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# 带有输入量化的分布式多无人船舶自适应模糊编队控制

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**摘要:** 针对复杂海洋环境下, 欠驱动水面无人船舶(USV)编队控制任务中存在的模型不确定性、参数摄动及输入量化等问题, 提出一种考虑输入量化的分布式欠驱动多无人船舶自适应模糊编队控制算法. 首先, 在USV运动学子系统, 设计基于扩张状态观测器(ESO)的分布式制导律, 分别实现对期望路径的跟踪、邻居USV速度信息的估计以及海流引起的运动学偏移的补偿; 其次, 在USV动力学子系统, 通过使用模糊逻辑系统实现对模型不确定及外界干扰的逼近, 采用一种线性解析模型描述输入量化过程, 所设计的自适应模糊量化控制器不需要量化参数的先验信息, 基于输入到状态稳定性理论证明闭环系统的稳定性; 最后, 通过仿真实验验证所提出的算法的有效性.

**关键词:** 欠驱动USV; 分布式编队; 扩张状态观测器; 输入量化; 自适应模糊控制

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## Fuzzy-adaptive distributed formation control of unmanned surface vehicles with input quantization

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**Abstract:** This paper addresses a distributed fuzzy-adaptive quantized formation control algorithm of multiple under-actuated unmanned surface vehicles (USVs) subject to model uncertainty, parameter perturbation and input quantization under complex marine environment. Specifically, at the kinematic level, an extended-state-observer-based (ESO) distributed guidance law is developed to track a time-varying trajectory where the ESO is adopted to estimate the unavailable velocity of neighboring USVs and compensate for the kinematic offset caused by ocean current. At the kinetic level, by using a linear analytical model to describe the quantization process and fuzzy logic system to identify the unknown kinetics, a fuzzy-adaptive quantized control law is developed where no information on the parameters of quantizers is required. The stability of the closed-loop control system is proven on the basis of input-to-state stability. Finally, the simulation results demonstrate the effectiveness of the proposed algorithm.

**Keywords:** under-actuated unmanned surface vehicles; distributed formation control; extended state observer; input quantization; adaptive fuzzy control

## 0 引言

随着海洋战略地位日渐凸显, 各国都加强了对海洋工程装备的投入, 力图增强各自开发勘探海洋资源的能力. 多无人水面船舶(USV)系统凭借其时空分

布带来的高效性以及冗余配置带来的可靠性, 为复杂海洋任务提供了新的解决思路<sup>[1]</sup>. 通过多USV协同实现集群控制, 不仅可以显著地减轻操作人员的负担, 还将使海洋作业变得更加持续、更具规模、更加

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智能. 多USV系统不体现在数量上的增加,而且通过彼此之间的协调与合作完成单USV难以胜任的任务. 编队控制作为多USV协同控制的一个典型问题,是研究其他协作问题的基础<sup>[2]</sup>. USV属于多智能体中的一种特定类型的运动体,对于USV编队控制的研究大多应用leader-follower法<sup>[3]</sup>、虚拟结构法<sup>[4]</sup>和基于图论法<sup>[5]</sup>等方法. 根据编队控制结构的不同,多USV编队控制可分为分布式控制<sup>[6]</sup>、集中式控制<sup>[7]</sup>和分散式控制<sup>[8]</sup>;根据控制目标的不同可分为编队目标跟踪<sup>[9]</sup>、编队轨迹跟踪<sup>[10]</sup>和编队路径跟踪<sup>[11]</sup>3类主要问题. 此外,USV在海上航行过程中具有典型的欠驱动特性,无法使用侧推器直接控制横向运动,相对全驱动无人船舶协同编队控制<sup>[12-13]</sup>而言,欠驱动无人船舶协同编队控制更为困难<sup>[14-15]</sup>.

近年来,控制系统中的量化问题受到了广泛的关注<sup>[16-23]</sup>. 在航海实践中,各传感器组件之间的信息传输是通过通信信道进行的. 信息要想在信道中传输,必须要经过量化与编码. 同时,考虑信道带宽受限,为了保证系统在给定的带宽内正常运行,可以使用量化技术降低通信速率. 量化器将信号从连续区域映射到离散区域,而相应地会引入量化误差,降低系统性能,甚至导致系统不稳定. 此外,量化控制输入更切合航海实践中船舶底层执行器的控制规律. 因此考虑带有输入量化的多USV编队控制具有重要意义.

在上述文献[3-23]中,主要存在两类问题亟待解决. 首先,文献[3-15]在考虑复杂海况影响时,均将其视为外界干扰,叠加到船舶动力学子系统中,这与航海实践是不相符的. 风浪的影响可以在动力学子系统中考虑,而海流造成的运动学偏移需要在运动学子系统中考虑. 其次,尚没有文献考虑多USV编队控制系统的输入量化问题. 而现有有关系统输入量化的研究大都将量化后的变量视作未量化变量的扰动,继而利用量化器产生扰动的界来分析量化对系统性能的影响<sup>[16-23]</sup>.

基于以上分析,本文提出一种考虑输入量化的分布式多无人船舶自适应模糊编队控制算法. 在USV运动学子系统,引入扩张状态观测器(ESO)实现对邻居USV速度信息的估计及海流引起的运动学偏移的补偿. 将所设计的欠驱动分布式路径跟踪引导律用作动力学子系统的期望输入. 在USV动力学子系统,引入模糊逻辑系统实现对系统模型不确定及外界干扰的逼近. 通过一种线性解析模型描述输入量化过程,在量化参数未知的假设下,所设计的自适应模糊量化控制器能够稳定跟踪运动学制导信号. 基于输入到状态稳定性理论证明闭环系统的稳定性,并通过

仿真实验验证所提出算法的有效性.

## 1 问题描述

本文中: $\|\cdot\|$ 为Euclidean范数, $\lambda_{\min}(\cdot)$ 和 $\lambda_{\max}(\cdot)$ 为对阵矩阵的最小及最大特征值.

考虑由 $N$ 个欠驱动USV构成的编队系统,基于牛顿力学和拉格朗日力学,欠驱动USV非线性数学模型<sup>[24]</sup>可表述为

$$\begin{cases} \dot{x}_i = u_i \cos \psi_i - v_i \sin \psi_i + u_{ci} \cos \psi_{ci}, \\ \dot{y}_i = u_i \sin \psi_i + v_i \cos \psi_i + u_{ci} \sin \psi_{ci}, \\ \dot{\psi}_i = r_i; \end{cases} \quad (1)$$

$$\begin{cases} m_{iu} \dot{u}_i = f_{iu}(u_i, v_i, r_i) + Q(\tau_{iu}) + \tau_{iuw}, \\ m_{iv} \dot{v}_i = f_{iv}(u_i, v_i, r_i) + \tau_{ivw}, \\ m_{ir} \dot{r}_i = f_{ir}(u_i, v_i, r_i) + Q(\tau_{ir}) + \tau_{irw}. \end{cases} \quad (2)$$

