具有时变输出约束的柔性机翼非线性自适应控制

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具有时变输出约束的柔性机翼非线性自适应控制

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摘　要: 具有弯曲和扭转变形的柔性翼系统由一个四阶偏微分方程(PDE)与一个二阶常微分方程(ODE)描述,其中各个通道的干扰和参考轨迹来自于一个未知外系统. 此时所有干扰和参考轨迹的系数均未知,且由于外系统初值未知,扰动和参考轨迹的时变状态也未知. 首先,将柔性翼系统的鲁棒输出调节问题转化为跟踪误差系统的镇定问题,此时来自于干扰和参考轨迹的时变未知项被进一步转化为未知系数与已知时变信号的组合;然后,提出非线性自适应控制, 其中非线性对数项用于保证时变输出约束, 自适应律用于估测未知干扰的系数; 最后, 基于 Barrier Lyapunov 函数证明闭环系统跟踪误差的收敛性, 以及跟踪误差的时变约束特性,通过Matlab数值仿真进一步验证该控制方法的有效性.

关键词: 时变约束；鲁棒输出调节；非线性自适应控制；Barrier Lyapunov 函数；柔性翼系统；渐近稳定

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Nonlinear adaptive control for a flexible wing with time-varying output constraints

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Abstract: A flexible wing system of bending and twisting deformations is described by a fourth-order partial differential equation (PDE) and a second-order partial differential equation (ODE), where the disturbances in all channels and references are supposed to be from an unknown exosystem. In this sense, all the disturbances and references are composed of uncertain coefficients and time-varying states due to unknown initial values. Firstly, the robust output regulation is firstly transformed into a stabilization problem of the tracking error system, where the unknown time-varying terms from disturbances and references are rewritten into unknown coefficients but known time-varying terms. Then, adaptive nonlinear controls are proposed where adaption laws are used to estimate the unknown coefficients and nonlinear terms are used to guarantee the output constraint. Based on a Barrier Lyapunov function, the tracking errors of the closed-loop system are proved to converge toward zero, and are further restrained by time-varying constraints. Two numerical simulations are provided to show the effectiveness of the adaptive controls.

Keywords: time-varying constraints；robust output regulation；nonlinear adaptive control；Barrier Lyapunov function；flexible wing system；asymptotic stability

0　引　言

扑翼飞行机器人集工业设计、材料设计、机械硬件设计、控制算法设计、图像识别与检测等方向于一体,飞行姿态丰富多样,应用领域广泛,具有重要的研究价值[1-3]. 柔性翼可以提高扑翼机器人的灵敏性,其多自由度的变形可以产生连续、多变的气动外

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力. 在数学上,柔性翼系统可描述为混合分布参数系统,其中四阶偏微分方程(partial differential equation, PDE)可描述柔性翼的弯曲运动,二阶PDE可描述柔性翼的扭转运动[4-7]. 文献[4]通过线性算子理论处理了边界控制作用下的柔性翼镇定问题;文献[5]通过边界控制处理了三维柔性翼在弯曲和扭转方向的镇定问题以及旋转角度的跟踪问题;类似地,文献[6]通过边界控制实现了刚柔耦合翼跟踪问题;文献[7]对柔性翼系统在弯曲和扭转方向进行解耦,并设计了自适应控制和基于观测器的控制用于处理输出调节问题. 本文主要考虑柔性翼的鲁棒输出调节以及时变约束问题,其中,柔性翼系统无K-V阻尼,且各个通道存在未知干扰,参考轨迹也未知.

输出调节不仅要求性能输出渐近跟踪参考轨迹,同时也要求闭环系统有界[8-9]. 当输出信号在整个调节过程中始终在一个上下限内时,可以进一步称为输出调节与约束问题[10-11]. 此时,鲁棒性可以是针对外界干扰等外界未知不确定性[12-14],也可以是针对系统内部不确定性[15]. 文献[16]设计基于观测器的鲁棒控制用于处理具有异端边界扰动的一维波方程鲁棒输出调节问题,此时需假设跟踪误差和参考轨迹已知,且要求外系统基于参考轨迹可检测;文献[17]通过自适应律处理谐波干扰和参考轨迹中未知的幅值和初始相位,但是该文章只考虑了输入干扰. 类似的问题可参见文献[18-19]. 以上文献只考虑部分干扰作用下的简单PDE,所设计的控制不具备对所有通道干扰的鲁棒性. 内模控制可以通过外系统的观测器引入内模结构,进而保证对外系统产生的任意干扰和参考轨迹的鲁棒性[20-23]. 实际上,输出受限对于一致有界的闭环系统更加重要. 文献[24-27]通过Barrier Lyapunov函数进一步保证了某些关键输出信号约束在指定的常值范围内;文献[28]中,自适应容错控制用于处理具有可变长度的加油软管镇定问题和常值输出约束问题;文献[29-30]实现了时变输出约束,其中文献[30]考虑了分布参数系统描述的柔性弦系统.

