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状态约束非线性系统自适应有限时间 命令滤波输出反馈控制

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摘要: 针对一类具有状态约束和不可测状态的非线性系统, 提出一种基于观测器的自适应神经网络有限时间命令滤波控制方案. 首先, 构造积分障碍李雅普诺夫函数来确保系统的状态变量满足时变约束条件. 其次, 考虑到大多数非线性系统的状态是不可测的, 设计一个自适应神经网络状态观测器对不可测状态进行估计. 然后, 在控制器设计过程中采用有限时间命令滤波控制方法, 避免了“微分爆炸”现象, 并进一步引入误差补偿机制消除了滤波误差. 此外, 为了解决输入死区问题, 将死区模型描述为线性输入和有界扰动的形式. 所提方法保证了系统状态不会超出约束边界, 闭环系统中所有信号在有限时间内是有界的, 且跟踪误差在有限时间内收敛到原点附近邻域内. 最后, 通过两个例子验证了所提方法的有效性.

关键词: 非线性系统; 观测器; 神经网络; 有限时间命令滤波控制; 障碍李雅普诺夫函数; 输入死区

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Adaptive finite time command filtered output feedback control of nonlinear systems with state constraints

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Abstract: This paper proposes an adaptive neural network finite time command filtered control scheme based on observers for a class of nonlinear systems with full state constraints and unmeasured states. Firstly, the integral barrier Lyapunov functions are constructed to guarantee that the state variables satisfy the time-varying constraint conditions. Secondly, considering that the states of most nonlinear systems are unmeasurable, an adaptive neural network state observer is designed to estimate the unmeasurable states. Then, the finite time command filtered control method is used in the process of controller design to avoid the phenomenon of ‘differential explosion’, and the error compensation mechanism is further introduced to eliminate the filtering errors. Besides, the dead zone model is described as the form of linear input and bounded disturbance to solve the input dead zone problem. The proposed method ensures that the system states do not exceed the constraint bounds, all signals in the closed-loop system are bounded in finite time, and the tracking error can converge to within the neighborhood near zero in finite time. Finally, the effectiveness of the proposed method is verified by two examples.

Keywords: nonlinear systems; observer; neural network; finite time command filtered control; barrier Lyapunov functions; input dead zone

0 引言

近年来, 基于反步法的自适应控制技术在处理不确定非线性系统方面得到了广泛的发展和应用^[1-3]. 其中, 模糊逻辑系统和神经网络作为处理未知函数

的两种有效方法, 在自适应反演设计中发挥着重要作用. 然而, 反步法存在虚拟控制信号连续求导产生的“微分爆炸”问题, 进一步限制了自适应反步法的发展. 为解决这一问题, 一些学者提出了动态面控

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制^[4-5]和命令滤波控制^[6-8]方法. 文献[9]提出了一种结合动态面控制技术的自适应神经网络控制方法, 其在控制器设计过程中引入了一阶滤波器, 消除了“微分爆炸”问题. 但是, 它没有考虑滤波误差的问题, 这将影响系统的控制性能. 文献[10]提出了一种自适应神经网络命令滤波跟踪控制方案, 该方案不仅消除了“微分爆炸”问题, 还引入了补偿信号来消除命令滤波器带来的误差. 此外, 文献[11]又将命令滤波控制和有限时间控制^[12-14]方法相结合, 可以满足更快的响应速度、更高的跟踪精度和更好的性能要求. 然而, 上述所有基于命令滤波的控制研究都没有考虑状态约束问题, 在实际系统中可能会造成状态变量超过约束边界, 导致设备故障甚至安全的问题.

为了保证系统状态不违反约束条件, 许多学者将各种类型障碍李雅普诺夫函数 (BLF) 引入到非线性系统来解决约束问题^[15-20]. 文献[21]提出了一种自适应神经网络反步控制方法, 其采用了一种对称对数型 BLF, 解决了系统的状态约束问题. 但是, 由于实际系统中的许多约束都是时变情况, 越来越多的学者开始将时变约束控制作为一个重点研究课题进行研究^[22-23]. 文献[24]提出了一种自适应神经网络有限时间控制方法, 其构造了一种非对称时变对数型 BLF, 可以确保所有状态都保持在约束范围内. 值得注意的是, 对数型 BLF 大都通过限制跟踪误差的大小, 间接保证系统状态处于约束范围之内. 为解决这一问题, 文献[25]采用了对称积分 BLF, 其可以直接将系统状态保持在约束范围内, 避免了对数型 BLF 通过对跟踪误差进行约束间接得到系统状态约束的过程, 降低了一定程度的保守性. 然而, 利用积分型 BLF 解决状态约束问题的文献大都没有考虑“微分爆炸”的问题.

此外, 为了处理具有不可测状态的非线性系统控制问题, 基于观测器的自适应控制方法受到了广泛的关注, 并取得了许多重要成果^[26-28]. 文献[29]提出了一种基于观测器的自适应模糊控制方法, 其不仅利用积分型 BLF 保证了所有状态都在约束范围内, 而且设计状态观测器来估计不可测状态. 为了避免“微分爆炸”的影响, 文献[30]结合了命令滤波控制方法, 并采用积分型 BLF 实现了输出约束. 然而, 上述研究中并没有将全状态约束、有限时间命令滤波控制以及输出反馈控制结合进行控制设计. 另外, 输入死区问题^[31-32]在非线性系统中也是值得考虑的问题.

综合上述分析, 本文针对一类具有输入死区和

状态约束的不确定非线性系统, 提出一种自适应神经网络有限时间命令滤波输出反馈控制方案, 主要有如下优势:

1) 与文献[33-34]的状态约束形式相比, 本文引入时变积分型 BLF, 可以使得系统状态约束在时变的范围内;

2) 与文献[35-36]的控制方法相比, 本文引入有限时间命令滤波控制方法, 不仅避免了“微分爆炸”现象, 减少了计算量, 而且可以使得闭环系统中所有信号在有限时间内都是有界的, 跟踪误差在有限时间内收敛到原点附近的邻域内;

3) 与文献[37-38]中要求状态变量必须是可测量的相比, 本文设计了状态观测器, 可以估计系统中存在的不可测状态变量, 且估计误差可以收敛到原点附近的邻域内.

1 问题描述与预备知识

1.1 系统模型

考虑如下具有严格反馈结构的非线性系统:

$$\begin{cases} \dot{x}_i(t) = f_i(\bar{x}_i) + x_{i+1} + d_i(t); \\ \dot{x}_n(t) = f_n(\bar{x}_n) + u + d_n(t); \\ y = x_1, i = 1, 2, \dots, n-1. \end{cases} \quad (1)$$

其中: $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$, $x = \bar{x}_n = [x_1, x_2, \dots, x_n]^T \in R^n$ 表示不可测的系统状态变量; $u, y \in R$ 分别是系统的输入和输出; $f_i(\cdot)$ 是未知光滑的非线性函数, 且满足 $f_i(0) = 0$; $d_i(\cdot)$ 是未知的外部扰动, 且满足 $|d_i(t)| \leq \bar{d}_i$, \bar{d}_i 是未知的正常数. 另外, 系统状态的时变约束条件为 $\forall t \geq 0, |x_i| < k_{bi}(t)$, $k_{bi}(t)$ 是连续时变函数.

1.2 死区描述

根据文献[39-40], 定义输入死区模型为

$$u = D(v) = \begin{cases} c_1(v - d_1), & v < d_1; \\ 0, & d_1 \leq v \leq d_2; \\ c_2(v - d_2), & v > d_2. \end{cases} \quad (2)$$

其中: v 表示死区的输入信号, $c_1 > 0$ 和 $c_2 > 0$ 分别表示曲线在负半轴和正半轴上的未知斜率, $d_1 < 0$ 和 $d_2 > 0$ 表示未知的断点.

将式(2)改写为

$$u = D(v) = cv + d. \quad (3)$$

其中

$$c = \begin{cases} c_1, & v < d_1; \\ 0, & d_1 \leq v \leq d_2; \\ c_2, & v > d_2. \end{cases} \quad d = \begin{cases} -c_1 d_1, & v < d_1; \\ 0, & d_1 \leq v \leq d_2; \\ -c_2 d_2, & v > d_2. \end{cases}$$

另外, 满足下述条件:

$$\begin{cases} d \leq \bar{d} = \max\{-c_1 d_1, c_2 d_2\}, \\ \gamma_1 = \min\{c_1, c_2\}, \gamma_2 = \max\{c_1, c_2\}. \end{cases} \quad (4)$$

1.3 径向基函数神经网络

径向基函数神经网络具有逼近特性, 在控制器设计过程中常被用来处理系统中的未知非线性函数. 对于定义在紧集 $\Omega_X \in R^q$ 上的任意未知光滑非线性函数 $f(X)$, 存在神经网络 $\theta^{*T} S(X)$ 满足如下等式:

$$f(X) = \theta^{*T} S(X) + \delta(X), |\delta(X)| \leq \bar{\delta}. \quad (5)$$

其中: $X \in \Omega_X$ 是输入向量; $\delta(X)$ 是逼近误差; $\bar{\delta}$ 是正常数; θ^* 是理想权重向量, 表示为

$$\theta^* = \arg \min_{\theta \in R^p} \{ \sup_{X \in \Omega_X} |f(X) - \theta^T S(X)| \}, \quad (6)$$

$\theta = [\theta_1, \theta_2, \dots, \theta_p]^T$ 是权重向量, 且 $p > 1$ 为神经网络节点数量; $S(X) = [S_1(X), S_2(X), \dots, S_p(X)]^T \in R^p$ 是基函数向量, 选择高斯函数作为径向基函数, 其表达式为

$$S_i(X) = \exp \left[\frac{-(X - c_i)^T (X - c_i)}{2b_i^2} \right], \quad (7)$$

$i = 1, 2, \dots, p$, c_i 和 b_i 分别表示高斯函数的中心和宽度.

