控制与决策
Control and Decision

离散切换线性系统的异步滤波器设计

郝现志, 黄金杰, 潘晓真

引用本文:
郝现志,黄金杰,潘晓真. 离散切换线性系统的异步滤波器设计[J]. 控制与决策, 2022, 37(7): 1785-1793.

在线阅读 View online: https://doi.org/10.13195/j.kzyjc.2021.0091

离散切换线性系统的异步滤波器设计

郝现志[1]，黄金杰[1,2,3†]，潘晓真[1]

(1. 哈尔滨理工大学 计算机科学与技术学院，哈尔滨 150080；2. 哈尔滨理工大学 自动化学院，哈尔滨 150080；3. 黑龙江省复杂智能系统与集成重点实验室，哈尔滨 150080)

摘 要: 利用模型依赖的平均驻留时间策略研究离散异步切换线性系统的指数 H_∞ 滤波问题. 考虑到在实际问题中,所设计的模型依赖的全阶滤波器的切换往往会滞后于其相应的子系统,将子系统的运行区间划分为与滤波器匹配的区间和不匹配的区间. 针对两种工作模态, 利用模型依赖的平均驻留时间切换策略和 μ 依赖的多 Lyapunov 函数方法完成滤波器的设计,并使得增广得到的异步滤波误差系统全局一致指数稳定且满足指数 H_∞ 性能指标. 该滤波器存在的充分条件在文中以线性矩阵不等式的形式给出. 最后通过数值仿真验证所提方法的有效性.

关键词: 切换系统；异步切换；H_∞ 滤波；模型依赖平均驻留时间

中图分类号: TP13 **文献标志码:** A

DOI: 10.13195/j.kzyjc.2021.0091

引用格式: 郝现志,黄金杰,潘晓真. 离散切换线性系统的异步滤波器设计[J]. 控制与决策, 2022, 37(7): 1785-1793.

Asynchronous filter design for discrete-time switched linear systems

HAO Xian-zhi[1]，*HUANG Jin-jie*[1,2,3†]，*PAN Xiao-zhen*[1]

(1. School of Computer Science and Technology，Harbin University of Science and Technology，Harbin 150080，China；2. School of Automation，Harbin University of Science and Technology，Harbin 150080，China；3. Heilongjiang Provincial Key Laboratory of Complex Intelligent System and Integration，Harbin 150080，China)

Abstract: This paper addresses the exponential H_∞ filtering problem of asynchronous switched linear discrete-time systems with mode-dependent average dwell time (MDADT). In practical situation, the switching of the designed mode-dependent full-order filter often lags behind its corresponding subsystems. Therefore, the running time interval of the subsystem is divided into two parts: the matched and the mismatched. For these two parts, the full-order filter is designed using the MDADT and the μ dependent multi-Lyapunov function method, such that the global uniform exponential stability and exponential H_∞ performance of the asynchronous filtering error system are guaranteed. The sufficient conditions for the existence of the designed filter are given in terms of LMIs. Finally, a numerical example is provided to illustrate the effectiveness of the proposed method.

Keywords: switched systems；asynchronous switching；H_∞ filtering；mode-dependent average dwell time

0 引 言

切换控制已成为现代控制领域中一个相对新颖且颇具活力的重要组成部分,具有非常广泛的用途,如发动机控制系统[1]、网络控制系统[2-3]、飞行控制[4] 和通讯工程[5] 等. 切换系统可以高效地描述很多实际的系统,这些系统往往在内部具有多个模态或者多个动态系统,如DC/DC变流器[6]、振荡器[7] 和混沌系统的电路设计[8] 等. 一般来说,切换系统的稳定性问题是该研究领域的主要关注点,Lyapunov 函数方法可以有效地分析切换系统的稳定性问题. 以往的研究中利用全局 Lyapunov 函数[9](CLF)、切换 Lyapunov 函数[10](SLF) 和 μ 依赖的多 Lyapunov 函数[11](MLF) 方法得到了许多关于切换系统稳定性质的重要结论. 全局 Lyapunov 函数和切换 Lyapunov 函数均不适合慢切换系统[12]. 采用全局 Lyapunov 函数主要是因为在任意切换模式下,子系统的运行时间可能会非常短暂,若为其设计对应的 Lyapunov 函数, 则容易造成计算资源浪费且起不到实质的作用. 而切换 Lyapunov 函数则在切换瞬间对其能量函数要求太过严苛. 在研究切换系统的稳定性方面,多 Lyapunov

收稿日期: 2021-01-17；**录用日期:** 2021-04-21.

基金项目: 国家自然科学基金项目 (61305001)；黑龙江省自然科学基金项目 (F201222).

责任编委: 虞文武.

†通讯作者. E-mail: jjhuangps@163.com.

函数法被普遍认为是更为有效的方法. 与一般系统不同的是,切换系统的可行解不仅取决于系统的初始状态,还与系统的切换信号有关. 切换信号作为一个重要因素在很大程度上决定了切换系统的动态性能. 切换系统按照切换信号的来源可分为自主切换系统和被控切换系统,例如切换LPV系统由参变量的变化决定系统的切换,可归类于自主切换系统[11]. 若自主切换系统的切换信号服从马尔科夫过程,则这种系统可称为马尔科夫跳跃系统[13]. 对于被控切换系统而言,切换信号直接影响系统的性能,因此需要确定合适的切换信号来稳定系统,这使得相应的设计问题变得更加复杂. 由不同切换信号引起的系统稳定性问题是切换系统领域的主要问题. 因此以往的文献中利用Lyapunov函数方法,再结合驻留时间[14-15](DT)、平均驻留时间[16-17](ADT)和模型依赖平均驻留时间[18-20](MDADT)等方法,得到了许多关于时序切换系统稳定性问题的有价值的研究. MDADT是一种非常重要的切换信号形式,它通过综合子系统的信息确定子系统的模型依赖的驻留时间,这个驻留时间不小于一个常数. 若选取全局的参数代替每个子系统的参数,则MDADT退化为ADT,而ADT中又包含了DT且它的极限形式就是任意切换,因此利用MDADT策略研究切换系统的稳定性问题是十分有意义的.

具有扰动输入的系统的状态估计在工程应用上是一个非常重要的问题,为了更好地控制系统,需要得到准确的状态估计. 近些年学者们利用不同的方法和标准对此问题做了大量研究,如卡尔曼滤波[21]、H_2/H_∞ 滤波[22-24]和 l_2-l_∞ 滤波[25-26]等方法. 当外部扰动输入无法精确获得时,卡尔曼滤波不再适用,通常采用行之有效的 H_∞ 滤波器方法进行处理. 另一方面,指数滤波方法在某些复杂系统的滤波过程中,能够达到高度期望的指数稳定性[27],因此研究切换系统的指数 H_∞ 滤波问题是非常有价值的. 在过往的文献中,利用多Lyapunov函数结合任意切换、平均驻留时间方法对切换线性系统的滤波器进行过很多有价值的研究. 而采用模型依赖平均驻留时间策略对对应的离散切换线性系统进行状态估计和滤波器设计却还没有得到充分的研究.

近年来,切换系统的状态估计和滤波在控制领域吸引了大量的关注和研究. 一个普遍的做法就是寻找一簇模型依赖的滤波器和一个可容许的切换信号,使得构造的滤波误差系统能够稳定且满足某一性能指标,其中大部分集中在同步切换的领域中[28],然而滤波器与子系统之间不可避免地存在异步的现象. 所谓的异步是指在滤波器的切换与子系统的切换之间存在一个时间滞后;同步切换即假定每个子系统对应的滤波器的切换是在子系统切换的瞬间同时完成的. 但在实际应用中,滤波器的切换很难恰好与子系统的切换一致,滤波器通常需要时间根据子系统的模态来选择正确的控制模态,因此滤波器的切换往往是滞后于子系统的切换. 使用同步切换设计的滤波器在实际应用中显然无法取得良好的控制效果. 也有部分文献考虑了异步切换的情况,但多数也是以ADT的方法进行研究[29-33]. 因此,结合异步滤波和MDADT进行研究具有一定的意义.

