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二阶多智能体系统在自适应间歇牵制控制下的部分分量一致性

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摘要: 部分分量一致指的是多智能体系统中所有状态变量的一些分量渐近趋于恒同这样一种现象, 它是一种比恒同一致弱的群体动力学行为. 基于二阶多智能体系统的动力学模型, 设计一种自适应间歇牵制控制协议, 通过置换矩阵方法将原偏差系统中待研究的状态分量转换为新偏差系统中前面部分的状态分量, 并运用矩阵理论和部分变元稳定性理论, 导出部分分量一致性准则, 从而确保在指数稳定意义下该多智能体系统的部分分量一致性得以实现. 最后, 通过数值模拟验证理论分析结果.

关键词: 二阶领导-跟随多智能体系统; 自适应控制; 间歇牵制控制; 部分分量一致性

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Partial component consensus of second-order multi-agent systems via adaptive intermittent pinning control

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Abstract: Partial component consensus refers to the phenomenon that some components of all state variables of multi-agent systems become asymptotically identical, so it is a dynamic behavior that is weaker than identical consensus. Based on a dynamical model of second-order multi-agent systems, this paper designs an adaptive intermittent pinning control protocol, converts the state components to be studied in the original error system into the state components of the previous part of the new error system through the permutation matrix method, and uses the matrix theory and partial variable stability theory to derive out the partial component consensus criterion, so as to ensure the consensus according to exponential trend. Finally, the numerical simulation verifies the theoretical analysis.

Keywords: second-order leader-following multi-agent systems; adaptive control; intermittent pinning control; partial component consensus

0 引言

多智能体系统的协同控制问题涉及数学、生物、物理、计算机网络及人工智能等领域. 该问题不仅能够直观地反映许多自然现象, 而且在交叉学科领域应用广泛, 例如跟踪^[1-2]、蜂拥控制^[3-4]、编队控制^[5]、一致性控制等^[6-8]. 一致性问题为协同控制中最基础和关键的研究方向之一, 即通过智能体内部耦合或借助控制器的作用, 随着时间的演化每个智能体可运行到

预定的位置.

由位置和速度两类状态变量所确定的二阶多智能体, 因其动力系统更精确和较好的鲁棒性而被广泛运用到人工智能系统、卫星导航和传感器网络等现代化网络系统. 各领域科研者争相探究, 从而越来越多的高水平研究成果得以产生^[8-10].

为了更好、更快和更优地实现多智能体系统的一致性, 研究者们提出了多种控制方案和控制技术. 这

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些协议可大致划分为连续控制和非连续控制,而前者比后者更经济适用. 因此,研究者们越来越偏爱于将非连续性控制应用到多智能体系统一致性研究中. 文献[11]先是探讨了一类非线性系统在固定有向网络上的分布一致问题,提出了一种新的自适应间歇控制协议. 之后,文献[12]在文献[11]的结果上进一步考虑了具有几类时滞且含未知参数的多智能体系统的一致问题.

许多自然现象和实际应用表明,有时没必要或很难将所有的状态分量趋于一致. 如阅兵盛典上,各类战斗机变换着队形以向世界展示中国的实力. 各架战斗机的状态变量在其运行方向上的分量可达成一致,而在其他方位上则不然,该类现象体现了研究系统部分分量一致的必要性和重要性. 根据社会发展的需求,系统部分分量一致的探索越来越深入. 文献[13]最早提出了多智能体系统达成部分分量一致的定义,并借助矩阵理论和部分变元稳定性理论导出了一致性判据;文献[14]得到了一阶领导-跟随多智能体系统在两类间歇牵制控制策略下的部分分量一致性准则;文献[15]考虑了自适应动态事件触发机制的有向图的异构线性多智能体系统的输出一致性问题;文献[16]针对二阶非线性多智能体系统,介绍了一种基于状态补偿的周期间歇控制算法,探究了非线性二阶多智能体系统的状态补偿以及二分一致性问题. 基于团队之前的工作^[13-14],考虑到线性系统与周期间歇控制分别是非线性系统与非周期间歇控制协议的特例^[15-16],本文将自适应间歇控制与非周期间歇牵制控制相结合的协议运用到多智能体系统的二阶部分分量一致性问题的研究^[17].

本文的创新和主要贡献有以下 3 个方面:

- 1) 采用自适应控制与非周期间歇牵制控制相结合的协议对二阶非线性多智能体系统的部分分量一致性进行分析;
- 2) 探究了二阶多智能体系统的部分分量指数稳定性;
- 3) 推广了二阶部分分量一致性判据来确保系统达成一致.