在本文中,柔性翼系统不仅存在弯曲、扭转加速度的高度耦合,还考虑了分布式未知干扰、边界未知干扰. 由于没有考虑K-V阻尼,本文需要高阶边界信号实现柔性翼系统跟踪误差的渐近收敛. 本文的创新点主要包括:

1) 柔性翼系统不仅存在弯曲、扭转之间的高度耦合,还考虑了分布式未知干扰、边界未知干扰. 由于没有考虑K-V阻尼,本文需要高阶边界信号实现柔性翼系统跟踪误差的渐近收敛.

2) 首先,设计状态反馈控制,其中未知输入干扰无法直接通过时域范围下的自适应律进行估测. 因此,本文进一步将其拆分为已知的时变项和未知的常值系数,然后设计自适应律,进而得到自适应控制.

3) 基于Barrier Lyapunov函数,证明闭环系统一致有界,跟踪误差收敛到零且整个调节过程中满足输出约束.

1 问题描述

与参考文献[4]的镇定问题不同,本文主要研究柔性翼的鲁棒输出调节问题,并将文献[4]的柔性翼系统一般化,不仅在各个通道考虑了未知干扰,还去掉了K-V阻尼. 此时,柔性翼系统可以描述为

$$\begin{cases} mw_{tt}(x,t) - mx_e c\theta_{tt}(x,t) + \mathrm{EI}_b w_{xxxx}(x,t) = \\ d_1^{\mathrm{T}}(x)p(t), \\ I_p\theta_{tt}(x,t) - mx_e c w_{tt}(x,t) - \mathrm{GJ}\theta_{xx}(x,t) = \\ d_2^{\mathrm{T}}(x)p(t), \\ w(0,t) = d_3^{\mathrm{T}}p(t),\ w_x(0,t) = d_4^{\mathrm{T}}p(t), \\ w_{xx}(L,t) = d_5^{\mathrm{T}}p(t),\ \theta(0,t) = d_6^{\mathrm{T}}p(t), \\ m_s w_{tt}(L,t) - m_s x_e c\theta_{tt}(L,t) - \mathrm{EI}_b w_{xxx}(L,t) = \\ u_1(t) + d_7^{\mathrm{T}}p(t), \\ I_{ps}\theta_{tt}(L,t) - m_s x_e c w_{tt}(L,t) + \mathrm{GJ}\theta_x(L,t) = \\ u_2(t) + d_8^{\mathrm{T}}p(t), \\ y_c(t) = (w(L,t),\ \theta(L,t)), \end{cases} \tag{1}$$

其中系统参数及物理变量描述如表1所示.

表1 系统参数及变量

参数	物理意义/条件
$w(x,t), \theta(x,t)$	弯曲和扭转位移
$L, \mathrm{EI}_b, \mathrm{GJ}$	柔性翼系统参数
$I_p, m, x_e c$	$m(I_p - mx_e^2c^2) > 0$
m_s, I_{ps}	$m_s(I_{ps} - m_s x_e^2 c^2) > 0$
$d_k \in \mathbb{C}^n, k = 1, 2, \cdots, 8$	未知扰动系数
$u_k, k = 1, 2$	控制输入
$y_c(t)$	性能输出

外系统$p(t)$描述为

$$\dot{p}(t) = Sp(t),\ p_0 = p(0). \tag{2}$$

其中: $p_0 \in \mathbb{C}^n$是未知的, $S \in \mathbb{C}^{n\times n}$已知.

本文旨在通过设计非线性自适应控制器处理未知干扰以及参考轨迹,同时调控性能输出$y_c(t)$跟踪参考轨迹$d_9^{\mathrm{T}}p(t)$、$d_{10}^{\mathrm{T}}p(t)$,其中d_9、$d_{10} \in \mathbb{C}^n$. 此时跟踪误差定义为

$$e_1(t)=w(L,t)-d_9^{\mathrm{T}}p(t),\ e_2(t)=\theta(L,t)-d_{10}^{\mathrm{T}}p(t). \tag{3}$$

控制目标可以总结如下:

1) 跟踪误差$e_k(t)(k=1,2)$渐近收敛到零,即$\lim\limits_{t\to\infty}|e_k(t)|=0$.

2) 跟踪误差满足时变输出约束,即$|e_k(t)|<C_k(t)$,其中$C_k(t)(k=1,2)$是已知的.

本文首先给出假设1,使得外系统可以产生谐波信号和基于时间的多项式信号. 此外,假设1还要求外系统特征值不同于传输零点,目的是保证鲁棒输出调节问题可解,即参考系统(6)的解有界且唯一.

假设1　对于外系统(2),假定所有特征值都位于虚轴上,且与式(1)的传输零点不同.

本文进一步给出如下引理用于证明闭环系统(4)的渐近稳定,这可以进一步证明基于自适应控制的闭环系统稳定性.