本文利用模型依赖平均驻留时间策略研究离散异步切换系统的指数 H_∞ 滤波问题. 本文的主要贡献是,针对现实情况中广泛存在的滤波器滞后问题,将滤波器与子系统组成的滤波误差系统划分为稳定的部分和可能不稳定的部分,将两个不同衰减指数的时间指数函数转化为引入中间参数的时间指数函数,通过限制两部分的时间比值保证系统的渐近稳定性. 系统中使用的参数均为模型依赖的,使得系统滤波器的设计更为灵活.

1 问题描述

考虑如下离散切换线性系统:

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(k) + B_{\sigma(k)}w(k),\\ y(k) &= C_{\sigma(k)}x(k) + D_{\sigma(k)}w(k),\\ z(k) &= E_{\sigma(k)}x(k) + F_{\sigma(k)}w(k). \end{aligned} \tag{1}$$

其中: $x(k) \in R^n$ 为状态变量; $w(k) \in R^m$ 为扰动输入; $y(k) \in R^y$ 为测量输出; $z(k) \in R^z$ 为期望输出; $\sigma(k):[0,\infty) \rightarrow M = \{1,2,\ldots,N\}$ 为切换信号,是分段右连续的常数函数, N 为子系统的个数; $A_{\sigma(k)}$、$B_{\sigma(k)}$、$C_{\sigma(k)}$、$D_{\sigma(k)}$、$E_{\sigma(k)}$、$F_{\sigma(k)}$ 为适当维数的常数矩阵. 系统的切换序列可以表示为 $\{(k_0,\sigma(k_0)),(k_1,\sigma(k_1)),\ldots,(k_l,\sigma(k_l)),\ldots\}$, $l = 0,1,\ldots$, k_0 为初始时间, k_l 为第 l 个切换瞬间. 当 $k \in [k_l,k_{l+1})$ 时,第 $\sigma(k_l)=i$ 个子系统运行, $i \in M$.

针对系统(1),设计如下全阶滤波器:

$$\begin{aligned} x_f(k+1) &= A_{fi}x_f(k) + B_{fi}y(k),\\ z_f(k) &= C_{fi}x_f(k) + D_{fi}y(k),\ i \in M. \end{aligned} \tag{2}$$

其中: $x_f(k)$ 为滤波器的状态变量, $z_f(k)$ 为系统(1)中 $z(k)$ 的估计函数, A_{fi}、B_{fi}、C_{fi}、D_{fi} 为待确定的滤波器系数.

相应的增广滤波误差系统可描述为

$$\tilde{x}(k+1)=\tilde{A}_i\tilde{x}(k)+\tilde{B}_iw(k),$$
$$\varepsilon(k)=\tilde{C}_i\tilde{x}(k)+\tilde{D}_iw(k),\ i\in M. \tag{3}$$

其中

$$\tilde{x}(k)=[x^{\mathrm{T}}(k)\ \ x_f^{\mathrm{T}}(k)]^{\mathrm{T}},$$
$$\varepsilon(k)=z(k)-z_f(k),$$
$$\tilde{A}_i=\begin{bmatrix}A_i & 0\\ B_{fi}C_i & A_{fi}\end{bmatrix},\ \tilde{B}_i=\begin{bmatrix}B_i\\ B_{fi}D_i\end{bmatrix},$$
$$\tilde{C}_i=[E_i-D_{fi}C_i\quad -C_{fi}],$$
$$\tilde{D}_i=F_i-D_{fi}D_i.$$

假设滤波器滞后相应子系统的时间$\Delta_l<\tau_l=(k_{l+1}-k_l)h, l=0,1,\ldots,h$为离散步长,这与实际情况相符合,$\sigma'(k_l)$为滞后的滤波器的切换信号,则相应的滤波器的时间切换序列为$\{(k_0+\Delta_0,\sigma'(k_0)),(k_1+\Delta_1,\sigma'(k_1)),\ldots,(k_l+\Delta_l,\sigma'(k_l)),\ldots\}$, $l=0,1,2,\ldots$. 假设在第l个切换瞬间(即$k=k_l$时),第i个子系统运行,即$\sigma(k_l)=i$. 当$k=k_{l-1}$时,$\sigma(k_{l-1})=j$. 由于滤波器的切换滞后于子系统,子系统的运行时间可以划分为与滤波器匹配的时间段$T^+(k_0,k)$和不匹配的时间段$T^-(k_0,k)$. 则由系统(3)可知,当$k\in[k_l,k_{l+1})$时,相应的异步切换系统可以描述为

$$\begin{cases}\tilde{x}(k+1)=\tilde{A}_{ij}\tilde{x}(k)+\tilde{B}_{ij}w(k),\\ \varepsilon(k)=\tilde{C}_{ij}\tilde{x}(k)+\tilde{D}_{ij}w(k),\ k\in[k_l,k_l+\Delta_l);\\ \tilde{x}(k+1)=\tilde{A}_i\tilde{x}(k)+\tilde{B}_iw(k),\\ \varepsilon(k)=\tilde{C}_i\tilde{x}(k)+\tilde{D}_iw(k),\ k\in[k_l+\Delta_l,k_{l+1}).\end{cases} \tag{4}$$

其中

$$\tilde{A}_{ij}=\begin{bmatrix}A_i & 0\\ B_{fj}C_i & A_{fj}\end{bmatrix},\ \tilde{B}_{ij}=\begin{bmatrix}B_i\\ B_{fj}D_i\end{bmatrix},$$
$$\tilde{C}_{ij}=[E_i-D_{fj}C_i\quad -C_{fj}],$$
$$\tilde{D}_{ij}=F_i-D_{fj}D_i,$$

其余符号与前文中描述一致.

定义1[12] 若存在正实数α、γ使得下面两个条件成立,则称系统具有指数的H_∞性能γ:

1) 当扰动为零时,系统指数稳定;

2) 在零初始条件下,扰动不为零,$w(k)\in l_2[0,\infty)$,系统满足

$$\sum_{s=k_0}^{\infty}(1-\alpha)^s\varepsilon^{\mathrm{T}}(s)\varepsilon(s)\leqslant\gamma^2\sum_{s=k_0}^{\infty}w^{\mathrm{T}}(s)w(s).$$

2 主要结论

首先考虑闭环系统(4)的稳定性问题.