1.4 相关假设与引理

为了完成控制系统的设计, 引入以下假设和引理.

假设 1^[25] 参考信号 x_d 及一阶导数 \dot{x}_d 是已知光滑且有界的, 存在正常数 Y_0 和连续时变函数 $Y_1(t)$ 、 $k_{c1}(t)$, 使得 $|x_d| \leq Y_1(t) < k_{c1}(t)$ 和 $|\dot{x}_d| \leq Y_0$ 成立.

假设 2^[25] 对于时变约束边界 $k_{bi}(t)$, $i = 1, 2, \dots, n$, 存在常数 K_{bi}^0 和 K_{bi}^1 , 使得 $|k_{bi}(t)| \leq K_{bi}^0$ 和 $|\dot{k}_{bi}(t)| \leq K_{bi}^1$ 成立.

假设 3^[28] 存在一组常数 Γ_i , $i = 1, 2, \dots, n$, 对于 $\forall X_1, X_2 \in R^i$, 下列不等式成立:

$$|f_i(X_1) - f_i(X_2)| \leq \Gamma_i \|X_1 - X_2\|, \quad (8)$$

其中 $\|X_1 - X_2\|$ 表示向量 $X_1 - X_2$ 的 2 范数.

通过假设 3 可知函数 f_i 满足 Lipschitz 条件.

引理 1^[12] 对于任意 $m, n \in R$, 存在正常数 p_1 、 p_2 和 p_3 满足以下不等式:

$$|m|^{p_1} |n|^{p_2} \leq \frac{p_1}{p_1 + p_2} p_3 |m|^{p_1 + p_2} + \frac{p_2}{p_1 + p_2} p_3^{-\frac{p_1}{p_2}} |n|^{p_1 + p_2}. \quad (9)$$

引理 2^[26] 对于任意 x_1, \dots, x_n 和 $0 < l \leq 1$, 有以下不等式成立:

$$\left(\sum_{j=1}^n |x_j| \right)^l \leq \sum_{j=1}^n |x_j|^l \leq n^{1-l} \left(\sum_{j=1}^n |x_j| \right)^l. \quad (10)$$

引理 3^[30] 对于 $\forall t \geq 0$, $|v + \pi| < k_c(t)$, 都有以下不等式成立:

$$\int_0^v \frac{\beta k_c^2(t)}{k_c^2(t) - (\beta + \pi)^2} d\beta \leq \frac{v^2 k_c^2(t)}{k_c^2(t) - (v + \pi)^2}, \quad (11)$$

其中 $k_c(t)$ 为连续函数.

引理 4^[28] 对于非线性系统 $\dot{x} = f(x)$, 如果存在一个连续正定函数 $V(x)$, 使得

$$\dot{V}(x) \leq -\kappa_1 V(x) - \kappa_2 V^\gamma(x),$$

则系统是有限时间稳定的. 其中: $\kappa_1 > 0$, $\kappa_2 > 0$, $0 < \gamma < 1$. 其稳定时间满足

$$T \leq t_0 + \frac{1}{\kappa_1(1-\gamma)} \ln \frac{\kappa_1 V^{1-\gamma}(t_0) + \kappa_2}{\kappa_2}.$$

根据文献 [38], 一阶 Levant 微分器描述如下:

$$\begin{cases} \dot{\varphi}_1 = \sigma_1, \\ \sigma_1 = -R_1 |\varphi_1 - \alpha_r|^{\frac{1}{2}} \text{sign}(\varphi_1 - \alpha_r) + \varphi_2, \\ \dot{\varphi}_2 = -R_2 \text{sign}(\varphi_2 - \sigma_1), \end{cases} \quad (12)$$

其中 α_r 是输入信号. 通过选择合适的参数 R_1 和 R_2 , 则下列引理成立.

引理 5^[38] 通过选择合适的参数 R_1 和 R_2 , 且没有噪声影响输入信号, 则在有限时间内可得以下等式:

$$\begin{cases} \varphi_1 = \alpha_{r0}, \\ \sigma_1 = \dot{\alpha}_{r0}. \end{cases} \quad (13)$$

注 1 根据引理 5 可知, 如果噪声不影响微分器的输入, 则 $\alpha_r = \alpha_{r0}$ 成立; 如果噪声影响微分器的输入, 则可以利用以下引理.

引理 6^[38] 考虑输入噪声满足 $|\alpha_r - \alpha_{r0}| \leq \kappa$, 则以下不等式在有限时间内成立:

$$\begin{cases} |\varphi_1 - \alpha_{r0}| \leq \mu_1 \kappa = \varpi_1, \\ |\sigma_1 - \dot{\alpha}_{r0}| \leq \lambda_1 \kappa^{\frac{1}{2}} = \varpi_2. \end{cases} \quad (14)$$

其中 $\mu_1 > 0$ 和 $\lambda_1 > 0$ 是完全取决于微分器设计参数的常数.

2 神经网络状态观测器设计

由于系统 (1) 的状态变量不全是可测的, 需要设计神经网络状态观测器来估计系统中的不可测状态. 系统 (1) 可以改写为如下形式:

$$\begin{cases} \dot{x} = Ax + Ly + \sum_{i=1}^n B_i (f_i(\hat{x}_i) + \Delta f_i + d_i(t)) + B_n u, \\ y = C^T x, \end{cases} \quad (15)$$

其中

$$L = [l_1, l_2, \dots, l_n]^T,$$

$$A = \begin{bmatrix} -l_1 & 1 & \dots & 0 \\ -l_2 & 0 & 1 & \vdots \\ \vdots & 0 & \ddots & 1 \\ -l_n & 0 & \dots & 0 \end{bmatrix}_{n \times n},$$

$$B_i = [\underbrace{0 \ \dots \ 0}_{i-1} \ 1 \ 0 \ \dots \ 0]_{n \times 1}^T,$$

$$B_n = [0 \ \dots \ 0 \ 1]_{n \times 1}^T,$$

$$C = [1 \ 0 \ \dots \ 0]_{n \times 1}^T$$

$$\hat{x}_i = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_i]^T,$$

$$\Delta f_i = f_i(\bar{x}_i) - f_i(\hat{x}_i),$$

$$i = 1, 2, \dots, n.$$

假设式 (15) 中的非线性函数可以用以下 RBF 神经网络近似:

$$\hat{f}_i(\hat{x}_i|\theta_i) = \theta_i^T S_i(\hat{x}_i), \quad i = 1, 2, \dots, n. \quad (16)$$

定义最优参数 θ_i^* 为

$$\theta_i^* = \arg \min_{\theta_i \in \Omega_i} [\sup_{\hat{x}_i \in U_i} |\hat{f}_i(\hat{x}_i|\theta_i) - f_i(\hat{x}_i)|], \quad (17)$$

其中 Ω_i 和 U_i 分别是 θ_i 和 \hat{x}_i 的紧致集, 且 θ_i 是最优参数 θ_i^* 的估计值, 满足 $\tilde{\theta}_i = \theta_i^* - \theta_i$. 定义最小逼近误差为

$$\delta_i = f_i(\hat{x}_i) - \hat{f}_i(\hat{x}_i|\theta_i^*), \quad (18)$$

其中 $|\delta_i| \leq \bar{\delta}_i$, 且 $\bar{\delta}_i$ 是正常数. 根据上述结论, 可得

$$f_i(\bar{x}_i) - f_i(\hat{x}_i|\theta_i) = \Delta f_i + \delta_i + \tilde{\theta}_i^T S_i(\hat{x}_i). \quad (19)$$

设计如下神经网络状态观测器:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Ly + \sum_{i=1}^n B_i \hat{f}_i(\hat{x}_i|\theta_i) + B_n u, \\ \hat{y} = C^T \hat{x}. \end{cases} \quad (20)$$

其中 $\hat{x} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]^T$. 通过选取合适的向量 L 可以使得矩阵 A 是 Hurwitz 矩阵. 因此, 给定一个正定对称矩阵 P , 则存在一个正定对称矩阵 Q , 使得 $A^T Q + Q A = -P$.

设 $e = x - \hat{x} = [e_1, e_2, \dots, e_n]^T$ 为系统的估计误差. 根据系统 (15)、式 (19) 以及观测器 (20), 得到下列估计误差系统:

$$\dot{e} = Ae + \sum_{i=1}^n B_i [\tilde{\theta}_i^T S_i(\hat{x}_i) + \Delta f_i + \delta_i + d_i(t)]. \quad (21)$$

为了评估该状态观测器 (20) 的性能, 选择以下 Lyapunov 函数:

$$V_0 = e^T Q e, \quad (22)$$

求导可得

$$\begin{aligned} \dot{V}_0 &= \dot{e}^T Q e + e^T Q \dot{e} = \\ &= -e^T P e + 2e^T Q \sum_{i=1}^n B_i [\tilde{\theta}_i^T S_i(\hat{x}_i) + \\ &= \Delta f_i + \delta_i + d_i(t)]. \end{aligned} \quad (23)$$

通过使用 Young's 不等式、假设 3 以及 $0 \leq S_i^T(\cdot) S_i(\cdot) \leq 1$, 可以得到下列不等式:

$$\begin{aligned} 2e^T Q \sum_{i=1}^n B_i \tilde{\theta}_i^T S_i(\hat{x}_i) &\leq n e^T e + \|Q\|^2 \sum_{i=1}^n \tilde{\theta}_i^T \tilde{\theta}_i, \\ 2e^T Q \sum_{i=1}^n B_i [\Delta f_i + \delta_i + d_i(t)] &\leq \\ (3n + \|Q\|^2 \sum_{i=1}^n \Gamma_i^2) e^T e &+ \|Q\|^2 \sum_{i=1}^n (\bar{\delta}_i^2 + \bar{d}_i^2). \end{aligned} \quad (24)$$