其中: $x_i$ 、 $y_i$ 、 $\psi_i$ 分别为惯性坐标系下USV的横向位置、纵向位置和艏摇角; $u_i$ 、 $v_i$ 、 $r_i$ 为附体坐标系下USV的前向速度、横向速度和艏摇速度; $u_{ci}$ 、 $\psi_{ci}$ 分别为海流的速度及流向; $m_{iu} = m_i - \dot{X}_{\dot{u}}$ 、 $m_{iv} = m_i - \dot{Y}_{\dot{v}}$ 、 $m_{ir} = I_z - \dot{N}_{\dot{r}}$ 为USV质量, $\dot{X}_{\dot{u}}$ 、 $\dot{Y}_{\dot{v}}$ 、 $\dot{N}_{\dot{r}}$ 为水动力导数项, $I_z$ 为关于 $z$ 轴的惯性矩; $f_{iu}(\cdot)$ 、 $f_{iv}(\cdot)$ 、 $f_{ir}(\cdot)$ 为向心力、科氏力和水动力阻尼构成的非线性项; $\tau_{iuw}$ 、 $\tau_{ivw}$ 、 $\tau_{irw}$ 为未知的环境干扰; $Q(\tau_{iu})$ 、 $Q(\tau_{ir})$ 为控制输入 $\tau_{iu}$ 、 $\tau_{ir}$ 的量化值,可由迟滞量化器表述,即

$$Q(\tau_i) = \begin{cases} \tau_i^q \operatorname{sign}(\tau_i), \frac{\tau_i^q}{1+\varsigma} \leq |\tau_i| \leq \tau_i^q, \dot{\tau}_i < 0 \text{ or} \\ \tau_i^q < |\tau_i| \leq \frac{\tau_i^q}{1-\varsigma}, \dot{\tau}_i > 0; \\ \tau_i^q(1+\varsigma) \operatorname{sign}(\tau_i), \tau_i^q < |\tau_i| \leq \frac{\tau_i^q}{1-\varsigma}, \dot{\tau}_i < 0 \text{ or} \\ \frac{\tau_i^q}{1-\varsigma} < |\tau_i| \leq \frac{\tau_i^q(1+\varsigma)}{1-\varsigma}, \dot{\tau}_i > 0; \\ 0, 0 \leq |\tau_i| < \frac{\tau_{i \min}}{1+\varsigma}, \dot{\tau}_i < 0 \text{ or} \\ \frac{\tau_{i \min}}{1+\varsigma} \leq |\tau_i| \leq \tau_{i \min}, \dot{\tau}_i > 0; \\ Q(\tau_i(t^-)), \dot{\tau}_i = 0. \end{cases}$$

这里: $\tau_i^q = \rho^{(1-q)} \tau_{i \min}$ ,  $q = 1, 2, \dots$ ,  $\rho = \frac{1-\varsigma}{1+\varsigma} \in (0, 1)$ ,  $\rho$ 为量化密度, $q$ 可以看成是量化级别; $0 < \varsigma < 1$ ;  $\tau_{i \min}$ 为量化死区; $Q(\tau_i(t^-))$ 为 $Q(\tau_i(t))$ 之前的状态.

经过量化器后,控制信号 $\tau_i$ 变成 $Q(\tau_i)$ ,这个过程中会产生量化误差,相当于丢失部分控制信息,继而导致控制系统性能下降. 为了保证USV动力学子系统能够有效跟踪制导输入,必须设计一种具有足够精度且较低通信速率的量化控制方案.

考虑一个时变参考轨迹

$$p_0(t) = [x_0(t), y_0(t)]^T \in \mathbb{R}^2. \quad (3)$$

其中:轨迹参数 $p_0(t)$ 为一个连续可导的向量,其导数为 $\dot{p}_0(t)$ .

以 $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ 表示多USV通讯拓扑结构.其中: $\mathcal{V} = \{n_0, n_1, \dots, n_N\}$ 是由所有节点构成的集合,表示编队中的 $N$ 艘船舶; $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V}\}$ 为所有边构成的集合,表示相邻船舶间的信息传输. $(n_j, n_i) \in \mathcal{E}$ 表示节点与节点间的信息互通,反之则不成立.定义 $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ 为第 $i$ 艘船舶的邻居集合, $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ 为编队系统的邻接矩阵,当 $(n_j, n_i) \in \mathcal{E}$ 时 $a_{ij} = 1$ ,反之 $a_{ij} = 0$ .

**假设1** 多USV之间的通信是无向的;对于每艘USV,存在一个虚拟领导者到该艘USV的有向通路.

本文的控制目标是在考虑输入量化的前提下,为每艘USV设计一个分布式的制导律及底层控制律,以期保持时变的特定编队任务队形,即

$$\lim_{t \rightarrow \infty} \|p_i(t) - p_{id}(t) - p_0(t)\| \leq l_{i0}. \quad (4)$$

其中: $l_{i0}$ 为一个正常数, $p_{id}(t) \in \mathbb{R}^2$ 为第 $i$ 艘船舶相对于参数化路径的位置偏差, $p_i = [x_i, y_i]^T$ .

## 2 控制器设计

多USV编队控制器设计过程可分为两个阶段:

1) 欠驱动分布式运动学制导律设计;2) 自适应模糊量化动力学控制器设计.

### 2.1 运动学制导律设计

首先,定义分布式多USV时变编队误差为

$$z_i = R_i^T(\psi_i) \left[ \sum_{j=1}^N a_{ij}(p_i - p_j - p_{ijd}) + a_{i0}(p_i - p_0(t) - p_{id}) \right], \quad (5)$$

其中 $p_{ijd} = p_{id} - p_{jd} \in \mathbb{R}^2$ 为USV间的相对位置偏差.如果USV $i$ 能够获取USV $j$ 的状态信息,则 $a_{ij} = 1$ ,否则 $a_{ij} = 0$ .旋转矩阵 $R_i(\psi_i)$ 可表示为

$$R_i(\psi_i) = \begin{bmatrix} \cos \psi_i & -\sin \psi_i \\ \sin \psi_i & \cos \psi_i \end{bmatrix}. \quad (6)$$

对式(5)进行求导可得

$$\begin{aligned} \dot{z}_i = & -r_i S z_i + \omega_i [u_i, v_i]^T - \sum_{j=1}^N a_{ij} R_i^T(\psi_i) \dot{p}_j + \\ & \sum_{j=1}^N a_{ij} R_i^T(\psi_i) [u_{ci} \cos \psi_{ci}, u_{ci} \sin \psi_{ci}]^T - \\ & a_{i0} R_i^T(\psi_i) \dot{p}_0(t) - \sum_{j=0}^N a_{ij} R_i^T(\psi_i) \dot{p}_{ijd}. \end{aligned} \quad (7)$$

其中: $\omega_i = \sum_{j=0}^N a_{ij}$ , $S$ 可定义为

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

为解决USV欠驱动问题,借鉴文献[11]的误差转移方程

$$\dot{\bar{z}}_i = z_i + [\delta_0, 0]^T \in \mathbb{R}^2, \quad (8)$$

其中 $\delta_0 \in R$ 为正的常值.将式(8)代入(7)可得

$$\dot{\bar{z}}_i = h_i [u_i, r_i]^T - a_{i0} R_i^T(\psi_i) \dot{p}_0(t) + \sigma_i. \quad (9)$$

其中

$$h_i = \text{diag}\{\omega_i, \delta_0\},$$

$$\sigma_i = -r_i S \bar{z}_i - \sum_{j=1}^N a_{ij} R_i^T(\psi_i) \dot{p}_j -$$

$$\sum_{j \in \bar{N}_i} a_{ij} R_i^T(\psi_i) \dot{p}_{ijd} + \sum_{j=1}^N a_{ij} R_i^T(\psi_i) \cdot$$

$$[u_{ci} \cos \psi_{ci}, u_{ci} \sin \psi_{ci}]^T + [0, \omega_i v_i]^T.$$

**假设2** 未知项 $\sigma_i$ 满足 $\|\dot{\sigma}_i\| \leq \sigma^*$ ,其中 $\sigma^*$ 为正常数.

航海实践中海流的流速、USV的速度及加速度均有上界,因此假设2是合理的.本文假设邻居USV的速度信息 $u_j$ 和 $v_j$ 不可测,海流的流向及流速不可测,因此 $\sigma_i$ 为未知项,采用ESO进行估计,即

$$\begin{cases} \dot{\tilde{z}}_i = \vartheta_i + \hat{\sigma}_i - k_{i1} \tilde{z}_i, \\ \dot{\hat{\sigma}}_i = -k_{i2} \tilde{z}_i. \end{cases} \quad (10)$$

