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钱伟, 张祥林, 赵运基, 费树岷

引用本文:

钱伟,张祥林,赵运基,费树岷. 随机饱和与测量缺失下非线性系统的分布式状态估计[J]. 控制与决策, 2023, 38(11): 3137–3146.

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随机饱和与测量缺失下非线性系统的分布式状态估计

钱伟^{1†}, 张祥林¹, 赵运基¹, 费树岷^{1,2}

(1. 河南理工大学 电气工程与自动化学院, 河南 焦作 454000; 2. 东南大学 自动化学院, 南京 210096)

摘要: 传感器网络环境中普遍存在的节点饱和、测量缺失、时滞等信息不完全现象, 必然导致系统整体性能变差. 研究随机饱和与测量缺失影响下非线性系统的分布式 H_∞ 状态估计问题. 通过两组已知概率的 Bernoulli 分布, 建立一个能够在统一框架内描述以上两种随机不完全信息的传感器模型. 每个传感器接收到的信号由采样间隔随时间变化的采样器分别采样, 并利用输入延迟的方法, 将采样周期转化为等价的有界时变时滞, 从而得到时滞随机非线性估计误差系统. 在此基础上构造合适的 Lyapunov-Krasovskii 泛函, 并选择与之有效配合的积分不等式, 得到具有较小保守性的分布式 H_∞ 状态估计器设计条件. 最后通过仿真分析验证所提方法的有效性.

关键词: 分布式 H_∞ 状态估计; 非均匀采样; 时滞; 随机饱和; 测量缺失; Lyapunov-Krasovskii 泛函

中图分类号: TP273 文献标志码: A

DOI: 10.13195/j.kzyjc.2022.0060

引用格式: 钱伟, 张祥林, 赵运基, 等. 随机饱和与测量缺失下非线性系统的分布式状态估计 [J]. 控制与决策, 2023, 38(11): 3137-3146.

Distributed state estimation for nonlinear systems with randomly occurring sensor saturations and missing measurement

QIAN Wei^{1†}, ZHANG Xiang-lin¹, ZHAO Yun-ji¹, FEI Shu-min^{1,2}

(1. School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, China; 2. School of Automation, Southeast University, Nanjing 210096, China)

Abstract: Incomplete information phenomena such as sensor saturation, missing measurement and time delay always exist in the sensor network, which leads to the performance deterioration of the system. In this paper, the problem of distributed H_∞ state estimation for nonlinear systems under the influence of random saturation and missing measurement is studied. Based on two sets of Bernoulli distributions with known probability, a sensor model which can describe the above two kinds of random incomplete information in a unified framework is established. The signals received by each sensor are sampled separately by a sampler whose sampling interval varies with time, and the sampling period is converted into an equivalent bounded time-varying delay by using the input delay method. Thus a stochastic nonlinear estimation error system with time-varying delay is obtained. On this basis, a suitable Lyapunov-Krasovskii functional is constructed, and several integral inequalities are selected to fit with it effectively. Then the design conditions of the distributed H_∞ state estimator with less conservatism are obtained. Finally, the effectiveness of the proposed method is demonstrated by simulation analysis.

Keywords: distributed H_∞ state estimation; non-uniformly sampling; time delays; randomly saturation; missing measurement; Lyapunov-Krasovskii functional

0 引言

传感器网络通常由大量具有传感计算和无线通信功能的传感器节点组成. 各节点通常部署在监测区域, 通过与邻接节点信息交互、协同工作执行某些特定任务, 常用于多智能体控制、智能交通、工业控

制、目标跟踪等领域^[1-6]. 在实际应用中, 由于信道容量有限、传输时间长等因素, 常常会引发饱和、时滞、测量丢失等不完全信息, 加上网络内部和外部的各种干扰, 均会造成系统整体性能恶化^[7-8]. 因此, 研究不完全信息影响下系统的分布式状态估计具有十分重

收稿日期: 2022-01-08; 录用日期: 2022-06-15.

基金项目: 国家自然科学基金项目(61973105); 河南省创新型科技团队项目(CXTD2016054); 中原高水平人才专项支持计划项目(ZYQR201912031).

责任编辑: 关新平.

[†]通讯作者. E-mail: qwei@hpu.edu.cn.

要的意义.

目前常见的分布式状态估计器设计方法多依赖于理想假设,即测量信号连续且通道容量不受限制,信息传输能够精准实现.然而,与传统的网络节点相比,传感器网络因其节点分布和数据交互的特点,会因通信带宽变化或外界干扰而引发测量缺失现象.在过去几年里,文献[5,9-11]已将这一现象引入传感器网络领域,文献[12-13]采用已知概率分布的随机过程描述传感器测量数据的随机缺失.此外,受限于当前安全要求、物理特性及技术等因素的影响,信号不可能被无限放大,因此,传感器饱和也是不可避免的,其存在会影响系统整体性能,甚至引发系统失稳^[3-4,8,14].需要指出的是,上述文献都是建立在只存在单一随机不完全信息或必定发生某种现象的基础上,但在实际中,不完全信息的发生是多重且不确定的,随着随机现象的增多,系统失稳的可能性必然提升,同时,使得状态估计器的设计变得困难.因此,建立更具一般性的传感器模型,使随机发生的传感器饱和与测量缺失能在统一框架下进行描述具有重要的理论价值和实际意义.另外,现有研究中对象多为线性系统,而不完全信息影响下非线性系统的分布式状态估计却研究尚浅.

在传感器网络中,由于传感器是数字设备,每个节点接收到的信号在传输到状态估计器处理之前必须进行数据采样^[15].传统的方法是利用周期/均匀采样方法将目标对象建模成一个离散时间系统,然而,这样一个离散时间模型并不能捕捉到实际系统的真实信号特点,特别是当采样周期与信号周期一致时,非常不利于信号重构.因此,非均匀采样被广泛应用,该方法不仅可以获得更精确的数据信号,而且适应性更强^[16].针对非均匀采样,文献[17]提出的输入延迟方法被广泛应用,其主要思想是将采样数据系统转换为具有有界时滞的连续时间系统.在过去的几年里,这种输入延迟方法使用得非常频繁^[15,18-20].例如,文献[18-19]分别就信道冗余的时变多速率系统和多传感器系统提出了分布式状态估计方案的数据采样方法,并且基于输入延迟方法的分布时滞系统的状态估计问题也在文献[20]中得到了解决.因此,本文的另一目标则是采用一种非均匀采样方法,以保证信号重构更加准确、估计性能更加优良.

基于上述讨论,本文旨在研究受随机饱和与测量丢失影响下非线性系统的分布式 H_∞ 状态估计问题.利用Bernoulli分布统一刻画传感器网络中随机发生的传感器饱和和缺失测量,并使每个传感器接

收到的信号包括来自系统及其邻接节点的测量值,然后将传感器输出信号由采样器分别采样,传输给相应的估计器,保证不同采样节点的采样间隔随时间变化,且独立有界.进一步,利用输入延迟的方法,将基于非均匀数据采样的 H_∞ 状态估计问题转化为具有多个有界时滞的 H_∞ 状态估计问题.在此基础上,充分利用非均匀采样带来的时滞信息构造Lyapunov-Krasovskii泛函,避免引入复杂的多重积分型和增广型泛函项,有效降低计算负担.同时选用合适的积分不等式,提出一种易于实现并具有较小保守性的分布式状态估计器设计方法.最后通过仿真实例验证该估计方法的有效性.

本文中的相关符号注释如下: \mathbf{R}^n 表示 n 维欧氏空间; $\mathbf{R}^{n \times m}$ 表示 $n \times m$ 的实矩阵; \mathcal{P}^T 表示 \mathcal{P} 的转置; $\mathcal{P} > 0$ ($\mathcal{P} < 0$)表示正(负)定矩阵;矩阵中的 $*$ 表示对称元素; I 表示一个具有适当维数的单位矩阵; \mathcal{E} 表示数学期望; $\text{col}\{\dots\}$ 表示括号中组成元素的列向量; $\text{diag}_N\{\cdot\}$ 表示由括号中的元素组成的块对角矩阵,并且 $\text{diag}_N^i\{\mathcal{P}_i\} = \text{diag}_N^i\{\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_N\}$; $\mathcal{P} \otimes \mathcal{Q}$ 表示矩阵 \mathcal{P} 与 \mathcal{Q} 的Kronecker积.