引理1　假设$\alpha,\ \beta,\ k_j>0,\ j=1,2,3,4$,且系统初值满足$|\varepsilon_w(L,0)|<C_1(0)$和$|\varepsilon_\theta(L,0)|<C_2(0)$,可知下面系统是指数稳定的,且满足输出约束:

$$\begin{cases}
m\varepsilon_{w,tt}(x,t)+\mathrm{EI}_b\varepsilon_{w,xxxx}(x,t)-mx_ec\varepsilon_{\theta,tt}(x,t)=0,\\
I_p\varepsilon_{\theta,tt}(x,t)-\mathrm{GJ}\varepsilon_{\theta,xx}(x,t)-mx_ec\varepsilon_{w,tt}(x,t)=0,\\
\varepsilon_w(0,t)=0,\ \varepsilon_{w,x}(0,t)=0,\\
\varepsilon_{w,xx}(L,t)=0,\ \varepsilon_\theta(0,t)=0,\\
m_s[\varepsilon_{w,tt}(L,t)-\alpha\varepsilon_{w,xxxt}(L,t)]-m_sx_ec[\varepsilon_{\theta,tt}(L,t)+\\
\beta\varepsilon_{\theta,xt}(L,t)]=\\
-k_1[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]-\\
\Big[\dfrac{m_s}{2}[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]-\\
\dfrac{m_sx_ec}{2}[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]\Big]\dfrac{\dot{A}_{ln}(t)}{A_{ln}(t)}-\\
\dfrac{k_2[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]}{A_{ln}(t)},\\
I_{ps}[\varepsilon_{\theta,tt}(L,t)+\beta\varepsilon_{\theta,xt}(L,t)]-m_sx_ec[\varepsilon_{w,tt}(L,t)-\\
\alpha\varepsilon_{w,xxxt}(L,t)]=\\
-k_3[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]-\\
\Big[\dfrac{I_{ps}}{2}[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]-\\
\dfrac{m_sx_ec}{2}[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]\Big]\dfrac{\dot{A}_{ln}(t)}{A_{ln}(t)}-\\
\dfrac{k_4[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]}{A_{ln}(t)}.
\end{cases} \tag{4}$$

其中

$$A_{ln}(t)=\ln\frac{2C_1^2(t)}{C_1^2(t)-[\varepsilon_w(L,t)]^2}+\ln\frac{2C_2^2(t)}{C_2^2(t)-[\varepsilon_\theta(L,t)]^2}.$$

证明　定义如下Lyapunov函数:

$$\begin{aligned}
V(t)=&\\
&\frac{\beta_1}{2}\int_0^L[m[\varepsilon_{w,t}(x,t)]^2+I_p[\varepsilon_{\theta,t}(x,t)]^2-\\
&2mx_ec\varepsilon_{w,t}(x,t)\varepsilon_{\theta,t}(x,t)]\mathrm{d}x+\frac{\beta_1}{2}\int_0^L\mathrm{EI}_b\cdot\\
&[\varepsilon_{w,xx}(x,t)]^2\mathrm{d}x+\frac{\beta_1}{2}\int_0^L\mathrm{GJ}[\varepsilon_{\theta,x}(x,t)]^2\mathrm{d}x+\\
&A_{ln}(t)\Big[\frac{\beta_1}{2}m_s[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]^2-\\
&\beta_1m_sx_ec[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)][\varepsilon_{\theta,t}(L,t)+\\
&\beta\varepsilon_{\theta,x}(L,t)]+\frac{\beta_1}{2}I_{ps}[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]^2\Big]+\\
&\beta_2\int_0^Lmx[\varepsilon_{w,t}(x,t)-x_ec\varepsilon_{\theta,t}(x,t)]\varepsilon_{w,x}(x,t)\mathrm{d}x+\\
&\beta_2\int_0^Lx\varepsilon_{\theta,x}(x,t)[I_p\varepsilon_{\theta,t}(x,t)-mx_ec\varepsilon_{w,t}(x,t)]\mathrm{d}x.
\end{aligned}$$

其中:$\beta_1,\beta_2>0$,且$V(t)$满足

$$\lambda_{V1}\kappa(t)\leqslant V(t)\leqslant\lambda_{V2}\kappa(t),$$

$$\begin{aligned}
\kappa(t)=&\int_0^L[\varepsilon_{w,t}(x,t)]^2\mathrm{d}x+\int_0^L[\varepsilon_{\theta,t}(x,t)]^2\mathrm{d}x+\\
&\int_0^L[\varepsilon_{w,xx}(x,t)]^2\mathrm{d}x+\int_0^L[\varepsilon_{\theta,x}(x,t)]^2\mathrm{d}x+\\
&[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]^2A_{ln}(t)+\\
&[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]^2A_{ln}(t).
\end{aligned}$$

此时,正数$\lambda_{Vk}>0,k=1,2$. 因此可证$V(t)$是正定的.

基于式(4)可得

$$\begin{aligned}
\dot{V}(t)\leqslant&\\
&-\Big(\frac{\beta_1\mathrm{EI}_b}{2\alpha}-\frac{\beta_2mL}{2}-\frac{\beta_2mx_ecL}{\delta_2}\Big)\cdot\\
&[\varepsilon_{w,t}(L,t)]^2-\Big(\frac{\beta_1\mathrm{GJ}}{2\beta}-\frac{\beta_2I_pL}{2}-\beta_2mx_ecL\delta_2\Big)\cdot\\
&[\varepsilon_{\theta,t}(L,t)]^2-\mathrm{EI}_b\Big(\frac{\beta_1\alpha}{2}-\frac{\beta_2L}{\delta_1}\Big)[\varepsilon_{w,xxx}(L,t)]^2-\\
&\frac{\mathrm{GJ}}{2}(\beta_1\beta-\beta_2L)[\varepsilon_{\theta,x}(L,t)]^2-\beta_1\Big(k_2-\frac{\mathrm{EI}_b}{2\alpha}\Big)\cdot\\
&[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]^2-\beta_1\Big(k_4-\frac{\mathrm{GJ}}{2\beta}\Big)\cdot\\
&[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]^2-\beta_2\mathrm{EI}_b\Big(\frac{3}{2}-L^2\delta_1\Big)\cdot\\
&\int_0^L[\varepsilon_{w,xx}(x,t)]^2\mathrm{d}x-\frac{\beta_2\mathrm{GJ}}{2}\int_0^L[\varepsilon_{\theta,x}(x,t)]^2\mathrm{d}x-
\end{aligned}$$