将式 (24) 代入 (23) 中得

$$\begin{aligned} \dot{V}_0 &\leq -(\lambda_{\min}(P) - 4n - \|Q\|^2 \sum_{i=1}^n \Gamma_i^2) e^T e + \\ &= \|Q\|^2 \sum_{i=1}^n (\tilde{\theta}_i^T \tilde{\theta}_i + \bar{\delta}_i^2 + \bar{d}_i^2) = \\ &= -l_0 e^T e + \|Q\|^2 \sum_{i=1}^n \tilde{\theta}_i^T \tilde{\theta}_i + C_0. \end{aligned} \quad (25)$$

其中

$$l_0 = \lambda_{\min}(P) - 4n - \|Q\|^2 \sum_{i=1}^n \Gamma_i^2,$$

$$C_0 = \|Q\|^2 \sum_{i=1}^n (\bar{\delta}_i^2 + \bar{d}_i^2).$$

注 2 由式 (25) 可知, 若 $l_0 > 0$ 且 $\|Q\|^2 \sum_{i=1}^n \tilde{\theta}_i^T \tilde{\theta}_i$ 是有界时, 则状态观测器 (20) 是稳定的. 然而, $\|Q\|^2 \sum_{i=1}^n \tilde{\theta}_i^T \tilde{\theta}_i$ 的有界性无法确定, 因此, 需要设计一个合适的控制器使闭环系统中所有信号有界.

3 控制器设计与稳定性分析

3.1 控制器设计

对于具有全状态约束的非线性系统 (1), 整个设计过程的控制方案流程如图 1 所示.

根据式 (12), 有限时间命令滤波器设计如下:

$$\begin{cases} \dot{\varphi}_{i,1} = \sigma_{i,1}, \\ \sigma_{i,1} = -R_1 |\varphi_{i,1} - \alpha_i|^{\frac{1}{2}} \text{sign}(\varphi_{i,1} - \alpha_i) + \varphi_{i,2}, \\ \dot{\varphi}_{i,2} = -R_2 \text{sign}(\varphi_{i,2} - \sigma_{i,1}), \\ i = 1, 2, \dots, n-1. \end{cases} \quad (26)$$

其中: α_i 是输入, $\pi_i(t) = \varphi_{i,1}(t)$ 和 $\dot{\pi}_i(t) = \sigma_{i,1}(t)$ 是输出. 基于有限时间命令滤波器的输出, 进一步将跟踪误差定义如下:

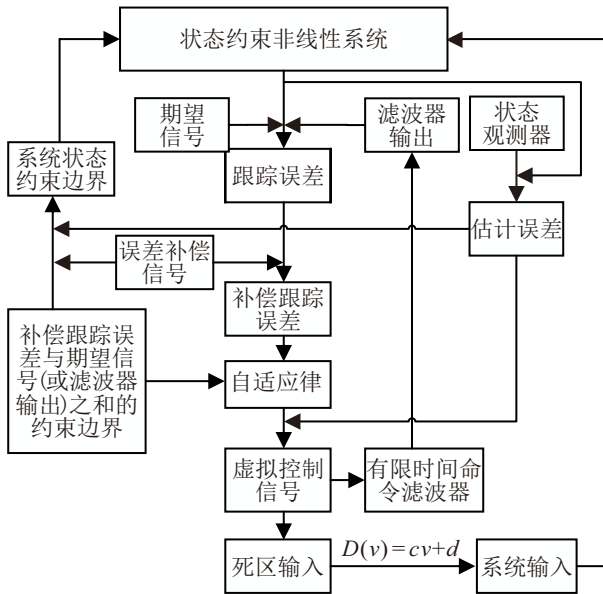


图1 控制方案流程图

$$\begin{cases} z_1 = x_1 - x_d; \\ z_i = \hat{x}_i - \pi_{i-1}, i = 2, 3, \dots, n. \end{cases} \quad (27)$$

其中: x_d 是期望的参考信号, π_{i-1} 是命令滤波器的输出。

注3 命令滤波器可能会产生滤波误差, 并且滤波误差 $|\pi_i - \alpha_i|$ ($i = 1, 2, \dots, n-1$) 会随着系统阶数的增大而增大。为了消除滤波误差的影响, 设计如下误差补偿机制:

$$\begin{cases} \dot{\xi}_i = -k_i \xi_i + \xi_{i+1} + (\pi_i - \alpha_i) - s_i \text{sign}(\xi_i); \\ \dot{\xi}_n = -k_n \xi_n - s_n \text{sign}(\xi_n), i = 1, 2, \dots, n-1. \end{cases} \quad (28)$$

其中: k_i 和 s_i 是正常数, 且 $\xi_i(0) = 0$ 。定义如下补偿跟踪误差:

$$v_i = z_i - \xi_i, i = 1, 2, \dots, n. \quad (29)$$

设计如下虚拟控制信号 α_i ($i = 1, 2, \dots, n$):

$$\begin{aligned} \alpha_1 = & -k_1 z_1 - \bar{k}_1(t)v_1 - \theta_1^T S_1(\hat{x}_1) - \\ & \frac{5}{2} \frac{v_1 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} + \\ & \frac{k_{c1}^2(t) - (v_1 + x_d)^2}{k_{c1}^2(t)} \psi_1 \dot{x}_d - \\ & \frac{k_{c1}^2(t) - (v_1 + x_d)^2}{k_{c1}^2(t)} I_1 \dot{k}_{c1}(t) - \\ & \omega_1 \left(\frac{k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} \right)^{\eta-1} v_1^{2\eta-1}, \\ \alpha_i = & -k_i z_i - \bar{k}_i(t)v_i - \theta_i^T S_i(\hat{x}_i) - \\ & \frac{v_i k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} + \\ & \frac{k_{ci}^2(t) - (v_i + \pi_{i-1})^2}{k_{ci}^2(t)} \psi_i \dot{\pi}_{i-1} - \end{aligned}$$

$$l_i e_1 - \frac{k_{ci}^2(t) - (v_i + \pi_{i-1})^2}{k_{ci}^2(t)} I_i \dot{k}_{ci}(t) -$$

$$\omega_i \left(\frac{k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} \right)^{\eta-1} v_i^{2\eta-1} - \frac{v_{i-1} k_{ci-1}^2(t) [k_{ci}^2(t) - (v_i + \pi_{i-1})^2]}{k_{ci}^2(t) [k_{ci-1}^2(t) - (v_{i-1} + \pi_{i-2})^2]},$$

$$\alpha_n = \frac{1}{\gamma_1} \left(-k_n z_n - \bar{k}_n(t)v_n - \theta_n^T S_n(\hat{x}) - \frac{3}{2} \frac{v_n k_{cn}^2(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} + \frac{k_{cn}^2(t) - (v_n + \pi_{n-1})^2}{k_{cn}^2(t)} \psi_n \dot{\pi}_{n-1} - \right.$$

$$l_n e_1 - \frac{k_{cn}^2(t) - (v_n + \pi_{n-1})^2}{k_{cn}^2(t)} I_n \dot{k}_{cn}(t) -$$

$$\omega_n \left(\frac{k_{cn}^2(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} \right)^{\eta-1} v_n^{2\eta-1} - \frac{v_{n-1} k_{cn-1}^2(t) [k_{cn}^2(t) - (v_n + \pi_{n-1})^2]}{k_{cn}^2(t) [k_{cn-1}^2(t) - (v_{n-1} + \pi_{n-2})^2]} \Big) = v. \quad (30)$$

其中: $\omega_i, \frac{1}{2} < \eta < 1$ 是正常数, $\pi_0 = x_d$ 。时变增益 $\bar{k}_i(t)$ 、变量 ψ_i 和 I_i 将在后面设计给出。

设计自适应律为

$$\dot{\theta}_i = \frac{r_i v_i k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} S_i(\hat{x}_i) - m_i \theta_i. \quad (31)$$

其中: $i = 1, 2, \dots, n$, 常数 $r_i > 0$ 和 $m_i > 0$ 是设计参数。

step 1: 补偿跟踪误差对时间求导得

$$\begin{aligned} \dot{v}_1 = & f_1(\hat{x}_1) + \Delta f_1 + z_2 + (\pi_1 - \alpha_1) + \\ & \alpha_1 + e_2 + d_1 - \dot{x}_d - \dot{\xi}_1. \end{aligned} \quad (32)$$

选择如下积分型 BLF 备选 V'_1 :

$$V'_1 = \int_0^{v_1} \frac{\beta k_{c1}^2(t)}{k_{c1}^2(t) - (\beta + x_d)^2} d\beta, \quad (33)$$

其中参考信号 x_d 满足 $|x_d| \leq Y_1(t) < k_{c1}(t)$ 。函数 V'_1 在集合 $|v_1 + x_d| < k_{c1}(t)$ 中是正定、连续可微的, 同时满足递减条件。令 $\beta = \tau v_1$, 可得如下不等式^[35]:

$$\begin{aligned} \frac{v_1^2}{2} \leq V'_1 \leq \\ v_1^2 \int_0^1 \frac{\tau k_{c1}^2(t)}{k_{c1}^2(t) - (\tau v_1 + \text{sign}(v_1) Y_1(t))^2} d\tau. \end{aligned} \quad (34)$$

当 $\beta + x_d = 0$ 时, $\int_0^{v_1} \frac{\beta k_{c1}^2(t)}{k_{c1}^2(t) - (\beta + x_d)^2} d\beta = \frac{v_1^2}{2}$ 。

注4 上述积分 BLF 是本文在命令滤波控制方法的基础上选用的一种处理全状态约束的 BLF。因为对数型 BLF 由补偿跟踪误差 v_i 构成, 且满足约束条件 $|v_i| < k_{ci}(t)$, 则需要通过 $x_1 = v_1 + x_d + \xi_1$ 和

$x_i = v_i + \pi_{i-1} + \xi_i + e_i$ ($i = 2, 3, \dots, n$) 进行转换, 间接保证系统状态 x_i 的约束条件. 本文选用的积分 BLF 满足补偿跟踪误差 v_i 与期望输出 x_d , 或滤波器输出 π_{i-1} 之和的约束边界条件 $|v_1 + x_d| < k_{c1}(t)$ 和 $|v_i + \pi_{i-1}| < k_{ci}(t)$ ($i = 2, \dots, n$), 再根据 $x_1 = v_1 + x_d + \xi_1$ 和 $x_i = v_i + \pi_{i-1} + \xi_i + e_i$, 可以得到 $|x_1| < k_{c1}(t) + |\xi_1|$ 和 $|x_i| < k_{ci}(t) + |\xi_i| + |e_i|$. 与前者相比少了一步的转换过程, 降低了一定的保守性.