1 问题描述

考虑如下受外部扰动影响的非线性连续时间网络化系统:

$$\dot{x}(t) = Ax(t) + f(x(t)) + B\omega(t), \quad z(t) = Mx(t). \quad (1)$$

其中: $x(t) \in \mathbf{R}^{n_x}$ 表示状态向量; $z(t) \in \mathbf{R}^{n_z}$ 表示待估计向量;系统矩阵 A 、 B 、 M 表示已知维数的常数矩阵;外部干扰 $\omega(t) \in L_2[0, \infty)$; $f(\cdot)$ 表示系统的非线性函数,且满足以下假设.

假设1 非线性函数 $f(x(t)) : \mathbf{R}^n \rightarrow \mathbf{R}^n$ 满足以下约束条件:

$$\|f(x(t))\|^2 \leq \|Ux(t)\|^2, \quad (2)$$

其中 $U \in \mathbf{R}^{n \times n}$ 表示常数矩阵.

本文考虑具有 N 个传感器节点的估计器配置模型,如图1所示.

在图1中:每个传感器节点 i 都可以从目标网络及邻接节点接收到信息.在信号传输过程中,由于通讯能力受限,可能遭遇随机发生的传感器饱和和测量缺失.因此,对 N 个具有饱和与测量缺失的传感器进行建模

$$y_i(t) = \alpha_i(t)C_i x(t) + (1 - \alpha_i(t))\beta_i(t)\sigma(C_i x(t)) + D_i v(t), \quad i = 1, 2, \dots, N. \quad (3)$$

其中: $\tau_i(t)$ 表示由非均匀采样引起的时变延迟且满足 $\tau_d < \tau_i(t) < \tau_D, \tau_d \triangleq \min_{i \in \mathcal{V}} \{\tau_i\}, \tau_D \triangleq \max_{i \in \mathcal{V}} \{\tau_i\}, \tau_s = \tau_D - \tau_d$, 且 $t \neq t_m^i (m = 0, 1, \dots, \infty)$.

注2 对于采样方法, 现有文献大多通过给出一个满足 $\tau_k < \bar{\tau}$ 的时间间隔来作为下一个采样时刻^[22]. 这种采样方法本身并不利于信号重构, 并且对于不同节点采用相同的采样间隔, 得到的数据信息也无法充分体现传感器网络中各节点以不同形式、不同时间随机发生的信息不完全现象. 因此, 本文针对不同的传感器节点, 采用由时变函数 $\tau_i(t)$ 决定的非均匀采样方式, 从而保证不同采样节点的采样间隔随时间变化, 且独立有界.

令状态估计误差 $\tilde{x}_i(t) = x(t) - \hat{x}_i(t)$, 非线性误差 $f(\tilde{x}_i(t)) = f(x(t)) - f(\hat{x}_i(t))$ 以及输出估计误差 $\tilde{z}_i(t) = z(t) - \hat{z}_i(t)$, 由式(1)和(11)可得

$$\left\{ \begin{aligned} \dot{\tilde{x}}_i(t) &= A\tilde{x}_i(t) + f(\tilde{x}_i(t)) + B\omega(t) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}(1 - \bar{\alpha}_i)\bar{\beta}_i C_j \tilde{x}_j(t - \tau_i(t)) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}(1 - \bar{\alpha}_i)\bar{\beta}_i \Psi(C_j \tilde{x}_j(t - \tau_i(t))) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}\bar{\alpha}_i C_j \tilde{x}_j(t - \tau_i(t)) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}[(1 - \alpha_i(t))\beta_i(t) - (1 - \bar{\alpha}_i)\bar{\beta}_i] \cdot \\ &\underline{L}_i C_j x(t - \tau_i(t)) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}[(1 - \alpha_i(t))\beta_i(t) - (1 - \bar{\alpha}_i)\bar{\beta}_i] \cdot \\ &\Psi(C_j x(t - \tau_i(t))) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}(\alpha_i(t) - \bar{\alpha}_i)C_j x(t - \tau_i(t)) - \\ &K_i \sum_{j \in \mathcal{N}_i} a_{ij}D_j v_j(t - \tau_i(t)), \\ \hat{z}_i(t) &= M\tilde{x}_i(t). \end{aligned} \right. \quad (12)$$

记

$$\begin{aligned} \bar{K} &\triangleq \text{diag}_N^i \{K_i\}, \bar{A} \triangleq \text{diag}_N \{A\}, \bar{B} \triangleq \text{col}_N \{B\}, \\ \bar{C} &\triangleq \text{col}_N \{C_i\}, \bar{C} \triangleq \text{diag}_N^i \{C_i\}, \bar{D} \triangleq \text{col}_N^i \{D_i\}, \\ \bar{U} &\triangleq \text{diag}_N \{U\}, \bar{M} \triangleq \text{diag}_N \{M\}, \\ \tilde{x}(t) &\triangleq \text{col}_N^i \{\tilde{x}_i(t)\}, \hat{z}(t) \triangleq \text{col}_N^i \{\hat{z}_i(t)\}, \\ \tilde{x}_\tau(t) &\triangleq \text{col}_N^i \{\tilde{x}_i(t - \tau_i(t))\}, \\ F(\tilde{x}(t)) &\triangleq \text{col}_N^i \{f(\tilde{x}_i(t))\}, \\ v_\tau(t) &\triangleq \text{col}_N^i \{v(t - \tau_i(t))\}, \\ \bar{L} &\triangleq \text{diag}_N^i \{\underline{L}_i I\}, \bar{L} \triangleq \text{diag}_N^i \{L_i I\}, \\ \bar{L}_\alpha &\triangleq \text{diag}_N^i \{\bar{\alpha}_i I\}, \bar{L}_{\alpha(t)} \triangleq \text{diag}_N^i \{\alpha_i(t) I\}, \end{aligned}$$

$$I_i \triangleq \underbrace{\{0, \dots, 0\}}_{i-1}, \underbrace{\{I, 0, \dots, 0\}}_{N-i},$$

$$\bar{L}_\beta \triangleq \text{diag}_N^i \{\bar{\beta}_i I\}, \bar{L}_{\beta(t)} \triangleq \text{diag}_N^i \{\beta_i(t) I\}.$$

估计误差系统(12)可以进一步改写为

$$\left\{ \begin{aligned} \dot{\tilde{x}}(t) &= \bar{A}\tilde{x}(t) + F(\tilde{x}(t)) + \bar{B}\omega(t) - \\ &\sum_{i=1}^N (I - \bar{L}_\alpha)\bar{L}_\beta \bar{L} \bar{K} I_i (\mathcal{W} \otimes I) \bar{C} \tilde{x}_\tau(t) - \\ &\sum_{i=1}^N (I - \bar{L}_\alpha)\bar{L}_\beta \bar{K} I_i (\mathcal{W} \otimes I) \Psi(\bar{C} \tilde{x}_\tau(t)) - \\ &\sum_{i=1}^N \bar{L}_\alpha \bar{K} I_i (\mathcal{W} \otimes I) \bar{C} \tilde{x}_\tau(t) - \\ &\sum_{i=1}^N [(I - \bar{L}_{\alpha(t)})\bar{L}_{\beta(t)} - (I - \bar{L}_\alpha)\bar{L}_\beta] \cdot \\ &\bar{L} \bar{K} I_i (\mathcal{W} \otimes I) \bar{C} x_\tau(t) - \\ &\sum_{i=1}^N [(I - \bar{L}_{\alpha(t)})\bar{L}_{\beta(t)} - (I - \bar{L}_\alpha)\bar{L}_\beta] \cdot \\ &\bar{K} I_i (\mathcal{W} \otimes I) \Psi(\bar{C} x_\tau(t)) - \\ &\sum_{i=1}^N (\bar{L}_{\alpha(t)} - \bar{L}_\alpha) \bar{K} I_i (\mathcal{W} \otimes I) \bar{C} x_\tau - \\ &\sum_{i=1}^N \bar{K} I_i (\mathcal{W} \otimes I) \bar{D} v_\tau(t), \\ \hat{z}(t) &= \bar{M}\tilde{x}(t). \end{aligned} \right. \quad (13)$$