$\beta_2 m\Big(\dfrac{1}{2} - x_e c\delta_3\Big)\int_0^L [\varepsilon_{w,t}(x,t)]^2 \mathrm{d}x - \beta_2\Big(\dfrac{I_p}{2} - \dfrac{m x_e c}{\delta_3}\Big)\int_0^L [\varepsilon_{\theta,t}(x,t)]^2 \mathrm{d}x - \beta_1 k_1[\varepsilon_{w,t}(L,t) - \alpha\varepsilon_{w,xxx}(L,t)]^2 A_{ln}(t) - \beta_1 k_3[\varepsilon_{\theta,t}(L,t) + \beta\varepsilon_{\theta,x}(L,t)]^2 A_{ln}(t).$

其中: $\delta_1 \sim \delta_3$ 是正数, 所有参数需保证 $\dot V(t) \leqslant -\lambda_{V3}\kappa(t) \leqslant -\dfrac{\lambda_{V3}}{\lambda_{V2}}V(t),\ V(t) \leqslant V(0)\mathrm{e}^{-\frac{\lambda_{V3}}{\lambda_{V2}}t}$, 且存在常数 $\lambda_{V3} > 0$. 因此, 闭环系统(4)指数稳定, 同时边界位移收敛到零且满足时变约束. □

2 非线性自适应控制设计

本节主要是对柔性翼系统设计非线性自适应控制, 具体思路如图1所示.

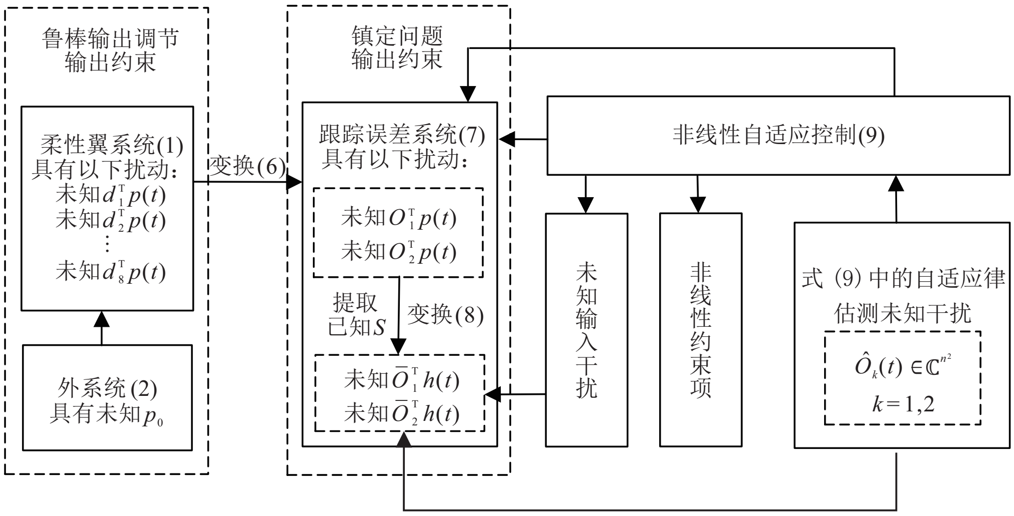

图1 非线性自适应控制设计

为了将柔性翼系统(1)的跟踪问题转化为镇定问题, 引入以下变换

$$\begin{cases}\varepsilon_w(x,t) = w(x,t) - H_w^{\mathrm T}(x)p(t),\\ \varepsilon_\theta(x,t) = \theta(x,t) - H_\theta^{\mathrm T}(x)p(t).\end{cases} \tag{5}$$

其中 $H_w(x), H_\theta(x) \in \mathbb{C}^n$ 定义为

$$\begin{cases}[mH_w^{\mathrm T}(x) - m x_e c H_\theta^{\mathrm T}(x)]S^2 + \mathrm{EI}_b H_w^{''''\mathrm T}(x) = d_1^{\mathrm T}(x),\\ [I_p H_\theta^{\mathrm T}(x) - m x_e c H_w^{\mathrm T}(x)]S^2 - \mathrm{GJ}H_\theta^{''\mathrm T}(x) = d_2^{\mathrm T}(x),\\ H_w^{\mathrm T}(0) = d_3^{\mathrm T},\ H_w^{'\mathrm T}(0) = d_4^{\mathrm T},\ H_w^{''\mathrm T}(L) = d_5^{\mathrm T},\\ H_\theta^{\mathrm T}(0) = d_6^{\mathrm T},\ H_w^{\mathrm T}(L) = d_9^{\mathrm T},\ H_\theta^{\mathrm T}(L) = d_{10}^{\mathrm T}.\end{cases} \tag{6}$$

基于假设1, 以上边值问题可以保证存在有界解.

