控制与决策
Control and Decision

具有输入量化和全状态约束的非严格反馈随机非线性系统的有限时间动态面控制

朱新峰, 丁文武, 张天平

引用本文:
朱新峰,丁文武,张天平. 具有输入量化和全状态约束的非严格反馈随机非线性系统的有限时间动态面控制[J]. 控制与决策, 2022, 37(10): 2575-2584.

在线阅读 View online: https://doi.org/10.13195/j.kzyjc.2021.0012

具有输入量化和全状态约束的非严格反馈随机非线性系统的有限时间动态面控制

朱新峰，丁文武，张天平†

(扬州大学 信息工程学院，江苏 扬州 225127)

摘　要: 研究具有输入量化和全状态约束的非严格反馈随机非线性系统的有限时间自适应跟踪控制. 首先, 利用双曲正切函数进行非线性映射, 消除全状态约束的限制, 将系统变换为无约束系统; 其次, 引入滞回量化器克服量化信号中的抖动和量化误差. 为实现有限时间控制, 提出概率意义下半全局有限时间稳定控制方法, 加快系统的收敛速度, 并在此基础上采用径向基函数神经网络逼近未知非线性函数; 接着, 基于动态面控制技术和高斯函数的性质, 对变换后的非严格反馈随机系统进行自适应控制设计, 所设计的控制器能够保证闭环系统中的所有信号在概率意义下有限时间稳定; 最后通过仿真实验表明所设计控制方案的有效性.

关键词: 非严格反馈；随机系统；全状态约束；量化输入；有限时间稳定；动态面控制

中图分类号: TP273　　　**文献标志码:** A

DOI: 10.13195/j.kzyjc.2021.0012

引用格式: 朱新峰, 丁文武, 张天平. 具有输入量化和全状态约束的非严格反馈随机非线性系统的有限时间动态面控制[J]. 控制与决策, 2022, 37(10): 2575-2584.

Finite-time dynamic surface control for nonstrict-feedback stochastic nonlinear systems with input quantization and full-state constraints

ZHU Xin-feng, DING Wen-wu, ZHANG Tian-ping†

(College of Information Engineering，Yangzhou University，Yangzhou 225127，China)

Abstract: This paper mainly studies the finite-time adaptive tracking control of a class of non-strict feedback stochastic nonlinear systems with input quantization and full-state constraints. First, the hyperbolic tangent function is used for nonlinear mapping, which eliminates the constraints of the full-state constraints and transforms the system into an unconstrained system. Second, a hysteresis quantizer is introduced to avoid the chattering and reduce the quantisation error in the quantized signal. Third, in order to achieve finite time control, a semi-global finite time stability criterion is proposed in the sense of probability, which speeds up the convergence speed of the system. On this basis, radial basis function neural networks are used to approximate the unknown nonlinear functions. Based on the dynamic surface control technology and the properties of the Gaussian function, adaptive control design is performed for the transformed non-strict feedback stochastic system. The designed controller can guarantee that all signals in the closed-loop system are semi-globally finite time stable in probability(SGFTSP). Simulation results show the effectiveness of the proposed control scheme.

Keywords: non-strict feedback；stochastic systems；full-state constraints；input quantization；finite-time stability；dynamic surface control

0 引　言

反推方法已广泛用于许多非线性系统的控制器设计[1-2], 这种方法使用中间状态作为虚拟控制信号, 由于设计过程中的每一步都需要虚拟控制信号及其导数, 在实际应用中导数计算相当复杂和繁琐, 从而引发“计算膨胀”. 文献[3]提出了动态面控制方法, 即在每个迭代步骤中建立一阶滤波器. 文献[4]采用积分型李亚普诺夫函数, 利用动态面方法解决了一类具有未知死区的非线性纯反馈系统. 在实际控制系统中, 约束的影响是决定系统性能的一个重要因素. 文

收稿日期: 2021-01-05；**录用日期:** 2021-07-19.
基金项目: 国家自然科学基金项目(62073283)；江苏省自然科学基金项目(BK20181218).
责任编委: 关新平.
†通讯作者. E-mail: tpzhang@yzu.edu.cn.

献[5]针对一类不确定单输入单输出(SISO)随机非线性系统,在输入饱和和状态不可测的条件下,提出了一种有效的预定性能控制方法. 与基于积分型或对数型障碍李亚普诺夫函数的自适应控制器设计相比,利用双曲正切函数作为非线性映射方法更为方便,并且没有初始条件的限制. 通过采用一对一的非线性映射方法消除非对称时变输出约束,文献[6]针对非仿射系统提出了自适应跟踪控制. 量化控制在电力系统等工业领域和网络控制系统中得到了广泛应用. 例如,在无线通信网络中为了节省有限的带宽资源,需要量化技术以降低信息传输过程中的通信速率. 量化器作为映射工具可将连续信号转换成离散信号. 为了避免抖振现象,滞回量化器得到了深入研究. 文献[7]提出了在输入未建模条件下,借助量化输入和积分型李亚普诺夫函数求解非线性系统的自适应量化动态控制方法. 文献[8]针对一类不确定非线性系统提出了一种新的自适应控制方案,该方案采用的输入量化器由对数量化器和均匀量化器组成. 文献[9]针对一类非线性随机单输入单输出系统,通过选择一个新的非线性量化输入,提出了一种新的自适应输出反馈控制策略.

上述研究内容都是基于无限时间跟踪控制范畴. 然而,在许多实际工程中,往往要求控制目标在有限的时间内收敛,而且有限时间控制可以使闭环系统具有更快的收敛速度、更高的跟踪精度和更强的抗干扰能力,因此,近年来有限时间控制的研究越来越受到重视[10]. 针对有限时间控制,将动态面控制技术与反推法相结合,设计了一种新的自适应控制器. 文献[11]研究了非线性下三角系统的有限时间镇定问题,证明了在系统中,通过赫尔德连续状态反馈可以获得全局有限时间镇定. 为了解决有限时间跟踪控制问题,文献[12]针对一类严格反馈非线性系统提出了一种自适应模糊控制器. 文献[13]对全状态约束的系统,通过构造一个障碍型李亚普诺夫函数,提出了一种具有参数不确定的随机非线性系统自适应有限时间控制器,解决了有限时间跟踪控制问题. 文献[14]借助于反推方法,提出了一个新的随机非线性系统的自适应有限时间控制器. 非严格反馈系统中每个子系统都包含所有的状态变量,因此,其设计过程要比严格反馈或纯反馈系统复杂得多. 为了克服上述困难,文献[15-18]引入一个关键引理,并通过对变量分离法的研究,获得了一些重要的研究结果. 对于非严格反馈系统,文献[19]提出了一种变量分离方法,将所有变量中的难点放在控制设计的最后一步,然后提出了一种自适应控制器,以保证系统的稳定性. 同时,系统中通常存在影响系统性能的各种干扰,随机干扰[20]会损害系统的性能,因此,研究随机系统的控制策略是十分必要的. 由于随机系统微分算子的计算过程比一般的求导过程复杂,文献[21]根据四阶李亚普诺夫函数提出一种自适应控制方法,解决了随机扰动的问题. 文献[22]针对具有状态约束的不确定纯反馈随机非线性系统,采用障碍李雅亚诺夫函数处理状态约束,然后利用反推技术递归建立自适应神经网络有限时间控制器,使得跟踪误差在有限时间内收敛到原点的邻域内. 文献[23]通过引入时变不对称障碍李亚普诺夫函数,将所有状态变量都被限制在预设范围内,并通过非线性分解补偿输入量化引起的系统效应,最后采用滑模变结构控制理论保证了在有限的时间内实现跟踪性能.

本文针对一类具有输入量化和全状态约束的非严格反馈非线性随机系统,提出一种有限时间自适应控制方法. 与文献[22-23]不同的是,针对全状态约束下的随机非严格反馈非线性系统,采用双曲正切函数将全状态约束的系统转换成无约束系统,提出有限时间自适应输入量化控制方法. 与文献[24]相比,不仅考虑了非严格反馈随机系统,还提出一种有限时间控制策略——基于动态面控制技术,对输入量化的随机非严格反馈系统设计有限时间控制器. 与文献[25]不同,本文提出了基于动态面控制方法的自适应神经控制方案,与传统的反推方案相比,避免了繁重的计算负担.

1 问题描述和基本假设

1.1 预备知识

考虑如下随机非线性系统:

$$\mathrm{d}x = f(x,t)\mathrm{d}t + h(t,x)\mathrm{d}B. \tag{1}$$

其中: $x \in R^n$ 为系统的状态变量, B 为标准的维纳过程, $f(\cdot): R^+ \times R^n \to R^n$ 和 $h(\cdot): R^+ \times R^n \to R^{n\times r}$ 满足局部李普希茨条件. 在系统(1),对于任意给定的 $V(t,x) \in C^{1,2}$,可以得到

$$\mathrm{d}V(t,x) = LV(t,x)\mathrm{d}t + \frac{\partial V(t,x)}{\partial x}h\mathrm{d}B. \tag{2}$$

定义 1[26] 对于任意给定的函数 $V(t,x) \in C^{1,2}$,微分算子 L 定义如下:

$$LV(t,x) = \frac{\partial V(t,x)}{\partial t} + \frac{\partial V(t,x)}{\partial x^{\mathrm{T}}}f + \frac{1}{2}\mathrm{tr}\Big[h^{\mathrm{T}}\frac{\partial^2 V(t,x)}{\partial x^{\mathrm{T}}\partial x}h\Big], \tag{3}$$

其中 $\mathrm{tr}(\cdot)$ 为矩阵的迹.