1 预备知识

1.1 稳定性理论

考虑 n 维非自治常微分方程组^[18]

$$\frac{dx}{dt} = f(t, x). \quad (1)$$

其中: $f(t, x) \in C[\mathbf{R}^+ \times \mathbf{R}^n, \mathbf{R}^n]$, $f(t, 0) \equiv 0$; $x = (x_1, x_2, \dots, x_m, x_{m+1}, \dots, x_n) \in \mathbf{R}^n$, $y = (x_1, x_2, \dots, x_m) \in \mathbf{R}^m$, $z = (x_{m+1}, \dots, x_n) \in \mathbf{R}^p$, $m + p = n$,

$$\|x\| = \left(\sum_{i=1}^n x_i^2 \right)^{\frac{1}{2}}, \|y\| = \left(\sum_{i=1}^m x_i^2 \right)^{\frac{1}{2}}, \|z\| = \left(\sum_{i=m}^n x_i^2 \right)^{\frac{1}{2}}.$$

定义 1^[18] 式(1)的零解关于 y 是全局指数稳定的,若对于任意的 $\varepsilon > 0, \forall t_0 \in \mathbf{R}^+, \exists \lambda > 0, \delta(t_0, \varepsilon) > 0$,使得 $\|x_0\| < \delta(\varepsilon)$,则关于式(1)的解 $\|x(t, t_0, x_0)\|$ 的部分解 $\|y(t, t_0, x_0)\|$ 满足

$$\|y(t, t_0, x_0)\| < \varepsilon e^{-\lambda(t-t_0)}, t \geq t_0.$$

1.2 矩阵论

引理 1^[19] 设矩阵 A, B, C, D 之间的克罗内克积符合以下运算规律:

- 1) $(A + B) \otimes C = A \otimes C + B \otimes C$;
- 2) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$;
- 3) $(kA) \otimes B = A \otimes (kB)$;
- 4) $(A \otimes B)^T = A^T \otimes B^T$.

其中 k 是任意常数.

引理 2^[20] 设两个矩阵分别为

$$A = (a_{ij}) \in \mathbf{R}^{N \times N}, B = (b_{ij}) \in \mathbf{R}^{n \times n}.$$

令 $P_i (i = 1, 2, \dots, r)$ 为初等行变换矩阵,有 nN 阶初等置换矩阵 $P = P_1 \dots P_i \dots P_r$,使得如下算式成立:

$$P(A \otimes B)P^{-1} = B \otimes A,$$

其中 \otimes 为克罗内克积.

2 主要内容

考虑由 N 个智能体构成的二阶多智能体系统

$$\dot{x}_i(t) = v_i(t),$$

$$\dot{v}_i(t) =$$

$$\begin{cases} f(x_i(t), v_i(t)) + c_1 H(t) \sum_{j \in N_i} a_{ij} \Gamma(x_j - x_i) + \\ c_2 H(t) \sum_{j \in N_i} a_{ij} \Gamma(v_j - v_i) + u_i(t), t \in [t_k, s_k]; \\ f(x_i(t), v_i(t)) + c_1 \sum_{j \in N_i} a_{ij} \Gamma(x_j - x_i) + \\ c_2 \sum_{j \in N_i} a_{ij} \Gamma(v_j - v_i), t \in (s_k, t_{k+1}). \end{cases}$$

(2)

其中: $i = 1, 2, \dots, N, x_i(t) \in \mathbf{R}^n$ 表示第 i 个智能体的位置状态变量; $v_i(t) \in \mathbf{R}^n$ 代表第 i 个智能体的速度状态变量; $f: \mathbf{R}^n \rightarrow \mathbf{R}^n$ 为一个连续映射,是第 i 个智能体的自身动力学;网络间的通信权重由连接矩阵 $A = (a_{ij})_{N \times N}$ 描述,当第 i 个智能体可以接收第 j 个智能体的信息时, $a_{ij} > 0$, 否则, $a_{ij} = 0$; c_1 和 c_2 分别表示第 i 个智能体关于位移和速度变量的内部耦合强度; $\Gamma = \text{diag}\{r_1, r_2, \dots, r_n\} \in \mathbf{R}^{n \times n}, r_k \geq$

0, $k = 1, 2, \dots, n$ 表示内部耦合矩阵; $\mathbf{H}(t)$ 是随系统动力学的自适应演化而变化的时变强度, 随着时间的推移逐渐收敛到一个常数. 现实生活中的一些控制系统即采用这种适应性时变强度进行控制. 如汽车启动时油门并非一直保持不变, 而是通过逐步加大踩踏力度使油门逐步加大, 在速度达到预期后保持油门即可. $u_i(t) \in \mathbf{R}^n$ 为控制输入, 在控制时刻 $[t_k, s_k]$ 牵制控制生效, 在休息间隔 (s_k, t_{k+1}) 控制失效, $k \in N$. 这里, 控制时间和休息时间都是非周期性的.