定理1 对于闭环切换系统(4),给定实数$0<\alpha_i<1,\beta_i>0,\mu_i>1$. 若存在一簇正定实对称矩阵$P_i\in R^{n\times n}$,实数$\tau_i>0,\zeta_i>0$和一个正实数$\gamma$,以及$i,j\in M, i\neq j$,有

$$\begin{bmatrix}-P_i & 0 & P_i\tilde{A}_i & P_i\tilde{B}_i\\ * & -I & \tilde{C}_i & \tilde{D}_i\\ * & * & -\tilde{\alpha}_iP_i & 0\\ * & * & * & -\gamma^2I\end{bmatrix}<0, \tag{5}$$

$$\begin{bmatrix}-P_i & 0 & P_i\tilde{A}_{ij} & P_i\tilde{B}_{ij}\\ * & -I & \tilde{C}_{ij} & \tilde{D}_{ij}\\ * & * & -\tilde{\beta}_iP_i & 0\\ * & * & * & -\gamma^2I\end{bmatrix}<0, \tag{6}$$

$$P_i\leqslant\mu_iP_j, \tag{7}$$

$$\tau_i>-\frac{\ln\mu_i}{\ln\zeta_i}, \tag{8}$$

$$\min_{f\in\varphi(i)}\frac{T^+_{\sigma(k_f)}}{T^-_{\sigma(k_f)}}\geqslant\frac{\ln\tilde{\beta}_i-\ln\zeta_i}{\ln\zeta_i-\ln\tilde{\alpha}_i}, \tag{9}$$

则对于满足式(8)的切换信号,闭环系统(4)全局一致指数稳定,且满足如下H_∞性能指标:

$$\gamma_s=\max\left\{\sqrt{\frac{1-\zeta_{\min}}{(1-\zeta_{\max})\tilde{\alpha}^{\theta}_{\min}}}\cdot\gamma\right\}.$$

其中

$$\tilde{\alpha}_i=1-\alpha_i,\ \tilde{\beta}_i=1+\beta_i,\ \zeta_i\in(1-\tilde{\alpha}_i,1),$$
$$i\in M,\ \theta=\max\{T^+_{\sigma(k_f)}|f\in\varphi(i),i\in M\},$$
$$f\in\varphi(i)=\{f|\sigma(k_f)=i,i\in M\}.$$

注1 α_i、β_i和μ_i均为模型依赖的系统参数,τ_i则为可容许的模型依赖的平均驻留时间. 接下来的讨论中,$N_{\sigma,i}(k_0,k)$表示第i个子系统在区间$[k_0,k]$上的切换次数,且有$N_{\sigma,i}(k_0,k)\leqslant T_i(k_0,k)/\tau_i$.

下面给出定理的证明.

证明 设闭环系统(4)的Lyapunov函数为

$$V_i(\tilde{x}(k))=\tilde{x}^{\mathrm{T}}(k)P_i\tilde{x}(k),\ i\in M. \tag{10}$$

当$w(k)=0$时,由式(5)和(6)可得

$$\Delta V_i(\tilde{x}(k))\leqslant\begin{cases}-\alpha_iV_i(\tilde{x}(k)),\ k\in[k_l+\Delta_l,k_{l+1});\\ \beta_iV_i(\tilde{x}(k)),\ k\in[k_l,k_l+\Delta_l).\end{cases} \tag{11}$$

由式(11)可得

$$V_{\sigma(k_l)}(\tilde{x}(k))\leqslant$$
$$\begin{cases}\tilde{\alpha}^{(k-k_l-\Delta_l)}_{\sigma(k_l)}V_{\sigma(k_l)}(\tilde{x}(k_l+\Delta_l)),\ k\in[k_l+\Delta_l,k_{l+1});\\ \tilde{\beta}^{(k-k_l)}_{\sigma(k_l)}V_{\sigma(k_l)}(\tilde{x}(k_l)),\ k\in[k_l,k_l+\Delta_l).\end{cases} \tag{12}$$

因此,当$k\in[k_l,k_{l+1})$时,由式(7)和(12)可得

$$V_{\sigma(k_l)}(\tilde{x}(k))\leqslant$$

$$(\tilde{\alpha}_{\sigma(k_l)}^{T^+(k_l,k_{l+1})}\cdot\tilde{\beta}_{\sigma(k_l)}^{T^-(k_l,k_{l+1})})\cdot V_{\sigma(k_l)}(\tilde{x}(k_l))\leqslant$$

$$\mu_{\sigma(k_l)}(\tilde{\alpha}_{\sigma(k_l)}^{T^+(k_l,k_{l+1})}\cdot\tilde{\beta}_{\sigma(k_l)}^{T^-(k_l,k_{l+1})})\cdot V_{\sigma(k_{l-1})}(\tilde{x}(k_l))\leqslant$$

$$\mu_{\sigma(k_l)}(\tilde{\alpha}_{\sigma(k_l)}^{T^+(k_l,k_{l+1})}\cdot\tilde{\beta}_{\sigma(k_l)}^{T^-(k_l,k_{l+1})})\cdot$$

$$(\tilde{\alpha}_{\sigma(k_{l-1})}^{T^+(k_{l-1},k_l)}\cdot\tilde{\beta}_{\sigma(k_{l-1})}^{T^-(k_{l-1},k_l)})\cdot$$

$$V_{\sigma(k_{l-1})}(\tilde{x}(k_{l-1}))\leqslant\ldots\leqslant$$

$$\mu_{\sigma(k_l)}\mu_{\sigma(k_{l-1})}\cdots\mu_{\sigma(k_1)}\cdot$$

$$(\tilde{\alpha}_{\sigma(k_l)}^{T^+(k_l,k_{l+1})}\cdot\tilde{\beta}_{\sigma(k_l)}^{T^-(k_l,k_{l+1})})\cdot$$

$$(\tilde{\alpha}_{\sigma(k_{l-1})}^{T^+(k_{l-1},k_l)}\cdot\tilde{\beta}_{\sigma(k_{l-1})}^{T^-(k_{l-1},k_l)})\cdots$$

$$(\tilde{\alpha}_{\sigma(k_0)}^{T^+(k_0,k_1)}\cdot\tilde{\beta}_{\sigma(k_0)}^{T^-(k_0,k_1)})V_{\sigma(k_0)}(\tilde{x}(k_0))=$$

$$\prod_{i\in M}\mu_i^{N_{\sigma,i}}\cdot\mathrm{e}^{\sum\limits_{i\in M,f\in\phi(i)}\ln\tilde{\alpha}_iT^+(k_f,k_{f+1})+\ln\tilde{\beta}_iT^-(k_f,k_{f+1})}\cdot$$

$$V_{\sigma(k_0)}(\tilde{x}(k_0)).\tag{13}$$

为得到系统的渐近稳定性,将式(13)中两参数的时间指数函数通过(9)转化为一个参数的时间指数函数,由(9)可知

$$(T^+(k_l,k_{l+1})+T^-(k_l,k_{l+1}))\ln\zeta_{\sigma(k_l)}\geqslant$$
$$T^+(k_l,k_{l+1})\ln\tilde{\alpha}_{\sigma(k_l)}+T^-(k_l,k_{l+1})\ln\tilde{\beta}_{\sigma(k_l)}.\tag{14}$$

由式(13)和(14)可知

$$V_{\sigma(k_l)}(\tilde{x}(k))\leqslant$$

$$\prod_{i\in M}\mu_i^{N_{\sigma,i}}\cdot\mathrm{e}^{\sum\limits_{i\in M,f\in\phi(i)}\ln\tilde{\alpha}_iT^+(k_f,k_{f+1})+\ln\tilde{\beta}_iT^-(k_f,k_{f+1})}\cdot$$

$$V_{\sigma(k_0)}(\tilde{x}(k_0))\leqslant$$

$$\mathrm{e}^{\sum\limits_{i\in M,f\in\phi(i)}(\ln\zeta_i+\frac{\ln\mu_i}{\tau_i})(k_{f+1}-k_f)}\cdot$$

$$V_{\sigma(k_0)}(\tilde{x}(k_0)).\tag{15}$$

由式(8)和(15)可知,当$k\to\infty$时,$V_{\sigma(k)}(k)$收敛到零. 因此当式(5)$\sim$(8)成立时,闭环系统(4)全局一致渐近稳定. 此处的切换信号仅与时间有关而与状态无关,因此系统的渐近稳定性与指数稳定性等价,在此不再赘述.