V_1' 的导数计算为

$$\begin{aligned} \dot{V}_1' &= \frac{v_1 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} \dot{v}_1 + \\ &\dot{x}_d \int_0^{v_1} \frac{\partial}{\partial x_d} \frac{\beta k_{c1}^2(t)}{k_{c1}^2(t) - (\beta + x_d)^2} d\beta + \\ &\dot{k}_{c1}(t) \int_0^{v_1} \frac{\partial}{\partial k_{c1}(t)} \frac{\beta k_{c1}^2(t)}{k_{c1}^2(t) - (\beta + x_d)^2} d\beta. \end{aligned} \quad (35)$$

式 (35) 中的积分项表示为

$$\begin{aligned} &\int_0^{v_1} \frac{\partial}{\partial x_d} \frac{\beta k_{c1}^2(t)}{k_{c1}^2(t) - (\beta + x_d)^2} d\beta = \\ &v_1 \left(\frac{k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} - \psi_1(v_1, x_d, k_{c1}) \right), \\ &\int_0^{v_1} \frac{\partial}{\partial k_{c1}(t)} \frac{\beta k_{c1}^2(t)}{k_{c1}^2(t) - (\beta + x_d)^2} d\beta = \\ &v_1 \left(\frac{-v_1 k_{c1}(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} + I_1(v_1, x_d, k_{c1}) \right). \end{aligned} \quad (36)$$

其中

$$\begin{aligned} \psi_1(v_1, x_d, k_{c1}) &= \\ &\frac{k_{c1}(t)}{2v_1} \ln \frac{(k_{c1}(t) + v_1 + x_d)(k_{c1}(t) - x_d)}{(k_{c1}(t) - v_1 - x_d)(k_{c1}(t) + x_d)}, \\ I_1(v_1, x_d, k_{c1}) &= \\ &\frac{-x_d k_{c1}(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} + \\ &\frac{k_{c1}(t)}{v_1} \ln \frac{k_{c1}^2(t) - x_d^2}{k_{c1}^2(t) - (v_1 + x_d)^2} + \\ &\frac{x_d}{2v_1} \ln \frac{(k_{c1}(t) - v_1 - x_d)(k_{c1}(t) + x_d)}{(k_{c1}(t) + v_1 + x_d)(k_{c1}(t) - x_d)}. \end{aligned}$$

注 5 利用洛必达法则可以得到: $\lim_{v_1 \rightarrow 0} \psi_1(v_1, x_d, k_{c1}) = \frac{k_{c1}^2(t)}{k_{c1}^2(t) - x_d^2}$, $\lim_{v_1 \rightarrow 0} I_1(v_1, x_d, k_{c1}) = 0$. 由假设 1 可知 $|x_d| \leq Y_1(t) < k_{c1}(t)$, 因此 $\psi_1(v_1, x_d, k_{c1})$ 和 $I_1(v_1, x_d, k_{c1})$ 定义在 $v_1 = 0$ 的邻域内是有界的.

将式 (28)、(30)、(32) 和 (36) 代入 (35), 并利用 Young's 不等式^[24] 得到

$$\begin{aligned} \dot{V}_1' &\leq \\ &\frac{v_1 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} [\tilde{\theta}_1^T S_1(\hat{x}_1) + \\ &v_2 - k_1 v_1 - \bar{k}_1(t) v_1] + \end{aligned}$$

$$\begin{aligned} &\frac{1}{2}(\bar{\delta}_1^2 + \bar{d}_1^2 + (1 + \Gamma_1^2)e^T e + s_1^2) - \\ &\omega_1 \left(\frac{v_1^2 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} \right)^\eta - \frac{v_1^2 k_{c1}(t) \dot{k}_{c1}(t)}{k_{c1}^2(t) - (v_1 + x_d)^2}, \end{aligned} \quad (37)$$

其中时变增益 $\bar{k}_i(t) = \sqrt{\left(\frac{\dot{k}_{ci}(t)}{k_{ci}(t)}\right)^2 + \vartheta_i}$, $\vartheta_i > 0$, $i = 1, 2, \dots, n$. 由文献 [35] 可知 $\bar{k}_i(t) + \frac{\dot{k}_{ci}(t)}{k_{ci}(t)} \geq 0$, 所以式 (37) 可以化简为

$$\begin{aligned} \dot{V}_1' &\leq \frac{v_1 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} [\tilde{\theta}_1^T S_1(\hat{x}_1) + v_2 - k_1 v_1] - \\ &\omega_1 \left(\frac{v_1^2 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} \right)^\eta + \\ &\frac{1}{2}(\bar{\delta}_1^2 + \bar{d}_1^2 + (1 + \Gamma_1^2)e^T e + s_1^2). \end{aligned} \quad (38)$$

考虑如下 Lyapunov 函数:

$$V_1 = V_0 + V_1' + \frac{1}{2r_1} \tilde{\theta}_1^2. \quad (39)$$

对 V_1 求导, 并代入自适应律 (31) 得

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + \dot{V}_1' - \frac{1}{r_1} \tilde{\theta}_1^T \dot{\theta}_1 \leq \\ &\dot{V}_0 - \frac{k_1 v_1^2 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} - \\ &\omega_1 \left(\frac{v_1^2 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} \right)^\eta + \frac{v_2 v_1 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} + \\ &\frac{1}{2}(\bar{\delta}_1^2 + \bar{d}_1^2 + (1 + \Gamma_1^2)e^T e + s_1^2) + \frac{m_1}{r_1} \tilde{\theta}_1^T \theta_1. \end{aligned} \quad (40)$$

step i ($2 \leq i \leq n - 1$): 补偿跟踪误差对时间求导得

$$\begin{aligned} \dot{v}_i &= \hat{f}_i(\hat{x}_i | \theta_i) - \tilde{\theta}_i^T S_i(\hat{x}_i) + \tilde{\theta}_i^T S_i(\hat{x}_i) + z_{i+1} + \\ &(\pi_i - \alpha_i) + \alpha_i + l_i e_1 - \dot{\pi}_{i-1} - \dot{\xi}_i. \end{aligned} \quad (41)$$

选择如下积分型 BLF 备选 V_i' :

$$V_i' = \int_0^{v_i} \frac{\beta k_{ci}^2(t)}{k_{ci}^2(t) - (\beta + \pi_{i-1})^2} d\beta, \quad (42)$$

其中滤波器输出信号 π_{i-1} ($i = 2, 3, \dots, n - 1$) 是连续可微函数, 满足 $|\pi_{i-1}| \leq Y_i(t) < k_{ci}(t)$, $Y_i(t)$ 是未知函数. 函数 V_i' 在集合 $|v_i + \pi_{i-1}| < k_{ci}(t)$ 中是正定、连续可微的, 且满足递减条件. 令 $\beta = \tau v_i$, 则有如下不等式成立:

$$\begin{aligned} \frac{v_i^2}{2} &\leq V_i' \leq \\ &v_i^2 \int_0^1 \frac{\tau k_{ci}^2(t)}{k_{ci}^2(t) - (\tau v_i + \text{sign}(v_i) Y_i(t))^2} d\tau. \end{aligned} \quad (43)$$

V_i' 的导数计算为

$$\begin{aligned} \dot{V}'_i = & \frac{v_i k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} \dot{v}_i + \\ & \dot{\pi}_{i-1} \int_0^{v_i} \frac{\partial}{\partial \pi_{i-1}} \frac{\beta k_{ci}^2(t)}{k_{ci}^2(t) - (\beta + \pi_{i-1})^2} d\beta + \\ & \dot{k}_{ci}(t) \int_0^{v_i} \frac{\partial}{\partial k_{ci}(t)} \frac{\beta k_{ci}^2(t)}{k_{ci}^2(t) - (\beta + \pi_{i-1})^2} d\beta. \end{aligned} \quad (44)$$

式(44)中的积分项表示为

$$\begin{aligned} & \int_0^{v_i} \frac{\partial}{\partial \pi_{i-1}} \frac{\beta k_{ci}^2(t)}{k_{ci}^2(t) - (\beta + \pi_{i-1})^2} d\beta = \\ & v_i \left(\frac{k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} - \psi_i(v_i, \pi_{i-1}, k_{ci}) \right), \\ & \int_0^{v_i} \frac{\partial}{\partial k_{ci}(t)} \frac{\beta k_{ci}^2(t)}{k_{ci}^2(t) - (\beta + \pi_{i-1})^2} d\beta = \\ & v_i \left(\frac{-v_i k_{ci}(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} + I_i(v_i, \pi_{i-1}, k_{ci}) \right). \end{aligned} \quad (45)$$

其中

$$\begin{aligned} \psi_i(v_i, \pi_{i-1}, k_{ci}) = & \frac{k_{ci}(t)}{2v_i} \ln \frac{(k_{ci}(t) + v_i + \pi_{i-1})(k_{ci}(t) - \pi_{i-1})}{(k_{ci}(t) - v_i - \pi_{i-1})(k_{ci}(t) + \pi_{i-1})}, \\ I_i(v_i, \pi_{i-1}, k_{ci}) = & \frac{-\pi_{i-1} k_{ci}(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} + \\ & \frac{k_{ci}(t)}{v_i} \ln \frac{k_{ci}^2(t) - \pi_{i-1}^2}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} + \\ & \frac{\pi_{i-1}}{2v_i} \ln \frac{(k_{ci}(t) - v_i - \pi_{i-1})(k_{ci}(t) + \pi_{i-1})}{(k_{ci}(t) + v_i + \pi_{i-1})(k_{ci}(t) - \pi_{i-1})}. \end{aligned}$$

与注5类似, $\psi_i(v_i, \pi_{i-1}, k_{ci})$ 和 $I_i(v_i, \pi_{i-1}, k_{ci})$ 定义在 $v_i = 0$ 的邻域内是有界的.