假设仅有一个领导智能体, 其动力学可以描述如下:

$$\begin{cases} \dot{x}_0(t) = v_0(t), \\ \dot{v}_0(t) = \mathbf{f}(x_0(t), v_0(t)). \end{cases} \quad (3)$$

探讨以下以非周期形式出现的自适应间歇控制输入:

$$u_i(t) = \begin{cases} -c_1 \mathbf{H}(t) b_i \mathbf{\Gamma}(x_i - x_0) - \\ c_2 \mathbf{H}(t) b_i \mathbf{\Gamma}(v_i - v_0), t \in [t_k, s_k]; \\ 0, t \in (s_k, t_{k+1}). \end{cases} \quad (4)$$

其中: b_i 为牵制控制增益, 若第 i 个跟随智能体能得知领导智能体的变量状况, 则规定 $b_i > 0$, 否则 b_i 为零, 令 $\mathbf{B} = \text{diag}(b_1, b_2, \dots, b_N)$.

规定多智能体系统在控制区间及休息区间的自适应更新率分别为

$$\dot{\mathbf{H}}(t) = \begin{cases} \frac{d_0}{2} e^{\alpha t} \hat{\mathbf{e}}^T(t) \mathbf{\Lambda} \left[\mathbf{\Gamma} \otimes \begin{bmatrix} c_1^2 \mathbf{D}^S & c_1 c_2 \mathbf{D}^S \\ c_1 c_2 \mathbf{D}^S & c_2^2 \mathbf{D}^S \end{bmatrix} \right] \hat{\mathbf{e}}(t), \\ t \in [t_k, s_k]; \\ 0, t \in (s_k, t_{k+1}). \end{cases} \quad (5)$$

其中: $\mathbf{L} = (a_{ij})_{N \times N}$, $\mathbf{D}^S = \frac{(\mathbf{L} + \mathbf{B}) + (\mathbf{L} + \mathbf{B})^T}{2}$, 符号 $(*)^s$ 表示一个矩阵的对称部分, $\mathbf{\Lambda} = \text{diag}(1, 1, \dots, 1, 0, \dots, 0) \in 2\mathbf{R}^{nN}$, 元素为1的个数为 l .

假设1^[21] 针对非周期性间歇控制而言, 假设存在两个标量 ϕ 和 δ 符合如下算式:

$$\begin{cases} \min\{s_k - t_k\} = \phi, \\ \max\{t_{k+1} - s_k\} = \delta. \end{cases} \quad (6)$$

假设2^[21] 考虑非周期性间歇控制协议, 存在 ϕ 和 δ 符合以下条件:

$$\begin{cases} \inf_k (s_k - t_k) = \phi, \\ \sup_k (t_{k+1} - t_k) = \gamma. \end{cases} \quad (7)$$

其中: $0 < \phi < \gamma < +\infty$, $k \in N$, 表示每个休息状态的时间区间长度应该小于或等于 $\gamma - \phi$, 故控制区间和休息时间区间都有界.

定义2^[21] 对于非周期间歇控制协议而言, 规定

$$\eta = \limsup_{k \rightarrow +\infty} \frac{t_{k+1} - s_k}{t_{k+1} - t_k}, \quad (8)$$

由假设1和假设2可得 $0 < \eta < 1$. 显然, 当 $\eta = 0$ 时, 此时的系统将处于连续控制, 所以考虑非连续控制, 并假设 η 处于0到1之间.

假设3^[21] 若假设1成立, 则定义 $\eta \leq \frac{\phi}{\gamma}$.

假设4^[22] 存在非负常数 $\varepsilon_1, \varepsilon_2$, 对于所有的 $x_i, y_i \in \mathbf{R}^n, i = 1, 2$, 使得非线性函数 \mathbf{f} 满足

$$\begin{aligned} \|\mathbf{f}(x_1, y_1) - \mathbf{f}(x_2, y_2)\| &\leq \\ \varepsilon_1 \|x_1 - x_2\| + \varepsilon_2 \|y_1 - y_2\|. \end{aligned} \quad (9)$$

3 主要结果

令状态偏差为

$$\mathbf{e}(t) = (\mathbf{e}_x^T(t), \mathbf{e}_v^T(t))^T. \quad (10)$$

其中

$$\begin{aligned} \mathbf{e}_x(t) &= x_i - x_0, \mathbf{e}_v(t) = v_i - v_0, \\ \mathbf{e}_x(t) &= (\mathbf{e}_{x1}^T(t), \mathbf{e}_{x2}^T(t), \dots, \mathbf{e}_{xN}^T(t))^T \in \mathbf{R}^{nN}, \\ \mathbf{e}_v(t) &= (\mathbf{e}_{v1}^T(t), \mathbf{e}_{v2}^T(t), \dots, \mathbf{e}_{vN}^T(t))^T \in \mathbf{R}^{nN}, \\ \mathbf{e}_{xq}(t) &= (\mathbf{e}_{xq1}^T(t), \mathbf{e}_{xq2}^T(t), \dots, \mathbf{e}_{xqn}^T(t))^T \in \mathbf{R}^n, \\ \mathbf{e}_{vq}(t) &= (\mathbf{e}_{vq1}^T(t), \mathbf{e}_{vq2}^T(t), \dots, \mathbf{e}_{vqn}^T(t))^T \in \mathbf{R}^n, \end{aligned}$$

这里 $q = 1, 2, \dots, n$.