其中: $\tilde{z}_i = \hat{z}_i - \bar{z}_i$ 为ESO的估计误差,且 $\vartheta_i = h_i [u_i, r_i]^T - a_{i0} R_i^T(\psi_i) \dot{p}_0(t)$ .由式(9)和(10)可得

$$\begin{cases} \dot{\tilde{z}}_i = \tilde{\sigma}_i - k_{i1} \tilde{z}_i, \\ \dot{\tilde{\sigma}}_i = -k_{i2} \tilde{z}_i - \dot{\hat{\sigma}}_i. \end{cases} \quad (11)$$

进一步可表达为

$$\dot{\tilde{E}}_{i1} = A_i \tilde{E}_{i1} + B_i, \quad (12)$$

其中 $\tilde{E}_{i1} = [\tilde{z}_i, \tilde{\sigma}_i]^T$ ,且有

$$A = \begin{bmatrix} -k_{i1} & 1 \\ -k_{i2} & 0 \end{bmatrix}, B = \begin{bmatrix} 0_2 \\ -\dot{\hat{\sigma}}_i \end{bmatrix}. \quad (13)$$

基于ESO的分布式运动学制导律可设计为

$$\begin{bmatrix} u_{id} \\ r_{id} \end{bmatrix} = h_i^{-1} (-\chi_i \tilde{z}_i + a_{i0} R_i^T(\psi_i) \dot{p}_0(t) - \tilde{\sigma}_i). \quad (14)$$

其中: $\chi_i = \text{diag}\{k_{ix}, k_{iy}\} \in \mathbb{R}^{2 \times 2}$ , $k_{ix} \in \mathbb{R}$ 及 $k_{iy} \in \mathbb{R}$ 为正的常数.将式(14)代入(9)可得运动学误差动态方程

$$\dot{\tilde{z}}_i = -\chi_i \tilde{z}_i - \tilde{\sigma}_i. \quad (15)$$

### 2.2 动力学控制器设计

针对USV动力学子系统,引入模糊逻辑系统实现对系统模型不确定及外界干扰的逼近. 通过一种线性解析模型描述输入量化过程,在量化参数未知的假设下实现对运动学制导律的稳定跟踪. 由式(2)可得

$$\begin{cases} m_{iu}\dot{u}_i = f_{iu}(u_i, v_i, r_i) + Q(\tau_{iu}) + \tau_{iuw}, \\ m_{ir}\dot{r}_i = f_{ir}(u_i, v_i, r_i) + Q(\tau_{ir}) + \tau_{irw}. \end{cases} \quad (16)$$

动力学子系统的控制目标为

$$\begin{cases} \lim_{t \rightarrow \infty} |e_{iu}| = |u_i - u_{id}| \leq \delta_1, \\ \lim_{t \rightarrow \infty} |e_{ir}| = |r_i - r_{id}| \leq \delta_2. \end{cases} \quad (17)$$

其中 $\delta_1, \delta_2$ 为小的正常数. 式(16)可被转换为

$$\begin{cases} \dot{u}_i = F_{iu} + m_{iu}^{-1}Q(\tau_{iu}) + d_{iu}, \\ \dot{r}_i = F_{ir} + m_{ir}^{-1}Q(\tau_{ir}) + d_{ir}. \end{cases} \quad (18)$$

其中

$$\begin{aligned} [d_{iu}, d_{ir}]^T &= [m_{iu}^{-1}\tau_{iuw}, m_{ir}^{-1}\tau_{irw}]^T, \\ [F_{iu}, F_{ir}]^T &= [m_{iu}^{-1}f_{iu}(u_i, v_i, r_i), m_{ir}^{-1}f_{ir}(u_i, v_i, r_i)]^T. \end{aligned}$$

令 $Q(\tau_{iu}) = q_{1iu}(t)\tau_{iu} + q_{2iu}(t)$ ,  $Q(\tau_{ir}) = q_{1ir}(t)\tau_{ir} + q_{2ir}(t)$ , 且有

$$\begin{cases} q_{1iu}(t) = \begin{cases} \frac{Q(\tau_{iu}(t))}{\tau_{iu}(t)}, & |\tau_{iu}(t)| \geq a; \\ 1, & |\tau_{iu}(t)| < a. \end{cases} \\ q_{1ir}(t) = \begin{cases} \frac{Q(\tau_{ir}(t))}{\tau_{ir}(t)}, & |\tau_{ir}(t)| \geq a; \\ 1, & |\tau_{ir}(t)| < a. \end{cases} \\ q_{2iu}(t) = \begin{cases} 0, & |\tau_{iu}(t)| \geq a; \\ Q(\tau_{iu}(t)) - \tau_{iu}(t), & |\tau_{iu}(t)| < a. \end{cases} \\ q_{2ir}(t) = \begin{cases} 0, & |\tau_{ir}(t)| \geq a; \\ Q(\tau_{ir}(t)) - \tau_{ir}(t), & |\tau_{ir}(t)| < a. \end{cases} \end{cases} \quad (19)$$

其中 $q_{1iu}(t)$ 和 $q_{1ir}(t)$ 未知. 由于量化过程符号不变, 由式(19)可知 $q_{1iu}(t) > 0, q_{1ir}(t) > 0$ . 当 $|\tau_{iu}(t)| < a, |\tau_{ir}(t)| < a$ 时,  $Q(\tau_{iu}(t)), Q(\tau_{ir}(t))$ 是有界的, 因此 $q_{2iu}(t), q_{2ir}(t)$ 是有界的, 可取 $|q_{2iu}(t)| \leq \bar{q}_{2j}, |q_{2ir}(t)| \leq \bar{q}_{2j}$ . 定义积分滑模面如下:

$$\begin{cases} s_{iu} = c_{iu} \int_0^t e_{iu} dt + e_{iu}(t), \\ s_{ir} = c_{ir} \int_0^t e_{ir} dt + e_{ir}(t). \end{cases} \quad (20)$$

其中:  $s_i = [s_{iu}, s_{ir}]^T, c_{iu} > 0, c_{ir} > 0$ . 对式(20)求导可得

$$\dot{s}_{iu} = c_{iu}e_{iu} + \dot{e}_{iu} =$$

$$\begin{aligned} & m_{iu}^{-1}Q(\tau_{iu}) + d_{iu} + F_{iu} - \dot{u}_{id} + c_{iu}e_{iu} = \\ & m_{iu}^{-1}q_{1iu}\tau_{iu} + m_{iu}^{-1}q_{2iu} + d_{iu} + F_{iu} - \dot{u}_{id} + c_{iu}e_{iu}, \end{aligned} \quad (21)$$

$$\begin{aligned} & \dot{s}_{ir} = c_{ir}e_{ir} + \dot{e}_{ir} = \\ & m_{ir}^{-1}Q(\tau_{ir}) + d_{ir} + F_{ir} - \dot{r}_{id} + c_{ir}e_{ir} = \\ & m_{ir}^{-1}q_{1ir}\tau_{ir} + m_{ir}^{-1}q_{2ir} + d_{ir} + F_{ir} - \dot{r}_{id} + c_{ir}e_{ir}. \end{aligned} \quad (22)$$

