881\,4\delta_{i,1}$. 根据$z_{i,1}$的定义,一致性误差$z_1$满足$z_1 = (L+B)(y - 1_N y_0)$,因此$\|y - 1_N y_0\| \leqslant \|z_1\|/\underline{\varsigma}(L+B)$,跟踪误差$y - 1_N y_0$有界,且通过选择任意小的$\delta_{i,1}$可以使得一致性误差任意小.

情况2　若$z_{i,m} \notin \Omega_{z_{i,m}}$,则

$$\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i,m}}{\delta_{i,m}}\Big)\Big)\times\sum_{k=1}^{m}\sum_{h=1}^{k}\Big(\varpi_{i,k,h}^2(x_{i,h}(t)) + \sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big) \leqslant 0,$$

且如果

$$\sum_{i=1}^{N}\sum_{m=1}^{n}\Big(\frac{k_{i,m}z_{i,m}^2}{\hbar_m} + \frac{\sigma_{i,m}\tilde{\theta}_{i,m}^2}{2\gamma_{i,m}}\Big) > C,$$

则$\dot{V} < 0$. 所以$\|z_1\| \leqslant \sqrt{C/k}, k = \min\{k_{i,m}, i = 1, 2, \ldots, N, m = 1, 2, \ldots, n\}$,一致性跟踪误差$y -$

$1_N y_0$ 有界,通过选择任意小的 $\delta_{i,1}$ 可以使得一致性误差任意小.

情况3 将集合 $z_{i,m}$ 分成 $z_{i_1,m_1} \in \Omega_{z_{i_1,m_1}}$ 和 $z_{i_2,m_2} \notin \Omega_{z_{i_2,m_2}}$ 两个子集,其中 $i_1 \in S_{i_1}, m_1 \in S_{m_1}$, $i_2 \in S_{i_2}, m_2 \in S_{m_2}$. 如果 $z_{i_1,m_1} \in \Omega_{z_{i_1,m_1}}$,则 $|z_{i_1,m_1}| < 0.881\,4\delta_{i_1,m_1}$,因此对于 $\forall i_1 \in S_{i_1}, z_{i_1,1}$ 有界. 当 $z_{i_2,m_2} \notin \Omega_{z_{i_2,m_2}}$ 时,根据情况2和引理2可知

$$\frac{1}{2}\Big(1-2\tanh^2\Big(\frac{z_{i_2,m_2}}{\delta_{i_2,m_2}}\Big)\Big)\times\sum_{k\in S_{m_2}}\sum_{h=1}^{k}\Big(\varpi_{i,k,h}^2(x_{i,h}(t))+\sum_{j\in N_j}\varpi_{j,k,h}^2(x_{j,h}(t))\Big)\leqslant 0,$$

对于 $\forall i_2 \in S_{i_2}, z_{i_2,1}$ 有界.

4 仿真与分析

通过仿真验证所设计的控制策略的有效性. 选取由3个跟随者和1个虚拟领导者组成的二阶多智能体系统进行仿真. 二阶多智能体系统模型为

$$\dot{x}_{i,1}=x_{i,2},$$
$$\dot{x}_{i,2}=u_i+g_{i,2}(x_i(t-\tau_{i,2}))+0.5\cos(t)x_{i,1}x_{i,2},$$
$$y_i=x_{i,1},\ i=1,2,3.$$

其中: $g_{i,2}(x_i) = x_{i,1}x_{i,2}$,时延为 $\tau_{i,2} = 0.1\,\mathrm{s}$, 虚拟领导者的期望轨迹为 $y_0 = 0.5(\sin(t) + \sin(0.5t))$. 跟随者的初始状态选取为: $x_{i,1}(0) = [0.1,0,0.3]$, $\hat{\theta}_{i,1}(0) = [0.2,0.1,-0.3]$. 控制器参数选取为: $k_{i,1} = 20, k_{i,2} = 80, k_{b,1} = 2, k_{b,2} = 2, c_{i,1,1} = 10, c_{i,2,2} = 10, \sigma_{i,1} = 2, \sigma_{i,2} = 2, \eta_i = 0.5, \varsigma_i = 1, m_1 = 1, \bar{m}_1 = 0.4$. 预定性能和状态约束的参数设定为: $\mu_0 = 1, \mu_\infty = 0.03, \nu_0 = 0.3, l_{\max} = 10, l_{\min} = 3, \beta_{i,1} = 1, \beta_{i,2} = 1.3$. 通信拓扑图如图1所示.

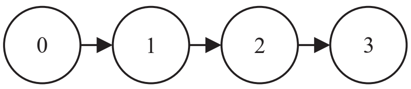
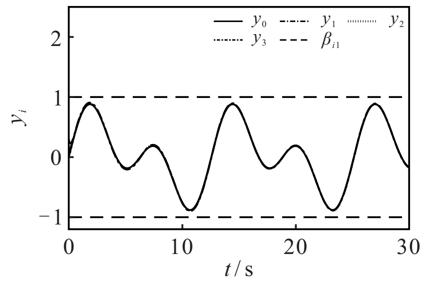
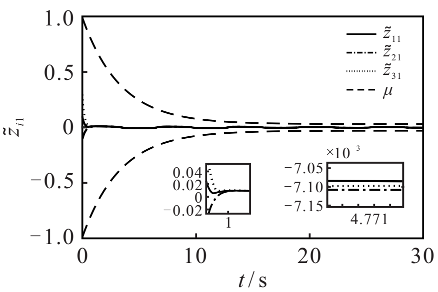