结合式(2)、(3)和(4), 可将偏差系统转化为如下矩阵形式:

$$\dot{\mathbf{e}}(t) = \begin{cases} \mathbf{F}(\mathbf{e}(t)) + \begin{bmatrix} 0_{Nn} & I_{Nn} \\ -c_1 \mathbf{H} \mathbf{D} \otimes \mathbf{\Gamma} & -c_2 \mathbf{H} \mathbf{D} \otimes \mathbf{\Gamma} \end{bmatrix} \mathbf{e}(t); \\ t \in [t_k, s_k]; \\ \mathbf{F}(\mathbf{e}(t)) + \begin{bmatrix} 0_{Nn} & I_{Nn} \\ -c_1 \mathbf{L} \otimes \mathbf{\Gamma} & -c_2 \mathbf{L} \otimes \mathbf{\Gamma} \end{bmatrix} \mathbf{e}(t), \\ t \in (s_k, t_{k+1}). \end{cases} \quad (11)$$

其中

$$\begin{aligned} \mathbf{F}(\mathbf{e}(t)) &= \begin{bmatrix} 0_{Nn} \\ \mathbf{F}(t, x, v) - \mathbf{1}_N \otimes \mathbf{f}(t, x_0, v_0) \end{bmatrix}, \\ \mathbf{1}_N^T &= (1, 1, \dots, 1)_{1 \times N}, \\ \mathbf{F}(t, x, v) &= (f^T(t, x_1, v_1), \dots, f^T(t, x_N, v_N))^T, \\ f(t, x_i, v_i) &= (f_1(t, x_i, v_i), \dots, f_n(t, x_i, v_i))^T. \end{aligned}$$

运用引理 2 将 Nn 阶偏差系统 (11) 置换为 Nn 阶新的偏差系统, 即令

$$\hat{e}(t) = \mathbf{P}e(t). \tag{12}$$

其中: $\hat{e}(t) = (\hat{e}_1(t)^T, \hat{e}_2(t)^T, \dots, \hat{e}_n(t)^T)^T$, $\hat{e}_i(t) = (\hat{e}_{xi}(t)^T, \hat{e}_{vi}(t)^T)^T$.

由以上操作, 新的偏差系统可转化为以下等式:

$$\dot{\hat{e}}(t) = \begin{cases} \hat{F}(\hat{e}(t)) + (I_n \otimes \Omega + \Gamma \otimes P_1)\hat{e}(t), & t \in [t_k, s_k]; \\ \hat{F}(\hat{e}(t)) + (I_n \otimes \Omega + \Gamma \otimes P_2)\hat{e}(t), & t \in (s_k, t_{k+1}). \end{cases} \tag{13}$$

其中

$$\begin{aligned} \Omega &= \begin{bmatrix} 0_N & I_N \\ 0_N & 0_N \end{bmatrix}, \quad P_1 = \begin{bmatrix} 0_N & 0_N \\ -c_1 \mathbf{H} \mathbf{D} & -c_2 \mathbf{H} \mathbf{D} \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 0_N & 0_N \\ -c_1 \mathbf{L} & -c_2 \mathbf{L} \end{bmatrix}, \\ \hat{F}(\hat{e}(t)) &= (0_N^T, \hat{F}_1^T, 0_N^T, \hat{F}_2^T, \dots, 0_N^T, \hat{F}_N^T)^T, \\ \hat{F}_i &= (f_i(t, x_1, v_1), \dots, f_i(t, x_N, v_N)) - 1_N^T f_i(t, x_0, v_0). \end{aligned}$$

定义 3^[23] 对于二阶多智能体系统而言, 其能够实现前 l 个分量一致, 对于任意初值, 第 i 个智能体均满足如下条件:

$$\begin{aligned} \lim_{t \rightarrow +\infty} \sum_{k=1}^l \|x_{ik}(t) - x_{0k}(t)\| &= 0, \\ \lim_{t \rightarrow +\infty} \sum_{k=1}^l \|v_{ik}(t) - v_{0k}(t)\| &= 0, \end{aligned} \tag{14}$$

其中 $i = 1, 2, \dots, N$.