令 $\varsigma(t) \triangleq [x^T(t), \tilde{x}^T(t)]^T, \bar{z}(t) \triangleq [z^T(t), \hat{z}^T(t)]^T$, 可得如下增广估计误差系统:

$$\left\{ \begin{aligned} \dot{\varsigma}(t) &= \mathcal{A}\varsigma(t) + \mathcal{C}\varsigma_\tau(t) + \mathcal{F}(\varsigma(t)) + \\ &\mathcal{J}\Psi(\bar{C}\varsigma_\tau(t)) + \mathcal{B}\omega(t) + \mathcal{D}v_\tau(t), \\ \bar{z}(t) &= \mathcal{M}\varsigma(t). \end{aligned} \right. \quad (14)$$

其中

$$\begin{aligned} \mathcal{A} &\triangleq \begin{bmatrix} A & 0 \\ 0 & \bar{A} \end{bmatrix}, \mathcal{B} \triangleq \begin{bmatrix} B \\ \bar{B} \end{bmatrix}, \mathcal{C} \triangleq \begin{bmatrix} 0 & 0 \\ \chi_1 & \chi_2 \end{bmatrix}, \\ \mathcal{D} &\triangleq \begin{bmatrix} 0 \\ \chi_3 \end{bmatrix}, \mathcal{F}(\varsigma(t)) \triangleq \begin{bmatrix} f(x(t)) \\ F(\tilde{x}(t)) \end{bmatrix}, \\ \mathcal{U} &\triangleq \begin{bmatrix} U & 0 \\ 0 & \bar{U} \end{bmatrix}, \mathcal{J} \triangleq \begin{bmatrix} 0 & 0 \\ \chi_3 & \chi_4 \end{bmatrix}, \mathcal{M} \triangleq \begin{bmatrix} M & 0 \\ 0 & \bar{M} \end{bmatrix}, \\ \varsigma_\tau(t) &\triangleq \text{col}_N^i \{\varsigma(t - \tau_i(t))\}, \\ \Psi(\bar{C}\varsigma_\tau(t)) &\triangleq \text{col}_N^i \{\Psi(C_i \varsigma(t - \tau_i(t)))\}, \\ \chi_1 &\triangleq - \sum_{i=1}^N (\bar{L}_{\alpha(t)} - \bar{L}_\alpha) \bar{K} I_i (\mathcal{W} \otimes I) \bar{C} - \\ &\sum_{i=1}^N [(I - \bar{L}_{\alpha(t)})\bar{L}_{\beta(t)} - (I - \bar{L}_\alpha)\bar{L}_\beta] \cdot \end{aligned}$$

$$\begin{aligned} & \bar{L}\bar{K}I_i(\mathcal{W} \otimes I)\bar{C}, \\ \chi_2 & \triangleq - \sum_{i=1}^N \bar{L}_\alpha \bar{K}I_i(\mathcal{W} \otimes I)\bar{C} - \\ & \sum_{i=1}^N (I - \bar{L}_\alpha) \bar{L}_\beta \bar{L}\bar{K}I_i(\mathcal{W} \otimes I)\bar{C}, \\ \chi_3 & \triangleq - \sum_{i=1}^N [(I - \bar{L}_{\alpha(t)}) \bar{L}_{\beta(t)} - (I - \bar{L}_\alpha) \bar{L}_\beta] \cdot \\ & \bar{K}I_i(\mathcal{W} \otimes I), \\ \chi_4 & \triangleq - \sum_{i=1}^N (I - \bar{L}_\alpha) \bar{L}_\beta \bar{K}I_i(\mathcal{W} \otimes I), \\ \chi_5 & \triangleq - \sum_{i=1}^N \bar{K}I_i(\mathcal{W} \otimes I)\bar{D}. \end{aligned}$$

本文的主要目的是设计形如式(1)的分布式状态估计器,以便同时满足以下两个要求:

1) 当 $v_\tau(t) = 0, \omega(t) = 0$ 时,估计误差系统(14)是均方渐近稳定的;

2) 在零初始条件下,对于所有非零的 $v_\tau(t)$ 和 $\omega(t), \bar{z}(t)$ 满足

$$\begin{aligned} & \mathcal{E} \left\{ \int_0^\infty \|\bar{z}(t)\|^2 dt \right\} \leq \\ & \mathcal{E} \left\{ \gamma^2 \int_0^\infty (\|\omega(t)\|^2 + \|v_\tau(t)\|^2) dt \right\}, \end{aligned} \quad (15)$$

其中 $\gamma > 0$ 表示给定的抗扰动水平.

定义 2^[23] 如果对于任意标量 $\kappa > 0$, 当满足 $\sup_{-d \leq s \leq 0} \mathcal{E}\{|\phi(s)|^2\} < \delta(\kappa)$ 时, 存在 $\varphi(\kappa) > 0$, 使得 $\mathcal{E}\{|x^2(t)|\} < \kappa$ 成立, 则估计误差系统在 $v_\tau(t) = 0$ 和 $\omega(t) = 0$ 的情况下均方稳定. 另外, 对于任意初始条件, 如果 $\lim_{t \rightarrow \infty} \mathcal{E}|x^2(t)| = 0$, 则估计误差系统被认为是均方渐近稳定的.

为得到主要结果, 需要用到以下引理.

引理 1^[24] 对于任意正定矩阵 $R \in \mathbf{R}^{n \times n}$, 标量 $h > 0$ 和向量值函数 $\dot{x} : [-h, 0]$, 有以下积分不等式:

$$- \int_{t-h}^t \dot{x}^T(s) R \dot{x}(s) ds \leq - \frac{2}{h} \zeta_1^T(t) \begin{bmatrix} R & -R \\ -R & R \end{bmatrix} \zeta_1(t),$$

其中 $\zeta_1^T(t) = \left[\frac{1}{h} \int_{t-h}^t x^T(s) ds, x^T(t-h) \right]$.

引理 2^[25] 对于任意向量函数 $x \in [0, d_M] \rightarrow \mathbf{R}^n$, 时变延迟 $d(t) \in [0, d_M]$, 对称矩阵 $V > 0$ 以及任意矩阵 T_1 满足 $\begin{bmatrix} V_1 & T_1 \\ * & V_1 \end{bmatrix} \geq 0$, 其中 $V_1 = \text{diag}\{V, 3V\}$,

可得

$$\begin{aligned} & - \int_{t-d(t)}^t \dot{x}^T(s) V \dot{x}(s) ds - \int_{t-d_M}^{t-d(t)} \dot{x}^T(s) V \dot{x}(s) ds \leq \\ & - \frac{1}{d_M} \zeta_2^T(t) \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^T \begin{bmatrix} V_1 & T_1 \\ * & V_1 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \zeta_2(t). \end{aligned}$$

其中

$$\begin{aligned} & \zeta_2(t) = \\ & [x^T(t), x^T(t-d(t)), x^T(t-h), v_1^T(t), v_2^T(t)]^T; \\ & e_i = [0_{n \times (i-1)n}, I, 0_{n \times (5-i)n}], \quad i = 1, 2, \dots, 5; \\ & W_1 = \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_4 \end{bmatrix}, \quad W_2 = \begin{bmatrix} e_2 - e_3 \\ e_2 + e_3 - 2e_5 \end{bmatrix}; \\ & v_1^T(t) = \frac{1}{d(t)} \int_{t-d(t)}^t x(s) ds, \\ & v_2^T(t) = \frac{1}{h-d(t)} \int_{t-d_M}^{t-d(t)} x(s) ds. \end{aligned}$$

2 主要结果

本节主要针对系统(1)和传感器网络(3), 在给定 H_∞ 性能水平 γ 下, 设计随机饱和和测量缺失影响下的分布式 H_∞ 状态估计器.