定义 2[27] 若存在紧集 D,对于任意 $x(t_0,B) =$

$x_0 \in D$,存在$\varepsilon > 0, p > 0$和时间常数$T(\varepsilon, x_0) < \infty$,使得$\forall t \geqslant t_0 + T$,有$E[\|x(t, B)\|^p] < \varepsilon$,则随机系统$\mathrm{d}x = f(x,t)\mathrm{d}t + h(t,x)\mathrm{d}B$的解在概率意义下半全局有限时间稳定.

引理1[27]　假设存在一个正定函数$V(t,x): R^n \to R^+$对于非线性系统$\dot{x} = f(x)$和K_∞类函数$\bar{\alpha}_1$、$\bar{\alpha}_2$,使得

$$\bar{\alpha}_1(\|x\|) \leqslant V(t,x) \leqslant \bar{\alpha}_2(\|x\|), \tag{4}$$

$$LV(t,x) \leqslant -aV^\beta(t,x) + b, \tag{5}$$

其中$a > 0$, $b > 0$和$0 < \beta < 1$为正常数. 对于任意$x_0 \in R^n$和$t > t_0$,随机系统在概率意义下半全局有限时间稳定.

引理2[25]　对于$\theta_1 > 0, \theta_2 > 0$, $\theta_3 > 0$和任意实数x、y,如下不等式成立:

$$|x|^{\theta_1}|y|^{\theta_2} \leqslant \frac{\theta_1}{\theta_1+\theta_2}\theta_3|x|^{\theta_1+\theta_2} + \frac{\theta_2}{\theta_1+\theta_2}\theta_3^{-\frac{\theta_1}{\theta_2}}|y|^{\theta_1+\theta_2}. \tag{6}$$

引理3[25]　对于$0 < q < 1$和$\xi_i \geqslant 0$,如下不等式成立:

$$\Big(\sum_{i=1}^N \xi_i\Big)^q \leqslant \sum_{i=1}^N {\xi_i}^q \leqslant N^{1-q}\Big(\sum_{i=1}^N \xi_i\Big)^q. \tag{7}$$

引理4[28]　对于$\forall \eta$, $\mu \in R^n$,可以推导出

$$\eta^\mathrm{T}\mu \leqslant \frac{1}{a}\|\eta\|^a + \frac{1}{b}\|\mu\|^b, \tag{8}$$

其中a, $b > 0$且$\dfrac{1}{a} + \dfrac{1}{b} = 1$.

引理5[29]　对于任意连续函数$f(x): R^n \to R$,定义其在一个紧集$\Omega_x \subset R^n$上,给定正常数$\bar{\delta}$,采用径向基函数神经网络$W^{*\mathrm{T}}\phi(x)$逼近$f(x)$,有

$$f(x) = W^{*\mathrm{T}}\phi(x) + \varepsilon(x). \tag{9}$$

其中: $\phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_l(x)]^\mathrm{T}$为一个基函数向量; $l > 1$为神经网络节点数量; $\varepsilon(x)$为近似误差,满足$|\varepsilon(x)| < \bar{\delta}$. 基函数$\phi_i(x)$为高斯函数,且满足

$$\phi_i(x) = \exp\Big[-\frac{(x-a_i)^\mathrm{T}(x-a_i)}{{b_i}^2}\Big],\ i = 1, 2, \ldots, l. \tag{10}$$

其中: a_i、b_i分别为基函数$\phi_i(x)$的中心和宽度, $a_i = [a_{i1}, a_{i2}, \ldots, a_{il}]^\mathrm{T}$. 理想权重$W^* = [{W_1}^*, {W_2}^*, \ldots, {W_l}^*]^\mathrm{T}$定义为如下形式:

$$W^* = \arg\min_{W\in R^l}\{\sup_{x\in\Omega_x}|f(x) - W^\mathrm{T}\phi(x)|\}. \tag{11}$$

引理6[28]　若$\phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_l(x)]^\mathrm{T}$为径向基函数神经网络的基函数向量, $x = [x_1, x_2, \ldots, x_n]^\mathrm{T}$为输入变量,则对于任意给定整数$c(1 \leqslant c \leqslant n)$和$\bar{x}_c = [x_1, x_2, \ldots, x_c]^\mathrm{T}$,下式成立:

$$\|\phi(x)\|^2 \leqslant \|\phi(\bar{x}_c)\|^2. \tag{12}$$

1.2　问题描述和基本假设

考虑如下全状态约束下输入量化的非严格反馈随机非线性系统:

$$\begin{aligned}
&\mathrm{d}x_1 = (x_2 + f_1(x) + d_1(t,x))\mathrm{d}t + h_1^\mathrm{T}(x_1)\mathrm{d}B,\\
&\mathrm{d}x_i = (x_{i+1} + f_i(x) + d_i(t,x))\mathrm{d}t + h_i^\mathrm{T}(\bar{x}_i)\mathrm{d}B,\\
&\mathrm{d}x_n = (q(u) + f_n(x) + d_n(t,x))\mathrm{d}t + h_n^\mathrm{T}(\bar{x}_n)\mathrm{d}B,\\
&y = x_1.
\end{aligned} \tag{13}$$

其中: $\bar{x}_i = [x_1, x_2, \ldots, x_i]^\mathrm{T} \in R^i, x = [x_1, x_2, \ldots, x_n]^\mathrm{T} \in R^n$为系统状态; $u \in R$为系统输入; $q(u)$为由量化器$q: R \to R$产生的量化输入; $y \in R$为系统输出; $f_i(\cdot)$、$h_i(\cdot)$为系统未知的连续函数; $d_i(t,x)$为未知的不确定干扰项; B为定义在完全概率空间(Ω, F, P)上的标准维纳过程, Ω为样本空间, F为σ-代数, P为概率测度. 所有状态$x_i(t)(i = 1, 2, \ldots, n)$依概率约束,即$\forall \varepsilon_0(0 < \varepsilon_0 < 1)$, $\inf\limits_{0\leqslant t<\infty} P\{-k_{i1} < x_i(t) < k_{i2}\} \geqslant 1 - \varepsilon_0$,其中$k_{i1}$、$k_{i2}$为正常数.

控制目标是设计控制输入u,使得输出在有限时间内跟踪给定的期望轨迹y_d,同时避免量化输入的抖动影响,并且每个状态x_i满足概率意义下的状态约束. 采用滞回量化器避免抖动,量化器$q(u)$定义为

$$q(u) = \begin{cases}
u_i\mathrm{sgn}(u),\ \dfrac{u_i}{1+\delta} < |u| \leqslant u_i,\ \dot{u} < 0,\ \text{or}\\
\quad u_i < |u| \leqslant \dfrac{u_i}{1-\delta},\ \dot{u} > 0;\\
u_i(1+\delta)\mathrm{sgn}(u),\ u_i < |u| \leqslant \dfrac{u_i}{1-\delta},\ \dot{u} < 0,\ \text{or}\\
\quad \dfrac{u_i}{1-\delta} < |u| \leqslant \dfrac{u_i(1+\delta)}{1-\delta},\ \dot{u} > 0;\\
0, 0 \leqslant q|u| < \dfrac{u_{\min}}{1+\delta},\ \dot{u} < 0,\ \text{or}\\
\quad \dfrac{u_{\min}}{1+\delta} \leqslant |u| \leqslant u_{\min},\ \dot{u} > 0;\\
q(u(t^-)),\ \dot{u} = 0.
\end{cases} \tag{14}$$

其中: $u_i = \eta^{1-i}u_{\min}, \delta = (1-\eta)/(1+\eta), 0 < \eta < 1$, $q(u) = \{0, \pm u_i, \pm u_i(1+\delta), i = 1, 2, \ldots\}, u_{\min} > 0$,参数$\eta$为量化密度. 存在两个函数$R(u)$和$S(t)$,使得系统输入$q(u)$满足

$$q(u) = R(u)u(t) + S(t). \tag{15}$$

$R(u)$、$S(t)$满足如下不等式:

$$1 - \delta \leqslant R(u) \leqslant 1 + \delta,\ |S(t)| \leqslant u_{\min}. \tag{16}$$

其中: $0 < \delta < 1, u_{\min}$为一个正常数.

假设1　期望轨迹向量$[y_d, \dot{y}_d, \ddot{y}_d] \in \Omega_d$是连续可量测的,其中$\Omega_d = \{[y_d, \dot{y}_d, \ddot{y}_d]^\mathrm{T}: {y_d}^2 + \dot{y}_d^2 + \ddot{y}_d^2 \leqslant B_0\} \in R^3, |y_d| \leqslant B_1, B_0$、$B_1$为已知的正常数,约束$k_{i1}$、$k_{i2}$满足$\min\{k_{i1}, k_{i2}\} > B_1 > 0$.

假设2 存在正常数$\bar{d}_i$满足$|d_i(t,x)|\leqslant\bar{d}_i$.