定理 1 假设系统满足假设 1 和假设 4, 且满足以下条件:

$$\begin{aligned} M_1 &= \mathbf{A} \left[I_n \otimes \begin{bmatrix} \nabla_1 I_N + \alpha c_1 c_2 \mathbf{D}^S & c_1 c_2 \mathbf{D}^S + \frac{\alpha c_1}{2} I_N \\ c_1 c_2 \mathbf{D}^S + \frac{\alpha c_1}{2} I_N & (\nabla_3 + \frac{\alpha c_2}{2}) I_N \end{bmatrix} \right] - h^* (\Gamma \otimes \Theta_1) \leq 0, \\ M_2 &= \mathbf{A} \left[I_n \otimes \begin{bmatrix} \nabla_2 I_N - \beta c_1 c_2 \mathbf{D}^S & c_1 c_2 \mathbf{D}^S - \frac{\beta c_1}{2} I_N \\ c_1 c_2 \mathbf{D}^S - \frac{\beta c_1}{2} I_N & (\nabla_3 - \frac{\beta c_2}{2}) I_N \end{bmatrix} \right] + \Gamma \otimes \Theta_2 \leq 0, \\ &- \sum_{r=0}^{\varsigma} \alpha(s_k - t_k) - \sum_{r=0}^{\varsigma} \beta(s_k - t_k) \rightarrow -\infty, \quad \varsigma \rightarrow +\infty, \end{aligned}$$

则系统在控制协议 (4) 下可实现部分分量一致. 其中: h^* 是个充分大的常数, $\nabla_1 = (c_1 \varepsilon_2 + c_2 \varepsilon_1)/2$, $\nabla_2 = \nabla_1 + c_1 \varepsilon_1$, $\nabla_3 = \nabla_1 + c_2 \varepsilon_2 + c_1$, $\alpha \geq 0, \beta \geq 0$, $\Theta_1 = \begin{bmatrix} c_1^2 \mathbf{D}^S & c_1 c_2 \mathbf{D}^S \\ c_1 c_2 \mathbf{D}^S & c_2^2 \mathbf{D}^S \end{bmatrix}$, $\Theta_2 = \begin{bmatrix} -c_1^2 \mathbf{L}^S & -c_1 c_2 \mathbf{L}^S \\ -c_1 c_2 \mathbf{L}^S & -c_2^2 \mathbf{L}^S \end{bmatrix}$.

证明 设 Lyapunov 函数

$$V(t) = \frac{1}{2} \hat{e}^T(t) \mathbf{A} (I_n \otimes Q) \hat{e}(t) + e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0}, \tag{15}$$

其中: $Q = \begin{bmatrix} 2c_1 c_2 \mathbf{D}^S & c_1 I_N \\ c_1 I_N & c_2 I_N \end{bmatrix}$, h^* 是个充分大的常数.

当 $t \in [t_k, s_k]$ 时, 控制器被激活, 考虑 $V(t)$ 沿着偏差系统 (13) 求导可得

$$\begin{aligned} \dot{V}(t) &= \hat{e}^T(t) \mathbf{A} (I_n \otimes Q) \dot{\hat{e}}(t) + e^{-\alpha t} \frac{\mathbf{H}(t) - h^*}{d_0} \dot{\mathbf{H}}(t) - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} = \\ &\hat{e}^T(t) \mathbf{A} (I_n \otimes Q) [\hat{F}(\hat{e}(t)) + (I_n \otimes \Omega + \Gamma \otimes P_1)\hat{e}(t)] + e^{-\alpha t} \frac{\mathbf{H}(t) - h^*}{d_0} \dot{\mathbf{H}}(t) - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} \leq \\ &\hat{e}^T(t) \mathbf{A} (I_n \otimes Q) \hat{F}(\hat{e}(t)) + \hat{e}^T(t) \mathbf{A} (I_n \otimes (Q\Omega)^s) \hat{e}(t) + \hat{e}^T(t) \mathbf{A} (\Gamma \otimes (QP_1)^s) \hat{e}(t) + \\ &e^{-\alpha t} \frac{\mathbf{H}(t) - h^*}{d_0} d_0 e^{\alpha t} \hat{e}^T(t) \mathbf{A} (\Gamma \otimes \Theta_1) \hat{e}(t) - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} = \\ &\sum_{i=1}^l c_1 \hat{e}_{xi}(t)^T \hat{F}_i(t, x, v) + \sum_{i=1}^l c_2 \hat{e}_{vi}(t)^T \hat{F}_i(t, x, v) + \hat{e}^T(t) \mathbf{A} (I_n \otimes (Q\Omega)^s) \hat{e}(t) - h^* \hat{e}^T(t) \mathbf{A} (\Gamma \otimes \Theta_1) \hat{e}(t) - \\ &\alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} \leq \\ &\sum_{i=1}^l c_1 (\varepsilon_1 \hat{e}_{xi}(t)^T \hat{e}_{xi}(t) + \varepsilon_2 \hat{e}_{xi}(t)^T \hat{e}_{vi}(t)) + \sum_{i=1}^l c_2 (\varepsilon_1 \hat{e}_{xi}(t)^T \hat{e}_{vi}(t) + \varepsilon_2 \hat{e}_{vi}(t)^T \hat{e}_{vi}(t)) + \\ &\hat{e}^T(t) \mathbf{A} (I_n \otimes (Q\Omega)^s) \hat{e}(t) \leq \\ &\sum_{i=1}^l c_1 \left(\frac{2\varepsilon_1 + \varepsilon_2}{2} \|\hat{e}_{xi}(t)\|^2 + \frac{\varepsilon_2}{2} \|\hat{e}_{vi}(t)\|^2 \right) + \sum_{i=1}^l c_2 \left(\frac{\varepsilon_1}{2} \|\hat{e}_{xi}(t)\|^2 + \frac{\varepsilon_1 + 2\varepsilon_2}{2} \|\hat{e}_{vi}(t)\|^2 \right) + \\ &\hat{e}^T(t) \mathbf{A} (I_n \otimes (Q\Omega)^s) \hat{e}(t) - h^* \hat{e}^T(t) \mathbf{A} (\Gamma \otimes \Theta_1) \hat{e}(t) - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} = \end{aligned}$$