若扰动$w(k)\neq0$,则由式(5)和(6)可知

$$\Delta V_i(\tilde{x}(k))\leqslant$$
$$\begin{cases}-\alpha_iV_i(\tilde{x}(k))-\varGamma(k),\ k\in[k_l+\varDelta_l,k_{l+1});\\ \beta_iV_i(\tilde{x}(k))-\varGamma(k),\ k\in[k_l,k_l+\varDelta_l).\end{cases}\tag{16}$$

其中$\varGamma(k)=\varepsilon^{\mathrm{T}}(k)\varepsilon(k)-\gamma^2w^{\mathrm{T}}(k)w(k)$. 由式(16)可得

$$V_{\sigma(k_l)}(\tilde{x}(k))\leqslant$$
$$\begin{cases}\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\varDelta_l)}V_{\sigma(k_l)}(\tilde{x}(k_l+\varDelta_l))-\\ \sum\limits_{s=k_l+\varDelta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\varGamma(s),\ k\in[k_l+\varDelta_l,\ k_{l+1});\\ \tilde{\beta}_{\sigma(k_l)}^{(k-k_l)}V_{\sigma(k_l)}(\tilde{x}(k_l))-\\ \sum\limits_{s=k_l}^{k-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\varDelta_l-s-1)}\varGamma(s),\ k\in[k_l,\ k_l+\varDelta_l).\end{cases}\tag{17}$$

由式(7)和(17)可得,当$k\in[k_l+\varDelta_l,k_{l+1})$时,有

$$V_{\sigma(k_l)}(\tilde{x}(k))\leqslant$$

$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\varDelta_l)}V_{\sigma(k_l)}(\tilde{x}(k_l+\varDelta_l))-$$

$$-\sum_{s=k_l+\varDelta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\varGamma(s)\leqslant$$

$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\varDelta_l)}\Big(\tilde{\beta}_{\sigma(k_l)}^{(\varDelta_l)}V_{\sigma(k_l)}(\tilde{x}(k_l))-$$

$$\sum_{s=k_l}^{k_l+\varDelta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\varDelta_l-s-1)}\varGamma(s)\Big)-$$

$$\sum_{s=k_l+\varDelta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\varGamma(s)\leqslant$$

$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\varDelta_l)}\Big(\tilde{\beta}_{\sigma(k_l)}^{(\varDelta_l)}\mu_{\sigma(k_l)}V_{\sigma(k_{l-1})}(\tilde{x}(k_l))-$$

$$\sum_{s=k_l}^{k_l+\varDelta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\varDelta_l-s-1)}\varGamma(s)\Big)-$$

$$\sum_{s=k_l+\varDelta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\varGamma(s)\leqslant\ldots\leqslant$$

$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\varDelta_l)}\Big(\tilde{\beta}_{\sigma(k_l)}^{(\varDelta_l)}\mu_{\sigma(k_l)}\Big(\ldots$$

$$\Big(\tilde{\alpha}_{\sigma(k_0)}^{(k_1-k_0-\varDelta_0)}\Big(\tilde{\beta}_{\sigma(k_0)}^{(\varDelta_0)}\mu_{\sigma(k_l)}V_{\sigma(k_0)}(x(k_0))-$$

$$-\sum_{s=k_0}^{k_0+\varDelta_0-1}\tilde{\beta}_{\sigma(k_0)}^{(k_0+\varDelta_0-s-1)}\varGamma(s)\Big)-$$

$$\sum_{s=k_0+\varDelta_0}^{k_1-1}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-s-1)}\varGamma(s)\Big)-\cdots\Big)-$$

$$\sum_{s=k_l}^{k_l+\varDelta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\varDelta_l-s-1)}\varGamma(s)\Big)-$$

$$\sum_{s=k_l+\varDelta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\varGamma(s).\tag{18}$$

为考察系统的扰动抑制性能,取零初始条件,即$V_{\sigma(k_0)}(\tilde{x}(k_0))=0$,又$V_{\sigma(k_l)}(\tilde{x}(k))\geqslant0$,由式(18)可得

$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\varDelta_l)}\ldots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\varDelta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\varDelta_l)}\ldots\tilde{\beta}_{\sigma(k_1)}^{(\varDelta_1)}\cdot$$

$$\mu_{\sigma(k_l)}\ldots\mu_{\sigma(k_1)}\cdot$$

$$\sum_{s=k_0}^{k_0+\Delta_0-1}\tilde{\beta}_{\sigma(k_0)}^{(k_0+\Delta_0-s-1)}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-k_0-\Delta_0)}\varepsilon^{\rm T}(s)\varepsilon(s)+$$
$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\cdot$$
$$\sum_{s=k_0+\Delta_0}^{k_1-1}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-s-1)}\varepsilon^{\rm T}(s)\varepsilon(s)+\cdots+$$
$$\sum_{s=k_l}^{k_l+\Delta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\Delta_l-s-1)}\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\varepsilon^{\rm T}(s)\varepsilon(s)+$$
$$\sum_{s=k_l+\Delta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\varepsilon^{\rm T}(s)\varepsilon(s)\leqslant$$
$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\cdot$$
$$\gamma^2\sum_{s=k_0}^{k_0+\Delta_0-1}\tilde{\beta}_{\sigma(k_0)}^{(k_0+\Delta_0-s-1)}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-k_0-\Delta_0)}w^{\rm T}(s)w(s)+$$
$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\cdot$$
$$\gamma^2\sum_{s=k_0+\Delta_0}^{k_1-1}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-s-1)}w^{\rm T}(s)w(s)+\cdots+$$
$$\gamma^2\sum_{s=k_l}^{k_l+\Delta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\Delta_l-s-1)}\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}w^{\rm T}(s)w(s)+$$
$$\gamma^2\sum_{s=k_l+\Delta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}w^{\rm T}(s)w(s). \tag{19}$$

设$(k-k_l-\Delta_l),(k_l-k_{l-1}-\Delta_{l-1}),\cdots,(k_1-k_0-\Delta_0)$的最大值为$\theta$,即$\max\{T^+_{\sigma(k_f)}|f\in\varphi(i),i\in M\}=\theta$. 由于$0<\alpha_i<1,\tilde{\alpha}_i=1-\alpha_i$,可知$\tilde{\alpha}_{\min}^{\theta}<\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}<1$,此处$\tilde{\alpha}_{\min}=1-\alpha_{\max}$,继而,式(19)可转化为

$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\cdot$$
$$\sum_{s=k_0}^{k_0+\Delta_0-1}\tilde{\beta}_{\sigma(k_0)}^{(k_0+\Delta_0-s-1)}\tilde{\alpha}_{\min}^{\theta}\varepsilon^{\rm T}(s)\varepsilon(s)+$$
$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\sum_{s=k_0+\Delta_0}^{k_1-1}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-s-1)}\tilde{\alpha}_{\min}^{\theta}\varepsilon^{\rm T}(s)\varepsilon(s)+$$
$$\cdots+\sum_{s=k_l}^{k_l+\Delta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\Delta_l-s-1)}\tilde{\alpha}_{\min}^{\theta}\varepsilon^{\rm T}(s)\varepsilon(s)+$$
$$\sum_{s=k_l+\Delta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}\tilde{\alpha}_{\min}^{\theta}\varepsilon^{\rm T}(s)\varepsilon(s)\leqslant$$
$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\gamma^2\sum_{s=k_0}^{k_0+\Delta_0-1}\tilde{\beta}_{\sigma(k_0)}^{(k_0+\Delta_0-s-1)}w^{\rm T}(s)w(s)+$$
$$\tilde{\alpha}_{\sigma(k_l)}^{(k-k_l-\Delta_l)}\cdots\tilde{\alpha}_{\sigma(k_1)}^{(k_2-k_1-\Delta_1)}\cdot\tilde{\beta}_{\sigma(k_l)}^{(\Delta_l)}\cdots\tilde{\beta}_{\sigma(k_1)}^{(\Delta_1)}\cdot$$
$$\mu_{\sigma(k_l)}\cdots\mu_{\sigma(k_1)}\gamma^2\sum_{s=k_0+\Delta_0}^{k_1-1}\tilde{\alpha}_{\sigma(k_0)}^{(k_1-s-1)}w^{\rm T}(s)w(s)+$$
$$\cdots+\gamma^2\sum_{s=k_l}^{k_l+\Delta_l-1}\tilde{\beta}_{\sigma(k_l)}^{(k_l+\Delta_l-s-1)}w^{\rm T}(s)w(s)+$$
$$\gamma^2\sum_{s=k_l+\Delta_l}^{k-1}\tilde{\alpha}_{\sigma(k_l)}^{(k-s-1)}w^{\rm T}(s)w(s). \tag{20}$$