令 $\Delta_{iu}(X) = d_{iu} + F_{iu}, \Delta_{ir}(X) = d_{ir} + F_{ir}$ , 可得

$$\begin{aligned} & s_{iu}\dot{s}_{iu} = \\ & s_{iu}m_{iu}^{-1}q_{1iu}\tau_{iu} + s_{iu}m_{iu}^{-1}q_{2iu} + s_{iu}d_{iu} + s_{iu}F_{iu} + \\ & s_{iu}(c_{iu}e_{iu} - \dot{u}_{id}) \leq \\ & s_{iu}m_{iu}^{-1}q_{1iu}\tau_{iu} + s_{iu}m_{iu}^{-1}\bar{q}_{2iu} + s_{iu}d_{iu} + s_{iu}F_{iu} + \\ & s_{iu}(c_{iu}e_{iu} - \dot{u}_{id}) \leq \\ & s_{iu}m_{iu}^{-1}q_{1iu}\tau_{iu} + \frac{1}{2}m_{iu}^{-1}s_{iu}^2 + \frac{1}{2}m_{iu}^{-1}\bar{q}_{2iu}^2 + \\ & s_{iu}(d_{iu} + F_{iu}) + s_{iu}(c_{iu}e_{iu} - \dot{u}_{id}) \leq \\ & s_{iu} \left[ -l_{iu}s_{iu} - \eta_{iu} \operatorname{sgn} s_{iu} + l_{iu}s_{iu} + \eta_{iu} \operatorname{sgn} s_{iu} + \right. \\ & \left. \frac{1}{2}m_{iu}^{-1}s_{iu} + \Delta_{iu}(X) + c_{iu}e_{iu} - \dot{u}_{id} \right] + \\ & m_{iu}^{-1}s_{iu}q_{1iu}\tau_{iu} + \frac{1}{2}m_{iu}^{-1}\bar{q}_{2iu}^2. \end{aligned} \quad (23)$$

由于 $\Delta_{iu}(X)$ 为未知项, 根据万能逼近定理<sup>[25]</sup>, 对于任意小的常数 $\varepsilon_N$ , 存在模糊逻辑系统 $W_{iu}^{*T}h_{iu}(X)$ , 使得 $\Delta_{iu}(X) = W_{iu}^{*T}h_{iu}(X) + \varepsilon_{iu}$ . 其中:  $W_{iu}^*$ 为模糊系统的理想权重;  $h_{iu}(X)$ 为模糊基向量;  $\varepsilon_{iu}$ 为模糊系统的逼近误差, 满足 $|\varepsilon_{iu}| \leq \varepsilon_N$ . 令 $\hat{\Delta}_{iu}(X)$ 为 $\Delta_{iu}(X)$ 的估计值, 且有

$$\hat{\Delta}_{iu}(X) = \hat{W}_{iu}^T h_{iu}(X). \quad (24)$$

其中:  $\hat{W}_{iu}$ 为理想权重 $W_{iu}^*$ 的估计值, 且有

$$W_{iu}^* = \arg \min_{W_{iu} \in \Omega} [\sup_{x \in \mathbb{R}^2} |\hat{\Delta}_{iu}(X | W_{iu}) - \Delta_{iu}(X)|].$$

定义

$$\begin{aligned} \kappa_{iu} &= l_{iu}s_{iu} + \eta_{iu} \operatorname{sgn} s_{iu} + \frac{1}{2}m_{iu}^{-1}s_{iu} + \hat{\Delta}_{iu}(X) + \\ & c_{iu}e_{iu} - \dot{u}_{id}. \end{aligned} \quad (25)$$

其中:  $l_{iu}, \eta_{iu}$ 为正的常值, 且满足 $\eta_{iu} \geq \varepsilon_N + \eta_d, \eta_d > 0$ . 由于

$$\begin{aligned} s_{iu}\kappa_{iu} &= l_{iu}s_{iu}^2 + \eta_{iu}|s_{iu}| + \frac{1}{2}m_{iu}^{-1}s_{iu}^2 + s_{iu}\hat{\Delta}_{iu}(X) + \\ & s_{iu}(c_{iu}e_{iu} - \dot{u}_{id}), \end{aligned} \quad (26)$$

可得

$$\begin{aligned} & s_{iu}\dot{s}_{iu} \leq \\ & -l_{iu}s_{iu}^2 - \eta_{iu}|s_{iu}| + s_{iu}\kappa_{iu} + s_{iu}\tilde{\Delta}_{iu}(X) + \end{aligned}$$

$$\begin{aligned}
 & m_{iu}^{-1} s_{iu} q_{1iu} \tau_{iu} + \frac{1}{2} m_{iu}^{-1} \bar{q}_{2iu}^2 \leq \\
 & -l_{iu} s_{iu}^2 - \eta_{iu} |s_{iu}| + s_{iu} \kappa_{iu} + \frac{1}{2} m_{iu}^{-1} \bar{q}_{2iu}^2 + \\
 & m_{iu}^{-1} s_{iu} q_{1iu} \tau_{iu} + s_{iu} [\tilde{W}_{iu}^T h_{iu}(X) + \varepsilon_{iu}] \leq \\
 & -l_{iu} s_{iu}^2 - \eta_d |s_{iu}| + s_{iu} \kappa_{iu} + s_{iu} \tilde{W}_{iu}^T h_{iu}(X) + \\
 & m_{iu}^{-1} s_{iu} q_{1iu} \tau_{iu} + \frac{1}{2} m_{iu}^{-1} \bar{q}_{2iu}^2. \tag{27}
 \end{aligned}$$

其中:  $\tilde{W}_{iu} = W_{iu} - \hat{W}_{iu}$ ,  $\tilde{\Delta}_{iu} = \Delta_{iu} - \hat{\Delta}_{iu}$ .

类似地,令  $\Delta_{ir}(X) = W_{ir}^{*T} h_{ir}(X) + \varepsilon_{ir}$ ,  $\hat{\Delta}_{ir}(X) = \hat{W}_{ir}^T h_{ir}(X)$ ,  $\kappa_{ir} = l_{ir} s_{ir} + \eta_{ir} \operatorname{sgn} s_{ir} + m_{ir}^{-1} s_{ir} / 2 + \hat{\Delta}_{ir}(X) + c_{ir} e_{ir} - \dot{r}_{id}$ . 其中:  $W_{ir}^*$  为模糊系统的理想权重;  $\hat{W}_{ir}$  为  $W_{ir}$  的估计值;  $\varepsilon_{ir}$  为模糊系统的逼近误差, 满足  $|\varepsilon_{ir}| \leq \varepsilon_N$ ;  $l_{ir}$  和  $\eta_{ir}$  为正的常值, 满足  $\eta_{ir} \geq \varepsilon_N + \eta_d$ ,  $\eta_d > 0$ . 由此可得

$$\begin{aligned}
 s_{ir} \dot{s}_{ir} \leq & -l_{ir} s_{ir}^2 - \eta_d |s_{ir}| + s_{ir} \kappa_{ir} + s_{ir} \tilde{W}_{ir}^T h_{ir}(X) + \\
 & m_{ir}^{-1} s_{ir} q_{1ir} \tau_{ir} + \frac{1}{2} m_{ir}^{-1} \bar{q}_{2ir}^2. \tag{28}
 \end{aligned}$$

其中:  $\hat{\Delta}_{ir}(X)$  为  $\Delta_{ir}(X)$  的估计值,  $\tilde{W}_{ir} = W_{ir} - \hat{W}_{ir}$ .