$$\begin{aligned} & \hat{e}^T(t)\mathbf{A} \left[I_n \otimes \begin{bmatrix} \nabla_2 I_N & 0_N \\ 0_N & (\nabla_3 - c_1)I_N \end{bmatrix} \right] \hat{e}(t) + \\ & \hat{e}^T(t)\mathbf{A}(I_n \otimes (Q\Omega)^s)\hat{e}(t) - h^*\hat{e}^T(t)\mathbf{A}(\Gamma \otimes \Theta_1)\hat{e}(t) - \\ & \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} - \frac{\alpha}{2} \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{e}(t) + \\ & \frac{\alpha}{2} \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{e}(t) = \\ & \hat{e}^T(t)M_1\hat{e}(t) - \alpha V \leq -\alpha V. \end{aligned} \tag{16}$$

当 $t \in (s_k, t_{k+1})$ 时, 控制器处于休息状态, 此时将 $V(t)$ 对 t 求导可得

$$\begin{aligned} \dot{V}(t) = & \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{e}(t) - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} = \\ & \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)[\hat{F}(\hat{e}(t)) + (I_n \otimes \Omega + \Gamma \otimes P_2)\hat{e}(t)] - \\ & - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} \leq \\ & \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{F}(\hat{e}(t)) + \hat{e}^T(t)\mathbf{A}(I_n \otimes (Q\Omega)^s)\hat{e}(t) + \\ & \hat{e}^T(t)\mathbf{A}(\Gamma \otimes (QP_2)^s)\hat{e}(t) - \alpha e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} \leq \\ & \hat{e}^T(t)\mathbf{A} \left[I_n \otimes \begin{bmatrix} \nabla_2 I_N & 0_N \\ 0_N & (\nabla_3 - c_1)I_N \end{bmatrix} \right] \hat{e}(t) + \\ & \hat{e}^T(t)\mathbf{A}(I_n \otimes (Q\Omega)^s)\hat{e}(t) + \\ & \hat{e}^T(t)\mathbf{A}(\Gamma \otimes (QP_2)^s)\hat{e}(t) + \\ & \beta e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} + \hat{e}^T(t)\mathbf{A}(I_n \otimes (Q\Omega)^s)\hat{e}(t) + \\ & \hat{e}^T(t)\mathbf{A}(\Gamma \otimes (QP_2)^s)\hat{e}(t) + \\ & \beta e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} + \frac{\beta}{2} \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{e}(t) - \\ & \frac{\beta}{2} \hat{e}^T(t)\mathbf{A}(\Gamma \otimes (QP_2)^s)\hat{e}(t) + \beta e^{-\alpha t} \frac{(\mathbf{H}(t) - h^*)^2}{2d_0} + \\ & \frac{\beta}{2} \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{e}(t) - \frac{\beta}{2} \hat{e}^T(t)\mathbf{A}(I_n \otimes Q)\hat{e}(t) = \\ & \hat{e}^T(t)M_2\hat{e}(t) + \beta V \leq \beta V. \end{aligned} \tag{17}$$

综上可得

$$\begin{cases} \dot{V}(t) \leq -\alpha V(t), & t \in [t_k, s_k], k \in N; \\ \dot{V}(t) \leq \beta V(t), & t \in (s_k, t_{k+1}), k \in N. \end{cases} \tag{18}$$

定理得证. \square

由定理1可知 $\alpha \geq 0, \beta \geq 0$, 所以可以考虑两种情况 $t \in [t_k, s_k]$ 或 $t \in (s_k, t_{k+1})$. 设 $[t_r, s_r] \subset [t_k, s_k], (s_r, t_{r+1}) \subset (s_k, t_{k+1}), r$ 为正整数. 以下方法类似于文献[14].