定理 1 给定抗扰动指标 $\gamma > 0$. 对于具有随机饱和和测量缺失的非线性系统(1)和传感器(3), 如果存在标量 $\varepsilon_1 > 0, \varepsilon_2 > 0$ 以及具有适当维数的矩阵 $\mathcal{P}_2 > 0, \mathcal{P}_3 > 0, \mathcal{Q}_1 > 0, \mathcal{R}_1 > 0, \mathcal{Q}_{2i} > 0, \mathcal{R}_{2i} > 0 (i \in \mathcal{V})$, 对角矩阵 $\mathcal{P}_1 = \text{diag}_N^i\{\mathcal{P}_1, P_{1im}\}$, $\bar{X} = \text{diag}_N^i\{X_i\}$ 和任意矩阵 $\mathcal{S}_i = \begin{bmatrix} \mathcal{S}_{i1} & \mathcal{S}_{i2} \\ \mathcal{S}_{i3} & \mathcal{S}_{i4} \end{bmatrix} (i \in \mathcal{V})$ 满足以下 LMIs, 则估计误差系统(14)是均方渐近稳定的, 且具有 H_∞ 性能指标:

$$\begin{bmatrix} \Xi & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 \\ [\tau_i(t)=\tau_d] & * & -\gamma^2 I & 0 & \mathcal{B}^T \mathcal{P}_1 & 0 \\ * & * & * & -\gamma^2 I & \mathcal{D}^T \mathcal{P}_1 & 0 \\ * & * & * & * & -2\mathcal{P}_1 + Y & 0 \\ * & * & * & * & * & -2\mathcal{P}_1 + Y \end{bmatrix} < 0, \quad (16)$$

$$\begin{bmatrix} \Xi & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 \\ [\tau_i(t)=\tau_D] & * & -\gamma^2 I & 0 & \mathcal{B}^T \mathcal{P}_1 & 0 \\ * & * & * & -\gamma^2 I & \mathcal{D}^T \mathcal{P}_1 & 0 \\ * & * & * & * & -2\mathcal{P}_1 + Y & 0 \\ * & * & * & * & * & -2\mathcal{P}_1 + Y \end{bmatrix} < 0, \quad (17)$$

$$\begin{bmatrix} \tilde{\mathcal{Q}}_{2i} & \mathcal{S}_i \\ * & \tilde{\mathcal{Q}}_{2i} \end{bmatrix} \geq 0, \quad i \in \mathcal{V}, \quad (18)$$

且估计器参数 $K_i = P_{1i}^{-1} X_i$.

其中

$$\Xi \triangleq [\Xi_{pq}]_{9 \times 9};$$

$$\Xi_{11} \triangleq \mathcal{P}_1 \mathcal{A} + \mathcal{A}^T \mathcal{P}_1 + \tau_d^2 \mathcal{R}_1 + \varepsilon_1 \mathcal{U}^T \mathcal{U} +$$

$$\sum_{i=1}^N \tau_s^2 \mathcal{R}_{2i} + \mathcal{M}^T \mathcal{M};$$

$$\Xi_{13} \triangleq -\mathcal{X}_{\delta_1} - \mathcal{X}_{\delta_2}; \quad \Xi_{15} \triangleq \mathcal{P}_1; \quad \Xi_{16} \triangleq \bar{\mathcal{X}}_{\delta_3} + \varepsilon_2 L_c;$$

$$\begin{aligned} \Xi_{22} &\triangleq -\mathcal{P}_2 + \mathcal{P}_3 - 2\mathcal{Q}_1 - 4 \sum_{i=1}^N \mathcal{Q}_{2i}; \\ \Xi_{23} &\triangleq -2\bar{\mathcal{Q}}_{2i} - \bar{\mathcal{S}}_{i1} - \bar{\mathcal{S}}_{i2} - \bar{\mathcal{S}}_{i3} - \bar{\mathcal{S}}_{i4}; \\ \Xi_{24} &\triangleq \sum_{i=1}^N (\mathcal{S}_{i1} + \mathcal{S}_{i3} - \mathcal{S}_{i2} - \mathcal{S}_{i4}); \\ \Xi_{29} &\triangleq 2\bar{\mathcal{S}}_{i2} + 2\bar{\mathcal{S}}_{i4}; \\ \Xi_{33} &\triangleq -8\hat{\mathcal{Q}}_{2i} + \text{sym}\{\hat{\mathcal{S}}_{i1}^T + \hat{\mathcal{S}}_{i2}^T - \hat{\mathcal{S}}_{i3}^T - \hat{\mathcal{S}}_{i4}^T\}; \\ \Xi_{34} &\triangleq -2\bar{\mathcal{Q}}_{2i}^T - \bar{\mathcal{S}}_{i1} + \bar{\mathcal{S}}_{i3} - \bar{\mathcal{S}}_{i2} + \bar{\mathcal{S}}_{i4}; \\ \Xi_{38} &\triangleq 6\hat{\mathcal{Q}}_{2i}^T + 2\hat{\mathcal{S}}_{i3}^T + \hat{\mathcal{S}}_{i4}^T; \\ \Xi_{39} &\triangleq 6\hat{\mathcal{Q}}_{2i}^T - 2\hat{\mathcal{S}}_{i2}^T + 2\hat{\mathcal{S}}_{i4}^T; \\ \Xi_{44} &\triangleq -\mathcal{P}_3 - 4 \sum_{i=1}^N \mathcal{Q}_{2i}; \quad \Xi_{48} \triangleq -2\bar{\mathcal{S}}_{i3}^T + 2\bar{\mathcal{S}}_{i4}^T; \\ \Xi_{55} &\triangleq -\varepsilon_1 I; \quad \Xi_{66} \triangleq -\varepsilon_2 I; \quad \Xi_{77} \triangleq -2\mathcal{Q}_1 - \tau_d^2 \mathcal{R}_1; \\ \Xi_{88} &\triangleq -12\hat{\mathcal{Q}}_{2i} - \tau_s(\tau_i(t) - \tau_d)\hat{\mathcal{R}}_{2i}; \\ \Xi_{99} &\triangleq -12\hat{\mathcal{Q}}_{2i} - \tau_s(\tau_D - \tau_i(t))\hat{\mathcal{R}}_{2i}; \\ \Pi_1 &\triangleq [\mathcal{B}^T \mathcal{P}_1, 0, 0, 0, 0, 0, 0, 0, 0]^T; \\ \Pi_2 &\triangleq [\mathcal{D}^T \mathcal{P}_1, 0, 0, 0, 0, 0, 0, 0, 0]^T; \\ \Pi_3 &\triangleq [\mathcal{P}_1 A, 0, \Pi_{33}, 0, \mathcal{P}_1, -\bar{\mathcal{X}}_{\delta_3}, 0, 0, 0]^T; \\ \Pi_4 &\triangleq [0, 0, \Pi_{43}, 0, 0, -\bar{\mathcal{X}}_{\rho_3}, 0, 0, 0]^T; \\ \Pi_{33} &\triangleq -\bar{\mathcal{X}}_{\delta_1} - \bar{\mathcal{X}}_{\delta_2}; \quad \Pi_{43} \triangleq -\bar{\mathcal{X}}_{\rho_1} - \bar{\mathcal{X}}_{\rho_2}; \\ L_c &\triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{L}\bar{C} \end{bmatrix}; \quad \bar{\mathcal{X}}_{\delta_1} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{X}_{\delta_1} \end{bmatrix}; \quad \bar{\mathcal{X}}_{\delta_2} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{X}_{\delta_2} \end{bmatrix}; \\ \bar{\mathcal{X}}_{\delta_3} &\triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{X}_{\delta_3} \end{bmatrix}; \quad \bar{\mathcal{X}}_{\rho_1} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{X}_{\rho_1} \end{bmatrix}; \\ \bar{\mathcal{X}}_{\rho_2} &\triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{X}_{\rho_2} \end{bmatrix}; \quad \bar{\mathcal{X}}_{\rho_3} \triangleq \begin{bmatrix} 0 & 0 \\ 0 & \bar{X}_{\rho_3} \end{bmatrix}; \\ \varphi_{2i} &\triangleq \bar{\beta}_i(1 - \bar{\alpha}_i) - \bar{\beta}_i^2(1 - \bar{\alpha}_i)^2; \\ \varphi_{1i} &\triangleq \bar{\alpha}_i(1 - \bar{\alpha}_i); \quad \bar{X}_{\delta_1} \triangleq \text{vec}_N^i\{\bar{X}I_i(\mathcal{W} \otimes I)\bar{\alpha}_i\bar{C}\}; \\ \bar{X}_{\delta_2} &\triangleq \text{vec}_N^i\{\bar{X}I_i(\mathcal{W} \otimes I)(1 - \bar{\alpha}_i)\bar{\beta}_i\bar{L}\bar{C}\}; \\ \bar{X}_{\delta_3} &\triangleq \text{vec}_N^i\{\bar{X}I_i(\mathcal{W} \otimes I)(1 - \bar{\alpha}_i)\bar{\beta}_i\}; \\ \bar{X}_{\rho_1} &\triangleq \text{vec}_N^i\{\bar{X}I_i(\mathcal{W} \otimes I)\sqrt{\varphi_{1i}}\bar{C}\}; \\ \bar{X}_{\rho_2} &\triangleq \text{vec}_N^i\{\bar{X}I_i(\mathcal{W} \otimes I)\sqrt{\varphi_{2i}}\bar{L}\bar{C}\}; \\ \bar{X}_{\rho_3} &\triangleq \text{vec}_N^i\{\bar{X}I_i(\mathcal{W} \otimes I)\sqrt{\varphi_{2i}}\}; \\ \hat{\mathcal{R}}_{2i} &\triangleq \text{diag}_N^i\{\mathcal{R}_{2i}\}; \quad \bar{\mathcal{R}}_{2i} \triangleq \text{vec}_N^i\{\mathcal{R}_{2i}\}; \\ \hat{\mathcal{Q}}_{2i} &\triangleq \text{diag}_N^i\{\mathcal{Q}_{2i}\}; \quad \bar{\mathcal{Q}}_{2i} \triangleq \text{vec}_N^i\{\mathcal{Q}_{2i}\}; \\ \hat{\mathcal{S}}_{ig} &\triangleq \text{diag}_N^i\{\mathcal{S}_{ig}\}; \quad \bar{\mathcal{S}}_{ig} \triangleq \text{vec}_N^i\{\mathcal{S}_{ig}\}; \\ \tilde{\mathcal{S}}_{ig} &\triangleq \text{col}_N^i\{\mathcal{S}_{ig}\}; \quad g = 1, 2, 3, 4; \end{aligned}$$