**注1** 由于  $q_{1iu}(t)$  和  $q_{1ir}(t)$  时变且未知, 需要进行自适应估计. 采用对其下界进行估计的方法, 防止估计值为零时引发的奇异问题, 定义时变增益  $\mu_{iu} = 1/q_{1iu \min}$ ,  $\mu_{ir} = 1/q_{1ir \min}$ , 其中  $q_{1iu \min}$ 、 $q_{1ir \min}$  分别为  $q_{1iu}(t)$ 、 $q_{1ir}(t)$  的下界.

设计自适应动力学控制律和自适应律为

$$\begin{cases} \tau_{iu} = -m_{iu}^{-1} \frac{s_{iu} \hat{\mu}_{iu}^2 \kappa_{iu}^2}{|s_{iu} \hat{\mu}_{iu} \kappa_{iu}| + \rho_{iu}}, \\ \tau_{ir} = -m_{ir}^{-1} \frac{s_{ir} \hat{\mu}_{ir}^2 \kappa_{ir}^2}{|s_{ir} \hat{\mu}_{ir} \kappa_{ir}| + \rho_{ir}}. \end{cases} \tag{29}$$

$$\begin{cases} \dot{\hat{\mu}}_{iu} = \gamma_2 s_{iu} \kappa_{iu} - \gamma_2 s_{iu} \hat{\mu}_{iu}, \\ \dot{\hat{\mu}}_{ir} = \gamma_4 s_{ir} \kappa_{ir} - \gamma_4 s_{ir} \hat{\mu}_{ir}. \end{cases} \tag{30}$$

$$\begin{cases} \dot{\hat{W}}_{iu} = \gamma_1 s_{iu} h_{iu}(X), \\ \dot{\hat{W}}_{ir} = \gamma_3 s_{ir} h_{ir}(X). \end{cases} \tag{31}$$

其中  $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \rho_{iu}, s_{iu}, \rho_{ir}, s_{ir}$  为正常值.

基于  $\tilde{z}_i, s_{iu}, s_{ir}, \tilde{\mu}_{iu}, \tilde{\mu}_{ir}, \tilde{W}_{iu}, \tilde{W}_{ir}$  构成的误差动态系统可表述为

$$\begin{cases} \dot{\tilde{z}}_i = -\chi_i \tilde{z}_i - \tilde{\sigma}_i, \\ \dot{s}_{iu} = m_{iu}^{-1} q_{1iu} \tau_{iu} + m_{iu}^{-1} q_{2iu} + \Delta_{iu} - \dot{u}_{id} + c_{iu} e_{iu}, \\ \dot{s}_{ir} = m_{ir}^{-1} q_{1ir} \tau_{ir} + m_{ir}^{-1} q_{2ir} + \Delta_{ir} - \dot{r}_{id} + c_{ir} e_{ir}, \\ \dot{\tilde{\mu}}_{iu} = \dot{\hat{\mu}}_{iu} - \dot{\mu}_{iu} = \gamma_2 s_{iu} \kappa_{iu} - \gamma_2 s_{iu} \hat{\mu}_{iu}, \\ \dot{\tilde{\mu}}_{ir} = \dot{\hat{\mu}}_{ir} - \dot{\mu}_{ir} = \gamma_4 s_{ir} \kappa_{ir} - \gamma_4 s_{ir} \hat{\mu}_{ir}, \\ \dot{\tilde{W}}_{iu} = \dot{\hat{W}}_{iu} - \dot{W}_{iu} = -\gamma_1 s_{iu} h_{iu}(X), \\ \dot{\tilde{W}}_{ir} = \dot{\hat{W}}_{ir} - \dot{W}_{ir} = -\gamma_3 s_{ir} h_{ir}(X). \end{cases} \tag{32}$$

**注2** 自适应律(30)可用于对量化参数进行估计, 使得所设计的动力学底层控制律(29)不依赖于量化信息, 无需预知量化程度, 量化参数可根据系统性能自行调整.

### 3 稳定性分析

**引理1** 子系统(12)可看作一个状态为  $\tilde{z}_i$  和  $\tilde{\sigma}_i$ 、输入为  $\dot{\sigma}_i$  的系统, 该系统是输入状态稳定的.

**证明** 考虑如下李雅普诺夫函数:

$$\dot{V}_1 = \sum_{i=1}^N \tilde{E}_{i1}^T P_i \tilde{E}_{i1}, \tag{33}$$

其中  $P_i$  为正定矩阵, 满足

$$A_i^T P_i + P_i^T A_i \leq -I.$$

对式(12)求导可得

$$\dot{V}_1 \leq \sum_{i=1}^N \{-E_{i1}^T E_{i1} + E_{i1}^T P_i B_i \dot{\sigma}_i\}. \tag{34}$$

由于

$$\|E_{i1}\| \geq \frac{\|P_i B_i\| \|\dot{\sigma}_i\|}{\bar{\theta}_{i1}}, \tag{35}$$

使得

$$\dot{V}_1 \leq -\sum_{i=1}^N (1 - \bar{\theta}_{i1}) \|E_{i1}\|^2. \tag{36}$$

其中:  $0 < \bar{\theta}_{i1} < 1$ , 子系统(12)是输入状态稳定的, 且

$$\begin{aligned}
 \|E_{i1}(t)\| \leq & \sqrt{\frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_i)}} \left\{ \max \left( \|E_{i1}(t_0)\| e^{-\varrho_{i1}(t-t_0)}, \frac{\|P_i B_i\| \|\sigma^*\|}{\bar{\theta}_{i1}} \right) \right\}, \\
 & \forall t \geq t_0, \tag{37}
 \end{aligned}$$

这里  $\varrho_{i1} = (1 - \bar{\theta}_{i1}) / \lambda_{\max}(P_i)$ .  $\square$

**引理2** 动力学误差子系统(32)可看作一个状态为  $s_{iu}, s_{ir}, \tilde{\mu}_{iu}, \tilde{\mu}_{ir}, \tilde{z}_i$ 、输入为  $q_{2iu}, q_{2ir}, \mu_{iu}, \mu_{ir}, \tilde{\sigma}_i$  的系统, 该系统是输入状态稳定的.

**证明** 构建如下李雅普诺夫函数:

$$\begin{aligned}
 V_2 = \sum_{i=1}^N \left\{ \frac{1}{2} s_{iu}^2 + \frac{1}{2\gamma_2 \mu_{iu}} \tilde{\mu}_{iu}^2 + \frac{1}{2\gamma_1} \tilde{W}_{iu}^T \tilde{W}_{iu} + \frac{1}{2} s_{ir}^2 + \right. \\
 \left. \frac{1}{2\gamma_4 \mu_{ir}} \tilde{\mu}_{ir}^2 + \frac{1}{2\gamma_3} \tilde{W}_{ir}^T \tilde{W}_{ir} + \frac{1}{2} \tilde{z}_i^T \tilde{z}_i \right\}. \tag{38}
 \end{aligned}$$

其中:  $\tilde{\mu}_{iu} = \hat{\mu}_{iu} - \mu_{iu}$ ,  $\tilde{\mu}_{ir} = \hat{\mu}_{ir} - \mu_{ir}$ ,  $\mu_{iu} > 0$ ,  $\mu_{ir} > 0$ .