2 自适应有限时间控制器设计

为处理全状态约束,引入如下可逆非线性映射:

$$s_i=T(x_i)=\frac{1}{2}\ln\frac{(k_{i1}+x_i)k_{i2}}{(k_{i2}-x_i)k_{i1}}. \quad (17)$$

易得

$$s_i+\ln\frac{k_{i1}}{k_{i2}}=\frac{1}{2}\ln\Big[\frac{\frac{k_{i1}+k_{i2}}{2}+\Big(\frac{k_{i1}-k_{i2}}{2}+x_i\Big)}{\frac{k_{i1}+k_{i2}}{2}-\Big(\frac{k_{i1}-k_{i2}}{2}+x_i\Big)}\Big].$$

由双曲正切函数$y=\tanh(x)$的反函数$x=\frac{1}{2}\ln\frac{1+y}{1-y}$,可得

$$x_i=\frac{k_{i1}+k_{i2}}{2}\cdot\tanh\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)-\frac{k_{i1}-k_{i2}}{2}, \quad (18)$$

则s_i的导数为

$$\dot{s}_i=\frac{2}{(k_{i1}+k_{i2})\Big[1-\tanh^2\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)\Big]}\dot{x}_i. \quad (19)$$

由式(2)、(3)和(17),可得

$$Ls_i=\dot{s}_i\dot{x}_i+\frac{1}{2}\cdot\frac{\partial^2 s_i}{\partial {x_i}^{\mathrm{T}}\partial x_i}h_i^{\mathrm{T}}h_i=\frac{2(x_{i+1}+f_i(x)+d_i(t,x))}{(k_{i1}+k_{i2})\Big[1-\tanh^2\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)\Big]}+\frac{4\tanh\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)h_i^{\mathrm{T}}h_i}{(k_{i1}+k_{i2})^2\Big[1-\tanh^2\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)\Big]^2}. \quad (20)$$

令

$$b_i(s_i)=\frac{2}{(k_{i1}+k_{i2})\Big[1-\tanh^2\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)\Big]}, \quad (21)$$

$$B_i(s_i)=\frac{4\tanh\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)}{(k_{i1}+k_{i2})^2\Big[1-\tanh^2\Big(s_i+\frac{1}{2}\ln\frac{k_{i1}}{k_{i2}}\Big)\Big]^2}, \quad (22)$$

$$H_i(\bar{s}_i)^{\mathrm{T}}=b_i(s_i)h_i(\bar{s}_i)^{\mathrm{T}}, \quad (23)$$

其中$\bar{s}_i=[s_1,s_2,\ldots,s_i]$. 进而可得

$$Ls_i=b_i(x_{i+1}+f_i(x)+d_i(t,x))+B_i(s_i)h_i^{\mathrm{T}}(\bar{s}_i)h_i(\bar{s}_i), \quad (24)$$

$$\mathrm{d}s_i=Ls_i(\bar{s}_i)\mathrm{d}t+b_ih_i(\bar{s}_i)^{\mathrm{T}}\mathrm{d}B. \quad (25)$$

其中b_i为$b_i(s_i)$的简写. 由此系统(13)可以改写为

$$\mathrm{d}s_1=(b_1s_2+F_1(\bar{s}_n)+b_1d_1(t,x))\mathrm{d}t+H_1^{\mathrm{T}}\mathrm{d}B,$$
$$\mathrm{d}s_i=(b_is_{i+1}+F_i(\bar{s}_n)+b_id_i(t,x))\mathrm{d}t+H_i^{\mathrm{T}}\mathrm{d}B,$$
$$\mathrm{d}s_n=(b_nq(u)+F_n(\bar{s}_n)+b_nd_n(t,x))\mathrm{d}t+H_n^{\mathrm{T}}\mathrm{d}B,$$
$$\hat{y}_d=\frac{1}{2}\ln\frac{(k_{11}+y_d)k_{12}}{(k_{12}-y_d)k_{11}}. \quad (26)$$

其中: $F_i(\bar{s}_n)=b_i(s_i)f_i(x)+B_i(s_i){h_i}^{\mathrm{T}}h_i$, $\bar{s}_n=[s_1,s_2,\ldots,s_n]^{\mathrm{T}}$. 为了使自适应控制设计更加容易,作如下坐标变换:

$$z_1=s_1-\hat{y}_d; \quad (27)$$

$$z_i=s_i-\beta_i,\ i=2,3,\ldots,n; \quad (28)$$

$$y_i=\beta_i-\alpha_{i-1}. \quad (29)$$

令$(\tilde{\cdot})=(\hat{\cdot})-(\cdot)$, $\lambda_i=\|{W_i}^*\|^2$, $\hat{\lambda}_i$是对λ_i的估计. 定义如下李亚普诺夫函数:

$$V_{z_i}=\frac{1}{4}z_i^4, \quad (30)$$

$$V_i=V_{z_i}+\frac{1}{2r_i}\tilde{\lambda}_i^2,\ i=1,2,\ldots,n. \quad (31)$$

其中$r_i>0$为设计常数.

step 1: 令$\beta_1=\hat{y}_d$,则$z_1=s_1-\beta_1$,可得

$$Lz_1=Ls_1-L\beta_1=b_1s_2+F_1(\bar{s}_n)+b_1d_1(t,x)-\dot{\beta}_1, \quad (32)$$

$$\mathrm{d}z_1=Lz_1\mathrm{d}t+H_1^{\mathrm{T}}(s_1)\mathrm{d}B. \quad (33)$$

由式(3)可得

$$LV_{z_1}=z_1^3(Lz_1)+\frac{3}{2}z_1^2H_1^{\mathrm{T}}(s_1)H_1(s_1)=z_1^3(b_1(s_1)s_2+F_1(\bar{s}_n)+b_1(s_1)d_1(t,x)-\dot{\beta}_1)+\frac{3}{2}z_1^2H_1^{\mathrm{T}}(s_1)H_1(s_1). \quad (34)$$

由引理4,得

$${z_1^3}b_1(s_1)d_1(t,x)\leqslant\frac{\bar{d}_1^4}{4}+\frac{3}{4}z_1^4(b_1(s_1))^{\frac{4}{3}}, \quad (35)$$

$$\frac{3}{2}{z_1}^2H_1^{\mathrm{T}}(s_1)H_1(s_1)\leqslant\frac{9}{16}z_1^4\|H_1\|^4+1. \quad (36)$$

将式(35)和(36)代入(34),得到

$$LV_{z_1}\leqslant z_1^3\Big(b_1(s_1)s_2+F_1(\bar{s}_n)+\frac{3}{4}z_1(b_1(s_1))^{\frac{4}{3}}-\dot{\beta}_1\Big)+\frac{4+\bar{d}_1^4}{4}+\frac{9}{16}z_1^4\|H_1\|^4. \quad (37)$$

设计一阶滤波器β_2如下:

$$\tau_2\dot{\beta}_2+\beta_2=\alpha_1,\ \beta_2(0)=\alpha_1(0), \quad (38)$$

其中α_1、β_2分别为一阶滤波器的输入和输出. 由式(28)和(29),得到$s_2=z_2+y_2+\alpha_1$,则有

$$LV_{z_1}\leqslant z_1^3\Big(b_1s_2+F_1(\bar{s}_n)+\frac{3}{4}z_1(b_1(s_1))^{\frac{4}{3}}-\dot{\beta}_1\Big)+\frac{4+\bar{d}_1^4}{4}+\frac{9}{16}{z_1}^4\|H_1\|^4\leqslant z_1^3(b_1y_2+b_1\alpha_1+p_1(S_1))+\frac{4+\bar{d}_1^4}{4}. \quad (39)$$

其中

$$p_1(S_1)=$$
$$F_1(\bar{s}_n)+b_1z_2+\frac{3}{4}z_1(b_1(s_1))^{\frac{4}{3}}-\dot{\beta}_1+\frac{9}{16}z_1\|H_1\|^4,$$
$$S_1=[s_1,s_2,\ldots,s_n,\beta_1,\dot{\beta}_1,\beta_2].$$

由于 $p_1(S_1)$ 是 S_1 的未知连续函数, 利用神经网络 ${W_1}^{*\mathrm{T}}\phi_1(S_1)$进行逼近,有

$$p_1(S_1)={W_1}^{*\mathrm{T}}\phi_1(S_1)+\varepsilon_1(S_1). \tag{40}$$

基于引理4,得到

$$z_1^3p_1(S_1)\leqslant$$
$$\frac{1}{2a_1^2}z_1^6\|{W_1}^*\|^2\|\phi_1(X_1)\|^2+\frac{a_1^2}{2}+\frac{3}{4}z_1^4+\frac{1}{4}\varepsilon_1^4, \tag{41}$$
$$z_1^3b_1(s_1)y_2\leqslant\frac{1}{4}b_1(s_1){y_2}^4+\frac{3}{4}b_1(s_1)z_1^4. \tag{42}$$

其中: $X_1=[s_1,\beta_2]^{\mathrm{T}}, a_1$为正的设计常数.