$$Y \triangleq \sum_{i=1}^N \frac{\tau_s^2 \hat{\mathcal{Q}}_{2i}}{2} + \frac{\tau_1^2}{2} \mathcal{Q}_1.$$

证明 考虑如下 Lyapunov-Krasovskii 泛函:

$$V(\varsigma(t)) = V_1(\varsigma(t)) + V_2(\varsigma(t)). \quad (19)$$

其中

$$V_1(\varsigma(t)) = \varsigma(t)\mathcal{P}_1\varsigma(t) + \int_{t-\tau_d}^t \varsigma^T(s)\mathcal{P}_2\varsigma(s)ds + \int_{t-\tau_D}^{t-\tau_d} \varsigma^T(s)\mathcal{P}_3\varsigma(s)ds, \quad (20)$$

$$\begin{aligned} V_2(\varsigma(t)) &= \tau_d \int_{t-\tau_d}^t \int_{\theta}^t \varsigma^T(s)\mathcal{R}_1\varsigma(s)d\theta ds + \\ &\tau_s \sum_{i=1}^N \int_{t-\tau_D}^{t-\tau_d} \int_{\theta}^t \varsigma^T(s)\mathcal{R}_{2i}\varsigma(s)d\theta ds + \\ &\tau_d \int_{t-\tau_d}^t \int_{\theta}^t \dot{\varsigma}^T(s)\mathcal{Q}_1\varsigma(s)d\theta ds + \\ &\tau_s \sum_{i=1}^N \int_{t-\tau_D}^{t-\tau_d} \int_{\theta}^t \dot{\varsigma}^T(s)\mathcal{Q}_{2i}\dot{\varsigma}(s)d\theta ds. \quad (21) \end{aligned}$$

误差系统的无穷小生成算子可表示为

$$\begin{aligned} LV(\varsigma(t)) &= \\ \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \{ \mathcal{E}\{V(\varsigma(t + \Delta)) | \varsigma(t)\} - V(\varsigma(t)) \}, \quad (22) \end{aligned}$$

则当 $\omega(t) = 0, v_\tau(t) = 0$ 时, 可得

$$\begin{aligned} LV_1(\varsigma(t)) &= \\ 2\varsigma^T(t)\mathcal{P}_1\dot{\varsigma}(t) + \varsigma^T(t)\mathcal{P}_2\varsigma(t) - \varsigma^T(t - \tau_D)\mathcal{P}_3\varsigma(t - \tau_D) + \varsigma^T(t - \tau_d)\mathcal{P}_3\varsigma(t - \tau_d) - \varsigma^T(t - \tau_d)\mathcal{P}_2\varsigma(t - \tau_d), \quad (23) \end{aligned}$$

$$\begin{aligned} LV_2(\varsigma(t)) &= \\ \tau_d^2 \dot{\varsigma}^T(t)\mathcal{Q}_1\dot{\varsigma}(t) - \tau_s \sum_{i=1}^N \int_{t-\tau_D}^{t-\tau_d} \dot{\varsigma}^T(s)\mathcal{R}_{2i}\dot{\varsigma}(s)ds - \\ \tau_d \int_{t-\tau_d}^t \dot{\varsigma}^T(s)\mathcal{Q}_1\dot{\varsigma}(s)ds + \sum_{i=1}^N \tau_s^2 \dot{\varsigma}^T(t)\mathcal{Q}_{2i}\dot{\varsigma}(t) - \\ \tau_s \sum_{i=1}^N \int_{t-\tau_D}^{t-\tau_d} \dot{\varsigma}^T(s)\mathcal{Q}_{2i}\dot{\varsigma}(s)ds + \tau_d^2 \dot{\varsigma}^T(t)\mathcal{R}_1\dot{\varsigma}(t) + \\ \tau_s^2 \sum_{i=1}^N \dot{\varsigma}^T(t)\mathcal{R}_{2i}\dot{\varsigma}(t) - \tau_d \int_{t-\tau_d}^t \dot{\varsigma}^T(s)\mathcal{R}_1\dot{\varsigma}(s)ds. \quad (24) \end{aligned}$$

利用引理1可得

$$\begin{aligned} -\tau_d \int_{t-\tau_d}^t \dot{\varsigma}^T(s)\mathcal{Q}_1\dot{\varsigma}(s)ds \leq \\ \begin{bmatrix} \eta_1(t) \\ \varsigma(t - \tau_d) \end{bmatrix}^T \begin{bmatrix} -2\mathcal{Q}_1 & 2\mathcal{Q}_1 \\ 2\mathcal{Q}_1 & -2\mathcal{Q}_1 \end{bmatrix} \begin{bmatrix} \eta_1(t) \\ \varsigma(t - \tau_d) \end{bmatrix}. \quad (25) \end{aligned}$$

根据引理2, 对于任意矩阵 S_i , 在 $\begin{bmatrix} \tilde{\mathcal{Q}}_{2i} & \mathcal{S}_i \\ * & \tilde{\mathcal{Q}}_{2i} \end{bmatrix} \geq 0$