此时, 跟踪误差系统 $(\varepsilon_w, \varepsilon_\theta)$ 可以表示为

$$\begin{aligned}&m\varepsilon_{w,tt}(x,t) - m x_e c\varepsilon_{\theta,tt}(x,t) + \mathrm{EI}_b\varepsilon_{w,xxxx}(x,t) = 0,\\ &I_p\varepsilon_{\theta,tt}(x,t) - m x_e c\varepsilon_{w,tt}(x,t) - \mathrm{GJ}\varepsilon_{\theta,xx}(x,t) = 0,\\ &\varepsilon_w(0,t) = 0,\ \varepsilon_{w,x}(0,t) = 0,\\ &\varepsilon_{w,xx}(L,t) = 0,\ \varepsilon_\theta(0,t) = 0,\\ &m_s\varepsilon_{w,tt}(L,t) - m_s x_e c\varepsilon_{\theta,tt}(L,t) - \mathrm{EI}_b\varepsilon_{w,xxx}(L,t) = u_1(t) - O_1^{\mathrm T}p(t),\\ &I_{ps}\varepsilon_{\theta,tt}(L,t) - m_s x_e c\varepsilon_{w,tt}(L,t) + \mathrm{GJ}\varepsilon_{\theta,x}(L,t) = u_2(t) - O_2^{\mathrm T}p(t),\\ &O_1^{\mathrm T} = -d_7^{\mathrm T} + m_s H_w^{\mathrm T}(L)S^2 - m_s x_e c H_\theta^{\mathrm T}(L)S^2 - \mathrm{EI}_b H_w^{'''\mathrm T}(L),\\ &O_2^{\mathrm T} = -d_8^{\mathrm T} + I_{ps}H_\theta^{\mathrm T}(L)S^2 - m x_e c H_w^{\mathrm T}(L)S^2 + \mathrm{GJ}H_\theta^{'\mathrm T}(L),\\ &e_1(t) = \varepsilon_w(L,t),\ e_2(t) = \varepsilon_\theta(L,t).\end{aligned} \tag{7}$$

与柔性翼(1)相比, $(\varepsilon_w, \varepsilon_\theta)$ 的系统只有在与控制同端的边界上存在未知干扰, 这极大地减少了控制设计的难度. 相应的控制目标等价为:

1) 跟踪误差系统(7)的边界位移 $\varepsilon_w(L,t), \varepsilon_\theta(L,t)$ 收敛到零.

2) 同时, 跟踪误差还进一步满足以下输出受限, 即 $|\varepsilon_w(L,t)| < C_1(t), |\varepsilon_\theta(L,t)| < C_2(t)$.

为了从 $p(t)$ 中提取更多的已知信号, 定义

$$\begin{cases}\bar O_k^{\mathrm T}h(t) = O_k^{\mathrm T}p(t) = O_k^{\mathrm T}\mathrm{e}^{St}p(0),\\ \bar O_k = \mathrm{vec}((O_k p(0)^{\mathrm T})^{\mathrm T}) \in \mathbb{C}^{n^2},\\ h(t) = \mathrm{vec}((\mathrm{e}^{St})^{\mathrm T}) \in \mathbb{C}^{n^2}.\end{cases} \tag{8}$$

其中: $\mathrm{vec}(\cdot)$ 为向量化算子, 具体定义为 $\mathrm{vec}(A) = (a_{11}, \ldots, a_{n1}, \ldots, a_{1n}, \ldots, a_{nn})^{\mathrm T}, A = (a_{ij})_{n\times n}, i,j = 1,2,\ldots,n$. 此时, $h(t)$ 便是从未知时变项 $O_k^{\mathrm T}p(t)(k = 1,2)$

提取出的已知时变信息,这进一步将时变未知性转化为未知的$\bar{O}_k^{\mathrm{T}}, k=1,2$.

对于以上柔性翼系统(7),提出以下自适应控制:

$$
\begin{aligned}
u_1(t) = & -k_1[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]- \\
& \mathrm{EI}_b\varepsilon_{w,xxx}(L,t)+m_s\alpha\varepsilon_{w,xxxt}(L,t)+ \\
& m_s x_e c\beta\varepsilon_{\theta,xt}(L,t)- \\
& \Big[\frac{m_s}{2}[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]- \\
& \frac{m_s x_e c}{2}[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]\Big]\frac{\dot{A}_{ln}(t)}{A_{ln}(t)}- \\
& \frac{k_2[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]}{A_{ln}(t)}+\hat{O}_1^{\mathrm{T}}(t)h(t), \\
u_2(t) = & -k_3[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]+ \\
& \mathrm{GJ}\varepsilon_{\theta,x}(L,t)-I_{ps}\beta\varepsilon_{\theta,xt}(L,t)- \\
& m_s x_e c\alpha\varepsilon_{w,xxxt}(L,t)- \\
& \Big[\frac{I_{ps}}{2}[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]- \\
& \frac{m_s x_e c}{2}[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]\Big]\frac{\dot{A}_{ln}(t)}{A_{ln}(t)}- \\
& \frac{k_4[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]}{A_{ln}(t)}+\hat{O}_2^{\mathrm{T}}(t)h(t), \\
\dot{\hat{O}}_1(t) = & \gamma_1[\varepsilon_{w,t}(L,t)-\alpha\varepsilon_{w,xxx}(L,t)]h(t)A_{ln}(t), \\
\dot{\hat{O}}_2(t) = & \gamma_2[\varepsilon_{\theta,t}(L,t)+\beta\varepsilon_{\theta,x}(L,t)]h(t)A_{ln}(t). \qquad (9)
\end{aligned}
$$