将式(28)、(30)、(41)和(45)代入(44), 并利用Young's不等式得到

$$\begin{aligned} \dot{V}'_i \leq & \frac{v_i k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} [\tilde{\theta}_i^T S_i(\hat{x}_i) + v_{i+1} - k_i v_i] - \\ & \omega_i \left(\frac{v_i^2 k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} \right)^\eta - \\ & \frac{v_{i-1} v_i k_{ci-1}^2(t)}{k_{ci-1}^2(t) - (v_{i-1} + \pi_{i-2})^2} + \frac{1}{2} (s_i^2 + \tilde{\theta}_i^2). \end{aligned} \quad (46)$$

考虑如下Lyapunov函数:

$$V_i = V_{i-1} + V'_i + \frac{1}{2r_i} \tilde{\theta}_i^2. \quad (47)$$

对 V_i 求导, 并代入自适应律(31)得

$$\begin{aligned} \dot{V}_i = & \dot{V}_{i-1} + \dot{V}'_i - \frac{1}{r_i} \tilde{\theta}_i^T \dot{\theta}_i \leq \\ & \dot{V}_0 - \sum_{j=1}^i \left[\frac{k_j v_j^2 k_{cj}^2(t)}{k_{cj}^2(t) - (v_j + \pi_{j-1})^2} + \right. \end{aligned}$$

$$\begin{aligned} & \left. \omega_j \left(\frac{v_j^2 k_{cj}^2(t)}{k_{cj}^2(t) - (v_j + \pi_{j-1})^2} \right)^\eta \right] + \sum_{j=1}^i \frac{m_j}{r_j} \tilde{\theta}_j^T \dot{\theta}_j + \\ & \frac{v_i v_{i+1} k_{ci}^2(t)}{k_{ci}^2(t) - (v_i + \pi_{i-1})^2} + \frac{1}{2} \sum_{j=2}^i \tilde{\theta}_j^2 + \frac{1}{2} \sum_{j=1}^i s_j^2 + \\ & \frac{1}{2} (\bar{\delta}_1^2 + \bar{d}_1^2 + (1 + \Gamma_1^2) e^T e). \end{aligned} \quad (48)$$

step n : 补偿跟踪误差对时间求导, 并代入式

(3)得

$$\begin{aligned} \dot{v}_n = & \hat{f}_n(\hat{x}|\theta_n) + \tilde{\theta}_n^T S_n(\hat{x}) - \tilde{\theta}_n^T S_n(\hat{x}) + \\ & cv + d + l_n e_1 - \dot{\pi}_{n-1} - \dot{\xi}_n. \end{aligned} \quad (49)$$

选择如下积分型BLF备选 V'_n :

$$V'_n = \int_0^{v_n} \frac{\beta k_{cn}^2(t)}{k_{cn}^2(t) - (\beta + \pi_{n-1})^2} d\beta, \quad (50)$$

其中滤波器输出信号 π_{n-1} 是连续可微函数, 满足 $|\pi_{n-1}| \leq Y_n(t) < k_{cn}(t)$, $Y_n(t)$ 是未知函数. 函数 V'_n 在集合 $|v_n + \pi_{n-1}| < k_{cn}(t)$ 中是正定、连续可微的, 且满足递减条件. 令 $\beta = \tau v_n$, 可得以下不等式:

$$\begin{aligned} \frac{v_n^2}{2} \leq V'_n \leq \\ v_n^2 \int_0^1 \frac{\tau k_{cn}^2(t)}{k_{cn}^2(t) - (\tau v_n + \text{sign}(v_n) Y_n(t))^2} d\tau. \end{aligned} \quad (51)$$

V'_n 的导数计算为

$$\begin{aligned} \dot{V}'_n = & \frac{v_n k_{cn}^2(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} \dot{v}_n + \\ & \dot{\pi}_{n-1} \int_0^{v_n} \frac{\partial}{\partial \pi_{n-1}} \frac{\beta k_{cn}^2(t)}{k_{cn}^2(t) - (\beta + \pi_{n-1})^2} d\beta + \\ & \dot{k}_{cn}(t) \int_0^{v_n} \frac{\partial}{\partial k_{cn}(t)} \frac{\beta k_{cn}^2(t)}{k_{cn}^2(t) - (\beta + \pi_{n-1})^2} d\beta, \end{aligned} \quad (52)$$

式(52)中的积分项表示为

$$\begin{aligned} & \int_0^{v_n} \frac{\partial}{\partial \pi_{n-1}} \frac{\beta k_{cn}^2(t)}{k_{cn}^2(t) - (\beta + \pi_{n-1})^2} d\beta = \\ & v_n \left(\frac{k_{cn}^2(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} - \psi_n(v_n, \pi_{n-1}, k_{cn}) \right), \\ & \int_0^{v_n} \frac{\partial}{\partial k_{cn}(t)} \frac{\beta k_{cn}^2(t)}{k_{cn}^2(t) - (\beta + \pi_{n-1})^2} d\beta = \\ & v_n \left(\frac{-v_n k_{cn}(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} + I_n(v_n, \pi_{n-1}, k_{cn}) \right). \end{aligned} \quad (53)$$

其中

$$\begin{aligned} \psi_n(v_n, \pi_{n-1}, k_{cn}) = \\ \frac{k_{cn}(t)}{2v_n} \ln \frac{(k_{cn}(t) + v_n + \pi_{n-1})(k_{cn}(t) - \pi_{n-1})}{(k_{cn}(t) - v_n - \pi_{n-1})(k_{cn}(t) + \pi_{n-1})}, \end{aligned}$$

$$I_n(v_n, \pi_{n-1}, k_{cn}) = \frac{-\pi_{n-1}k_{cn}(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} + \frac{k_{cn}(t)}{v_n} \ln \frac{k_{cn}^2(t) - \pi_{n-1}^2}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} + \frac{\pi_{n-1}}{2v_n} \ln \frac{(k_{cn}(t) - v_n - \pi_{n-1})(k_{cn}(t) + \pi_{n-1})}{(k_{cn}(t) + v_n + \pi_{n-1})(k_{cn}(t) - \pi_{n-1})}.$$

与注 5 类似, $\psi_n(v_n, \pi_{n-1}, k_{cn})$ 和 $I_n(v_n, \pi_{n-1}, k_{cn})$ 定义在 $v_n = 0$ 的邻域内是有界的.

将式 (28)、(30)、(49) 和 (53) 代入 (52), 并利用 Young's 不等式得到

$$\dot{V}'_n = \frac{v_n k_{cn}^2(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} [\tilde{\theta}_n^T S_n(\hat{x}) - k_n v_n] - \omega_n \left(\frac{v_n^2 k_{cn}^2(t)}{k_{cn}^2(t) - (v_n + \pi_{n-1})^2} \right)^\eta - \frac{v_{n-1} v_n k_{cn-1}^2(t)}{k_{cn-1}^2(t) - (v_{n-1} + \pi_{n-2})^2} + \frac{1}{2} (\bar{d}^2 + s_n^2 + \tilde{\theta}_n^T \tilde{\theta}_n). \quad (54)$$

考虑如下 Lyapunov 函数:

$$V_n = V_{n-1} + V'_n + \frac{1}{2r_n} \tilde{\theta}_n^2. \quad (55)$$

对 V_n 求导, 代入自适应律 (31), 并利用 Young's 不等式得

$$\begin{aligned} \dot{V}_n &= \dot{V}_{n-1} + \dot{V}'_n - \frac{1}{r_n} \tilde{\theta}_n^T \dot{\tilde{\theta}}_n \leq \\ &\dot{V}_0 - \sum_{j=1}^n \left[\frac{k_j v_j^2 k_{c_j}^2(t)}{k_{c_j}^2(t) - (v_j + \pi_{j-1})^2} + \omega_j \left(\frac{v_j^2 k_{c_j}^2(t)}{k_{c_j}^2(t) - (v_j + \pi_{j-1})^2} \right)^\eta \right] + \frac{1}{2} \sum_{j=2}^n \tilde{\theta}_j^2 + \\ &\sum_{j=1}^n \left(\frac{m_j}{2r_j} \theta_j^{*2} - \frac{m_j}{2r_j} \tilde{\theta}_j^2 \right) + \frac{1}{2} \sum_{j=1}^n s_j^2 + \\ &\frac{1}{2} (\bar{\delta}_1^2 + \bar{d}_1^2 + (1 + \Gamma_1^2) e^T e + \bar{d}^2). \quad (56) \end{aligned}$$

3.2 稳定性分析

定理 1 考虑系统 (1) 满足假设 1 ~ 假设 3, 给定参考信号 x_d , 若采用状态观测器 (20)、有限时间命令滤波器 (26)、误差补偿机制 (28)、虚拟控制信号 α_i 和控制律 (30) 及自适应更新律 (31), 则可以保证: 1) 跟踪误差 z_i 在有限时间内收敛到原点附近的一个邻域内; 2) 闭环系统中的所有信号在有限时间内都是有界的; 3) 所有状态都满足 $|x_i| < k_{bi}(t), \forall t \geq 0$.