条件下可得

$$-\tau_s \sum_{i=1}^N \int_{t-\tau_D}^{t-\tau_d} \dot{\varsigma}^T(s)\mathcal{Q}_{2i}\dot{\varsigma}(s)ds =$$

$$\begin{aligned}
 & - \sum_{i=1}^N \tau_s \int_{t-\tau_i(t)}^{t-\tau_d} \zeta^T(s) \mathcal{Q}_{2i} \dot{\zeta}(s) ds - \\
 & \sum_{i=1}^N \tau_s \int_{t-\tau_D}^{t-\tau_i(t)} \zeta^T(s) \mathcal{Q}_{2i} \dot{\zeta}(s) ds \leq \\
 & \begin{bmatrix} W_{1i} \\ W_{2i} \end{bmatrix}^T \begin{bmatrix} -\tilde{\mathcal{Q}}_{2i} & -\mathcal{S}_i \\ * & -\tilde{\mathcal{Q}}_{2i} \end{bmatrix} \begin{bmatrix} W_{1i} \\ W_{2i} \end{bmatrix}. \tag{26}
 \end{aligned}$$

其中

$$\begin{aligned}
 W_{1i} &= \begin{bmatrix} \zeta(t-\tau_d) - \zeta(t-\tau_i(t)) \\ \zeta(t-\tau_d) + \zeta(t-\tau_i(t)) - 2\eta_2(t) \end{bmatrix}, \\
 W_{2i} &= \begin{bmatrix} \zeta(t-\tau_i(t)) - \zeta(t-\tau_D) \\ \zeta(t-\tau_i(t)) + \zeta(t-\tau_D) - 2\eta_3(t) \end{bmatrix}.
 \end{aligned}$$

此外, 由 Jensen 积分不等式可得

$$-\tau_d \int_{t-\tau_d}^t \zeta^T(s) R_1 \zeta(s) ds \leq -\eta_1^T(t) \tau_d^2 R_1 \eta_1(t), \tag{27}$$

$$\begin{aligned}
 & -\tau_s \sum_{i=1}^N \int_{t-\tau_D}^{t-\tau_d} \zeta^T(s) \mathcal{R}_{2i} \zeta(s) ds \leq \\
 & -\sum_{i=1}^N \eta_2^T(t) \tau_s (\tau_i(t) - \tau_d) \mathcal{R}_{2i} \eta_2(t) + \\
 & \eta_3^T(t) \tau_s (\tau_D - \tau_i(t)) \mathcal{R}_{2i} \eta_3(t). \tag{28}
 \end{aligned}$$

其中

$$\begin{aligned}
 \eta_1(t) &= \frac{1}{\tau_d} \int_{t-\tau_d}^t \zeta(s) ds, \\
 \eta_2(t) &= \left[\frac{1}{\tau_1(t) - \tau_d} \int_{t-\tau_1(t)}^{t-\tau_d} \zeta(s) ds, \dots, \right. \\
 & \quad \frac{1}{\tau_{i-1}(t) - \tau_d} \int_{t-\tau_{i-1}(t)}^{t-\tau_d} \zeta(s) ds, \\
 & \quad \left. \frac{1}{\tau_i(t) - \tau_d} \int_{t-\tau_i(t)}^{t-\tau_d} \zeta(s) ds \right], \\
 \eta_3(t) &= \left[\frac{1}{\tau_D - \tau_1(t)} \int_{t-\tau_D}^{t-\tau_1(t)} \zeta(s) ds, \dots \right. \\
 & \quad \frac{1}{\tau_D - \tau_{i-1}(t)} \int_{t-\tau_D}^{t-\tau_{i-1}(t)} \zeta(s) ds, \\
 & \quad \left. \frac{1}{\tau_D - \tau_i(t)} \int_{t-\tau_D}^{t-\tau_i(t)} \zeta(s) ds \right].
 \end{aligned}$$

注 3 由于本文采用的估计器结构包含了传感器网络拓扑信息, 并且每个传感器的采样间隔是随时间变化的, 泛函的构造和积分项的放缩方法对结果的复杂程度和保守性有着至关重要的影响. 为此, 在构造泛函时引入单重和双重积分项, 充分利用非均匀采样带来的时滞信息, 并且没有引入复杂的多重积分型和增广型泛函项, 有效避免了不必要的计算负担. 同时, 在处理 LV_2 中积分项 $\tau_s \int_{t-\tau_D}^{t-\tau_d} \zeta(s) \mathcal{Q}_{2i} \dot{\zeta}(s) ds$ 时, 进行时滞分割, 得到了 $\tau_s \int_{t-\tau_i(t)}^{t-\tau_d} \zeta(s) \mathcal{Q}_{2i} \dot{\zeta}(s) ds$ 和 $\tau_s \int_{t-\tau_D}^{t-\tau_i(t)} \zeta(s) \mathcal{Q}_{2i} \dot{\zeta}(s) ds$, 然后采用引理 2 进行估计,

产生 $\eta_2(t)$ 和 $\eta_3(t)$ 两个受网络节点个数影响且随时间变化的向量, 通过引入自由矩阵 \mathcal{S}_i , 建立 $\eta_2(t)$ 、 $\eta_3(t)$ 与状态向量 $\zeta(t-\tau_d)$ 、 $\zeta(t-\tau_i(t))$ 、 $\zeta(t-\tau_D)$ 之间的交叉关系. 与现有分布式状态估计中普遍采用 Jensen 不等式相比^[15,20], 提高了泛函导数的放缩精度, 有助于降低保守性; 与时滞系统稳定性分析中采用的基于 Free-matrix 和 Auxiliary-function 方法相比^[8], 大大减少了引入的决策变量, 降低了复杂程度.

此外, 由假设 2 和式 (23) ~ (28) 很容易得出

$$\begin{aligned}
 LV(\zeta(t)) &\leq \\
 & \varpi^T(t) \tilde{\Phi} \varpi(t) - \varepsilon_1 \mathcal{F}^T(\zeta(t)) \mathcal{F}(\zeta(t)) - \\
 & \varepsilon_2 \Psi^T(\bar{C}_{\zeta_\tau(t)}) (\Psi(\bar{C}_{\zeta_\tau(t)}) - L_c \zeta(t)) + \\
 & \mathcal{H}_\alpha^T(t) Y \mathcal{H}_\alpha(t) + \mathcal{H}_{\alpha(t)}^T(t) Y \mathcal{H}_{\alpha(t)}(t) = \\
 & \varpi^T(t) \tilde{\Xi} \varpi(t) + \mathcal{H}_\alpha^T(t) Y \mathcal{H}_\alpha(t) + \mathcal{H}_{\alpha(t)}^T(t) Y \mathcal{H}_{\alpha(t)}(t). \tag{29}
 \end{aligned}$$

其中

$$\tilde{\Xi} \triangleq$$

$$\begin{bmatrix}
 \tilde{\Xi}_{11} & 0 & \tilde{\Xi}_{13} & 0 & \tilde{\Xi}_{15} & \tilde{\Xi}_{16} & 0 & 0 & 0 \\
 * & \tilde{\Xi}_{22} & \tilde{\Xi}_{23} & \tilde{\Xi}_{24} & 0 & 0 & 2\mathcal{Q}_1 & 6\tilde{\mathcal{Q}}_{2i} & \tilde{\Xi}_{29} \\
 * & * & \tilde{\Xi}_{33} & \tilde{\Xi}_{34} & 0 & 0 & 0 & \tilde{\Xi}_{38} & \tilde{\Xi}_{39} \\
 * & * & * & \tilde{\Xi}_{44} & 0 & 0 & 0 & \tilde{\Xi}_{48} & 6\tilde{\mathcal{Q}}_{2i} \\
 * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 & 0 \\
 * & * & * & * & * & -\varepsilon_2 I & 0 & 0 & 0 \\
 * & * & * & * & * & * & \tilde{\Xi}_{77} & 0 & 0 \\
 * & * & * & * & * & * & * & \tilde{\Xi}_{88} & -4\hat{\mathcal{S}}_{i4} \\
 * & * & * & * & * & * & * & * & \tilde{\Xi}_{99}
 \end{bmatrix},$$