情况1 当 $t \in [t_k, s_k]$ 且 $t \rightarrow \infty$ 时, 通过计算式(18)可得

$$V(t) \leq V(0)e^\chi, \tag{19}$$

其中

$$\chi = - \sum_{r=0}^{r=k-1} \int_{t_r}^{s_r} \alpha dt - \int_{t_k}^t \alpha dt + \sum_{r=0}^{r=k-1} \int_{s_r}^{t_{r+1}} \beta dt.$$

若 $t - t_k \rightarrow +\infty$, 则 $t, k \rightarrow +\infty$, 由定理1可知 $\alpha \geq 0$, 所以可以导出以下不等式:

$$\begin{aligned} \chi \leq & - \sum_{r=0}^{r=k-1} \int_{t_r}^{s_r} \alpha dt + \sum_{r=0}^{r=k-1} \int_{s_r}^{t_{r+1}} \beta dt = \\ & - \sum_{r=0}^{r=k-1} \alpha(s_r - t_r) + \sum_{r=0}^{r=k-1} \beta(t_{r+1} - s_r) \leq \\ & - \sum_{r=0}^{\zeta} \alpha(s_k - t_k) + \sum_{r=0}^{\zeta} \beta(t_{k+1} - s_k), \end{aligned} \tag{20}$$

最终得到

$$V(t) \leq V(0)e^{\{-\sum_{r=0}^{\zeta} \alpha(s_k - t_k) + \sum_{r=0}^{\zeta} \beta(t_{k+1} - s_k)\}}, \tag{21}$$

综合式(19)~(21)以及定理1, 可以得出 $\lim_{t \rightarrow +\infty} V(t) = 0$, 则 $\lim_{t \rightarrow +\infty} \hat{e}_r(t) = 0, r = 1, 2, \dots, l$.

情况2 对于 $t \in (s_k, t_{k+1}), t \rightarrow +\infty$, 有

$$\begin{aligned} \lambda = & - \sum_{r=0}^{r=k} \int_{t_r}^{s_r} \alpha dt - \int_{s_r}^t \alpha dt + \sum_{r=0}^{r=k-1} \int_{s_r}^{t_{r+1}} \beta dt \leq \\ & - \sum_{r=0}^{r=k} \int_{t_r}^{s_r} \alpha dt + \sum_{r=0}^{r=k-1} \int_{s_r}^{t_{r+1}} \beta dt \leq \\ & - \sum_{r=0}^{\zeta} \int_{t_k}^{s_k} \alpha dt + \sum_{r=0}^{\zeta} \int_{s_k}^{t_{k+1}} \beta dt = \\ & - \alpha \sum_{r=0}^{\zeta} (s_k - t_k) + \beta \sum_{r=0}^{\zeta} (t_{k+1} - s_k). \end{aligned} \tag{22}$$

最后推算出

$$\begin{aligned} V(t) \leq & V(0)e^\lambda \leq \\ & V(0)e^{\{-\sum_{r=0}^{\zeta} \alpha(s_k - t_k) + \sum_{r=0}^{\zeta} \beta(t_{k+1} - s_k)\}}. \end{aligned} \tag{23}$$

与情况1类似, 可得当 $\zeta \rightarrow +\infty, t \rightarrow +\infty$ 时, 有 $\lim_{t \rightarrow +\infty} V(t) = 0$, 即 $\lim_{t \rightarrow +\infty} \hat{e}_1(t) = \lim_{t \rightarrow +\infty} \hat{e}_2(t) = \lim_{t \rightarrow +\infty} \hat{e}_3(t) = \dots = \lim_{t \rightarrow +\infty} \hat{e}_l(t) = 0$, 即偏差系统的前 l 个分量随着时间推移将以指数形式趋于零, 意味着跟随者的全部状态变量的前 l 个分量与领导者的运行轨迹渐近趋于一致.

4 数值模拟

假设多智能系统是由1个领导者智能体与8个跟随智能体组成, 每个智能体的自身动力学为

$$f(x_i(t), v_i(t)) = \begin{bmatrix} 2v_{i2} + 0.1|v_{i1} + 1| - 0.1|v_{i1} - 1| \\ 0.2x_{i1} + 0.3v_{i2} \\ -2x_{i2} + 0.6v_{i3} \end{bmatrix}. \tag{24}$$

通过数值计算可得 $\varepsilon_1 = \varepsilon_2 = 3$, 可以满足假设4,

给出拉普拉斯矩阵如下:

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -3 & 6 & -2 & 0 & 0 & 0 & 0 \\ 0 & -4 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & -4 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & -6 & 0 & 0 & 6 \end{bmatrix},$$

$$B = \text{diag}(5, 0, 0, 0, 0, 0, 0, 0),$$

$$\Gamma = \text{diag}(1, 1, 0).$$

考虑二阶多智能体系统在自适应率(5)下的部分分量一致性,在[0, 10]时间内,适应性牵制控制协议在以下的间歇时间段内工作:

$$[0, 1.4] \cup [2.1, 3.6] \cup [4.5, 6.3] \cup [7.2, 8] \cup [9.4, 10].$$

显然,对于 $\forall k \in N^+$,有 $s_k - t_k \geq 0.6, t_{k+1} - t_k \leq 2.7$,然后利用定义2算出最大的休息率 $\eta = 0.78$.用Matlab进行数值模拟得出每个多智能体的位移和速度两个状态变量的3个分量的运动轨迹,由图1~图3可知,在自适应间歇控制下位移的第1个和第2个分量随着时间的演化与领导者智能体保持一致,即趋于一致,而第3个分量未能达成一致;类似地,图4~图6中展示了多智能体速度的第1和第2个分量可与领导多智能体逐渐收敛,而第3个分量逐渐发散;图7描述了不同牵制控制强度的设置对自适应控制增益的影响对比图.显然可知,不同的适应性增益随着时