证明 根据引理 1 可得

$$-p_1(e^T Q e)^\eta + p_1(e^T Q e)^\eta \leq$$

$$\begin{aligned} &-p_1(e^T Q e)^\eta + p_1(1 - \eta)\eta^{1-\eta} + p_1 e^T Q e, \\ &-p_2 \left(\sum_{j=1}^n \frac{1}{2r_j} \tilde{\theta}_j^2 \right)^\eta + p_2 \left(\sum_{j=1}^n \frac{1}{2r_j} \tilde{\theta}_j^2 \right)^\eta \leq \\ &-p_2 \left(\sum_{j=1}^n \frac{1}{2r_j} \tilde{\theta}_j^2 \right)^\eta + p_2(1 - \eta)\eta^{1-\eta} + p_2 \sum_{j=1}^n \frac{1}{2r_j} \tilde{\theta}_j^2, \quad (57) \end{aligned}$$

其中 p_1 和 p_2 是正设计参数.

将式 (25)、(57) 代入 (56) 可得

$$\begin{aligned} \dot{V}_n &\leq \\ &- \left[l_0 - \frac{1 + \Gamma_1^2}{2} \right] e^T e - \sum_{j=1}^n \left[\frac{k_j v_j^2 k_{c_j}^2(t)}{k_{c_j}^2(t) - (v_j + \pi_{j-1})^2} + \omega_j \left(\frac{v_j^2 k_{c_j}^2(t)}{k_{c_j}^2(t) - (v_j + \pi_{j-1})^2} \right)^\eta \right] - \left(\frac{m_1}{2r_1} - \|Q\|^2 \right) \tilde{\theta}_1^2 - \\ &\sum_{j=2}^n \left(\frac{m_j}{2r_j} - \frac{1}{2} - \|Q\|^2 \right) \tilde{\theta}_j^2 + \sum_{j=1}^n \left(\frac{m_j}{2r_j} \theta_j^{*2} \right) + \\ &\frac{1}{2} \sum_{j=1}^n s_j^2 + C_0 + \frac{1}{2} (\bar{\delta}_1^2 + \bar{d}_1^2 + \bar{d}^2) \leq \\ &- \left[\frac{l_0}{\lambda_{\max}(Q)} - \frac{1 + \Gamma_1^2}{2\lambda_{\max}(Q)} - p_1 \right] e^T Q e - p_1 (e^T Q e)^\eta - \\ &\sum_{j=1}^n \left[\frac{k_j v_j^2 k_{c_j}^2(t)}{k_{c_j}^2(t) - (v_j + \pi_{j-1})^2} + \omega_j \left(\frac{v_j^2 k_{c_j}^2(t)}{k_{c_j}^2(t) - (v_j + \pi_{j-1})^2} \right)^\eta \right] - \\ &\sum_{j=2}^n \left(\frac{m_j - p_2}{2r_j} - \frac{1}{2} - \|Q\|^2 \right) \tilde{\theta}_j^2 - \\ &\left(\frac{m_1 - p_2}{2r_1} - \|Q\|^2 \right) \tilde{\theta}_1^2 - \\ &p_2 \left(\sum_{j=1}^n \frac{\tilde{\theta}_j^2}{2r_j} \right)^\eta + (p_1 + p_2)(1 - \eta)\eta^{1-\eta} + \\ &\sum_{j=1}^n \left(\frac{m_j}{2r_j} \theta_j^{*2} \right) + \frac{1}{2} \sum_{j=1}^n s_j^2 + C_0 + \frac{1}{2} (\bar{\delta}_1^2 + \bar{d}_1^2 + \bar{d}^2). \quad (58) \end{aligned}$$

根据引理 2 和引理 3, 可将式 (58) 化简为

$$\dot{V}_n \leq -aV_n - bV_n^\eta + \rho. \quad (59)$$

其中

$$\begin{aligned} a &= \min \left\{ \frac{l_0}{\lambda_{\max}(Q)} - \frac{1 + \Gamma_1^2}{2\lambda_{\max}(Q)} - p_1, m_1 - p_2 - 2r_1 \|Q\|^2, \right. \\ &\left. m_i - p_2 - r_i - 2r_i \|Q\|^2, k_1, k_i, i = 2, 3, \dots, n \right\}, \\ b &= \min \{ p_1, \omega_i, p_2, i = 1, 2, \dots, n \}, \\ \rho &= (p_1 + p_2)(1 - \eta)\eta^{1-\eta} + \sum_{j=1}^n \left(\frac{m_j}{2r_j} \theta_j^{*2} \right) + \\ &\frac{1}{2} \sum_{j=1}^n s_j^2 + C_0 + \frac{1}{2} (\bar{\delta}_1^2 + \bar{d}_1^2 + \bar{d}^2). \end{aligned}$$

存在一个常数 $0 < \Lambda < 1$, 使得式 (59) 改写为

$$\dot{V}_n \leq -\Lambda a V_n - (1 - \Lambda) a V_n - b V_n^\eta + \rho, \quad (60)$$

或者

$$\dot{V}_n \leq -a V_n - \Lambda b V_n^\eta - (1 - \Lambda) b V_n^\eta + \rho. \quad (61)$$

由式 (60) 可知, 如果 $V_n > \frac{\rho}{(1 - \Lambda)a}$, 则有 $\dot{V}_n \leq -\Lambda a V_n - b V_n^\eta$, 所以在有限时间内 $(e, v_i, \tilde{\theta}_i) (i = 1, 2, \dots, n)$ 将收敛到如下区域中:

$$(e, v_i, \tilde{\theta}_i) \in \left\{ V_n \leq \frac{\rho}{(1 - \Lambda)a} \right\}. \quad (62)$$

到达区域 (62) 所需的时间满足

$$T_2 \leq \frac{1}{\Lambda a(1 - \eta)} \ln \frac{\Lambda a V_n^{1-\eta}(0) + b}{b}.$$

根据式 (61) 可知, 如果 $V_n > \frac{\rho}{(1 - \Lambda)b}$, 则有 $\dot{V}_n \leq -a V_n - \Lambda b V_n^\eta$, 所以在有限时间内, $(e, v_i, \tilde{\theta}_i) (i = 1, 2, \dots, n)$ 将收敛到如下区域中:

$$(e, v_i, \tilde{\theta}_i) \in \left\{ V_n \leq \left(\frac{\rho}{(1 - \Lambda)b} \right)^{\frac{1}{\eta}} \right\}, \quad (63)$$

到达区域 (63) 所需的时间满足

$$T_3 \leq \frac{1}{a(1 - \eta)} \ln \frac{a V_n^{1-\eta}(0) + \Lambda b}{\Lambda b}.$$

由式 (62) 和 (63) 及式 (34)、(43)、(51) 可知, 最终 $v_i (i = 1, 2, \dots, n)$ 会收敛到区域

$$|v_i| \leq \min \left\{ \sqrt{\frac{2\rho}{(1 - \Lambda)a}}, \sqrt{2 \left(\frac{\rho}{(1 - \Lambda)b} \right)^{\frac{1}{\eta}}} \right\}. \quad (64)$$

到达区域 (64) 的时间满足 $T_4 = \max\{T_2, T_3\}$.

选择补偿系统的 Lyapunov 备选函数 \bar{V} 为

$$\bar{V} = \frac{1}{2} \sum_{i=1}^n \xi_i^2. \quad (65)$$

对 \bar{V} 求导可得

$$\begin{aligned} \dot{\bar{V}} &= \sum_{i=1}^n \xi_i \dot{\xi}_i = \\ &= -k_1 \xi_1^2 + \xi_1 \xi_2 - s_1 \xi_1 \text{sign}(\xi_1) + \xi_1 (\pi_1 - \alpha_1) - \\ &= k_2 \xi_2^2 + \xi_2 \xi_3 - s_2 \xi_2 \text{sign}(\xi_2) + \xi_2 (\pi_2 - \alpha_2) + \\ &= \dots - k_n \xi_n^2 - s_n \xi_n \text{sign}(\xi_n) = \\ &= -\sum_{i=1}^n k_i \xi_i^2 + \sum_{i=1}^{n-1} \xi_i \xi_{i+1} - \sum_{i=1}^n s_i |\xi_i| + \sum_{i=1}^{n-1} \xi_i (\pi_i - \alpha_i). \end{aligned} \quad (66)$$

由引理 5 和引理 6 可知, 在有限时间 T_{i2} 内, 可以得到 $|\pi_i - \alpha_i| \leq \varpi_{i1}$. 当 $t \geq \max\{T_{i2}\}$ 时, 由 Young's 不等式和引理 2 可得

$$\dot{\bar{V}} \leq -\left(k_1 - \frac{1}{2}\right) \xi_1^2 - \sum_{i=2}^{n-1} (k_i - 1) \xi_i^2 - \left(k_n -$$

$$\begin{aligned} \frac{1}{2}\right) \xi_n^2 - \sum_{i=1}^n s_i |\xi_i| + \sum_{i=1}^{n-1} |\xi_i| \varpi_{i1} + |\xi_n| \varpi_{n1} \leq \\ -\bar{a} \bar{V} - s_0 \bar{V}^{\frac{1}{2}} + \sqrt{2n} \bar{\varpi}_1 \bar{V}^{\frac{1}{2}} \leq \\ -\bar{a} \bar{V} - (s_0 - \sqrt{2n} \bar{\varpi}_1) \bar{V}^{\frac{1}{2}}. \end{aligned} \quad (67)$$

其中: $\bar{a} = 2 \min \left\{ k_1 - \frac{1}{2}, k_i - 1, k_n - \frac{1}{2}, i = 2, 3, \dots, n - 1 \right\} > 0$, $s_0 = \sqrt{2} \min\{s_i, i = 1, 2, \dots, n\}$, $\bar{\varpi}_1 = \max\{\varpi_{i1}, i = 1, 2, \dots, n\}$.