其中: $\alpha,\ \beta,\ k_j>0,\ j=1,2,3,4; \gamma_1,\gamma_2\in\mathbb{C}^{n^2\times n^2}$为正定矩阵. 定义估测误差为$\tilde{O}_k(t)=\bar{O}_k-\hat{O}_k(t), k=1,2$.

注1 与文献[31]不同,本文的系统模型没有K-V阻尼,不是通过PDE和ODE观测器处理未知干扰,而是通过自适应律进行处理,同时实现了鲁棒输出调节和输出受限.

定理1 假设$\alpha,\beta,k_j>0, j=1,2,3,4,\gamma_1,\gamma_2\in\mathbb{C}^{n^2\times n^2}$是正定矩阵并且系统初值满足$w(0,0)=d_3^{\mathrm{T}}p(0), w_x(0,0)=d_4^{\mathrm{T}}p(0), w_{xx}(L,0)=d_5^{\mathrm{T}}p(0), \theta(0,0)=d_6^{\mathrm{T}}p(0), |w(L,0)-d_9^{\mathrm{T}}p(0)|<\bar{C}_1(0)$,以及$|\theta(L,0)-d_{10}^{\mathrm{T}}p(0)|<\bar{C}_2(0)$,通过自适应控制(9)并基于引理1,闭环系统具有以下性质:

1) 柔性翼闭环系统状态有界,且跟踪误差渐近收敛到零.

2) 闭环系统的输出调节对分布式干扰、边界干扰、参考轨迹都具有鲁棒性.

3) 柔性翼闭环系统的跟踪误差进一步限制在指定的时变轨迹内.

证明 根据引理1,对于自适应控制(9)下的闭环系统,定义如下Lyapunov函数:

$$
\begin{aligned}
& E(t)= \\
& V(t)+\frac{\beta_1}{2}\tilde{O}_1^{\mathrm{T}}(t)\gamma_1^{-1}\tilde{O}_1(t)+\frac{\beta_1}{2}\tilde{O}_2^{\mathrm{T}}(t)\gamma_2^{-1}\tilde{O}_2(t).
\end{aligned}
$$

类似于引理1,可以证明$E(t)$的正定性. 同时,求解$\dot{E}(t)$可得$\dot{E}(t)\leqslant-\dfrac{\lambda_{V3}}{\lambda_{V2}}V(t)$. 根据$E(t)$的正定性,上式表明$E(t)$是有界的,同理可得$\|\tilde{O}_k^{\mathrm{T}}(t)\|^2$的有界性. 这也进一步保证了$\dot{V}(t)$的有界性. 根据Barbalat's引理[32]可以证明Lyapunov函数$V(t)$渐近收敛到零. □

3 数值仿真

本小节将通过两组数值仿真展示所提出的控制器的有效性. 此时,系统参数和初始条件选择如下:

$$
\begin{cases}
m=0.2,\ L=1,\ x_e c=0.025,\ I_p=0.05, \\
\mathrm{EI}_b=0.03,\ \mathrm{GJ}=0.13,\ m_s=0.2,\ I_{ps}=0.5, \\
w(x,0)=\dfrac{x}{5L},\ \theta(x,0)=\dfrac{\pi x}{10L},\ w_t(x,0)=0, \\
\theta_t(x,0)=0,\ C_k(t)=0.4\mathrm{e}^{-0.2t}+0.1,\ k=1,2.
\end{cases}
$$

同时外系统的参数设为$S=[0,1;1,0]$, $p(0)=(0.3, 0.4)^{\mathrm{T}}$.

在本文中,控制器参数选为

$$
\begin{cases}
k_1=0.5,\ k_2=0.62,\ k_3=0.06, \\
k_4=0.8, \alpha=0.5,\ \beta=2.5, \\
\gamma_1=0.2I_{4\times4},\ \gamma_2=0.3I_{4\times4}, \\
\hat{O}_1^{\mathrm{T}}(0)=(-0.01,-0.01,-0.003,-0.004), \\
\hat{O}_2^{\mathrm{T}}(0)=(-0.004,-0.006,0,0).
\end{cases} \qquad (10)
$$