结合式(39)～(42),得

$$LV_{z_1}\leqslant$$
$$\frac{1}{2a_1^2}z_1^6\|{W_1}^*\|^2\|\phi_1\|^2+\frac{a_1^2}{2}+\frac{3}{4}z_1^4+\frac{1}{4}\varepsilon_1^4+$$
$$\frac{1}{4}b_1(s_1)y_2^4+\frac{3}{4}b_1(s_1)z_1^4+z_1^3b_1(s_1)\alpha_1+\frac{4+\bar{d}_1^4}{4}\leqslant$$
$$\frac{1}{2a_1^2}z_1^6\lambda_1\|\phi_1\|^2+\frac{a_1^2}{2}+\frac{3}{4}z_1^4+\frac{1}{4}b_1(s_1){y_2}^4+$$
$$\frac{1}{4}\varepsilon_1^4+\frac{3}{4}b_1(s_1)z_1^4+z_1^3b_1(s_1)\alpha_1+\frac{4+\bar{d}_1^4}{4}, \tag{43}$$

其中ϕ_1表示$\phi_1(X_1)$.

设计虚拟控制α_1如下:

$$\alpha_1=-\frac{1}{b_1}\Big(c_1z_1^{4\beta-3}+\frac{1}{2a_1^2}z_1^3\hat{\lambda}_1\|\phi_1\|^2+\frac{3}{4}z_1\Big)-\frac{3}{4}z_1. \tag{44}$$

将式(44)代入(43),得到

$$LV_{z_1}\leqslant-c_1z_1^{4\beta}+\frac{4+\bar{d}_1^4}{4}-\frac{1}{2a_1^2}z_1^6\tilde{\lambda}_1\|\phi_1\|^2+$$
$$\frac{1}{4}b_1y_2^4+\frac{a_1^2}{2}+\frac{1}{4}\varepsilon_1^4. \tag{45}$$

由式(45)可得

$$LV_1=LV_{z_1}+\frac{1}{r_1}\tilde{\lambda}_1\dot{\hat{\lambda}}_1. \tag{46}$$

设计自适应律

$$\dot{\hat{\lambda}}_1=r_1\Big(\frac{1}{2a_1^2}z_1^6\|\phi_1\|^2-\sigma_1\hat{\lambda}_1\Big), \tag{47}$$

其中$r_1,\sigma_1>0$为设计常数. 将式(45)和(47)代入(46),得到

$$LV_1\leqslant$$
$$-c_1z_1^{4\beta}-\frac{1}{2a_1^2}z_1^6\tilde{\lambda}_1\|\phi_1\|^2+\frac{1}{4}b_1y_2^4+\frac{a_1^2}{2}+$$
$$\frac{1}{4}\varepsilon_1^4+\frac{4+\bar{d}_1^4}{4}+\frac{1}{2a_1^2}\tilde{\lambda}_1z_1^6\|\phi_1\|^2-\sigma_1\tilde{\lambda}_1\hat{\lambda}_1\leqslant$$
$$-c_1z_1^{4\beta}+\frac{1}{4}b_1y_2^4+\frac{a_1^2}{2}+\frac{1}{4}{\varepsilon_1}^4-\sigma_1\tilde{\lambda}_1\hat{\lambda}_1+\frac{4+\bar{d}_1^4}{4}. \tag{48}$$

由式(2)和(3),可得

$$L\alpha_1=\frac{\partial\alpha_1}{\partial s_1}(b_1s_2+F_1(\bar{s}_n)+b_1d_1(t,x))+$$
$$\frac{\partial\alpha_1}{\partial\beta_1}\dot{\beta}_1+\frac{\partial\alpha_1}{\partial\hat{\lambda}_1}\dot{\hat{\lambda}}_1+\frac{1}{2}\frac{\partial^2\alpha_1}{\partial {s_1}^{\mathrm{T}}\partial s_1}H_1^{\mathrm{T}}H_1, \tag{49}$$
$$\mathrm{d}\alpha_1=L\alpha_1\mathrm{d}t+\frac{\partial\alpha_1}{\partial s_1}H_1^{\mathrm{T}}(s_1)\mathrm{d}B, \tag{50}$$
$$Ly_2=L\beta_2-L\alpha_1=-\frac{y_2}{\tau_2}-L\alpha_1, \tag{51}$$
$$\mathrm{d}y_2=Ly_2\mathrm{d}t-\frac{\partial\alpha_1}{\partial s_1}H_1^{\mathrm{T}}(s_1)\mathrm{d}B, \tag{52}$$
$$\frac{1}{4}Ly_2^4=y_2^3Ly_2+\frac{3}{2}y_2^2\Big(\frac{\partial\alpha_1}{\partial s_1}\Big)^2H_1^{\mathrm{T}}H_1\leqslant$$
$$y_2^3\Big(-\frac{y_2}{\tau_2}-L\alpha_1\Big)+\frac{3}{2}y_2^2\Big(\frac{\partial\alpha_1}{\partial s_1}\Big)^2H_1^{\mathrm{T}}H_1\leqslant$$
$$-\frac{y_2^4}{\tau_2}+\frac{7}{4}y_2^4+\frac{1}{4}(L\alpha_1)^4+\frac{9}{16}\Big(\frac{\partial\alpha_1}{\partial s_1}\Big)^4\|H_1\|^4. \tag{53}$$

存在一个非负的光滑函数$\kappa_2(\cdot)$满足如下不等式:

$$\frac{1}{4}(L\alpha_1)^4+\frac{9}{16}\Big(\frac{\partial\alpha_1}{\partial s_1}\Big)^4\|H_1\|^4\leqslant$$
$$\kappa_2(\bar{z}_n^{\mathrm{T}},\bar{y}_n^{\mathrm{T}},\bar{\hat{\lambda}}_{n-1}^{\mathrm{T}},\dot{y}_d,\ddot{y}_d), \tag{54}$$
$$\frac{1}{4}Ly_2^4\leqslant-\frac{y_2^4}{\tau_2}+\frac{7}{4}y_2^4+\kappa_2. \tag{55}$$

step $i(2\leqslant i\leqslant n-1)$: 令$z_i=s_i-\beta_i$,有

$$Lz_i=Ls_i-L\beta_i=$$
$$b_is_{i+1}+F_i(\bar{s}_n)+b_id_i(t,x)-\dot{\beta}_i, \tag{56}$$
$$\mathrm{d}z_i=Lz_i\mathrm{d}t+H_i^{\mathrm{T}}(s_i)\mathrm{d}B. \tag{57}$$

由式(3)可得

$$LV_{z_i}=z_i^3(Lz_i)+\frac{3}{2}z_i^2H_i^{\mathrm{T}}(s_i)H_i(s_i)=$$
$$z_i^3(b_i(s_i)s_{i+1}+F_i(\bar{s}_n)+b_i(s_i)d_i(t,x)-\dot{\beta}_i)+$$
$$\frac{3}{2}z_i^2H_i^{\mathrm{T}}(s_i)H_i(s_i). \tag{58}$$

基于引理4,可以得到

$$z_i^3b_i(s_i)d_i(t,x)\leqslant\frac{\bar{d}_i^4}{4}+\frac{3}{4}z_i^4(b_i(s_i))^{\frac{4}{3}}, \tag{59}$$
$$\frac{3}{2}z_i^2H_i^{\mathrm{T}}(s_i)H_i(s_i)\leqslant\frac{9}{16}z_i^4\|H_i\|^4+1. \tag{60}$$

将式(59)和(60)代入(58),得到

$$LV_{z_i}\leqslant qz_i^3\Big(b_i(s_i)s_{i+1}+F_i(\bar{s}_n)+\frac{3}{4}z_i(b_i(s_i))^{\frac{4}{3}}-$$
$$\dot{\beta}_i\Big)+\frac{4+\bar{d}_i^4}{4}+\frac{9}{16}z_i^4H_i^4. \tag{61}$$

设计一阶滤波器β_{i+1}如下:

$$\tau_{i+1}\dot{\beta}_{i+1}+\beta_{i+1}=\alpha_i,\beta_{i+1}(0)=\alpha_i(0), \tag{62}$$

其中α_i、β_{i+1}分别为一阶滤波器的输入与输出. 由式

(28)和(29),有 $s_{i+1} = z_{i+1} + y_{i+1} + \alpha_i$,则有

$$\begin{aligned}LV_{z_i} \leqslant\ & z_i^3\Big(b_i(s_i)s_{i+1} + F_i(\bar{s}_n) + \frac{3}{4}z_i(b_i(s_i))^{\frac{4}{3}} - \\ & \dot{\beta}_i\Big) + \frac{4+\bar{d}_i^4}{4} + \frac{9}{16}z_i^4H_i^4 \leqslant \\ & {z_i}^3(b_i(s_i)(z_{i+1}+y_{i+1}+\alpha_i) + F_i(\bar{s}_n) + \\ & \frac{3}{4}z_i^3(b_i(s_i))^{\frac{4}{3}} - \dot{\beta}_i) + \frac{4+\bar{d}_i^4}{4} + \frac{9}{16}z_i^4H_i^4 \leqslant \\ & z_i^3b_iy_{i+1} + z_i^3b_i\alpha_i + {z_i}^3p_i(S_i) + \frac{4+\bar{d}_i^4}{4}. \end{aligned} \tag{63}$$