由式(14)和(20)可得

$$\sum_{s=k_0}^{k-1}\tilde{\alpha}_{\min}^{\theta}\cdot{\rm e}^{(k-s-1)\ln\zeta_{\min}}\cdot$$
$${\rm e}^{\sum\limits_{i\in M,f\in\varphi(i)}N_{\sigma,i}(s,k_{f+1})\ln\mu_i}\varepsilon^{\rm T}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\sum_{s=k_0}^{k-1}{\rm e}^{(k-s-1)\ln\zeta_{\max}}\cdot$$
$${\rm e}^{\sum\limits_{i\in M,f\in\varphi(i)}N_{\sigma,i}(s,k_{f+1})\ln\mu_i}w^{\rm T}(s)w(s). \tag{21}$$

对式(21)两端同时乘以${\rm e}^{\sum\limits_{i\in M,f\in\varphi(i)}-N_{\sigma,i}(k_f,k_{f+1})\ln\mu_i}$可得

$$\sum_{s=k_0}^{k-1}\tilde{\alpha}_{\min}^{\theta}\cdot{\rm e}^{(k-s-1)\ln\zeta_{\min}-\sum\limits_{i\in M,f\in\varphi(i)}N_{\sigma,i}(k_f,s)\ln\mu_i}\cdot$$
$$\varepsilon^{\rm T}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\sum_{s=k_0}^{k-1}{\rm e}^{(k-s-1)\ln\zeta_{\max}-\sum\limits_{i\in M,f\in\varphi(i)}N_{\sigma,i}(k_f,s)\ln\mu_i}\cdot$$
$$w^{\rm T}(s)w(s). \tag{22}$$

易知$N_{\sigma,i}(k_f,s)\leqslant\dfrac{s-k_f}{\tau_i}$,由此可推出

$$N_{\sigma,i}(k_f,s)\ln\mu_i\leqslant-\ln\zeta_i(s-k_f). \tag{23}$$

由式(22)和(23)可得

$$\tilde{\alpha}_{\min}^{\theta}\sum_{s=k_0}^{k-1}\zeta_{\min}^{(k-s-1)}\prod_{i\in M,f\in\varphi(i)}\zeta_i^{(s-k_f)}\varepsilon^{\rm T}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\sum_{s=k_0}^{k-1}\zeta_{\max}^{(k-s-1)}w^{\rm T}(s)w(s). \tag{24}$$

对式(24)在$[k_0,\infty)$上进行累加可得

$$\tilde{\alpha}_{\min}^{\theta}\sum_{k=k_0}^{\infty}\sum_{s=k_0}^{k-1}\zeta_{\min}^{(k-s-1)}\prod_{i\in M,f\in\varphi(i)}\zeta_i^{(s-k_f)}\varepsilon^{\rm T}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\sum_{k=k_0}^{\infty}\sum_{s=k_0}^{k-1}\zeta_{\max}^{(k-s-1)}w^{\rm T}(s)w(s). \tag{25}$$

由式(25)可得

$$\tilde{\alpha}_{\min}^{\theta}\sum_{s=k_0}^{\infty}\sum_{k=s+1}^{\infty}\zeta_{\min}^{(k-s-1)}\zeta_{\min}^{s}\varepsilon^{\mathrm{T}}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\sum_{s=k_0}^{\infty}\sum_{k=s+1}^{\infty}\zeta_{\max}^{(k-s-1)}w^{\mathrm{T}}(s)w(s). \tag{26}$$

利用等比数列求和公式可得 $\sum_{k=s+1}^{\infty}\zeta_{\min}^{(k-s-1)}=\frac{1}{1-\zeta_{\min}}$, $\sum_{k=s+1}^{\infty}\zeta_{\max}^{(k-s-1)}=\frac{1}{1-\zeta_{\max}}$,因此式(26)可化为

$$\tilde{\alpha}_{\min}^{\theta}\sum_{s=k_0}^{\infty}\frac{1}{1-\zeta_{\min}}\zeta_{\min}^{s}\varepsilon^{\mathrm{T}}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\sum_{s=k_0}^{\infty}\frac{1}{1-\zeta_{\max}}w^{\mathrm{T}}(s)w(s), \tag{27}$$

即

$$\sum_{s=k_0}^{\infty}\zeta_{\min}^{s}\varepsilon^{\mathrm{T}}(s)\varepsilon(s)\leqslant$$
$$\gamma^2\frac{1-\zeta_{\min}}{\tilde{\alpha}_{\min}^{\theta}(1-\zeta_{\max})}\sum_{s=k_0}^{\infty}w^{\mathrm{T}}(s)w(s). \tag{28}$$

因此可得闭环系统(4)全局一致指数稳定,且满足指数H_∞性能指标

$$\gamma_s=\max\left\{\sqrt{\frac{1-\zeta_{\min}}{(1-\zeta_{\max})\tilde{\alpha}_{\min}^{\theta}}}\cdot\gamma\right\}.$$ □

接下来解决所设计滤波器的求解问题.

定理2 对于闭环系统(4),给定实数$0<\alpha_i<1$, $\beta_i>0,\mu_i>1$,若存在正定实对称矩阵P_{i1}、P_{i3},矩阵P_{i2}、R_i、S_i、T_i、A_{Fi}、B_{Fi}、C_{Fi}、D_{Fi},实数$\tau_i>0$, $\zeta_i>0$和一个正实数γ、i、$j\in M,i\neq j$,有

$$\begin{bmatrix}\varXi_i^{11} & \varXi_i^{12} & 0 & \varXi_i^{14} & A_{Fi} & \varXi_i^{16}\\ * & \varXi_i^{22} & 0 & \varXi_i^{24} & A_{Fi} & \varXi_i^{26}\\ * & * & -I & \varXi_i^{34} & -C_{Fi} & \varXi_i^{36}\\ * & * & * & -\tilde{\alpha}_iP_{i1} & -\tilde{\alpha}_iP_{i2} & 0\\ * & * & * & * & -\tilde{\alpha}_iP_{i3} & 0\\ * & * & * & * & * & -\gamma^2I\end{bmatrix}<0, \tag{29}$$

$$\begin{bmatrix}\varXi_{ij}^{11} & \varXi_{ij}^{12} & 0 & \varXi_{ij}^{14} & A_{Fj} & \varXi_{ij}^{16}\\ * & \varXi_{ij}^{22} & 0 & \varXi_{ij}^{24} & A_{Fj} & \varXi_{ij}^{26}\\ * & * & -I & \varXi_{ij}^{34} & -C_{Fj} & \varXi_{ij}^{36}\\ * & * & * & -\tilde{\beta}_iP_{i1} & -\tilde{\beta}_iP_{i2} & 0\\ * & * & * & * & -\tilde{\beta}_iP_{i3} & 0\\ * & * & * & * & * & -\gamma^2I\end{bmatrix}<0, \tag{30}$$

$$\begin{bmatrix}\varTheta^{11} & \varTheta^{12} & R_i^{\mathrm{T}} & T_i\\ * & \varTheta^{22} & S_i^{\mathrm{T}} & T_i\\ * & * & -\mu_iP_{j1} & -\mu_iP_{j2}\\ * & * & * & -\mu_iP_{j3}\end{bmatrix}\leqslant 0, \tag{31}$$

$$\tau_i>-\frac{\ln\mu_i}{\ln\zeta_i}, \tag{32}$$

$$\min_{f\in\varphi(i)}\frac{T_{\sigma(k_f)}^{+}}{T_{\sigma(k_f)}^{-}}\geqslant\frac{\ln\tilde{\beta}_i-\ln\zeta_i}{\ln\zeta_i-\ln\tilde{\alpha}_i}, \tag{33}$$