对式(38)求导可得

$$\begin{aligned}
 \dot{V}_2 = \sum_{i=1}^N \left\{ s_{iu} \dot{s}_{iu} + \frac{1}{\gamma_2 \mu_{iu}} \tilde{\mu}_{iu} \dot{\tilde{\mu}}_{iu} - \frac{1}{\gamma_1} \tilde{W}_{iu}^T \dot{\tilde{W}}_{iu} + \right. \\
 \left. s_{ir} \dot{s}_{ir} + \frac{1}{\gamma_4 \mu_{ir}} \tilde{\mu}_{ir} \dot{\tilde{\mu}}_{ir} - \frac{1}{\gamma_3} \tilde{W}_{ir}^T \dot{\tilde{W}}_{ir} + \tilde{z}_i^T \dot{\tilde{z}}_i \right\}. \tag{39}
 \end{aligned}$$

将式(27)及(28)代入(39)可得

$$\begin{aligned} \dot{V}_2 \leq & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - \eta_d|s_{iu}| + s_{iu}\kappa_{iu} + m_{iu}^{-1}s_{iu}q_{1iu}\tau_{iu} + \right. \\ & \frac{1}{2}m_{iu}^{-1}\bar{q}_{2iu}^2 + \frac{1}{\gamma_2\mu_{iu}}\tilde{\mu}_{iu}\dot{\mu}_{iu} + \tilde{W}_{iu}^T[s_{iu}h_{iu}(X) - \\ & \left. \frac{1}{\gamma_1}\dot{W}_{iu}] - l_{ir}s_{ir}^2 - \eta_d|s_{ir}| + s_{ir}\kappa_{ir} + \right. \\ & m_{ir}^{-1}s_{ir}q_{1ir}\tau_{ir} + \frac{1}{2}m_{ir}^{-1}\bar{q}_{2ir}^2 + \frac{1}{\gamma_4\mu_{ir}}\tilde{\mu}_{ir}\dot{\mu}_{ir} + \\ & \left. \tilde{W}_{ir}^T[s_{ir}h_{ir}(X) - \frac{1}{\gamma_3}\dot{W}_{ir}] + \bar{z}_i^T\dot{\bar{z}}_i \right\}. \end{aligned} \quad (40)$$

将控制律(29)、自适应律(30)、(31)代入(39)可得

$$\begin{aligned} \dot{V}_2 \leq & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - \eta_d|s_{iu}| + s_{iu}\kappa_{iu} + \frac{1}{2}m_{iu}^{-1}\bar{q}_{2iu}^2 - \right. \\ & q_{1iu}\frac{s_{iu}^2\hat{\mu}_{iu}^2\kappa_{iu}^2}{|s_{iu}\hat{\mu}_{iu}\kappa_{iu}| + \rho_{iu}} + \frac{1}{\gamma_2\mu_{iu}}\tilde{\mu}_{iu}(\gamma_2s_{iu}\kappa_{iu} - \\ & \gamma_2s_{iu}\hat{\mu}_{iu}) - l_{ir}s_{ir}^2 - \eta_d|s_{ir}| + s_{ir}\kappa_{ir} + \\ & \left. \frac{1}{2}m_{ir}^{-1}\bar{q}_{2ir}^2 - q_{1ir}\frac{s_{ir}^2\hat{\mu}_{ir}^2\kappa_{ir}^2}{|s_{ir}\hat{\mu}_{ir}\kappa_{ir}| + \rho_{ir}} + \right. \\ & \left. \frac{1}{\gamma_4\mu_{ir}}\tilde{\mu}_{ir}(\gamma_4s_{ir}\kappa_{ir} - \gamma_4s_{ir}\hat{\mu}_{ir}) + \bar{z}_i^T\dot{\bar{z}}_i \right\}. \end{aligned} \quad (41)$$

注意到

$$|a| - \frac{a^2}{\bar{\rho} + |a|} = \frac{\bar{\rho}|a|}{\bar{\rho} + |a|} < \bar{\rho},$$

可得

$$\begin{cases} -\frac{(s_{iu}\hat{\mu}_{iu}\kappa_{iu})^2}{\rho_{iu} + |s_{iu}\hat{\mu}_{iu}\kappa_{iu}|} \leq \rho_{iu} - s_{iu}\hat{\mu}_{iu}\kappa_{iu}, \\ -\frac{(s_{ir}\hat{\mu}_{ir}\kappa_{ir})^2}{\rho_{ir} + |s_{ir}\hat{\mu}_{ir}\kappa_{ir}|} \leq \rho_{ir} - s_{ir}\hat{\mu}_{ir}\kappa_{ir}. \end{cases} \quad (42)$$

考虑到  $q_{1iu} \geq q_{1iu \min} = 1/\mu_{iu} > 0$ ,  $q_{1ir} \geq q_{1ir \min} = 1/\mu_{ir} > 0$ ,  $-\eta_{iu}|s_{iu}| + \varepsilon_{iu}s_{iu} \leq 0$ ,  $-\eta_{ir}|s_{ir}| + \varepsilon_{ir}s_{ir} \leq 0$ , 可得

$$\begin{cases} -q_{1iu}\frac{s_{iu}^2\hat{\mu}_{iu}^2\kappa_{iu}^2}{|s_{iu}\hat{\mu}_{iu}\kappa_{iu}| + \rho_{iu}} \leq \frac{1}{\mu_{iu}}(\rho_{iu} - s_{iu}\hat{\mu}_{iu}\kappa_{iu}), \\ -q_{1ir}\frac{s_{ir}^2\hat{\mu}_{ir}^2\kappa_{ir}^2}{|s_{ir}\hat{\mu}_{ir}\kappa_{ir}| + \rho_{ir}} \leq \frac{1}{\mu_{ir}}(\rho_{ir} - s_{ir}\hat{\mu}_{ir}\kappa_{ir}). \end{cases} \quad (43)$$

基于式(43), 不等式(41)可写为

$$\begin{aligned} \dot{V}_2 \leq & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - \eta_d|s_{iu}| + \frac{1}{2}m_{iu}^{-1}\bar{q}_{2iu}^2 + \frac{1}{\mu_{iu}}\rho_{iu} - \right. \\ & \frac{1}{\mu_{iu}}\tilde{\mu}_{iu}s_{iu}\hat{\mu}_{iu} - l_{ir}s_{ir}^2 - \eta_d|s_{ir}| + \frac{1}{2}m_{ir}^{-1}\bar{q}_{2ir}^2 + \\ & \left. \frac{1}{\mu_{ir}}\rho_{ir} - \frac{1}{\mu_{ir}}\tilde{\mu}_{ir}s_{ir}\hat{\mu}_{ir} + \bar{z}_i^T\dot{\bar{z}}_i \right\}. \end{aligned} \quad (44)$$

由于

$$\begin{cases} -\tilde{\mu}_{iu}\hat{\mu}_{iu} \leq -\frac{1}{2}\tilde{\mu}_{iu}^2 + \frac{1}{2}\mu_{iu}^2, \\ -\tilde{\mu}_{ir}\hat{\mu}_{ir} \leq -\frac{1}{2}\tilde{\mu}_{ir}^2 + \frac{1}{2}\mu_{ir}^2. \end{cases}$$

且

$$\begin{cases} -\frac{1}{\mu_{iu}}\tilde{\mu}_{iu}s_{iu}\hat{\mu}_{iu} = -\frac{s_{iu}}{2\mu_{iu}}\tilde{\mu}_{iu}^2 + \frac{s_{iu}}{2}\mu_{iu}, \\ -\frac{1}{\mu_{ir}}\tilde{\mu}_{ir}s_{ir}\hat{\mu}_{ir} = -\frac{s_{ir}}{2\mu_{ir}}\tilde{\mu}_{ir}^2 + \frac{s_{ir}}{2}\mu_{ir}. \end{cases} \quad (45)$$