其中

$$p_i(S_i) =$$
$$F_i(\bar{s}_n) + b_iz_{i+1} + \frac{3}{4}z_i(b_i(s_i))^{\frac{4}{3}} - \dot{\beta}_i + \frac{9}{16}z_i\|H_i\|^4,$$
$$S_i = [s_1,\ldots,s_n,\beta_i,\dot{\beta}_i,\beta_{i+1}]^{\mathrm{T}}.$$

由于 $p_i(S_i)$ 是 S_i 的未知连续的函数. 用神经网络 ${W_i}^{*\mathrm{T}}\phi_i(S_i)$ 进行逼近,有

$$p_i(S_i) = {W_i}^{*\mathrm{T}}\phi_i(S_i) + \varepsilon_i(S_i). \tag{64}$$

基于引理4,可以得到

$$\begin{aligned}& z_i^3p_i(S_i) = z_i^3{W_i}^{*\mathrm{T}}\phi_i(S_i) + z_i^3\varepsilon_i(S_i) \leqslant \\ & \frac{1}{2a_i^2}z_i^6\|{W_i}^*\|^2\|\phi_i(X_i)\|^2 + \frac{a_i^2}{2} + \frac{3}{4}z_i^4 + \frac{1}{4}\varepsilon_i^4, \end{aligned} \tag{65}$$

$$z_i^3b_i(s_i)y_{i+1} \leqslant \frac{1}{4}b_i(s_i)y_{i+1}^4 + \frac{3}{4}b_i(s_i)z_i^4. \tag{66}$$

其中: $X_i = [s_1, s_2, \ldots, s_i, \beta_i]^{\mathrm{T}}$, a_i 为正设计常数. 结合式(63)～(66),得

$$\begin{aligned}& LV_{z_i} \leqslant \\ & \frac{1}{2a_i^2}z_i^6\|{W_i}^*\|^2\|\phi_i\|^2 + \frac{a_i^2}{2} + \frac{3}{4}z_i^4 + \frac{1}{4}{\varepsilon_i}^4 + \\ & \frac{1}{4}b_i(s_i)y_{i+1}^4 + \frac{3}{4}b_i(s_i)z_i^4 + {z_i}^3b_i(s_i)\alpha_i + \frac{4+\bar{d}_i^4}{4} \leqslant \\ & \frac{1}{2{a_i}^2}z_i^6\lambda_i\|\phi_i\|^2 + \frac{a_i^2}{2} + \frac{3}{4}z_i^4 + \frac{1}{4}b_i(s_i)y_{i+1}^4 + \\ & \frac{1}{4}\varepsilon_i^4 + \frac{3}{4}b_i(s_i)z_i^4 + {z_i}^3b_i(s_i)\alpha_i + \frac{4+\bar{d}_i^4}{4}, \end{aligned} \tag{67}$$

其中 ϕ_i 表示 $\phi_i(X_i)$.

设计虚拟控制 α_i 如下:

$$\alpha_i = -\frac{1}{b_i}\Big(c_iz_i^{4\beta-3} + \frac{1}{2a_i^2}z_i^3\hat{\lambda}_i\|\phi_i\|^2 + \frac{3}{4}z_i\Big) - \frac{3}{4}z_i. \tag{68}$$

将式(68)代入(67),得到

$$\begin{aligned}LV_{z_i} \leqslant\ & \frac{1}{2a_i^2}z_i^6\lambda_i\|\phi_i\|^2 + \frac{a_i^2}{2} + \frac{1}{4}\varepsilon_i^4 + \frac{1}{4}b_i(s_i)y_{i+1}^4 - \\ & c_i{z_i}^{4\beta} - \frac{1}{2{a_i}^2}{z_i}^6\hat{\lambda}_i\|\phi_i\|^2 + \frac{4+\bar{d}_i^4}{4} \leqslant \\ & -c_i{z_i}^{4\beta} + \frac{1}{4}b_i(s_i)y_{i+1}^4 - \frac{1}{2a_i^2}z_i^6\tilde{\lambda}_i\|\phi_i\|^2 + \frac{a_i^2}{2} + \\ & \frac{1}{4}\varepsilon_i^4 + \frac{4+\bar{d}_i^4}{4}. \end{aligned} \tag{69}$$

由式(69)可得

$$LV_i = LV_{z_i} + \frac{1}{r_i}\tilde{\lambda}_i\dot{\hat{\lambda}}_i. \tag{70}$$

设计 $\hat{\lambda}_i$ 的自适应律

$$\dot{\hat{\lambda}}_i = r_i\Big(\frac{1}{2a_i^2}z_i^6\|\phi_i\|^2 - \sigma_i\hat{\lambda}_i\Big), \tag{71}$$

其中 $r_i, \sigma_i > 0$ 为设计常数. 将式(69)和(71)代入(70),得到

$$\begin{aligned}LV_i \leqslant & -c_iz_i^{4\beta} + \frac{1}{4}b_iy_{i+1}^4 + \frac{a_i^2}{2} + \frac{1}{4}\varepsilon_i^4 + \\ & \frac{4+\bar{d}_i^4}{4} - \sigma_i\tilde{\lambda}_i\hat{\lambda}_i. \end{aligned} \tag{72}$$

由式(2)和(3),可得

$$\begin{aligned}& L\alpha_i = \\ & \sum_{j=1}^{i}\frac{\partial\alpha_j}{\partial s_j}(b_j(s_j)s_{j+1} + F_j(\bar{s}_n) + b_j(s_j)d_j(t,x)) + \\ & \frac{\partial\alpha_i}{\partial\beta_i}\dot{\beta}_i + \frac{\partial\alpha_i}{\partial\hat{\lambda}_i}\dot{\hat{\lambda}}_i + \frac{1}{2}\sum_{p,q=1}^{i}\frac{\partial^2\alpha_i}{\partial{s_i}^{\mathrm{T}}\partial s_i}H_p^{\mathrm{T}}H_q, \end{aligned} \tag{73}$$

$$Ly_{i+1} = L\beta_{i+1} - L\alpha_i = -\frac{y_{i+1}}{\tau_{i+1}} - L\alpha_i, \tag{74}$$

$$\mathrm{d}y_{i+1} = Ly_{i+1}\mathrm{d}t - \sum_{j=1}^{i}\frac{\partial\alpha_j}{\partial s_j}H_j^{\mathrm{T}}(\bar{s}_j)\mathrm{d}B, \tag{75}$$

$$\begin{aligned}& \frac{1}{4}Ly_{i+1}^4 = y_{i+1}^3Ly_{i+1} + \frac{3}{2}y_{i+1}^2\Big(\frac{\partial\alpha_i}{\partial s_i}\Big)^2H_i^{\mathrm{T}}H_i \leqslant \\ & y_{i+1}^3\Big(-\frac{y_{i+1}}{\tau_{i+1}} - L\alpha_i\Big) + \frac{3}{2}y_{i+1}^2\Big(\frac{\partial\alpha_i}{\partial s_i}\Big)^2H_i^{\mathrm{T}}H_i \leqslant \\ & -\frac{y_{i+1}^4}{\tau_{i+1}} + \frac{7}{4}y_{i+1}^4 + \frac{1}{4}(L\alpha_i)^4 + \frac{9}{16}\Big(\frac{\partial\alpha_i}{\partial s_i}\Big)^4\|H_i\|^4. \end{aligned} \tag{76}$$

存在一个非负的光滑函数 $\kappa_{i+1}(\cdot)$,满足

$$\frac{1}{4}(L\alpha_i)^4 + \frac{9}{16}\Big(\frac{\partial\alpha_i}{\partial s_i}\Big)^4\|H_i(s_i)\|^4 \leqslant \kappa_{i+1}, \tag{77}$$

$$\frac{1}{4}Ly_{i+1}^4 \leqslant -\frac{y_{i+1}^4}{\tau_{i+1}} + \frac{7}{4}y_{i+1}^4 + \kappa_{i+1}. \tag{78}$$

step n: 令 $z_n = s_n - \beta_n$,由式(3)可得

$$\begin{aligned}LV_{z_n} =\ & {z_n}^3\dot{z}_n + \frac{3}{2}z_n^2H_n^{\mathrm{T}}(s_n)H_n(s_n) = \\ & z_n^3(b_n(s_n)q(u) + F_n(\bar{s}_n) + \\ & b_n(s_n)d_n(t,x)) + \frac{3}{2}z_n^2\|H_n\|^2 \leqslant \\ & z_n^3\Big(b_nq(u) + F_n(\bar{s}_n) + \frac{3}{4}(b_n(s_n))^{\frac{4}{3}}\Big) + \\ & \frac{9}{16}z_n^4\|H_n\|^4 + \frac{4+\bar{d}_n^4}{4} = \\ & z_n^3b_n(s_n)q(u) + z_n^3p_n(s_n) + \frac{4+\bar{d}_n^4}{4}. \end{aligned} \tag{79}$$

其中 $p_n(S_n) = F_n(\bar{s}_n) + \frac{3}{4}z_n(b_n(s_n))^{\frac{4}{3}} + \frac{9}{16}z_n\|H_n\|^4$.