则对于满足式(32)的切换信号,闭环系统(4)全局一致指数稳定,且满足H_∞性能指标

$$\gamma_s=\max\left\{\sqrt{\frac{1-\zeta_{\min}}{(1-\zeta_{\max})\tilde{\alpha}_{\min}^{\theta}}}\cdot\gamma\right\}.$$

其中:$\tilde{\alpha}_i=1-\alpha_i,\tilde{\beta}_i=1+\beta_i,\zeta_i\in(1-\tilde{\alpha}_i,1),i\in M,f\in\varphi(i)=\{f|\sigma(k_f)=i,i\in M\},\varXi_i^{11}=P_{i1}-\mathrm{He}(R_i),\varXi_i^{12}=P_{i2}-S_i-T_i,\varXi_i^{14}=R_i^{\mathrm{T}}A_i+B_{Fi}C_i$, $\varXi_i^{16}=R_i^{\mathrm{T}}B_i+B_{Fi}D_i,\varXi_i^{22}=P_{i3}-\mathrm{He}(T_i),\varXi_i^{24}=S_i^{\mathrm{T}}A_i+B_{Fi}C_i,\varXi_i^{26}=S_i^{\mathrm{T}}B_i+B_{Fi}D_i,\varXi_i^{34}=E_i-D_{Fi}C_i,\varXi_i^{36}=F_i-D_{Fi}D_i,\varXi_{ij}^{11}=P_{i1}-\mathrm{He}(R_j)$, $\varXi_{ij}^{12}=P_{i2}-S_j-T_j,\varXi_{ij}^{14}=R_j^{\mathrm{T}}A_i+B_{Fj}C_i,\varXi_{ij}^{16}=R_j^{\mathrm{T}}B_i+B_{Fj}D_i,\varXi_{ij}^{22}=P_{i3}-\mathrm{He}(T_j),\varXi_{ij}^{24}=S_j^{\mathrm{T}}A_i+B_{Fj}C_i,\varXi_{ij}^{26}=S_j^{\mathrm{T}}B_i+B_{Fj}D_i,\varXi_{ij}^{34}=E_i-D_{Fj}C_i$, $\varXi_{ij}^{36}=F_i-D_{Fj}D_i,\varTheta^{11}=P_{i1}-\mu_i\mathrm{He}(R_i),\varTheta^{12}=P_{i2}-\mu_iS_i-\mu_iT_i,\varTheta^{22}=P_{i3}-\mu_i\mathrm{He}(T_i)$,He(·)表示该矩阵与它的转置相加得到的和.

同时可以获得滤波器增益

$$\begin{bmatrix}A_{fi} & B_{fi}\\ C_{fi} & D_{fi}\end{bmatrix}=\begin{bmatrix}T_i^{-1} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}A_{Fi} & B_{Fi}\\ C_{Fi} & D_{Fi}\end{bmatrix}. \tag{34}$$

证明 对于正定矩阵P_i和矩阵G_i易知

$$(P_i-G_i)^{\mathrm{T}}P_i^{-1}(P_i-G_i)\geqslant 0,$$
$$(P_j-G_i)^{\mathrm{T}}P_j^{-1}(P_j-G_i)\geqslant 0. \tag{35}$$

继而可得

$$P_i-\mathrm{He}(G_i)\geqslant -G_i^{\mathrm{T}}P_i^{-1}G_i,$$
$$P_j-\mathrm{He}(G_i)\geqslant -G_i^{\mathrm{T}}P_j^{-1}G_i. \tag{36}$$

若令$P_i=\begin{bmatrix}P_{i1} & P_{i2}\\ * & P_{i3}\end{bmatrix}$, $G_i=\begin{bmatrix}R_i & S_i\\ T_i^{\mathrm{T}} & T_i^{\mathrm{T}}\end{bmatrix}$,则式(29)和(30)分别等价于

$$\begin{bmatrix}P_i-\mathrm{He}(G_i) & 0 & G_i^{\mathrm{T}}\tilde{A}_i & G_i^{\mathrm{T}}\tilde{B}_i\\ * & -I & \tilde{C}_i & \tilde{D}_i\\ * & * & -\tilde{\alpha}P_i & 0\\ * & * & * & -\gamma^2I\end{bmatrix}<0, \tag{37}$$

$$\begin{bmatrix}P_i-\mathrm{He}(G_j) & 0 & G_j^{\mathrm{T}}\tilde{A}_{ij} & G_j^{\mathrm{T}}\tilde{B}_{ij}\\ * & -I & \tilde{C}_{ij} & \tilde{D}_{ij}\\ * & * & -\tilde{\beta}_iP_i & 0\\ * & * & * & -\gamma^2I\end{bmatrix}<0. \tag{38}$$

由式(36)~(38)可得

$$\begin{bmatrix}-G_i^{\mathrm{T}}P_i^{-1}G_i & 0 & G_i^{\mathrm{T}}\tilde{A}_i & G_i^{\mathrm{T}}\tilde{B}_i\\ * & -I & \tilde{C}_i & \tilde{D}_i\\ * & * & -\tilde{\alpha}P_i & 0\\ * & * & * & -\gamma^2I\end{bmatrix}<0, \tag{39}$$

$$\begin{bmatrix} -G_j^{\mathrm{T}}P_i^{-1}G_j & 0 & G_j^{\mathrm{T}}\tilde{A}_{ij} & G_j^{\mathrm{T}}\tilde{B}_{ij} \\ * & -I & \tilde{C}_{ij} & \tilde{D}_{ij} \\ * & * & -\tilde{\beta}_iP_i & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0. \tag{40}$$

利用$\mathrm{diag}\{G_i^{-1}P_i\ \ I\ \ I\ \ I\}$和$\mathrm{diag}\{G_j^{-1}P_i\ \ I\ \ I\ \ I\}$分别进行全等变换可得式(5)和(6). 由式(7)和(36)可知

$$P_i - \mu_i[G_i + G_i^{\mathrm{T}} - G_i^{\mathrm{T}}P_j^{-1}G_i] \leqslant 0. \tag{41}$$

对式(41)使用Schur补引理可得

$$\begin{bmatrix} P_i - \mu_i \mathrm{He}(G_i) & G_i^{\mathrm{T}} \\ * & -\mu_iP_j \end{bmatrix} \leqslant 0. \tag{42}$$

由如上定义可知,式(42)等价于(31). □

3　数值仿真

考虑离散切换系统的两个子系统

$$A_1 = \begin{bmatrix} -0.51 & 0.24 & -0.22 \\ -0.41 & 0.45 & -0.07 \\ 0.8 & -0.99 & -0.32 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.2 \\ 1.4 \\ -0.55 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.3 \\ 0.4 \\ -0.17 \end{bmatrix}^{\mathrm{T}},\ E_1 = \begin{bmatrix} -0.1 \\ 0.01 \\ 0.3 \end{bmatrix}^{\mathrm{T}},\ D_1 = 0.3,$$

$$F_1 = 0.3;$$

$$A_2 = \begin{bmatrix} -0.8 & 0.16 & 0.06 \\ -0.21 & -0.74 & 0.14 \\ 0.8 & -0.21 & -0.64 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.1 \\ 0.9 \\ 0.43 \end{bmatrix}^{\mathrm{T}},$$

$$C_2 = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.12 \end{bmatrix}^{\mathrm{T}},\ E_2 = \begin{bmatrix} -0.2 \\ 0.04 \\ 0.05 \end{bmatrix}^{\mathrm{T}},\ D_2 = -1.1,$$

$$F_2 = 0.4.$$

要求找到一簇合适的模型依赖的全阶滤波器和可容许的切换信号使得所研究的滤波误差系统指数稳定并满足H_∞性能.