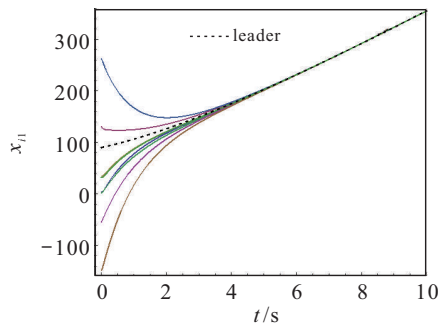


图1 智能体位移的第1个分量的轨迹

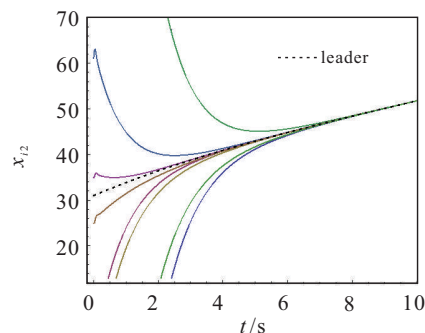


图2 智能体位移的第2个分量的轨迹

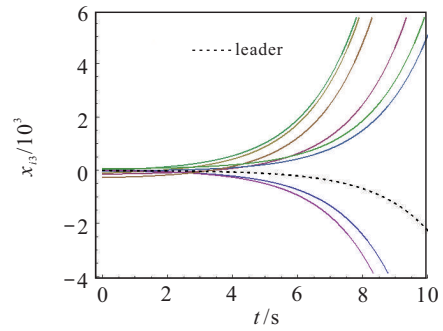


图3 智能体位移的第3个分量的轨迹

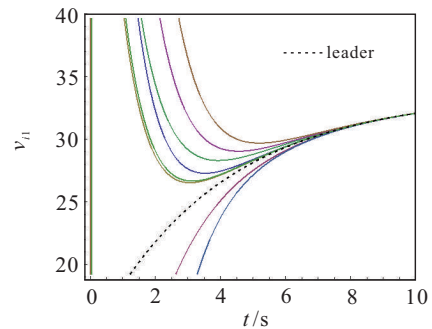


图4 智能体速度第1个分量随时间演化曲线

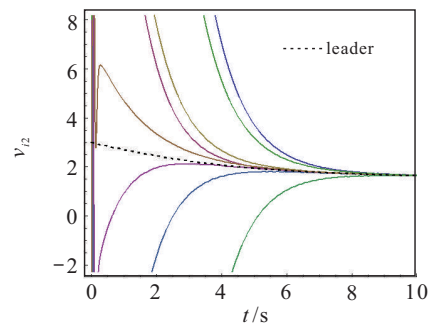


图5 智能体速度的第2个分量随时间演化曲线

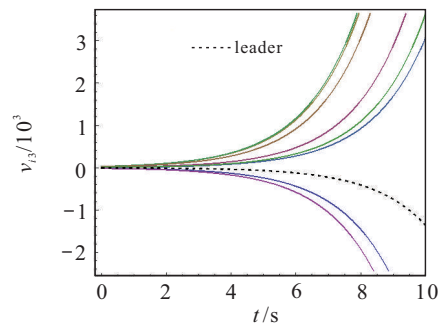


图6 智能体速度的第3个分量随时间演化曲线

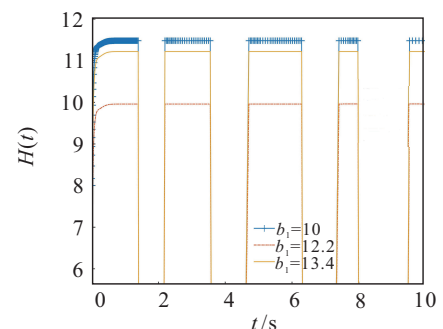


图7 网络系统在不同牵制强度下的适应性增益的轨迹

间的推移最终分别趋于一个定值,即均有界。

5 结论

本文主要借助自适应间歇控制,探究了此控制下多智能体系统的二阶部分分量一致性问题,将自适应牵制控制和非周期间歇控制协议相互联合,根据一系列理论推导得出达成部分分量一致的充分性准则,最终通过举例模拟证实了理论结果的正确性。下一步,将考虑具有通讯延迟或自身动力学时滞的情况。

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