$$\tilde{\Xi}_{11} \triangleq \mathcal{P}_1 \mathcal{A} + \mathcal{A}^T \mathcal{P}_1 + \varepsilon_1 \mathcal{U}^T \mathcal{U} + \tau_d^2 \mathcal{R}_1 + \sum_{i=1}^N \tau_s^2 \mathcal{R}_{2i},$$

$$\begin{aligned}
 \varpi(t) &\triangleq [\zeta^T(t), \zeta^T(t-\tau_d), \zeta_\tau^T(t), \zeta^T(t-\tau_D), \\
 & \mathcal{F}^T(\zeta(t)), \Psi^T(\bar{C}_{\zeta_\tau(t)}), \eta_1^T(t), \eta_2^T(t), \eta_3^T(t)]^T,
 \end{aligned}$$

$$\mathcal{H}_\alpha(t) \triangleq$$

$$\mathcal{A}_\zeta(t) + \mathcal{F}(\zeta(t)) - \begin{bmatrix} 0 & 0 \\ 0 & \chi_2 \end{bmatrix} \zeta_\tau(t) - \begin{bmatrix} 0 & 0 \\ 0 & \chi_4 \end{bmatrix} \Psi(\bar{C}_{\zeta_\tau(t)}),$$

$$\mathcal{H}_{\alpha(t)}(t) \triangleq$$

$$\mathcal{A}_\zeta(t) + \mathcal{F}(\zeta(t)) - \begin{bmatrix} 0 & 0 \\ \chi_1 & 0 \end{bmatrix} \zeta_\tau(t) - \begin{bmatrix} 0 & 0 \\ \chi_3 & 0 \end{bmatrix} \Psi(\bar{C}_{\zeta_\tau(t)}).$$

由 Schur 补引理和式 (16)、(17) 可得 $LV(\zeta(t)) < 0$. 由此可以说明, 估计误差系统 (14) 在 $\omega(t) = 0$, $v_\tau(t) = 0$ 情况下均方渐近稳定.

接下来, 对所有非零 $\omega(t) \in L_2[0, \infty)$ 和 $v_\tau(t) \in L_2[0, \infty)$, 估计误差系统 (14) 的 H_∞ 性能可以由式 (29) 得到, 即

$$\begin{aligned} & \mathcal{E}\{LV(\zeta(t)) + \|\bar{z}(t)\|^2 - \gamma^2\|\omega(t)\|^2 - \gamma\|v_\tau(t)\|^2\} \leq \\ & \mathcal{E}\{\xi^T(t)\tilde{\Phi}\xi(t) + \zeta^T(t)Y\zeta(t)\}. \end{aligned} \quad (30)$$

其中

$$\begin{aligned} \xi(t) & \triangleq [\varpi^T(t), \omega^T(t), v_\tau^T(t)]^T, \\ \tilde{\Phi} & \triangleq \begin{bmatrix} \Xi & \Pi_1 & \Pi_2 \\ * & -\gamma^2 I & 0 \\ * & * & -\gamma^2 I \end{bmatrix}. \end{aligned}$$

运用Schur补引理,并结合 $X_i = P_{1i}^{-1}K_i$ 可得

$$\begin{bmatrix} \Xi & \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 \\ * & -\gamma^2 I & 0 & \tilde{B}^T \mathcal{P}_1 & 0 \\ * & * & -\gamma^2 I & \mathcal{D}^T \mathcal{P}_1 & 0 \\ * & * & * & -\mathcal{P}_1 Y^{-1} \mathcal{P}_1 & 0 \\ * & * & * & * & -\mathcal{P}_1 Y^{-1} \mathcal{P}_1 \end{bmatrix} < 0. \quad (31)$$

进一步,结合定理1可以推导出

$$\mathcal{E}\{LV(\zeta(t)) + \|\bar{z}(t)\|^2 - \gamma^2\|\omega(t)\|^2 - \gamma^2\|v_\tau(t)\|^2\} < 0. \quad (32)$$

在零初始条件下,对于所有非零 $v_\tau(t), \omega(t)$ 可得

$$\begin{aligned} & \mathcal{E}\left\{ \int_0^t (\|\bar{z}(s)\|^2 - \gamma^2\|\omega(s)\|^2 - \gamma^2\|v_\tau(s)\|^2) ds \right\} \leq \\ & \mathcal{E}\left\{ \int_0^t (\|\bar{z}(s)\|^2 - \gamma^2\|\omega(s)\|^2 - \right. \\ & \left. \gamma^2\|v_\tau(s)\|^2 + \mathcal{L}V(\zeta(s))) ds - V(\zeta(t)) + V(\zeta(0)) \right\} \leq \\ & \mathcal{E}\left\{ \int_0^t (\|\bar{z}(s)\|^2 - \gamma^2\|\omega(s)\|^2 - \right. \\ & \left. \gamma^2\|v_\tau(s)\|^2 ds + \mathcal{L}V(\zeta(s))) ds \right\} < 0. \end{aligned} \quad (33)$$

由此可得,式(15)成立.同时,如果LMI(31)成立,则结合不等式 $-\mathcal{P}_1 Y^{-1} \mathcal{P}_1 < -2\mathcal{P}_1 + Y$,定理1中的LMI(16)和(17)也得以满足. □

3 仿真分析

本节通过仿真分析说明所提出的分布式 H_∞ 状态估计器设计方案的有效性.

传感器网络的拓扑为 $\mathcal{G} = (\mathcal{V}, \mathcal{O}, \mathcal{W})$,其中节点集为 $\mathcal{V} = \{1, 2, 3, 4\}$,边集为 $\mathcal{O} = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3), (4, 3), (4, 4)\}$,邻接矩阵为 $\mathcal{W} =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \text{考虑具有以下参数的非线性系统(1)和}$$

传感器网络(3),有

$$A = \begin{bmatrix} -0.6 & 0.2 \\ 0 & -0.8 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, M = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}^T,$$

$$C_1 = [0.1, 0], C_2 = [0.2, 0.1],$$

$$C_3 = [0.5, 0.7], C_4 = [0.1, 0.2],$$

$$D_1 = 1, D_2 = 0.5, D_3 = 0.7, D_4 = 0.5,$$

并且非线性函数 $f(x(t))$ 为

$$f(x(t)) = \left[\frac{0.2x_1(t)}{2x_2^2(t) + 1}, 0.1 \sin(x_1(t))x_2(t) \right]^T.$$

容易看出,当 $U = \text{diag}\{0.2, 0.15\}$ 时,条件(2)成立.在本例中,概率 $\bar{\alpha}_i (i = 1, 2, 3, 4)$ 分别为 0.9, 0.8, 0.85, 0.7. $\bar{\beta}_i (i = 1, 2, 3, 4)$ 分别为 0.1, 0.15, 0.1, 0.2. 扇形有界条件取自文献[5],且 $\underline{L}_i (i = 1, 2, 3, 4)$ 分别为 0.3, 0.4, 0.2, 0.1, $L_i (i = 1, 2, 3, 4)$ 分别为 0.7, 0.8, 0.9, 0.7. 饱和水平为 $v_{\max} = 0.3$. 通过选取不同的 τ ,最小 H_∞ 性能指标可由定理1计算得到,如表1所示.