设计控制律

$$u(t) = \frac{1}{b_n(1-\delta)}\Big(-c_nz_n^{4\beta-3} - \frac{1}{2a_n^2}z_n^3\hat{\lambda}_n\|\phi_n\|^2 -$$

$$\frac{3}{4}b_n(s_n)z_n - \frac{3}{4}z_n\Big). \tag{80}$$

由于$p_n(S_n)$是关于S_n的未知连续函数,使用神经网络$W_n^{*\mathrm{T}}\phi_n(S_n)$进行逼近,有

$$p_n(S_n) = W_n^{*\mathrm{T}}\phi_n(S_n) + \varepsilon_n(S_n), \tag{81}$$

其中$S_n = [s_1, s_2, \dots, s_n, \beta_n, \dot{\beta}_n]^{\mathrm{T}}$. 基于引理4, 可以得到

$$\begin{aligned} & z_n^3 p_n(S_n) = z_n^3 W_n^{*\mathrm{T}}\phi_n(S_n) + z_n^3\varepsilon_n(S_n) \leqslant \\ & \frac{1}{2a_n^2}z_n^6\|W_n^*\|^2\|\phi_n(X_n)\|^2 + \frac{a_n^2}{2} + \frac{3}{4}z_n^4 + \frac{1}{4}\varepsilon_n^4, \end{aligned} \tag{82}$$

$$\begin{aligned} & z_n^3 b_n(s_n)R(u)u(t) \leqslant \\ & -c_n z_n^{4\beta} - \frac{1}{2a_n^2}z_n^6\hat{\lambda}_n\|\phi_n\|^2 - \frac{3}{4}b_n(s_n)z_n^4 - \frac{3}{4}z_n^4, \end{aligned} \tag{83}$$

$$z_n^3 b_n(s_n)u_{\min} \leqslant \frac{3}{4}z_n^4 b_n(s_n) + \frac{1}{4}b_n(s_n)u_{\min}^4. \tag{84}$$

其中: $X_n = [s_1, s_2, \dots, s_n, \beta_n]^{\mathrm{T}}$, a_n为正设计常数. 将式(82) ∼ (84)代入(79), 得到

$$\begin{aligned} LV_{z_n} \leqslant & \frac{1}{2a_n^2}z_n^6\lambda_n^2\|\phi_n\|^2 + \frac{a_n^2}{2} + \frac{1}{4}\varepsilon_n^4 - c_n z_n^{4\beta} + \\ & \frac{4+\bar{d}_n^4}{4} - \frac{1}{2a_n^2}z_n^6\hat{\lambda}_n\|\phi_n\|^2 + \frac{1}{4}b_n(s_n)u_{\min}^4 \leqslant \\ & -c_n z_n^{4\beta} - \frac{1}{2a_n^2}z_n^6\tilde{\lambda}_n\|\phi_n\|^2 + \frac{1}{4}b_n(s_n)u_{\min}^4 + \\ & \frac{a_n^2}{2} + \frac{\varepsilon_n^4}{4} + \frac{4+\bar{d}_n^4}{4}, \end{aligned} \tag{85}$$

其中ϕ_n表示$\phi_n(X_n)$. 由式(31)得

$$LV_n = LV_{z_n} + \frac{1}{r_n}\tilde{\lambda}_n\dot{\hat{\lambda}}_n. \tag{86}$$

设计$\hat{\lambda}_n$自适应律

$$\dot{\hat{\lambda}}_n = r_n\Big(\frac{1}{2a_n^2}z_n^6\|\phi_n\|^2 - \sigma_n\hat{\lambda}_n\Big), \tag{87}$$

其中$r_n, \sigma_n > 0$为设计常数. 将式(85)和(87)代入(86), 得到

$$\begin{aligned} LV_n \leqslant & LV_{z_n} + \frac{1}{r_n}\tilde{\lambda}_n\cdot r_n\Big(\frac{1}{2a_n^2}z_n^6\|\phi_n\|^2 - \sigma_n\hat{\lambda}_n\Big) \leqslant \\ & -c_n z_n^{4\beta} + \frac{1}{4}b_n(s_n)u_{\min}^4 + \frac{a_n^2}{2} + \frac{\varepsilon_i^4}{4} - \\ & \sigma_n\tilde{\lambda}_n\hat{\lambda}_n + \frac{4+\bar{d}_n^4}{4}. \end{aligned} \tag{88}$$

3　稳定性分析

定义总的李亚普诺夫函数V和紧集Ω_n如下:

$$V(X) = \sum_{i=1}^{n}\Big(V_i + \frac{1}{4}y_i^4\Big) = \sum_{i=1}^{n}\Big(\frac{1}{4}z_i^4 + \frac{1}{2r_i}\tilde{\lambda}_i^2 + \frac{1}{4}y_i^4\Big),$$

$$\Omega_n = \{[\bar{z}_n^{\mathrm{T}}, \bar{y}_n^{\mathrm{T}}, \bar{\hat{\lambda}}_n^{\mathrm{T}}]^{\mathrm{T}} : V \leqslant p\} \subset R^{p_n}. \tag{89}$$

其中$y_1 = 0$, $p_n = 3n-1$, $p \geqslant 1$为一个任意正常数, $X = [\bar{z}_n^{\mathrm{T}}, \bar{y}_n^{\mathrm{T}}, \bar{\hat{\lambda}}_n^{\mathrm{T}}]^{\mathrm{T}}$.

定理1　在假设1和假设2下,考虑非严格反馈随机非线性系统(13)、虚拟控制器(44)和(68)、实际控制器(80)、自适应律(47)、(71)和(87). 对于有界初始条件$V(X(0)) \leqslant p/n_0$,存在正常数c_i、τ_i,使得闭环系统中的所有信号概率意义下有界,且状态变量满足概率约束条件. 此外,存在一个设定时间T^*,有

$$E[V^\beta(X)] \leqslant \frac{b}{(1-\rho)a},\ \forall t \geqslant T^*.$$

其中: $n_0 \geqslant 10$; $0 < \rho < 1$; b、T^*稍后给出; 常数c_i、τ_i满足下述不等式:

$$\begin{cases} \dfrac{1}{\tau_j} > \dfrac{7}{4} + \dfrac{a^{\frac{1}{\beta}}}{4},\ j = 2, 3, \dots, n; \\ a = \min\limits_{1\leqslant i\leqslant n}\{(4c_i)^\beta, (\sigma_i r_i)^\beta\}. \end{cases} \tag{90}$$

证明　对于任意给定的正常数$p \geqslant 1$, 若$V \leqslant p$, 则$\bar{z}_n$、$\bar{y}_n$、$\bar{\hat{\lambda}}_n$有界. 因为$z_1 = s_1 - \hat{y}_d$, $z_i = s_i - \beta_i$. 由式(44)可知α_1有界,进一步由式(38)得β_2有界,故$s_2 = z_2 + \beta_2$有界. 以此类推, $\alpha_2, \beta_3, s_3, \dots, s_n$有界. 于是由式(18)得到$x_i$满足时变约束,即

$$\begin{aligned} & LV \leqslant \\ & -\sum_{i=1}^{n}c_i z_i^{4\beta} + \sum_{i=1}^{n-1}\Big(-\frac{1}{\tau_{i+1}} + \frac{7}{4} + \frac{b_i(s_i)}{4}\Big)y_{i+1}^4 + \\ & \sum_{i=1}^{n}\frac{4+\bar{d}_i^4}{4} + \sum_{i=1}^{n}\Big(\frac{a_i^2}{2} + \frac{\varepsilon_n^4}{4}\Big)\sum_{i=1}^{n}\frac{\sigma_i}{2}(\lambda_i^2 - \tilde{\lambda}_i^2) + \\ & \sum_{i=1}^{n-1}\kappa_{i+1} + \frac{1}{4}b_n(s_n)u_{\min}^4. \end{aligned} \tag{91}$$

基于引理2, 可以得到

$$\Big(\sum_{i=1}^{n}\frac{\sigma_i}{2}\tilde{\lambda}_i^2\Big)^\beta \leqslant \sum_{i=1}^{n}\frac{\sigma_i}{2}\tilde{\lambda}_i^2 + \beta^{\frac{\beta}{1-\beta}}\Big(\frac{1-2\beta}{1-\beta}\Big), \tag{92}$$

$$\begin{aligned} & \Big(\sum_{i=1}^{n-1}\Big[\frac{1}{\tau_{i+1}} - \frac{7}{4}\Big]y_{i+1}^4\Big)^\beta \leqslant \\ & \sum_{i=1}^{n-1}\Big[\frac{1}{\tau_{i+1}} - \frac{7}{4}\Big]y_{i+1}^4 + \beta^{\frac{\beta}{1-\beta}}\Big(1 - \frac{\beta}{1-\beta}\Big). \end{aligned} \tag{93}$$

由引理3得

$$LV \leqslant -aV^\beta + b. \tag{94}$$

其中

$$\begin{aligned} b = & \sum_{i=1}^{n}\Big(\frac{a_i^2}{2} + \frac{\varepsilon_i^4}{4} + \frac{\sigma_i}{2}\lambda_i^2\Big) + \frac{1}{4}M_n u_{\min}^4 + \\ & \sum_{i=1}^{n-1}\frac{1}{4}M_i + \sum_{i=1}^{n-1}K_{i+1} + \\ & 2\beta^{\frac{\beta}{1-\beta}}\Big(1 - \frac{\beta}{1-\beta}\Big) + \sum_{i=1}^{n}\frac{4+\bar{d}_i^4}{4}, \end{aligned}$$

K_{i+1}为$\kappa_{i+1}(\cdot)$在紧集Ω_n上的最大值, M_n为$b_n(s_n)$在紧集Ω_n上的最大值, M_i为$b_i(s_i)y_{i+1}^4$在紧集Ω_n上的最大值, $1 \leqslant i \leqslant n-1$, 且有

$$\mathrm{d}(EV)/\mathrm{d}t = E[LV] \leqslant -aE[V^\beta] + b. \tag{95}$$

对于任意$0<\rho<1$,令

$$\Omega_X=\left\{X \mid E[V^{\beta}(X)]\leqslant\frac{b}{(1-\rho)a}\right\},$$
$$T^*=\frac{1}{(1-\beta)a\rho}E[V^{1-\beta}(X_0)].$$

类似于文献[27]的讨论,易得对于任意$t\geqslant T^*$,有$X(t)\in\Omega_X$,即V在有限时间内有界.