对于柔性翼系统(1),选择两组未知的干扰和参考轨迹,具体如下:

$$
\begin{cases}
d_k^{\mathrm{T}}(x)=(0,0),\ k=1,2,\ d_3^{\mathrm{T}}=(0.1,0.2), \\
d_4^{\mathrm{T}}=(0.5,0),\ d_5^{\mathrm{T}}=(0.2,0), \\
d_6^{\mathrm{T}}=(0.2,0.3),\ d_7^{\mathrm{T}}=(1,1),\ d_8^{\mathrm{T}}=(1,1), \\
d_9^{\mathrm{T}}=(0.5,0.2),\ d_{10}^{\mathrm{T}}=(0.1,0.4).
\end{cases} . \qquad (11)
$$

$$
\begin{cases}
d_k^{\mathrm{T}}(x)=(0,0),\ k=1,2,\ d_3^{\mathrm{T}}=(0.2,0.1), \\
d_4^{\mathrm{T}}=(0,0.5),\ d_5^{\mathrm{T}}=(0,0.2), \\
d_6^{\mathrm{T}}=(0.3,0.2),\ d_7^{\mathrm{T}}=(0.4,0.7), \\
d_8^{\mathrm{T}}=(-0.8,0.5),\ d_9^{\mathrm{T}}=(0.2,0.5), \\
d_{10}^{\mathrm{T}}=(0.4,0.1).
\end{cases} \qquad (12)
$$

在相同的控制参数下,基于式(10)的自适应控制(9)作用于具有不同干扰和参考轨迹的柔性翼系统. 图2～图5用于描述式(11)作用下的闭环系统.

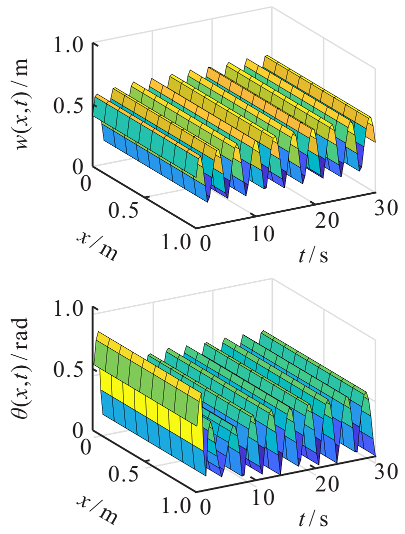

图 2 柔性翼系统弯曲和扭转位移: $w(x,t),\theta(x,t)$

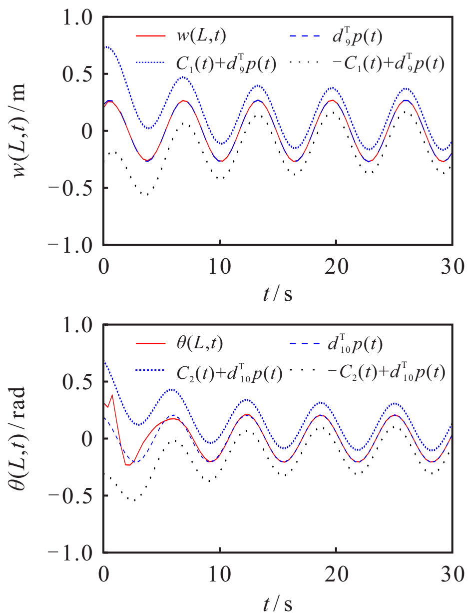

图 3 输出调节和时变输出约束

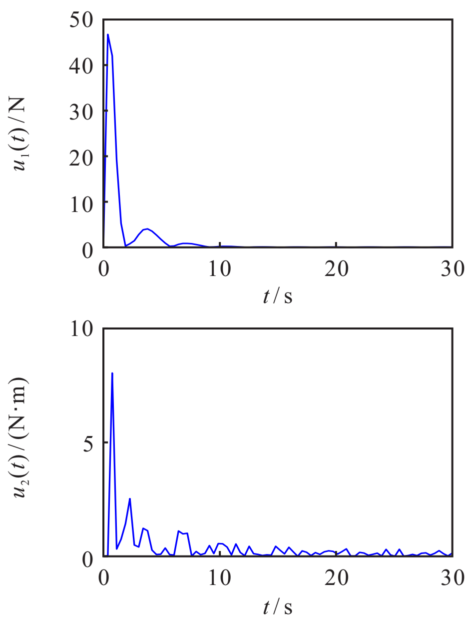

图 4 控制器 $u_k(t), k=1,2$

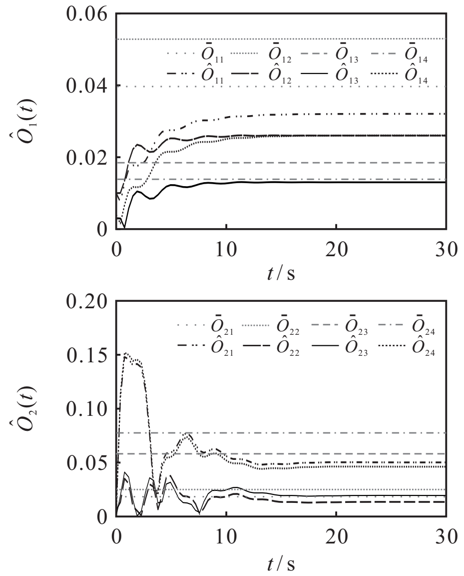

图 5 自适应律 $\hat{O}_k(t)=(\hat{O}_{k1},\hat{O}_{k2},\hat{O}_{k3},\hat{O}_{k4}), k=1,2$

图2和图3分别描述了闭环系统状态的有界性、跟踪误差的收敛性以及受限的特性; 图4和图5刻画了自适应控制的有界性, 即使在图5中自适应律无法估测未知$O_k(t)$, 自适应控制也可以保证在式(11)的作用下闭环系统的鲁棒输出调节. 此外, 相同的自适应控制也用于处理第2组不同的干扰和参考轨迹(12), 如图6～图9所示, 自适应控制仍然可以处理柔性翼的鲁棒输出调节.