式(44)可写为

$$\begin{aligned} \dot{V}_2 \leq & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - \eta_d|s_{iu}| + \frac{1}{2}m_{iu}^{-1}\bar{q}_{2iu}^2 + \frac{1}{\mu_{iu}}\rho_{iu} - \right. \\ & \frac{1}{2\mu_{iu}}s_{iu}\tilde{\mu}_{iu}^2 + \frac{1}{2}s_{iu}\mu_{iu} - l_{ir}s_{ir}^2 - \eta_d|s_{ir}| + \\ & \frac{1}{2}m_{ir}^{-1}\bar{q}_{2ir}^2 + \frac{1}{\mu_{ir}}\rho_{ir} - \frac{1}{2\mu_{ir}}s_{ir}\tilde{\mu}_{ir}^2 + \\ & \left. \frac{1}{2}s_{ir}\mu_{ir} + \bar{z}_i^T\dot{\bar{z}}_i \right\} \leq \\ & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - \frac{1}{2\mu_{iu}}s_{iu}\tilde{\mu}_{iu}^2 - \eta_d|s_{iu}| + D_{iu} - \right. \\ & \left. l_{ir}s_{ir}^2 - \frac{1}{2\mu_{ir}}s_{ir}\tilde{\mu}_{ir}^2 - \eta_d|s_{ir}| + D_{ir} + \bar{z}_i^T\dot{\bar{z}}_i \right\}. \end{aligned} \quad (46)$$

其中:  $D_{iu} = m_{iu}^{-1}\bar{q}_{2iu}^2/2 + \rho_{iu}/\mu_{iu} + s_{iu}\mu_{iu}/2$ ,  $D_{ir} = m_{ir}^{-1}\bar{q}_{2ir}^2/2 + \rho_{ir}/\mu_{ir} + s_{ir}\mu_{ir}/2$ .

将动力学误差动态(15)代入(46)可得

$$\begin{aligned} \dot{V}_2 \leq & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - \frac{1}{2\mu_{iu}}s_{iu}\tilde{\mu}_{iu}^2 - \eta_d|s_{iu}| + D_{iu} - l_{ir}s_{ir}^2 - \right. \\ & \frac{1}{2\mu_{ir}}s_{ir}\tilde{\mu}_{ir}^2 - \eta_d|s_{ir}| + D_{ir} - \bar{z}_i^T\chi_i\bar{z}_i - \bar{z}_i^T\tilde{\sigma}_i \left. \right\} \leq \\ & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - l_{ir}s_{ir}^2 - \frac{1}{2\mu_{iu}}s_{iu}\tilde{\mu}_{iu}^2 - \frac{1}{2\mu_{ir}}s_{ir}\tilde{\mu}_{ir}^2 - \right. \\ & \left. \bar{z}_i^T\chi_i\bar{z}_i + D_{iu} + D_{ir} - \bar{z}_i^T\tilde{\sigma}_i \right\} \leq \\ & \sum_{i=1}^N \left\{ -l_{iu}s_{iu}^2 - l_{ir}s_{ir}^2 - \frac{1}{2\mu_{iu}}s_{iu}\tilde{\mu}_{iu}^2 - \frac{1}{2\mu_{ir}}s_{ir}\tilde{\mu}_{ir}^2 - \right. \\ & \left. \left( \lambda_{\min}(\chi_i) - \frac{1}{2} \right) \|\bar{z}_i\|^2 + D_{iu} + D_{ir} + \frac{1}{2}\|\tilde{\sigma}_i\|^2 \right\}. \end{aligned} \quad (47)$$

定义  $\bar{z} = [z_1^T, z_2^T, \dots, z_N^T]^T$ ,  $s_u = [s_{1u}, s_{2u}, \dots, s_{Nu}]^T$ ,  $s_r = [s_{1r}, s_{2r}, \dots, s_{Nr}]^T$ ,  $\tilde{\mu}_u = [\tilde{\mu}_{1u}, \tilde{\mu}_{2u}, \dots, \tilde{\mu}_{Nu}]^T$ ,  $\tilde{\mu}_r = [\tilde{\mu}_{1r}, \tilde{\mu}_{2r}, \dots, \tilde{\mu}_{Nr}]^T$ ,  $\bar{h}_2 = \min_{i=1,2,\dots,N} (l_{iu}, l_{ir}, s_{iu}/2\mu_{iu}, s_{ir}/2\mu_{ir}, \lambda_{\min}(\chi_i) - 1/2)$ ,  $E_2 = [s_u, s_r, \tilde{\mu}_u,$

$\tilde{\mu}_r, \tilde{z}$ , 可得

$$\dot{V}_2 \leq -(1 - \bar{\theta}_2)\bar{h}_2\|E_2\|^2 - \bar{\theta}_2\bar{h}_2\|E_2\|^2 + \sum_{i=1}^N \left( D_{iu} + D_{ir} + \frac{1}{2}\|E_{i1}\|^2 \right). \quad (48)$$

注意到

$$\|E_2\| \geq \sum_{i=1}^N \left( \sqrt{\frac{2D_{iu} + 2D_{ir} + \|E_{i1}\|^2}{2\bar{\theta}_2\bar{h}_2}} \right), \quad (49)$$

使得

$$\dot{V}_2 \leq -(1 - \bar{\theta}_2)\bar{h}_2\|E_2\|^2, \quad (50)$$

其中  $0 < \bar{\theta}_2 < 1$ . 因此动力学误差子系统(32)是输入状态稳定的, 且有

$$\|E_2(t)\| \leq \sqrt{\frac{\lambda_{\max}(P_c)}{\lambda_{\min}(P_c)}} \left\{ \max \left( \|E_2(t_0)\| e^{-\varrho_2(t-t_0)}, \sum_{i=1}^N \left( \sqrt{\frac{2D_{iu} + 2D_{ir} + \|E_{i1}\|^2}{2\bar{\theta}_2\bar{h}_2}} \right) \right) \right\}, \quad \forall t \geq t_0. \quad (51)$$

其中  $\varrho_2 = 2\bar{h}_2(1 - \bar{\theta}_2)/\lambda_{\max}(P_c)$ , 且有

$$P_c = \text{diag}\{1, 1/(2\gamma_2\mu_{1u}), \dots, 1/(2\gamma_2\mu_{Nu}), 1/(2\gamma_4\mu_{1r}), \dots, 1/(2\gamma_4\mu_{Nr}), 1/(2\gamma_1), 1/(2\gamma_3)\}. \quad (52)$$

由此引理2得证.  $\square$

以下定理给出闭环系统稳定性.

**定理1** 考虑由USV运动学子系统(1)、动力学子系统(2)、运动学制导律(14)、动力学底层控制律(29)、自适应律(30)、(31)构成的闭环系统, 当假设1和假设2成立时, 级联系统是输入到状态稳定的, 分布式编队控制闭环系统的误差是一致最终有界的.