对于任意给定整数$n_0\geqslant 10$,若$V=p$, $a\geqslant b/p^{\beta}$,则$\mathrm{d}EV/\mathrm{d}t\leqslant 0$. 如果$V(X(0))\leqslant p/n_0$,则$E[V(X)]\leqslant p/n_0$. 根据切比雪夫不等式,可以得到

$$P\{V\leqslant p\}\geqslant P\{V^{\beta}(X)\leqslant p\}=$$
$$1-\frac{EV^{\beta}(X)}{p}\geqslant 1-\frac{p^{\beta-1}}{n_0},$$

因此V是依概率有界的且$V\leqslant p$成立的概率不小于$\left(1-\dfrac{p^{\beta-1}}{n_0}\right)$,进而$\tilde{\lambda}_1,\tilde{\lambda}_2,\dots,\tilde{\lambda}_n$均方半全局一致终结有界,$z_1,z_2,\dots,z_n,y_2,y_3,\dots,y_n$四阶矩意义下半全局一致终结有界. 再进一步,因为$x_i=T(s_i)$,可得

$$\inf_{0\leqslant t<\infty}P\{-k_{i1}<x_i(t)<k_{i2}\}\geqslant 1-\frac{p^{\beta-1}}{n_0},$$

所有信号都是依概率有界且状态变量依概率约束的条件得到满足. 根据上述讨论,对于任意$t\geqslant T^*$,有$E[V^{\beta}(X)]\leqslant\dfrac{b}{(1-\rho)a}$,所以闭环系统在概率意义下半全局有限时间稳定.□

4 仿真结果

为了验证所提出方法的有效性,下面给出一个仿真例子.

考虑一个具有量化输入和随机扰动的单连杆机械臂系统,其动态方程[30]可以描述为

$$\begin{cases}M\ddot{q}+B\dot{q}+N\sin(q)=I+D_I,\\ L\dot{I}+RI+K_b\dot{q}=V_d.\end{cases}\tag{96}$$

其中: q、$\dot{q}$和$\ddot{q}$分别为连杆角位置、角速度和角加速度,I为电机电枢电流和机电产生的转矩,V_d为控制输入,转矩$D_I=x_1\cos(x_2x_3t)$. 令$x_1=q,x_2=\dot{q}$, $x_3=I$,则系统可以改写为

$$\begin{cases}\dot{x}_1=x_2,\\ \dot{x}_2=(-Bx_2+x_3-N\sin(x_1)+D_I)/M,\\ \dot{x}_3=(V_d-K_bx_2-Rx_3)/L,\\ y=x_1.\end{cases}\tag{97}$$

参数的物理意义见文献[31],参数设计为: $M=1$, $B=1$, $N=2$, $L=1$, $R=1$, $K_b=2$. 具有随机扰动的随机系统可以描述为

$$\begin{aligned}&\mathrm{d}x_1=(x_2+d_1(t,x))\mathrm{d}t+0.15\cos(x_1)\mathrm{d}B,\\ &\mathrm{d}x_2=(x_3-2\sin(x_1)-x_2+d_2(t,x))\mathrm{d}t+\\ &\qquad\sin(0.4x_1x_2)\mathrm{d}B,\\ &\mathrm{d}x_3=(q(u)-2x_2-x_3+d_3(t,x))\mathrm{d}t+\\ &\qquad 0.1\sin(x_3)\mathrm{d}B,\\ &y=x_1.\end{aligned}\tag{98}$$

其中: $d_1(t,x)=0.1\sin(x_1x_2x_3t)$, $d_2(t,x)=x_1\cos(x_2x_3t)$, $d_3(t,x)=x_3\cos(x_1x_2t)$. 期望信号为$y_d=0.3(\sin(t)+\sin(0.3t))$. 自适应控制算法中的设计参数如表1所示. 选择$\beta=0.9998$, $x(0)=[0.9,1.1,-0.8]^{\mathrm{T}}$, $\hat{\lambda}(0)=[1.5,1,0.5]^{\mathrm{T}}$. $a_{1j}=0.1c(j-l_1/2)$, $j=1,2,\dots,l_1$, $c=1,2$; $a_{2j}=0.1c(j-l_2/2)$, $j=$

表1 设计参数

parameters	values	parameters	values
c_1	2	σ_1	0.1
c_2	0.1	σ_2	0.1
c_3	5	σ_3	0.1
a_1	10	τ_1	0.001
a_2	10	τ_2	0.001
a_3	10	r_1	8
r_2	8	r_3	8
k_{11}	2.2	k_{12}	2.5
k_{21}	2	k_{22}	2.5
k_{31}	2.5	k_{32}	2.1
l_1	20	l_2	20
l_3	20	$u_{\min}$	0.02

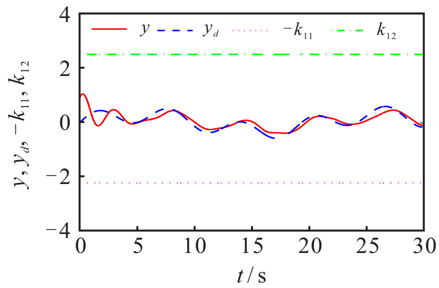


图1 输出y、y_d和约束$-k_{11}$、k_{12}

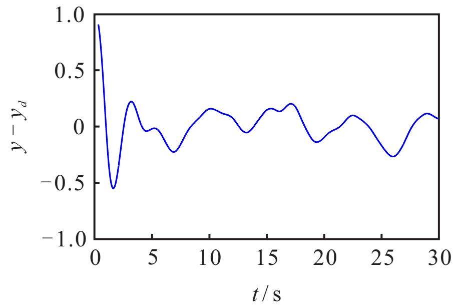


图2 跟踪误差$y-y_d$

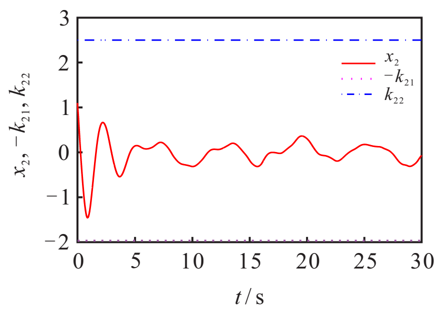


图3 状态x_2和约束$-k_{21}$、k_{22}

$1,2,\ldots,l_2$, $c=1,2,3$; $a_{3j}=0.1c(j-l_3/2)$, $j=1,2,\ldots,l_3$, $c=1,2,3,4$. 仿真结果如图1～图7所示.

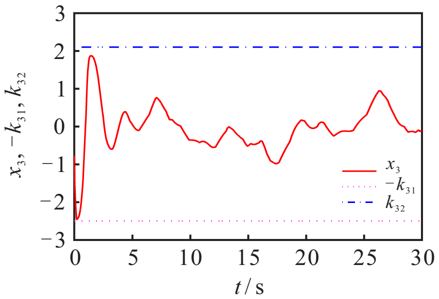


图 4　状态x_3和约束$-k_{31}$、k_{32}

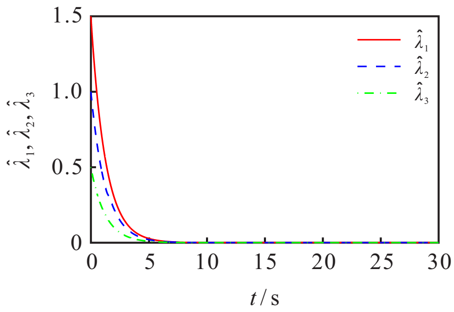
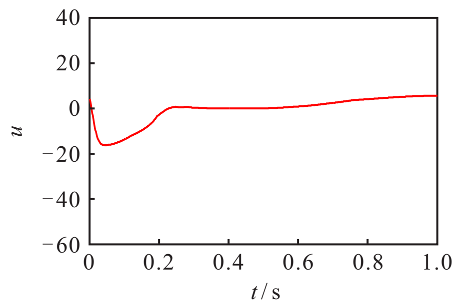
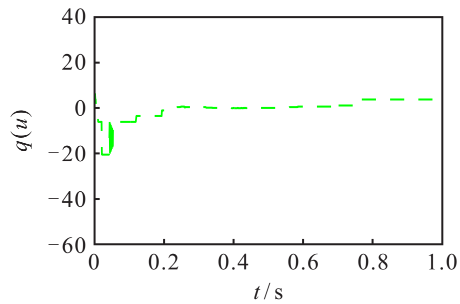