求解定理2时,若令$\mu_1 = \mu_2 = 1.5$,则系统H_∞性能与参数的关系如表1所示.

表1　$\mu_1 = \mu_2 = 1.5$时系统H_∞性能与参数的关系

α_1	α_2	β_1	β_2	γ
0.05	0.04	0.03	0.02	0.85
0.05	0.05	0.02	0.02	0.74
0.05	0.05	0.02	0.01	0.73
0.06	0.05	0.02	0.01	0.738
0.1	0.08	0.06	0.03	0.443
0.08	0.08	0.05	0.03	0.44

选取$\alpha_1 = \alpha_2 = 0.08, \beta_1 = 0.05, \beta_2 = 0.03$作为系统的控制参数,可解的滤波器增益为

$$A_{f1} = \begin{bmatrix} -0.3138 & 0.0084 & -0.0893 \\ 0.2725 & -0.5641 & -0.1646 \\ -0.1122 & 0.0793 & -0.1654 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} 0.6591 \\ 0.9424 \\ -0.8075 \end{bmatrix},\ C_{f1} = \begin{bmatrix} 0.0574 \\ -0.0670 \\ -0.0490 \end{bmatrix}^{\mathrm{T}},$$

$$D_{f1} = -0.1950;$$

$$A_{f2} = \begin{bmatrix} -0.1364 & -0.0503 & -0.0463 \\ 0.6654 & -0.8536 & -0.1434 \\ -0.4904 & 0.2296 & -0.2889 \end{bmatrix},$$

$$B_{f2} = \begin{bmatrix} 1.4665 \\ 2.7551 \\ -2.3275 \end{bmatrix},\ C_{f2} = \begin{bmatrix} 0.1221 \\ -0.0922 \\ -0.0453 \end{bmatrix}^{\mathrm{T}},$$

$$D_{f2} = -0.2776.$$

若选取$\zeta_1 = 0.92, \zeta_2 = 0.93$,则可得$\tau_1 \geqslant 4.86$, $\tau_2 \geqslant 5.58$. 若选取$\tau = 5\,\mathrm{s}, \tau_2 = 8\,\mathrm{s}$,则由相应的容许的滤波器的滞后时间可以得到$\varDelta_1 = 1\,\mathrm{s}, \varDelta_2 = 2\,\mathrm{s}$. 设扰动输入为

$$w(k) = \begin{cases} 0.8\sin t,\ 0 \leqslant k \leqslant 20; \\ 0,\ \text{otherwise}. \end{cases}$$

切换信号如图1所示.

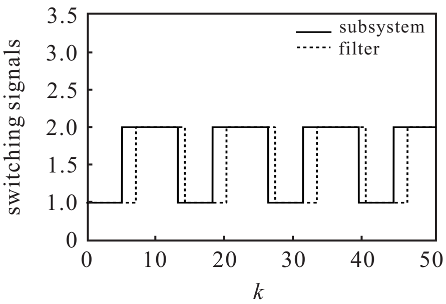

图1　异步切换信号

设初始状态

$$\tilde{x}(0) = [x^{\mathrm{T}}(0)\ \ x_f^{\mathrm{T}}(0)]^{\mathrm{T}} = [1\ \ -1\ \ 2\ \ 0\ \ 0\ \ 0]^{\mathrm{T}},$$

异步滤波误差系统在异步切换信号下的状态响应如图2～图4所示.

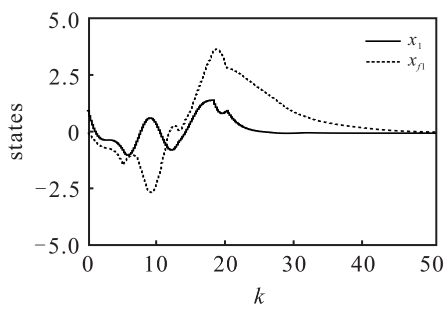

图2　异步滤波误差系统的状态响应: 状态1

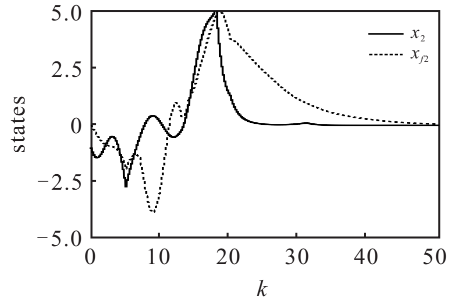

图3 异步滤波误差系统的状态响应:状态2

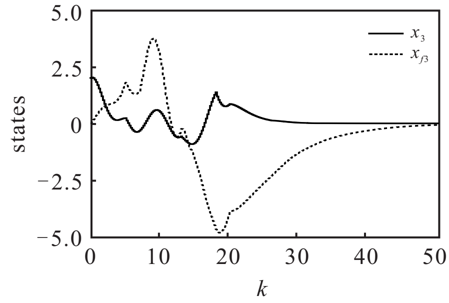

图4 异步滤波误差系统的状态响应:状态3

由图2∼图4可以看出,x_{f1}、x_{f2}和x_{f3}均有较好的估计效果,而发生的状态估计偏离则是因为所设计的滤波器与子系统不匹配造成的. 被估计函数$z(k)$和滤波器的估计函数$z_{f(k)}$的响应曲线如图5所示.

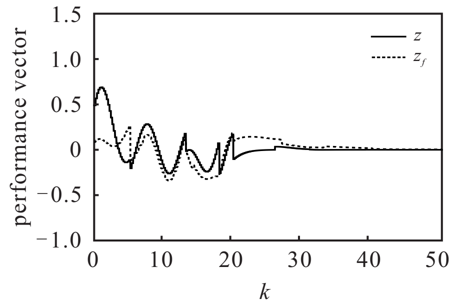

图5 被估计信号和滤波器的估计

由图5可以看出,滤波器对原系统的状态估计具有较高的准确性. 异步滤波误差系统的误差响应如图6所示,在扰动消失后迅速归零.

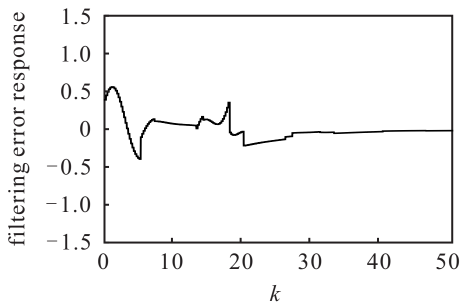

图6 异步滤波误差系统的误差响应

4 结 论

本文利用模型依赖平均驻留时间策略为离散异步切换线性系统设计了指数收敛的H_∞滤波器,解决了在实际应用中滤波器的切换滞后于子系统的问题. 将子系统运行时间分为与对应的滤波器匹配的区间和不匹配的区间,利用μ依赖不连续的多Lyapunov函数和MDADT方法分析异步切换滤波误差系统的指数稳定性,设计了模型依赖的指数收敛的H_∞滤波器,使得异步滤波误差系统满足指数H_∞性能指标. 数值仿真验证了所设计的异步切换滤波器是有效的.