通过选择合适的 s_i , 使得 $s_0 - \sqrt{2n} \bar{\varpi}_1 > 0$, 则由引理 4 可知, ξ_i 将在有限时间内趋于稳定. 当 $T_1 = \max\{T_{i2}\} + \frac{2}{\bar{a}} \ln \frac{\bar{a} \sqrt{\bar{V}}(0) + s_0 - \sqrt{2n} \bar{\varpi}_1}{s_0 - \sqrt{2n} \bar{\varpi}_1}$ 时, 可以得到 $\lim_{t \rightarrow T_1} \xi_i(t) = 0$.

基于以上分析, 当时间 $t \geq \max\{T_1, T_4\}$ 时, 可得 $|z_i| \leq |v_i| + |\xi_i| \leq$

$$\min \left\{ \sqrt{\frac{2\rho}{(1 - \Lambda)a}}, \sqrt{2 \left(\frac{\rho}{(1 - \Lambda)b} \right)^{\frac{1}{\eta}}} \right\}. \quad (68)$$

综上所述, $z_i (i = 1, 2, \dots, n)$ 和 v_i 将在有限时间内收敛到原点附近的邻域内, 补偿信号 ξ_i 在有限时间内也是有界的. 根据假设 1 和 $x_1 = v_1 + x_d + \xi_1$, 可得 x_1 是有界的且满足 $|x_1| = |v_1 + x_d + \xi_1| \leq |v_1 + x_d| + |\xi_1| < k_{c1}(t) + |\xi_1| = k_{b1}(t)$, 所以系统状态 x_1 满足约束条件. 因为估计误差 $e = [e_1, e_2, \dots, e_n]^T$ 是有界的, 又根据 $x_2 - \hat{x}_2 = e_2$ 和 $\hat{x}_2 = v_2 + \pi_1 + \xi_2$, 可得 $|x_2| < k_{c2}(t) + |\xi_2| + |e_2| = k_{b2}(t)$, 所以系统状态 x_2 满足约束条件. 相似地, $|x_i| < k_{bi}(t) (i = 3, 4, \dots, n)$ 也可得到证明. 由假设 2 和 $k_{c1}(t) = k_{b1}(t) - |\xi_1|$ 可知, $k_{c1}(t)$ 和 $\dot{k}_{c1}(t)$ 是有界的, 又根据 $\theta_i = \theta_i^* - \tilde{\theta}_i$ 得到 θ_i 也是有界的. 因为虚拟控制信号 α_1 是 $x_1, \xi_1, x_d, \dot{x}_d, k_{c1}, \dot{k}_{c1}, \theta_1, \psi_1$ 以及 I_1 的函数, 所以 α_1 一定是有界的, 可知 π_1 也一定是有界的. 相似地, 可以得出 α_i 和 $\pi_i (i = 2, 3, \dots, n - 1)$ 都一定是有界. 在此基础上, 很容易得到闭环系统中的所有信号在有限时间内都是有界的, 系统状态都满足约束条件. □

注 6 在本文的证明过程中, 利用了引理 5 和引理 6 中的结论 $|\pi_i - \alpha_i| \leq \varpi_{i1}$. 注意, 如果滤波器 (26) 的输入 α_i 不受噪声的影响, 则 $\varpi_{i1} = 0$; 反之, 可以从引理 6 得到结论.

注 7 为了保证 $a > 0$, 观测器增益 L 和参数 $p_1, p_2, \Gamma_i, m_i, r_i (i = 1, 2, \dots, n)$ 的选取应满足 $\frac{l_0}{\lambda_{\max}(Q)} - \frac{1 + \Gamma_1^2}{2\lambda_{\max}(Q)} - p_1 > 0, m_1 - p_2 - 2r_1 \|Q\|^2 > 0, m_i -$

$p_2 - r_i - 2r_i \|Q\|^2 > 0$, 且控制增益满足 $k_1 > 1/2$, $k_i > 1 (i = 2, 3, \dots, n-1)$, $k_n > 1/2$. 另外, 当选择较大的参数 k_i 、 ω_i 、 r_i 以及较小的参数 s_i 时, 可以保证跟踪误差较小的收敛半径.

4 仿真结果与分析

在本节中, 分别用数值案例和实际案例来验证所提控制方法的有效性.

案例 1 (数值案例) 考虑以下具有严格反馈结构的非线性系统:

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + d_1(t), \\ \dot{x}_2 = u + f_2(x_2) + d_2(t), \\ y = x_1. \end{cases} \quad (69)$$

其中: $f_1 = \sin(x_1^2) \cos(x_1)$, $f_2 = x_1 \sin(x_2)(1 + x_2^2)$, $d_1 = 0.01 \cos(2t)$, $d_2 = 0.02 \sin(t)$; 系统的初始状态为 $x_1(0) = x_2(0) = 0$, 系统状态满足的约束条件为 $|x_1| < k_{b1}(t) = 0.1 \sin(t) + 0.5$, $|x_2| < k_{b2}(t) = 0.1 \sin(t) + 0.4$; 期望信号 $x_d = 0.3 \sin(t)$. 死区模型可以表示为

$$u = D(v) = \begin{cases} 0.8v + 0.4, & v < -0.5; \\ 0, & -0.5 \leq v \leq 0.75; \\ 1.2v - 0.9, & v > 0.75. \end{cases} \quad (70)$$

根据式 (20), 状态观测器被设计为

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \theta_1^T S_1(\hat{x}_1) + l_1(y - \hat{y}), \\ \dot{\hat{x}}_2 = u + \theta_2^T S_2(\hat{x}_2) + l_2(y - \hat{y}), \\ \hat{y} = \hat{x}_1. \end{cases} \quad (71)$$

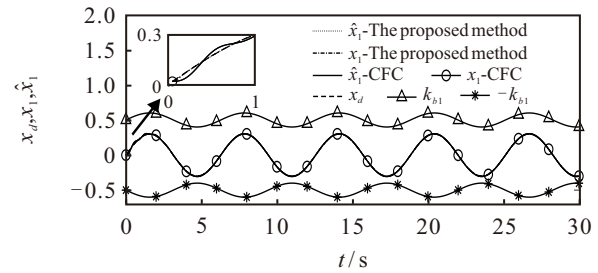
其中状态观测器的初始状态为 $\hat{x}_1(0) = \hat{x}_2(0) = 0$. 为了逼近系统的非线性函数, 选取高斯函数为神经网络的基函数, 其表达式为

$$S_i(\hat{X}) = \exp \left[\frac{-(\hat{X} - c_i)^T (\hat{X} - c_i)}{2b_i^2} \right]. \quad (72)$$

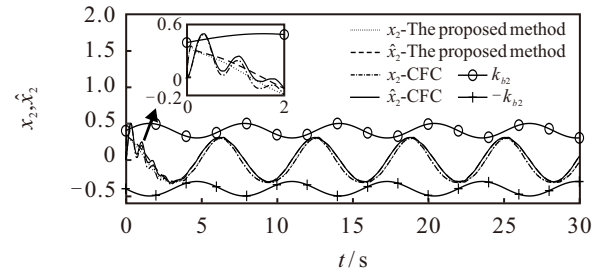
其中: $i = 1, 2, \dots, p$, 节点数量 $p = 7$; 高斯函数的中心 c_i 分布区间为 $[-3, 3]$, 高斯函数的宽度 $b_i = 1$.

虚拟控制信号 α_1 和控制律 v 由式 (30) 给出, 补偿误差信号由式 (28) 给出, 自适应律由式 (31) 给出. 控制器设计过程的相关参数如下: $r_1 = r_2 = 0.2$, $m_1 = m_2 = 0.1$, $\gamma_1 = 0.8$, $k_1 = 10$, $k_2 = 50$, $R_1 = 10$, $R_2 = 20$, $w_1 = w_2 = 1$, $\eta = 0.97$, $\beta_1 = 0.1$, $\beta_2 = 0.2$, $s_1 = 0.01$, $s_2 = 0.02$, $l_1 = 10$, $l_2 = 200$, $\theta_1(0) = \theta_2(0) = 0$, $\xi_1(0) = \xi_2(0) = 0$.

为了证明本文所提方案的有效性, 将所提方法 (具有状态约束、有限时间控制、命令滤波控制) 与不考虑状态约束且采用渐近收敛控制的命令滤波控制 (CFC) 方法进行比较, 两种方法选用的各项参数一致, 仿真结果如图 2 ~ 图 4 所示.