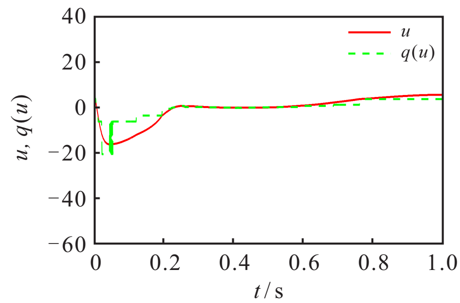
图 5　自适应调节参数$\hat{\lambda}_1$、$\hat{\lambda}_2$和$\hat{\lambda}_3$



(a)　输入u



(b)　量化输入信号$q(u)$



(c)　两者比较

图 6　输入u、量化输入信号$q(u)(t\in[0,1])$

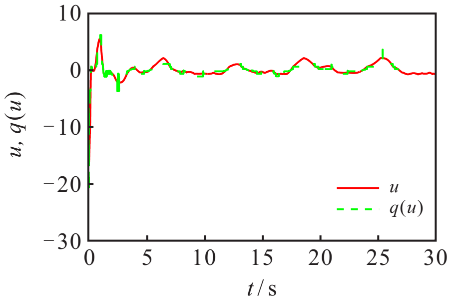


图 7　输入u、量化输入信号$q(u)$

5　结　论

本文将非线性映射技术与滞回量化器相结合,解决了系统的全状态约束问题和量化信号中的抖动现象. 借助高斯函数的性质和变量分离方法,克服了非严格反馈结构带来的困难. 采用径向基函数神经网络逼近未知非线性函数,针对输入量化和全状态约束下的随机非严格反馈非线性系统,提出了一种自适应有限时间动态面控制方法. 所设计的控制器能够保证闭环系统中所有信号依概率有界,且跟踪误差的4阶矩在有限时间内有界. 仿真结果表明了所提出方法的有效性.

参考文献(References)

[1] Khalil H K, Grizzle J W. Nonlinear systems[M]. Upper Saddle River: Prentice Hall, 2002: 551-623.

[2] Zhang T, Ge S S, Hang C C. Adaptive neural network control for strict-feedback nonlinear systems using backstepping design[J]. Automatica, 2000, 36(12): 1835-1846.

[3] Swaroop D, Hedrick J K, Yip P P, et al. Dynamic surface control for a class of nonlinear systems[J]. IEEE Transactions on Automatic Control, 2000, 45(10): 1893-1899.

[4] Zhang T P, Ge S S. Adaptive dynamic surface control of nonlinear systems with unknown dead zone in pure feedback form[J]. Automatica, 2008, 44(7): 1895-1903.

[5] Sui S, Tong S C, Li Y M. Observer-based fuzzy adaptive prescribed performance tracking control for nonlinear stochastic systems with input saturation[J]. Neurocomputing, 2015, 158: 100-108.

[6] Meng W C, Yang Q M, Si J, et al. Adaptive neural control of a class of output-constrained nonaffine systems[J]. IEEE Transactions on Cybernetics, 2016, 46(1): 85-95.

[7] Xia X N, Fang Y, Zhang T P. Adaptive quantized DSC of output-constrained uncertain nonlinear systems with quantized input and input unmodeled dynamics[J]. Journal of the Franklin Institute, 2020, 357(9): 5199-5225.

[8] Xing L T, Wen C Y, Zhu Y, et al. Output feedback control for uncertain nonlinear systems with input quantization[J]. Automatica, 2016, 65: 191-202.

[9] Huo X, Ma L, Zhao X D, et al. Observer-based fuzzy

adaptive stabilization of uncertain switched stochastic nonlinear systems with input quantization[J]. Journal of the Franklin Institute, 2019, 356(4): 1789-1809.

[10] Fridman E, Dambrine M. Control under quantization, saturation and delay: An LMI approach[J]. Automatica, 2009, 45(10): 2258-2264.

[11] Huang X Q, Lin W, Yang B. Global finite-time stabilization of a class of uncertain nonlinear systems[J]. Automatica, 2005, 41(5): 881-888.

[12] Xia J W, Zhang J, Sun W, et al. Finite-time adaptive fuzzy control for nonlinear systems with full state constraints[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2019, 49(7): 1541-1548.

[13] Zhang J, Xia J W, Sun W, et al. Finite-time tracking control for stochastic nonlinear systems with full state constraints[J]. Applied Mathematics and Computation, 2018, 338: 207-220.

[14] Sun Y M, Mao B W, Zhou S W, et al. Finite-time adaptive control for non-strict feedback stochastic nonlinear systems[J]. IEEE Access, 2019, 7: 179758-179764.

[15] Chen B, Liu X P, Ge S S, et al. Adaptive fuzzy control of a class of nonlinear systems by fuzzy approximation approach[J]. IEEE Transactions on Fuzzy Systems, 2012, 20(6): 1012-1021.

[16] Ni J K, Wu Z H, Liu L, et al. Fixed-time adaptive neural network control for nonstrict-feedback nonlinear systems with deadzone and output constraint[J]. ISA Transactions, 2020, 97: 458-473.

[17] Du P H, Pan Y N, Chadli M, et al. Asymptotic tracking control for constrained nonstrict-feedback MIMO nonlinear systems via parameter compensations[J]. International Journal of Robust and Nonlinear Control, 2020, 30(8): 3365-3381.

[18] Ba D S, Li Y X, Tong S C. Fixed-time adaptive neural tracking control for a class of uncertain nonstrict nonlinear systems[J]. Neurocomputing, 2019, 363: 273-280.

[19] Yang H J, Shi P, Zhao X D, et al. Adaptive output-feedback neural tracking control for a class of nonstrict-feedback nonlinear systems[J]. Information Sciences, 2016, 334/335: 205-218.

[20] Yu Q, Wang X Y, Zong G D, et al. Adaptive neural tracking control for a class of uncertain nonstrict-feedback nonlinear systems[J]. Journal of the Franklin Institute, 2017, 354(15): 6503-6519.

[21] Li Y M, Tong S C, Li T S. Observer-based adaptive fuzzy tracking control of MIMO stochastic nonlinear systems with unknown control directions and unknown dead zones[J]. IEEE Transactions on Fuzzy Systems, 2015, 23(4): 1228-1241.

[22] Liu Y C, Zhu Q D. Adaptive neural network finite-time tracking control of full state constrained pure feedback stochastic nonlinear systems[J]. Journal of the Franklin Institute, 2020, 357(11): 6738-6759.

[23] Hua C C, Jiang A Q, Li K. Adaptive neural network finite-time tracking quantized control for uncertain nonlinear systems with full-state constraints and applications to QUAVs[J]. Neurocomputing, 2021, 440: 264-274.

[24] Xia X N, Zhang T P. Robust adaptive quantized DSC of uncertain pure-feedback nonlinear systems with time-varying output and state constraints[J]. International Journal of Robust and Nonlinear Control, 2018, 28(10): 3357-3375.

[25] Sun Y M, Chen B, Lin C, et al. Finite-time adaptive control for a class of nonlinear systems with nonstrict feedback structure[J]. IEEE Transactions on Cybernetics, 2018, 48(10): 2774-2782.

[26] Wu Z J, Xie X J, Zhang S Y. Adaptive backstepping controller design using stochastic small-gain theorem[J]. Automatica, 2007, 43(4): 608-620.

[27] Sui S, Chen C L P, Tong S C. Fuzzy adaptive finite-time control design for nontriangular stochastic nonlinear systems[J]. IEEE Transactions on Fuzzy Systems, 2019, 27(1): 172-184.

[28] Sun Y M, Chen B, Lin C, et al. Adaptive neural control for a class of stochastic nonlinear systems by backstepping approach[J]. Information Sciences, 2016, 369: 748-764.

[29] Sanner R M, Slotine J J E. Gaussian networks for direct adaptive control[J]. IEEE Transactions on Neural Networks, 1992, 3(6): 837-863.

[30] Wang D, Huang J. Neural network-based adaptive dynamicsur face control for a class of uncertain nonlinear systemsinstrict-feedback form[J]. IEEE Transactionson Neural Networks, 2005, 16(1): 195-202.

[31] Li T S, Wang D, Feng G, et al. A DSC approach to robust adaptive NN tracking control for strict-feedback nonlinear systems[J]. IEEE Transactions on Systems, Man, and Cybernetics — Part B: Cybernetics, 2010, 40(3): 915-927.

作者简介

朱新峰 (1978−), 男, 副教授, 博士, 从事智能控制、最优控制等研究, E-mail: xfzhu@yzu.edu.cn;

丁文武 (1995−), 男, 硕士生, 从事自适应控制、智能控制的研究, E-mail: wenwuding1996@126.com;

张天平 (1964−), 男, 教授, 博士生导师, 从事鲁棒自适应控制、智能控制及非线性控制等研究, E-mail: tpzhang@yzu.edu.cn.

(责任编辑：郑晓蕾)