**证明** 基于文献[7]提出的级联系统稳定性理论, 结合引理1和引理2可得, 由子系统(12)和子系统(32)构成的级联系统是输入状态稳定的. 由式(37)和(51)可得  $\|E_2(t)\|$  是有界的, 即

$$\|E_2(t)\| \leq \sqrt{\frac{\lambda_{\max}(P_c)}{\lambda_{\min}(P_c)}} \sum_{i=1}^N \left\{ \sqrt{\frac{D_{iu} + D_{ir}}{\bar{\theta}_2\bar{h}_2}} + \sqrt{\frac{\|P_i B_i\|^2 \sigma^{*2}}{2\bar{\theta}_2\bar{h}_2\bar{\theta}_{i1}^2}} \right\}. \quad (53)$$

由此定理1得证.  $\square$

### 4 仿真实验

考虑由5艘USV构成的编队控制系统, 领导者沿着时变的参数化轨迹  $p_0(t) = [2.5t, 0.2t]^T$  航行. USVs

参数<sup>[10]</sup>选取如下:  $m_{iu} = 25.8 \text{ kg}, m_{iv} = 33.8 \text{ kg}, m_{ir} = 2.76 \text{ kg}\cdot\text{m}^2, f_{iu}(\cdot) = -5.87u^3 - 1.33|u|u - 0.72u + m_{iv}vr + 1.0948r^2, f_{iv}(\cdot) = -36.5|v|v - 0.8896v - 0.805v|r| - m_{iu}ur, f_{ir}(\cdot) = -0.75|r|r - 1.90r + 0.08|v|r + (m_{iu} - m_{iv})uv - 1.0948ur$ . USVs初始状态设为  $p_1 = [0, 0]^T, p_2 = [-10, 10]^T, p_3 = [-10, -10]^T, p_4 = [-20, 20]^T, p_5 = [-20, -20]^T$ . 期望协同姿态设为  $p_{1d} = [0, 0]^T, p_{2d} = [-10, 10]^T, p_{3d} = [-10, -10]^T, p_{4d} = [-20, 20]^T, p_{5d} = [-20, -20]^T$ . 选取模糊集  $\mu_{F_i^j} = \exp[-(x+l)^2/2], l \in N, l \in [-5, 5]$ . 根据式(14)、(28)、(47)依次选取控制器参数  $c_{iu} = c_{ir} = 10, l_{iu} = l_{ir} = 30, \eta_{iu} = \eta_{ir} = 2, \rho_{iu} = \rho_{ir} = 0.02, \varsigma_{iu} = \varsigma_{ir} = 0.2, \gamma_1 = \gamma_3 = 3, \gamma_2 = \gamma_4 = 2$ . 结合时变参数轨迹及USVs物理参数特性, 选取量化密度参数及量化死区参数  $\varsigma = 0.2, \tau_{i\min} = 0.01$ .

仿真结果如图1~图5所示. 图1为5艘USV编队跟踪时变路径几何示意图. 由图1可知, 从给定初始位置出发, 在引导律及控制律的作用下USVs间能够保持预定的间距及几何队形. 图2为有无量化输入的编队横向跟踪误差对比曲线. 图3为有无量化输入的编队纵向跟踪误差对比曲线.

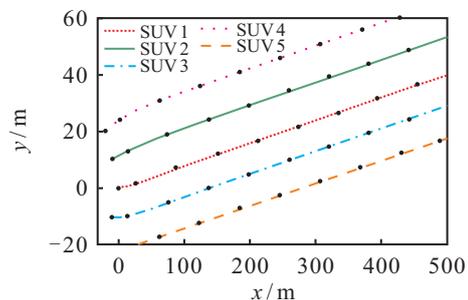


图1 多USV编队轨迹

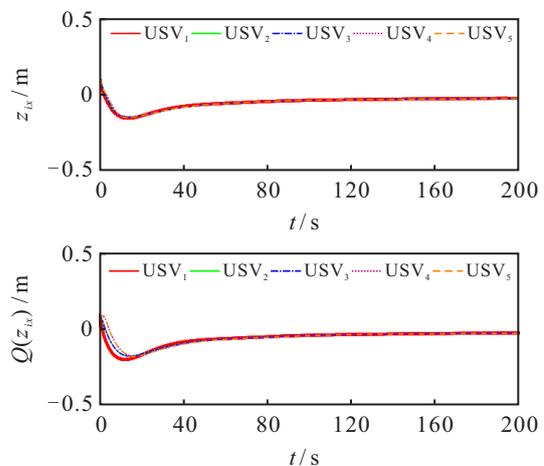


图2 编队横向跟踪误差对比

由图2和图3可知, 在假设海流信息未知、邻居速度信息不可获取的前提下, 基于ESO的估计及补

偿能够实现 USVs 的协同控制. 图4为有无量化输入的动力学控制输入推进力  $\tau_{iu}$  及  $Q(\tau_{iu})$  对比曲线. 图5为有无量化输入的动力学控制输入推进力矩  $\tau_{ir}$  及  $Q(\tau_{ir})$  对比曲线.

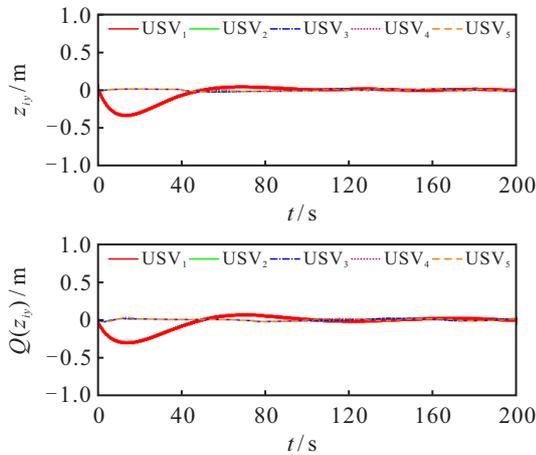


图3 编队纵向跟踪误差对比

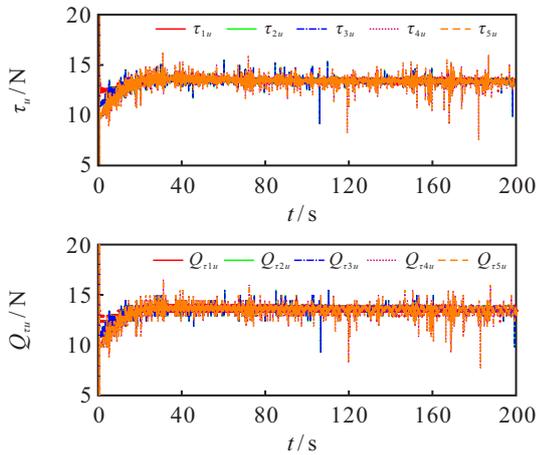


图4 控制输入推进力对比

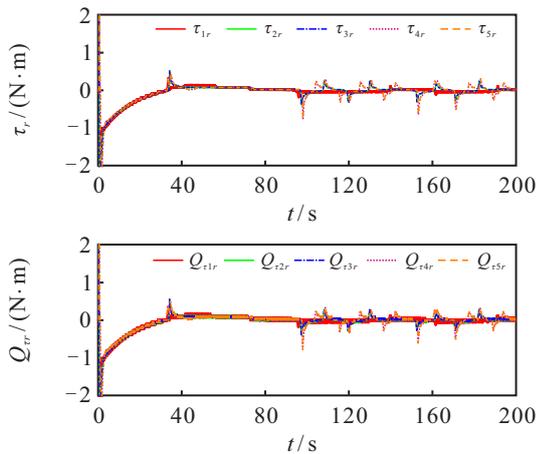


图5 控制输入转舵力矩对比

通过对比图2~图5可知,在外界干扰、系统不确定性、量化误差等多重影响下,闭环系统的编队跟踪误差收敛到一个较小的残差集.在考虑带有量化输入的情况时,在保证编队系统跟踪性能的同时,并未

显著牺牲过渡过程的控制品质,进一步说明了本文所提算法的有效性.

### 5 结论

本文探讨了带有模型不确定性及输入量化的欠驱动 USV 分布式编队控制问题.在运动学子系统中,设计了基于 ESO 的分布式协同制导律,实现了海流引起的运动学偏移的补偿及邻居速度信息的估计.在动力学子系统中,基于模糊逻辑系统实现了对系统不确定项及外界扰动项的逼近.所构造的时变线性解析模型能够描述量化过程,所构造的模糊自适应量化控制器能够有效跟踪运动学制导律,而不需要量化参数的先验信息.仿真实验验证了所提方法的有效性.

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