参考文献(References)

[1] Yang D, Zhao J. H_∞ output tracking control for a class of switched LPV systems and its application to an aero-engine model[J]. International Journal of Robust and Nonlinear Control, 2017, 27(12): 2102-2120.

[2] Donkers M C F, Hetel L, Heemels W P M H, et al. Stability analysis of networked control systems using a switched linear systems approach[J]. IEEE Transactions on Automatic Control, 2011, 56(9): 2101-2115.

[3] 齐迹, 李艳辉. 网络环境下连续切换系统观测器和控制器设计[J]. 控制与决策, 2019, 34(12): 2649-2655.
(Qi J, Li Y H. Observer and controller design for continuous switched systems in the network environment[J]. Control and Decision, 2019, 34(12): 2649-2655.)

[4] He X, Dimirovski G M, Zhao J. Control of switched LPV systems using common Lyapunov function method and an F-16 aircraft application[C]. 2010 IEEE International Conference on Systems, Man, and Cybernetics. Istanbul, 2010: 386-392.

[5] Wong W S, Brockett R W. Systems with finite communication bandwidth constraints. II. stabilization with limited information feedback[J]. IEEE Transactions on Automatic Control, 1999, 44(5): 1049-1053.

[6] Ma Y, Kawakami H, Tse C K. Bifurcation analysis of switched dynamical systems with periodically moving borders[J]. IEEE Transactions on Circuits and Systems I: Regular Papers, 2004, 51(6): 1184-1193.

[7] Vega F, Rachidi F, Giri D, et al. A new set of electrodes for coaxial, quarter wave, and switched oscillators[J]. IEEE Transactions on Plasma Science, 2013, 41(9): 2545-2550.

[8] Mitsubori K, Saito T. Dependent switched capacitor chaos generator and its synchronization[J]. IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, 1997, 44(12): 1122-1128.

[9] Liberzon D. Switching in systems and control[M]. Boston: Birkhäuser Boston, 2003: 21-24.

[10] Daafouz J, Riedinger P, Iung C. Stability analysis and control synthesis for switched systems: A switched Lyapunov function approach[J]. IEEE Transactions on

Automatic Control, 2002, 47(11): 1883-1887.

[11] Lu B, Wu F. Switching LPV control designs using multiple parameter-dependent Lyapunov functions[J]. Automatica, 2004, 40(11): 1973-1980.

[12] Zhang L X, Boukas E K, Shi P. Exponential H_∞ filtering for uncertain discrete-time switched linear systems with average dwell time: A μ-dependent approach[J]. International Journal of Robust and Nonlinear Control, 2008, 18(11): 1188-1207.

[13] Lee J W, Dullerud G E. Uniform stabilization of discrete-time switched and Markovian jump linear systems[J]. Automatica, 2006, 42(2): 205-218.

[14] Battilotti S, De Santis A. Dwell-time controllers for stochastic systems with switching Markov chain[J]. Automatica, 2005, 41(6): 923-934.

[15] Bolzern P, Colaneri P, De Nicolao G. Markov jump linear systems with switching transition rates: Mean square stability with dwell-time[J]. Automatica, 2010, 46(6): 1081-1088.

[16] Jiang Z Y, Yan P. Asynchronous switching control of discrete impulsive switched delay systems with average dwell time[J]. IET Control Theory & Applications, 2018, 12(7): 992-999.

[17] Zhao X D, Zhang L X, Shi P, et al. Stability of switched positive linear systems with average dwell time switching[J]. Automatica, 2012, 48(6): 1132-1137.

[18] Lu Q G, Zhang L X, Karimi H R, et al. H_∞ control for asynchronously switched linear parameter-varying systems with mode-dependent average dwell time[J]. IET Control Theory & Applications, 2013, 7(5): 673-683.

[19] Zhang J, Han Z, Zhu F, et al. Stability and stabilization of positive switched systems with mode-dependent average dwell time[J]. Nonlinear Analysis: Hybrid Systems, 2013, 9: 42-55.

[20] Zhao X D, Zhang L X, Shi P, et al. Stability and stabilization of switched linear systems with mode-dependent average dwell time[J]. IEEE Transactions on Automatic Control, 2012, 57(7): 1809-1815.

[21] Zhou J M, Gu G X, Xiang C. Distributed Kalman filtering over wireless sensor networks in the presence of data packet drops[J]. IEEE Transactions on Automatic Control, 2018, 64(4): 1603-1610.

[22] Xu T B, Zhang Q L, Zhao F. H_∞ filtering problem of singular systems with uncertainties in the difference matrix[J]. International Journal of Control, Automation and Systems, 2018, 16(3): 1207-1216.

[23] Morais C F, Palma J M, Peres P L D, et al. An LMI approach for H_2 and H_∞ reduced-order filtering of uncertain discrete-time Markov and Bernoulli jump linear systems[J]. Automatica, 2018, 95: 463-471.

[24] 李艳辉, 柳桐. 具有分布时滞和随机测量数据丢失的离散切换系统H_∞滤波[J]. 控制与决策, 2017, 32(8): 1486-1492.
(Li Y H, Liu T. H_∞ filtering for discrete switched systems with distributed delays and random measurement data missing[J]. Control and Decision, 2017, 32(8): 1486-1492.)

[25] Wu Z G, Shi P, Su H Y, et al. Asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities[J]. Automatica, 2014, 50(1): 180-186.

[26] Li L, Jia Y M. Robust l_2-l_∞ filtering for stochastic systems with discrete and distributed time-varying delays[J]. Asian Journal of Control, 2012, 14(4): 1047-1058.

[27] Wang Z D, Burnham K J. Robust filtering for a class of stochastic uncertain nonlinear time-delay systems via exponential state estimation[J]. IEEE Transactions on Signal Processing, 2001, 49(4): 794-804.

[28] Shen H, Huang Z G, Cao J D, et al. Exponential H_∞ filtering for continuous-time switched neural networks under persistent dwell-time switching regularity[J]. IEEE Transactions on Cybernetics, 2020, 50(6): 2440-2449.

[29] Zhang L X, Cui N G, Liu M, et al. Asynchronous filtering of discrete-time switched linear systems with average dwell time[J]. IEEE Transactions on Circuits and Systems I: Regular Papers, 2011, 58(5): 1109-1118.

[30] Mahmoud M S, Shi P. Asynchronous H_∞ filtering of discrete-time switched systems[J]. Signal Processing, 2012, 92(10): 2356-2364.

[31] Xiang W M, Xiao J. H_∞ filtering for switched nonlinear systems under asynchronous switching[J]. International Journal of Systems Science, 2011, 42(5): 751-765.

[32] Xiang W M, Xiao J, Mahmoud M S. Filtering for switched discrete-time systems under asynchronous switching: A dwell-time dependent Lyapunov functional method[J]. International Journal of Adaptive Control and Signal Processing, 2015, 29(8): 971-990.

[33] Xiang Z R, Liang C Y, Chen Q W. Robust L_2-L_∞ filtering for switched systems under asynchronous switching[J]. Communications in Nonlinear Science and Numerical Simulation, 2011, 16(8): 3303-3318.

作者简介

郝现志 (1991−), 男, 博士生, 从事切换控制、鲁棒控制的研究, E-mail: hxz_man@163.com;

黄金杰 (1967−), 男, 教授, 博士生导师, 从事增益调度、复杂工业过程优化控制及其应用等研究, E-mail: jjhuangps@163.com;

潘晓真 (1989−), 女, 博士生, 从事参数变量控制、事件触发控制的研究, E-mail: panxiaozhenvvip@163.com.

(责任编辑: 闫 妍)