表1 不同 τ 下的 H_∞ 性能指标 γ_{\min}

$\tau_d = 0.1$	τ_D			
	0.2	0.4	0.8	1
γ	0.0057	0.0184	0.0583	0.0875

本例在文献[5]中也进行了分析,当 $\tau_d = 0.1, \tau_D = 0.2$ 时,取其给出的最小 $\gamma = 1.0214$,求解LMI(16)~(18)获得如下一组可行解:

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.0017 \\ -0.0001 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0036 \\ 0.0016 \end{bmatrix}, \\ K_3 &= \begin{bmatrix} 0.0009 \\ 0.0011 \end{bmatrix}, K_4 = \begin{bmatrix} 0.0095 \\ 0.0152 \end{bmatrix}. \end{aligned}$$

当只受饱和影响,即 $\alpha_i(t) \equiv 1$ 时,保持其他参数不变,可得

$$\begin{aligned} K_1^* &= \begin{bmatrix} 0.0608 \\ -0.0028 \end{bmatrix}, K_2^* = \begin{bmatrix} 0.0530 \\ 0.0252 \end{bmatrix}, \\ K_3^* &= \begin{bmatrix} 0.1147 \\ 0.1413 \end{bmatrix}, K_4^* = \begin{bmatrix} 0.0565 \\ 0.094 \end{bmatrix}. \end{aligned}$$

在仿真分析中,初始状态 $x(0) = [-0.3, 0.2]^T$, $\hat{x}_i(0) (i = 1, 2, 3, 4)$ 在均匀分布区间 $[-0.3, 0.2]^T$ 中随机获取,外部扰动为

$$\omega(t) = e^{-0.2t} \sin t, v(t) = \frac{\sin(10t + 1)}{3t + 1}.$$

仿真结果如图2~图8所示.

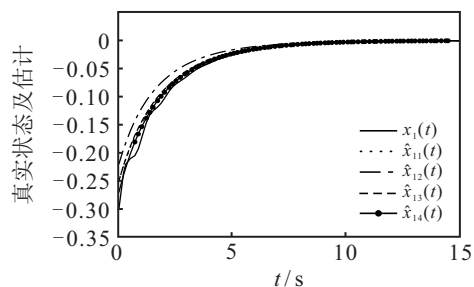


图2 真实状态 $x_1(t)$ 及其估计 $\hat{x}_{1i}(t) (i = 1, 2, 3, 4)$

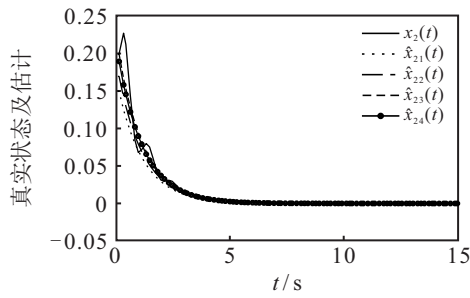


图3 真实状态 $x_2(t)$ 及其估计 $\hat{x}_{2i}(t)(i = 1, 2, 3, 4)$

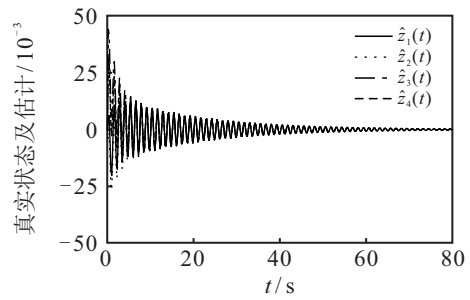


图8 大扰动条件下的估计误差 $\hat{z}_i(t)(i = 1, 2, 3, 4)$

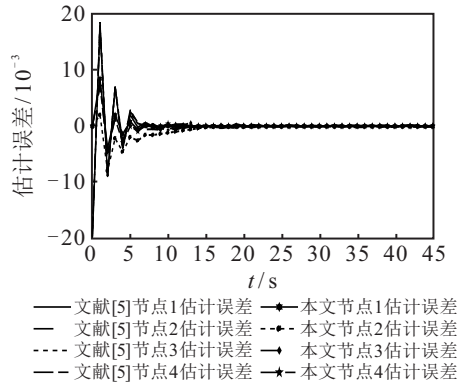


图4 有随机饱和与测量缺失的估计误差 $\hat{z}_i(t)(i = 1, 2, 3, 4)$

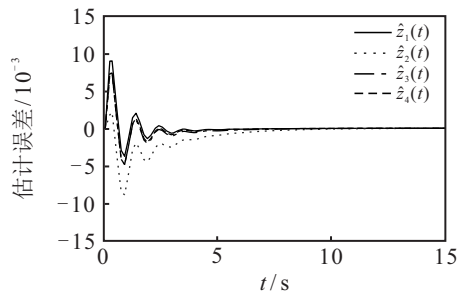


图5 仅受饱和影响下的估计误差 $\hat{z}_i(t)(i = 1, 2, 3, 4)$

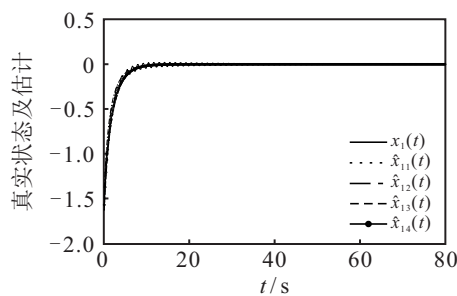


图6 大扰动条件下真实状态 $x_1(t)$ 及其估计 $\hat{x}_{1i}(t)(i = 1, 2, 3, 4)$

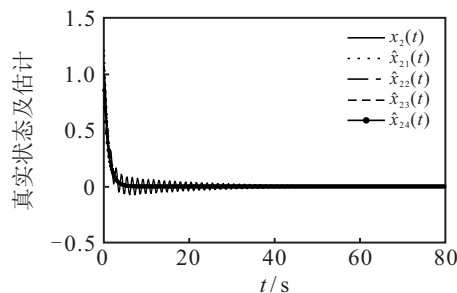


图7 大扰动条件下真实状态 $x_2(t)$ 及其估计 $\hat{x}_{2i}(t)(i = 1, 2, 3, 4)$

由图2和图3可以看出,在充分考虑传感器网络存在随机饱和与测量缺失的情况下,各节点估计值 $\hat{x}_i(t)$ 与系统状态 $x(t)$ 轨迹基本一致,各传感器节点均能实现对系统状态的跟踪.另外,从图4可以得到,各节点的估计误差 $\hat{z}_i(t)(i = 1, 2, 3, 4)$ 范围一直保持在 $-0.01 \sim 0.01$ 之间,并最终趋于稳定,与文献[5]中的 $-0.02 \sim 0.02$ 相比,范围缩小了1倍.此外,由图4和图5不难看出,系统在受随机饱和与测量缺失的影响以及仅受饱和的影响两种情况下,估计误差均能趋于稳定.

为了进一步体现本文算法效果,保持上述算例不变,在初始状态 $x(0) = [-1.5, 1.0]^T$ 且外部扰动增大为 $\omega(t) = e^{-0.01t} \sin t$ 时,由图6~图8可以看出,状态整体波动较大,但最终依旧能趋于稳定.以上结果均能说明,本文提出的具有随机饱和与测量缺失的分布式 H_∞ 状态估计算法是有用的,并且降低了现有结论的保守性.

4 结论

本文研究了随机饱和与测量缺失影响下非线性系统的分布式 H_∞ 状态估计问题,提出了一种更具一般性的传感器模型.利用输入延迟的方法,将基于非均匀采样方法的分布式 H_∞ 状态估计问题转化为有界时滞的分布式 H_∞ 状态估计问题,并利用 Lyapunov 泛函方法设计了满足性能要求的分布式状态估计器,降低了结论的保守性.最后通过仿真分析进行了验证.在接下来的研究中将进一步优化模型,使其能在统一框架下包含更多随机发生的信息不完全现象,并探索分布式状态估计降低结论保守性的新方法.

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作者简介

钱伟(1978—), 男, 教授, 博士生导师, 从事鲁棒控制、智能控制等研究, E-mail: qwei@hpu.edu.cn;

张祥林(1993—), 男, 助理工程师, 硕士, 从事分布式状态估计与滤波的研究, E-mail: zxlin727@126.com;

赵运基(1980—), 男, 副教授, 博士, 从事机器视觉、模式识别等研究, E-mail: auyjz@hpu.edu.cn;

费树岷(1961—), 男, 教授, 博士生导师, 从事非线性系统分析与控制、鲁棒控制等研究, E-mail: smfei@seu.edu.cn.