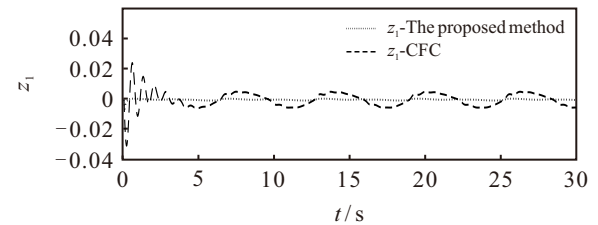


(a) 系统状态和估计值以及期望信号

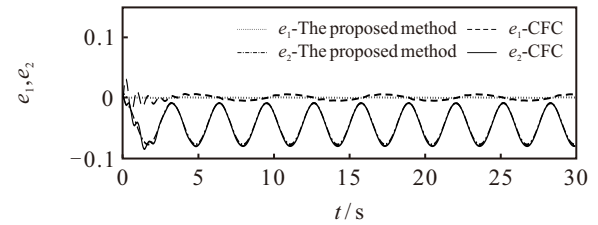


(b) 系统状态和估计值

图2 系统状态及其估计值的轨迹 (案例 1)



(a) 跟踪误差



(b) 估计误差

图3 两种方法下跟踪误差和估计误差 (案例 1)

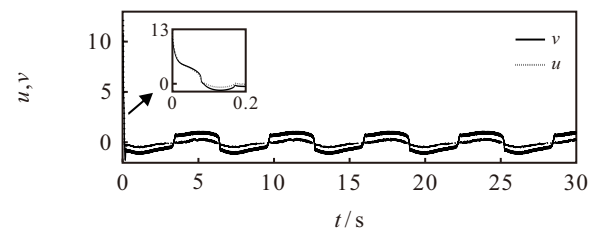


图4 所提方法下死区输入和系统输入的轨迹 (案例 1)

图 2 给出了期望信号 x_d 以及两种方法下系统状态 $x_i (i = 1, 2)$ 和状态估计 \hat{x}_i 的轨迹. 可以看出, 系统状态 x_1 满足约束条件, 且所提控制方法的 x_1 和 \hat{x}_1 可以在有限时间内趋于稳定. 同时, 系统状态 x_2 和状态估计 \hat{x}_2 的轨迹, 在所提控制方法下不仅收敛速度更快, 而且满足约束条件. 图 3 给出了两种方法下跟踪误差 z_1 和估计误差 $e_i (i = 1, 2)$. 可以看出, 所提控制方法的跟踪误差 z_1 更小, 且在有限时间内趋于原点

附近, 总体上跟踪效果更好. 同时, 在所提控制方法下, 估计误差 e_1 和 e_2 的轨迹具有更快的收敛速度, 且误差更小. 从图4可以看出, 在所提方法下, 系统输入信号 u 和死区输入信号 v 都是有界的. 从上述仿真结果可知, 闭环系统中的所有信号都是有界的, 设计的控制器性能可以达到预期的控制效果.

案例2 (实际案例) 考虑文献[41]中的单连杆机械手模型, 其动力学模型描述为

$$J\ddot{q} + B\dot{q} + Mgl \sin(q) = u(t). \quad (73)$$

令 $x_1 = q$ 和 $x_2 = \dot{q}$ 分别表示连杆的角度和角速度, J 是电机的总转动惯量, B 是整体的阻尼系数, M 是连杆的质量, l 是关节轴到质心的距离, g 是重力加速度, u 是控制输入. 若系统(73)存在外部扰动, 则可以将式(73)改写为

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\frac{B}{J}x_2 - \frac{Mgl \sin(x_1)}{J} + \frac{u}{J} + \frac{d(t)}{J}, \\ y = x_1. \end{cases} \quad (74)$$

上述系统选取的物理参数为 $J = 1, B = 2, M = 2, g = 10, l = 1$, 则 $f_1 = 0, f_2 = -2x_2 - 20 \sin(x_1), d = 0.01 \cos(t)$; 系统的初始状态为 $x_1(0) = x_2(0) = 0$, 系统状态满足的约束条件和期望信号同案例1, 死区模型同式(70).

根据式(20), 状态观测器被设计为

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + l_1(y - \hat{y}), \\ \dot{\hat{x}}_2 = u + \theta_2^T S_2 + l_2(y - \hat{y}), \\ \hat{y} = \hat{x}_1. \end{cases} \quad (75)$$

其中状态观测器的初始状态为 $\hat{x}_1(0) = \hat{x}_2(0) = 0$. 为了逼近系统的非线性函数, 选取高斯函数为神经网络的基函数, 其表达式和参数同式(72).

对于系统(74), 设计虚拟控制信号 α_1 为

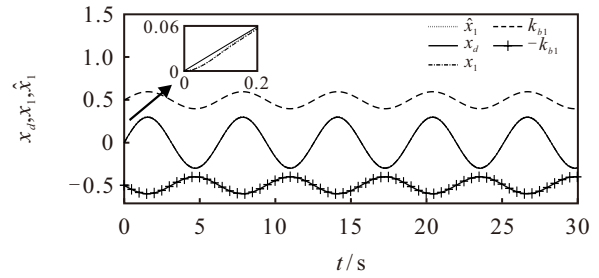
$$\begin{aligned} \alpha_1 = & -k_1 z_1 - \bar{k}_1(t)v_1 - \\ & \frac{k_{c1}^2(t) - (v_1 + x_d)^2}{k_{c1}^2(t)} I_1 \dot{k}_{c1}(t) + \\ & \frac{k_{c1}^2(t) - (v_1 + x_d)^2}{k_{c1}^2(t)} \psi_1 \dot{x}_d - \\ & \frac{5}{2} \frac{v_1 k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} - \\ & \omega_1 \left(\frac{k_{c1}^2(t)}{k_{c1}^2(t) - (v_1 + x_d)^2} \right)^{\eta-1} v_1^{2\eta-1}. \end{aligned} \quad (76)$$

控制律 v 同式(30), 补偿误差信号同式(28), 自适应律同式(31).

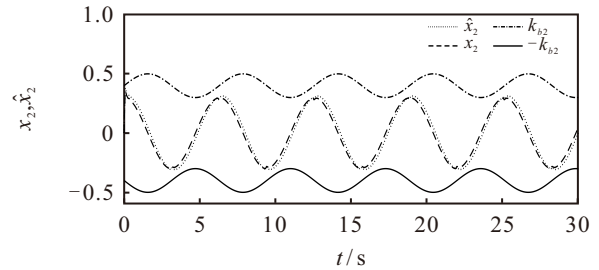
控制器设计过程的相关参数如下: $r_1 = r_2 = 0.2, m_1 = m_2 = 0.1, \gamma_1 = 0.8, k_1 = 10, k_2 = 20,$

$R_1 = 10, R_2 = 20, \omega_1 = \omega_2 = 1, \eta = 0.97, \beta_1 = 0.1, \beta_2 = 0.2, s_1 = 0.01, s_2 = 0.02, l_1 = 10, l_2 = 1200,$
 $\theta_1(0) = \theta_2(0) = 0, \xi_1(0) = \xi_2(0) = 0.$

对于非线性系统(74), 仿真结果如图5和图6所示. 可以看出, 所提控制方法在理论案例和实际案例中的仿真结果相似, 系统状态都满足规定的状态约束范围, 且系统输出 x_1 可以很好地跟踪期望信

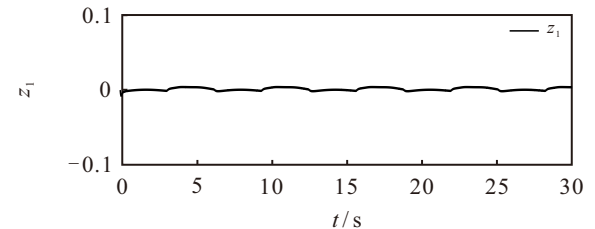


(a) 系统状态和估计值以及期望信号

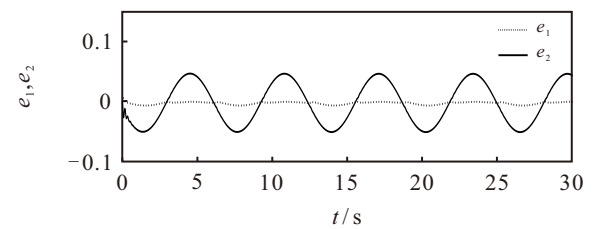


(b) 系统状态和估计值

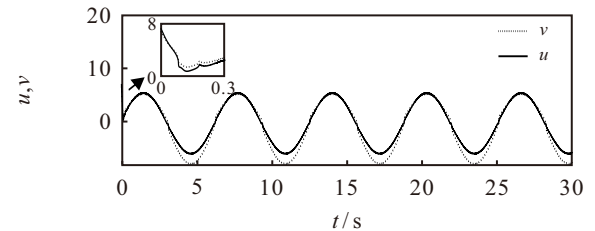
图5 所提方法下系统状态及其估计值的轨迹 (案例2)



(a) 跟踪误差



(b) 估计误差



(c) 死区输入和系统输入

图6 所提方法下跟踪误差、估计误差以及死区输入和系统输入的轨迹 (案例2)

号 x_d , 跟踪误差可以在有限时间内收敛到原点附近邻域内; 估计误差很小, 且可以在有限时间内收敛到原点附近的邻域内; 系统输入 u 和死区输入 v 都是有界的. 从上述仿真结果可知, 闭环系统中的所有信号都是有界的, 本文设计的控制器性能在理论案例和实际案例中都可以达到预期的控制效果.

5 结论

本文设计了不确定非线性系统在全状态约束下的自适应神经网络有限时间命令滤波输出反馈控制方案. 所设计的控制方案不仅解决了状态不可测和状态约束的问题, 也解决了输入死区问题. 另外, 通过利用命令滤波技术, 避免了由于虚拟控制信号连续求导而引起的“微分爆炸”现象, 而滤波器产生的滤波误差由设计的误差补偿机制来消除. 最后, 两个案例的仿真结果验证了所提控制方案的有效性, 也表明了闭环系统中的所有信号都是有界的.

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