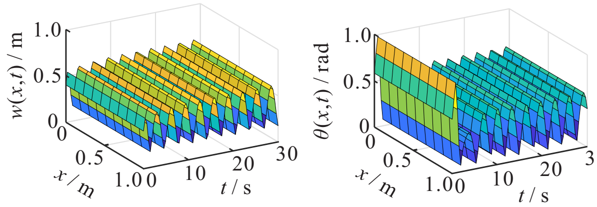

图 6 基于式(12)的柔性翼系统弯曲和扭转位移: $w(x,t),\theta(x,t)$

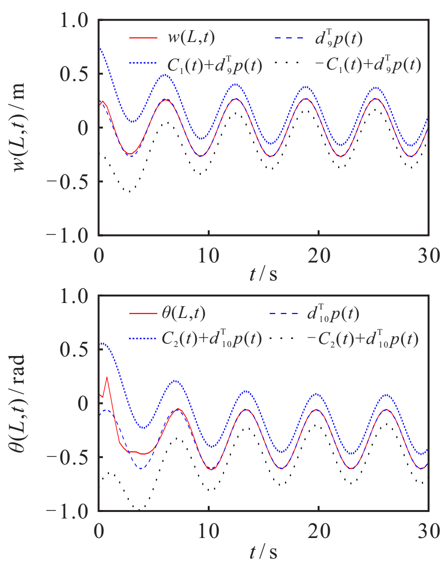

图 7 基于式(12)的输出调节和时变输出约束

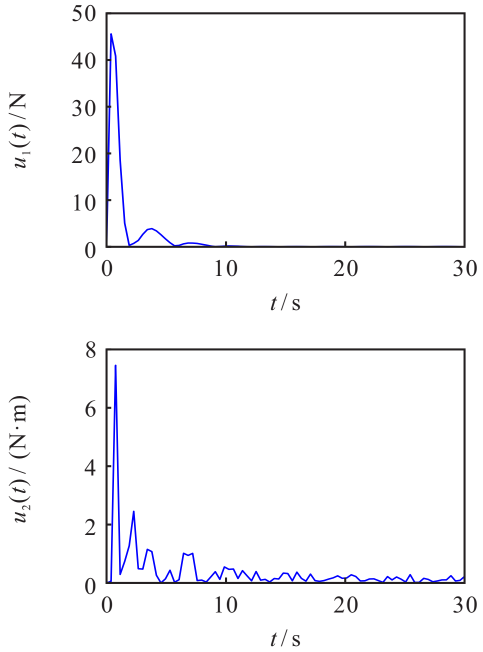

图 8　基于式(12)的控制器$u_k(t), k = 1, 2$

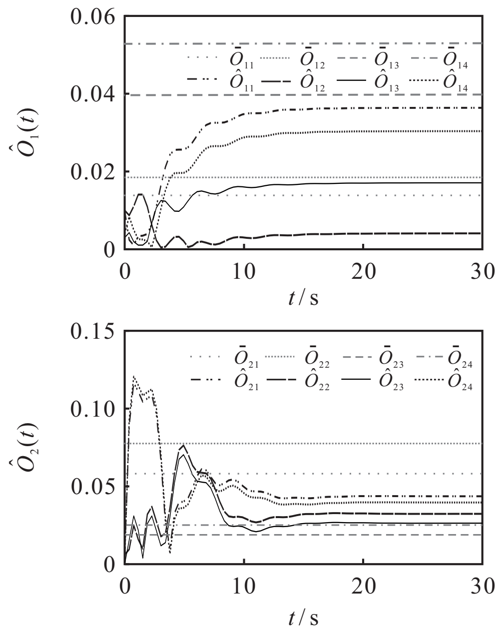

图 9　基于式(12)的自适应律
$\hat{O}_k(t) = (\hat{O}_{k1}, \hat{O}_{k2}, \hat{O}_{k3}, \hat{O}_{k4}), k = 1, 2$

4　结　论

本文考虑了柔性翼系统在时变输出约束下的鲁棒输出调节问题,其中柔性翼各个通道存在未知干扰,且系统无K-V阻尼. 由于干扰和参考轨迹产生于外系统,外系统矩阵已知导致外系统产生的所有信号具有已知的时变信号,但是幅值和相位未知. 本文提取出了已知时变信号,进而促使未知变量从时变函数变为一个自适应律可以估测的常值函数. 基于Barrier Lyapunov函数,本文提出了自适应非线性函数,并证明了闭环系统跟踪误差的收敛性,且满足时变输出约束.

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