图1 通信拓扑图



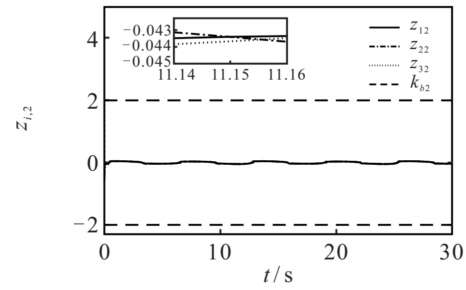
(a) 输出



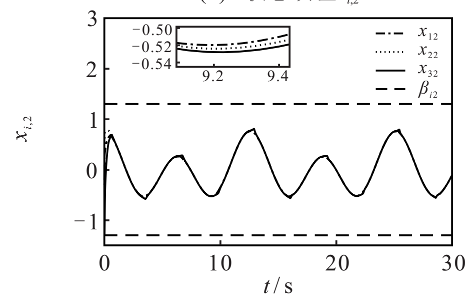
(b) 跟踪误差

图2 领导者和跟随者的输出 y_i 和跟踪误差 $\tilde{z}_{i,1}$

为了更好地体现控制策略的优越性,对不考虑预定性能以及不考虑事件触发机制的两种情形也进行仿真,仿真结果如图2～图6所示.

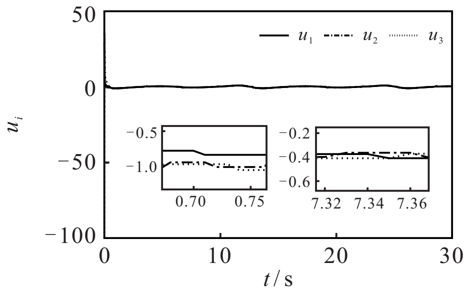


(a) 状态误差$z_{i,2}$

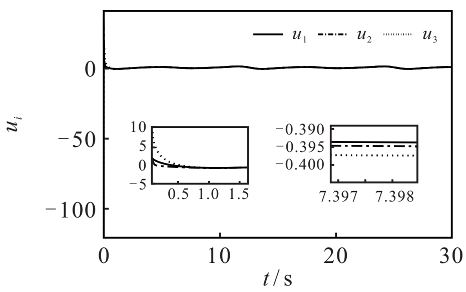


(b) 状态$x_{i,2}$

图3 状态误差 $z_{i,2}$ 和状态 $x_{i,2}$



(a) 考虑事件触发机制



(b) 不考虑事件触发机制

图4 事件触发机制和不考虑事件触发机制的控制输入 u_i

图2为跟随者的跟踪效果,由图2可知,3个跟随者在较短时间内实现了对期望轨迹的跟踪,所提出控制策略实现了多智能体系统的一致跟踪,并且跟踪误差满足预定性能. 图3为状态误差 $z_{i,2}$ 和状态 $x_{i,2}$,由

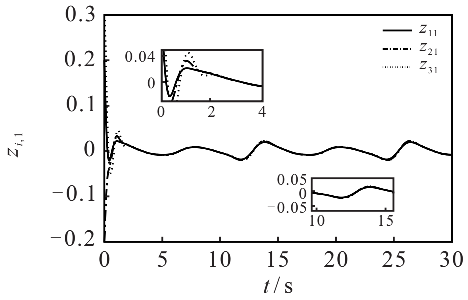


图 5　不考虑预定性能的跟踪误差 $z_{i,1}$

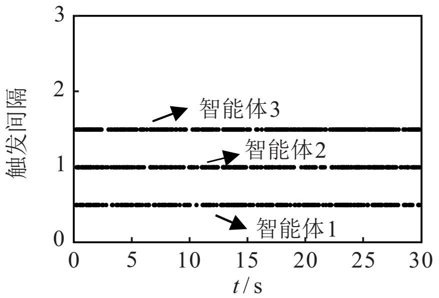


图 6　各智能体触发间隔

图3可知,系统的所有状态误差均满足预先设定的约束条件. 图4(a)和图4(b)分别为有事件触发机制下的控制输入和无事件触发机制下的控制输入,控制输入在起始阶段有较大的波动,图4(a)局部放大图显示控制信号仅在满足触发条件的时刻更新,由图4(b)可知控制信号光滑且有界. 图5为不考虑预定性能的输出误差,对比图2(b)和图5,两种情形下的输出误差均收敛于原点的有界邻域,图5中的输出误差虽然在预定区域内,但是稳态误差大于考虑预定性能时的稳态误差. 图6为控制输入的事件触发时间间隔图,结合表1可知,3个跟随者的事件触发次数不同,减少了事件触发控制器的更新次数,且无Zeno行为发生.

表 1　3个跟随者智能体的事件触发次数和触发比率

智能体编号	1	2	3
采样次数		3 000	
触发次数	253	304	379
触发率/%	8.4	10.1	12.6

5　结　论

本文研究了带有状态时延、外界干扰以及全状态约束的多智能体系统的一致跟踪控制问题,提出了自适应事件触发控制策略. 通过引入性能函数并对误差进行转换,使输出误差满足预定性能,并采用障碍Lyapunov函数确保了所有状态满足约束条件. 利用LK泛函消除了状态时延的影响,设计的事件触发控制策略减少了通信带宽和计算资源. 最后,对比仿真实例验证了所提出策略的有效性. 在未来工作中,将研究多智能体系统在实际工程系统中的应用,如多无人机编队控制、多机器人控制等.

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